Consumption and Saving

Lecture 3

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maximization problem:

\[
\max_{\{C_t\}^N_{t=0}} \mathbb{E}_0 \left[ \sum_{t=0}^N \beta^t u(C_t, Z_t) + \beta^{N+1} V_{N+1}(W_{N+1}) \right]
\]

s.t.

\[
W_{t+1} = R(W_t + Y_t - C_t) \\
W_{N+1} \geq 0
\]

with

\[
u(C, Z) = v(Z) \frac{C^{1-\rho}}{1 - \rho}
\]
Labor Income Process during Working Age

labor income process during working age:

\[ Y_t = P_t U_t \]

with

\[ P_t = GP_{t-1} N_t \]

and

\[ \log U_t = \begin{cases} -\infty & \text{with probability } p \\ \sim N \left( -\frac{\sigma_u^2}{2}, \sigma_u^2 \right) & \text{with probability } 1 - p \end{cases} \]

\[ \log N_t \sim N \left( -\frac{\sigma_N^2}{2}, \sigma_N^2 \right) \]
Retirement Value Function

- retirement value function:
  \[ V_{T+1}(X_{T+1}, H_{T+1}, Z_{T+1}) = \kappa \nu (Z_{T+1})(X_{T+1} + H_{T+1})^{1-\rho} \]

  \( X \) is cash at hand

- if there is no uncertainty in retirement than time of death and asset return, then
  \[ C_{T+1} = \gamma_1 (X_{T+1} + H_{T+1}) \]
Bellman Equation

\[ V_\tau (X_\tau , P_\tau , Z_\tau ) = \max_{C_\tau , \ldots , C_T} E_\tau \left\{ \sum_{t=\tau}^{T} \beta^{t-\tau} \nu (Z_t) \frac{C_t^{1-\rho}}{1-\rho} + \beta^{T+1-\tau} \kappa \nu (Z_{t+1}) (X_{T+1} + hP_{T+1})^{1-\rho} \right\} \]

s.t.

\[ X_{t+1} = R (X_t - C_t) + Y_{t+1} \text{ and } X_{T+1} \geq 0 \]

\[ \bullet \text{ dividing by } P_t \]

\[ c_{T+1} = \gamma_0 + \gamma_1 x_{T+1} \]

\[ \gamma_0 = \gamma_1 h \]
Panel A: $\beta = 0.960$, $\rho = 0.514$, $\gamma_1 = 0.071$, $\gamma_0 = 0.001$.

Panel B: $\beta = 0.960$, $\rho = 0.514$, $\gamma_1 = 0.077$, $\gamma_0 = 0.594$.

Figure 1.—A tale of two households.

Gourinchas & Parker, 2002
Panel A implies $\gamma_1 = 0.071, \gamma_0 = 0.001, h=0.014$
Panel B implies $\gamma_1 = 0.077, \gamma_0 = 0.077, h=7.7$
Method of Simulated Moments

• moment condition:

\[ E \left[ \zeta \left( \ln C_i; \psi_0 \right) \right] = 0 \]

with \( t \)-the element being

\[ \zeta_t \left( \ln C_i; \psi_0 \right) = \ln C_{i,t} - \ln C_t (\psi_0) \]
Method of Simulated Moments (cont.)

- estimated moments from data:
  \[
  \ln \overline{C}_t = \frac{1}{I_t} \sum_I \ln C_{i,t}
  \]

- simulated moments: unconditional expectation of consumption at each age
  \[
  \ln \widehat{C}_t (\theta, \widehat{\chi}) = \frac{1}{L} \sum_L \ln C_t (\widehat{s}_{l,t}, \theta, \widehat{\chi})
  \]
  \[
  \implies \ln \widehat{C}_t (\theta, \widehat{\chi}) \text{ as } L \to \infty
  \]


Method of Simulated Moments

• empirical counterpart of moment condition

\[ g_t (\theta, \hat{\chi}) = \frac{1}{I_t} \sum_{i} \hat{\zeta} (\ln C_{i,t}, \theta, \hat{\chi}) \]

\[ = \frac{1}{I_t} \sum_{i} \ln C_{i,t} - \ln \hat{C}_t (\theta, \hat{\chi}) \]

\[ = \ln \overline{C}_t - \ln \hat{C}_t (\theta, \hat{\chi}) \]

• minimize

\[ g (\theta, \hat{\chi})' W g (\theta, \hat{\chi}) \]
Two step procedure

1. use $W = I$, get consistent estimates $\hat{\theta}_1$, construct

$$\hat{\Omega} = \left[ \ln \overline{C}_t - \ln \hat{C}_t \left( \hat{\theta}_1, \hat{\chi} \right) \right] \left[ \ln \overline{C}_t - \ln \hat{C}_t \left( \hat{\theta}_1, \hat{\chi} \right) \right]'$$

