Consumption and Saving

Lecture 3

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Gourinchas and Parker (2002)

maximization problem:

$$\max_{\{C_t\}_{t=0}^{N}} E_0 \left[\sum_{t=0}^{N} \beta^t u\left(C_t, Z_t\right) + \beta^{N+1} V_{N+1}\left(W_{N+1}\right) \right]$$

s.t.

$$\begin{aligned} W_{t+1} &= R\left(W_t + Y_t - C_t\right) \\ W_{N+1} &\geq 0 \end{aligned}$$

with

$$u\left(C, Z\right) = v\left(Z\right) \frac{C^{1-\rho}}{1-\rho}$$

Labor Income Process during Working Age

labor income process during working age:

$$Y_t = P_t U_t$$

with

$$P_t = G P_{t-1} N_t$$

and

$$\log U_t = \begin{cases} -\infty & \text{with probability } p \\ \sim N\left(-\frac{\sigma_u^2}{2}, \sigma_u^2\right) & \text{with probability } 1-p \end{cases}$$

$$\log N_t \sim N\left(-\frac{\sigma_N^2}{2}, \sigma_N^2\right)$$

Retirement Value Function

- retirement value function:

$$V_{T+1}\left(X_{T+1}, H_{T+1}, Z_{T+1}\right) = \kappa v\left(Z_{T+1}\right)\left(X_{T+1} + H_{T+1}\right)^{1-\rho}$$

  X is cash at hand

- if there is no uncertainty in retirement than time of death and asset return, then

$$C_{T+1} = \gamma_1\left(X_{T+1} + H_{T+1}\right)$$

Bellman Equation

$$V_\tau\left(X_\tau, P_\tau, Z_\tau\right) = \max_{C_\tau,\ldots,C_T} E_\tau \left\{ \begin{array}{c} \sum_{t=\tau}^{T} \beta^{t-\tau} v\left(Z_t\right) \frac{C_t^{1-\rho}}{1-\rho} + \beta^{T+1-\tau} \\ \kappa v\left(Z_{t+1}\right) \left(X_{T+1} + hP_{T+1}\right)^{1-\rho} \end{array} \right\}$$

s.t.

$$X_{t+1} = R\left(X_t - C_t\right) + Y_{t+1} \text{ and } X_{T+1} \geq 0$$

- dividing by P_t

$$\begin{aligned} c_{T+1} &= \gamma_0 + \gamma_1 x_{T+1} \\ \gamma_0 &= \gamma_1 h \end{aligned}$$