2. use $W = \hat{\Omega}^{-1}$, get consistent and efficient estimates $\hat{\theta}_2$
Asymptotic properties

asymptotic properties of \( \hat{\theta}_2 \):

\[
\sqrt{N} \left( \hat{\theta} - \theta \right) \rightarrow N \left( 0, \Sigma \right)
\]

with

\[
\widehat{\Sigma} = \left( 1 + \frac{1}{nS} \right) \left[ \frac{\partial \left( \ln \bar{C}_t - \ln \hat{C}_t(\theta, \hat{\chi}) \right)}{\partial \theta} \right]^{-1} \Omega^{-1} \left( \frac{\partial \left( \ln \bar{C}_t - \ln \hat{C}_t(\theta, \hat{\chi}) \right)}{\partial \theta} \right)
\]
Figure 6. Value of the Objective Function (10), College Graduates, PSID. The minimum is .989 and 4.26.

Cagetti (2003)
this is sample wealth graph for quite extreme assumptions on life cycle income growth; beta=0.96 is red line; other lines: beta varying from 0.92 to 0.98
this is sample wealth graph with same calibration; red line rho=3; other lines rho=1.5 to rho=5
Consumption Profile from CEX

• estimate

\[ \ln \tilde{C}_i = f_i \pi_1 + a_i \pi_2 + b_i \pi_3 + U_i \pi_4 + \text{Re} t_i \pi_5 + \varepsilon_i \]

• construct household-level consumption that represents consumption of the observed household with "average" characteristics

\[ \ln C_i = \bar{f}_i \hat{\pi}_1 + a_i \hat{\pi}_2 + \bar{U} \hat{\pi}_4 + \hat{\varepsilon}_i \]

• average these consumption data across households:

\[ \ln C_a = \bar{f} \hat{\pi}_1 + a \hat{\pi}_2 + \bar{U} \hat{\pi}_4 \]
Figure 2.—Household consumption and income over the life cycle.

Gourinchas/Parker (2002)
FIGURE 3.—Household consumption and income over the life cycle, by education group.

G/P(2002)
### TABLE III

**Structural Estimation Results**

<table>
<thead>
<tr>
<th>MSM Estimation</th>
<th>Robust Weighting</th>
<th>Optimal Weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor ($\beta$)</td>
<td>0.9598</td>
<td>0.9569</td>
</tr>
<tr>
<td>S.E.(A)</td>
<td>(0.0101)</td>
<td></td>
</tr>
<tr>
<td>S.E.(B)</td>
<td>(0.0179)</td>
<td>(0.0150)</td>
</tr>
<tr>
<td>Discount Rate ($\beta^{-1} - 1$)(%)</td>
<td>4.188</td>
<td>4.507</td>
</tr>
<tr>
<td>S.E.(A)</td>
<td>(1.098)</td>
<td></td>
</tr>
<tr>
<td>S.E.(B)</td>
<td>(1.949)</td>
<td>(1.641)</td>
</tr>
<tr>
<td>Risk Aversion ($\rho$)</td>
<td>0.5140</td>
<td>1.3969</td>
</tr>
<tr>
<td>S.E.(A)</td>
<td>(0.1690)</td>
<td></td>
</tr>
<tr>
<td>S.E.(B)</td>
<td>(0.1707)</td>
<td>(0.1137)</td>
</tr>
<tr>
<td>Retirement Rule:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>0.0015</td>
<td>5.68 $10^{-6}$</td>
</tr>
<tr>
<td>S.E.(A)</td>
<td>(3.84)</td>
<td></td>
</tr>
<tr>
<td>S.E.(B)</td>
<td>(3.85)</td>
<td>(16.49)</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.0710</td>
<td>0.0613</td>
</tr>
<tr>
<td>S.E.(A)</td>
<td>(0.1215)</td>
<td></td>
</tr>
<tr>
<td>S.E.(B)</td>
<td>(0.1244)</td>
<td>(0.0511)</td>
</tr>
<tr>
<td>$\chi^2$(A)</td>
<td>175.25</td>
<td></td>
</tr>
<tr>
<td>$\chi^2$(B)</td>
<td>174.10</td>
<td>185.67</td>
</tr>
</tbody>
</table>

*Note: MSM estimation for entire group. Standard errors calculated without (A) and with (B) correction for first stage estimation. Cell size is 36,691 households. The last row reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71. Efficient estimates are calculated with a weighting matrix $\hat{\Omega}$ computed from the robust estimates.*

G/P(2002)
Panel A: Baseline Estimation

\[ \beta = 0.960, \quad \rho = 0.514, \quad \gamma_1 = 0.071, \quad \gamma_0 = 0.001 \]

Panel B: Various \( \beta \)

\( \beta = 0.95 \)

\( \beta = 0.97 \)

**Figure 5.**—The fitted consumption profile.
Target cash-on-hand (normalized)  

Parameters: $\beta = 0.960$, $\rho = 0.514$, $\gamma_1 = 0.0071$, $\gamma_0 = 0.001$

**Figure 6.**—Normalized target cash-on-hand by age.
Figure 7.—The role of risk in saving and wealth accumulation.
### TABLE IV
ESTIMATES FROM THE STOCHASTIC MODEL BY EDUCATION AND OCCUPATION

<table>
<thead>
<tr>
<th>Group</th>
<th>Discount Factor (\beta)</th>
<th>Discount Rate (\beta^{-1} - 1)</th>
<th>Risk Aversion (\rho)</th>
<th>Retirement Rule (\gamma_0)</th>
<th>(\gamma_1)</th>
<th>(\chi^2)</th>
<th>(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Education</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Some High School</td>
<td>0.962</td>
<td>3.94</td>
<td>0.282</td>
<td>0.209</td>
<td>0.072</td>
<td>53.60</td>
<td>4,270</td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
<td>(8.92)</td>
<td>(1.481)</td>
<td>(5.04)</td>
<td>(2.360)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High School Graduate</td>
<td>0.949</td>
<td>5.30</td>
<td>0.869</td>
<td>3.79 (10^{-3})</td>
<td>0.059</td>
<td>59.12</td>
<td>12,445</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(1.64)</td>
<td>(0.220)</td>
<td>(20.05)</td>
<td>(0.049)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Some College</td>
<td>0.960</td>
<td>4.15</td>
<td>0.394</td>
<td>0.351</td>
<td>0.043</td>
<td>84.21</td>
<td>9,653</td>
</tr>
<tr>
<td></td>
<td>(0.159)</td>
<td>(17.29)</td>
<td>(2.344)</td>
<td>(4.095)</td>
<td>(3.156)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>College Graduate</td>
<td>0.930</td>
<td>7.48</td>
<td>2.290</td>
<td>1.55 (10^{-8})</td>
<td>0.049</td>
<td>111.70</td>
<td>6,350</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(6.97)</td>
<td>(0.423)</td>
<td>(54.60)</td>
<td>(0.075)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graduate School</td>
<td>0.944</td>
<td>5.93</td>
<td>1.694</td>
<td>1.06 (10^{-7})</td>
<td>0.057</td>
<td>87.26</td>
<td>5,973</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(9.77)</td>
<td>(0.843)</td>
<td>(18.23)</td>
<td>(0.076)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Occupation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Managerial and Prof.</td>
<td>0.946</td>
<td>5.71</td>
<td>1.672</td>
<td>5.20 (10^{-8})</td>
<td>0.050</td>
<td>115.62</td>
<td>12,693</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(6.69)</td>
<td>(0.524)</td>
<td>(22.78)</td>
<td>(0.067)</td>
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</tr>
<tr>
<td>Tech., Sales, Admin.</td>
<td>0.953</td>
<td>4.90</td>
<td>1.059</td>
<td>2.13 (10^{-7})</td>
<td>0.049</td>
<td>64.02</td>
<td>6,548</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(4.11)</td>
<td>(0.339)</td>
<td>(39.42)</td>
<td>(0.064)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precision Prod., Craft</td>
<td>0.953</td>
<td>4.97</td>
<td>0.990</td>
<td>0.003</td>
<td>0.054</td>
<td>52.86</td>
<td>4,469</td>
</tr>
<tr>
<td></td>
<td>(0.333)</td>
<td>(36.77)</td>
<td>(3.895)</td>
<td>(18.49)</td>
<td>(0.997)</td>
<td></td>
<td></td>
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<tr>
<td>Operators, Laborers</td>
<td>0.953</td>
<td>4.90</td>
<td>0.867</td>
<td>3.14 (10^{-6})</td>
<td>0.049</td>
<td>57.58</td>
<td>6,063</td>
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<tr>
<td></td>
<td>(0.489)</td>
<td>(53.80)</td>
<td>(4.846)</td>
<td>(1365.28)</td>
<td>(2.35)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: MSM estimation in levels. Standard errors calculated with correction for first stage estimation. The next to last column reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71.*

G/P(2002)
FIGURE 3.—Household consumption and income over the life cycle, by education group.
G/P(2002)