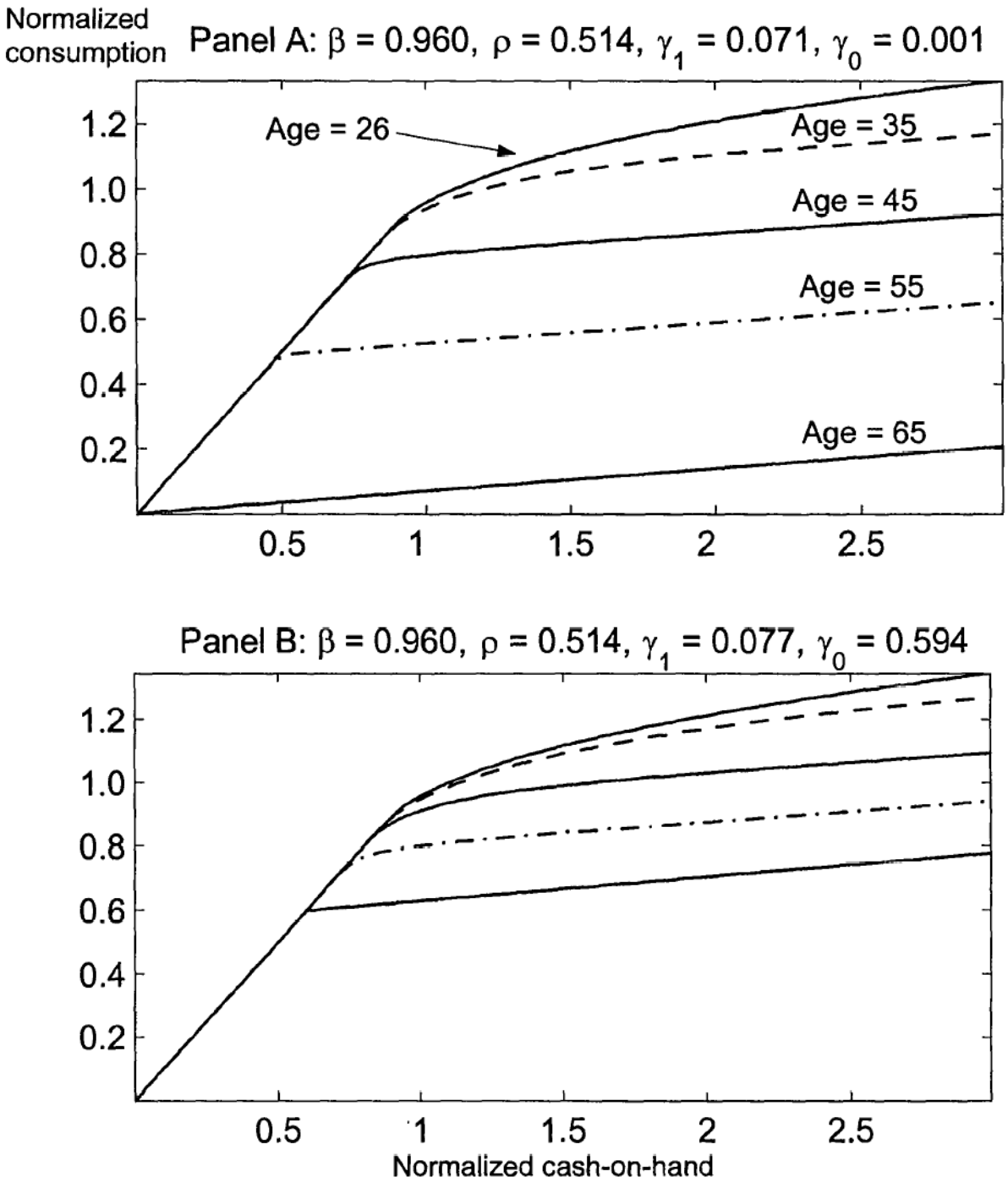


FIGURE 1.—A tale of two households.

Gourinchas & Parker, 2002

Panel A implies $\gamma_1 = 0.071, \gamma_0 = 0.001$, h=0.014

Panel B implies $\gamma_1 = 0.077, \gamma_0 = 0.077$, h=7.7

Method of Simulated Moments

- moment condition:

$$E\left[\zeta\left(\ln C_i; \psi_0\right)\right] = 0$$

with t-the element being

$$\zeta_t\left(\ln C_i; \psi_0\right) = \ln C_{i,t} - \ln C_t\left(\psi_0\right)$$

Method of Simulated Moments (cont.)

- estimated moments from data:

$$\ln \overline{C}_t = \frac{1}{I_t} \sum_{I} \ln C_{i,t}$$

- simulated moments: unconditional expectation of consumption at each age

$$\begin{aligned} \ln \widehat{C}_t \left(\theta, \widehat{\chi}\right) &= \frac{1}{L} \sum_{L} \ln C_t \left(\widehat{s}_{l,t}, \theta, \widehat{\chi}\right) \\ &\rightsquigarrow \ln C_t \left(\theta, \widehat{\chi}\right) \ as \ L \to \infty \end{aligned}$$

Method of Simulated Moments

- empirical counterpart of moment condition

$$\begin{aligned} g_t\left(\theta, \widehat{\chi}\right) &= \frac{1}{I_t}\sum_{I}\widehat{\zeta}\left(\ln C_{i,t}, \theta, \widehat{\chi}\right) \\ &= \frac{1}{I_t}\sum_{I}\ln C_{i,t} - \ln \widehat{C}_t\left(\theta, \widehat{\chi}\right) \\ &= \ln \overline{C}_t - \ln \widehat{C}_t\left(\theta, \widehat{\chi}\right) \end{aligned}$$

- minimize

$$g\left(\theta, \widehat{\chi}\right)' W g\left(\theta, \widehat{\chi}\right)$$

Two step procedure

1. use $W = I$, get consistent estimates $\widehat{\theta}_1$, construct

$$\widehat{\Omega} = \left[\ln \overline{C}_t - \ln \widehat{C}_t\left(\widehat{\theta}_1, \widehat{\chi}\right)\right]\left[\ln \overline{C}_t - \ln \widehat{C}_t\left(\widehat{\theta}_1, \widehat{\chi}\right)\right]'$$

2. use $W = \widehat{\Omega}^{-1}$, get consistent and efficient estimates $\widehat{\theta}_2$

Asymptotic properties

asymptotic properties of $\widehat{\theta}_2$:

$$\sqrt{N}\left(\widehat{\theta}-\theta\right)\rightarrow N\left(0,\Sigma\right)$$

with

$$\widehat{\Sigma}=\left(1+\frac{1}{nS}\right)\left[\left(\frac{\partial\left(\ln\overline{C}_t-\ln\widehat{C}_t(\theta,\widehat{\chi})\right)}{\partial\theta}\right)'\widehat{\Omega}^{-1}\left(\frac{\partial\left(\ln\overline{C}_t-\ln\widehat{C}_t(\theta,\widehat{\chi})\right)}{\partial\theta}\right)\right]^{-1}$$

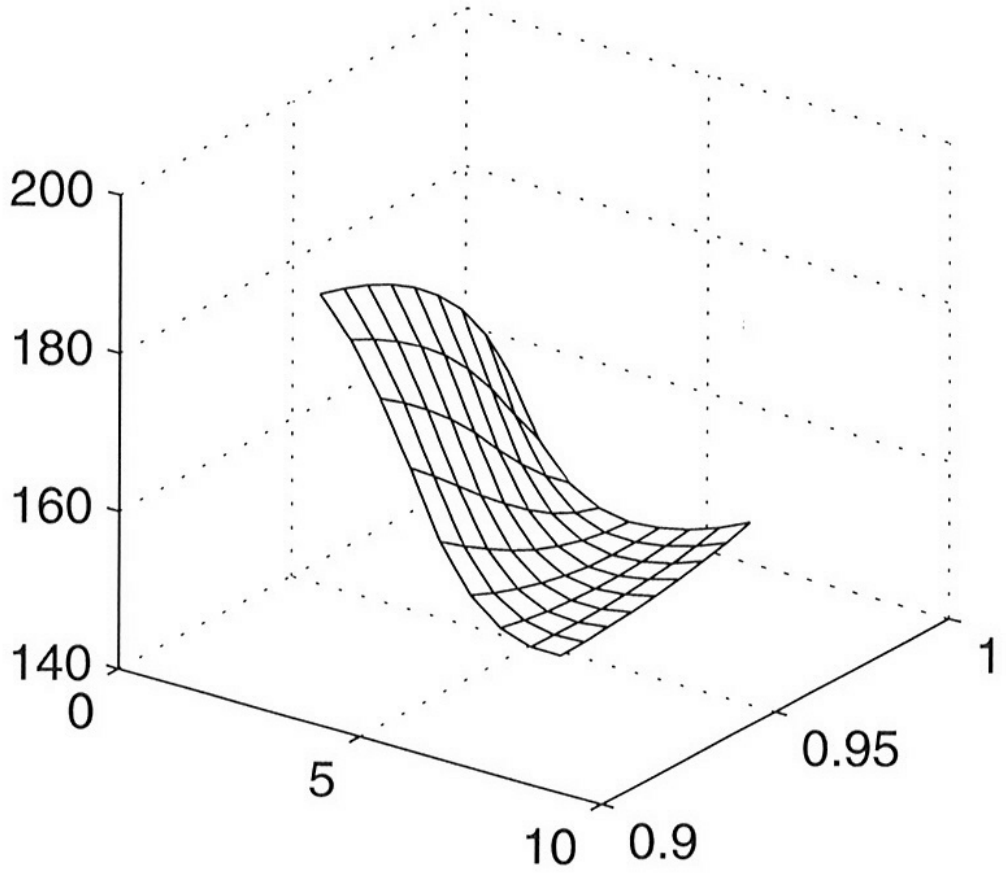
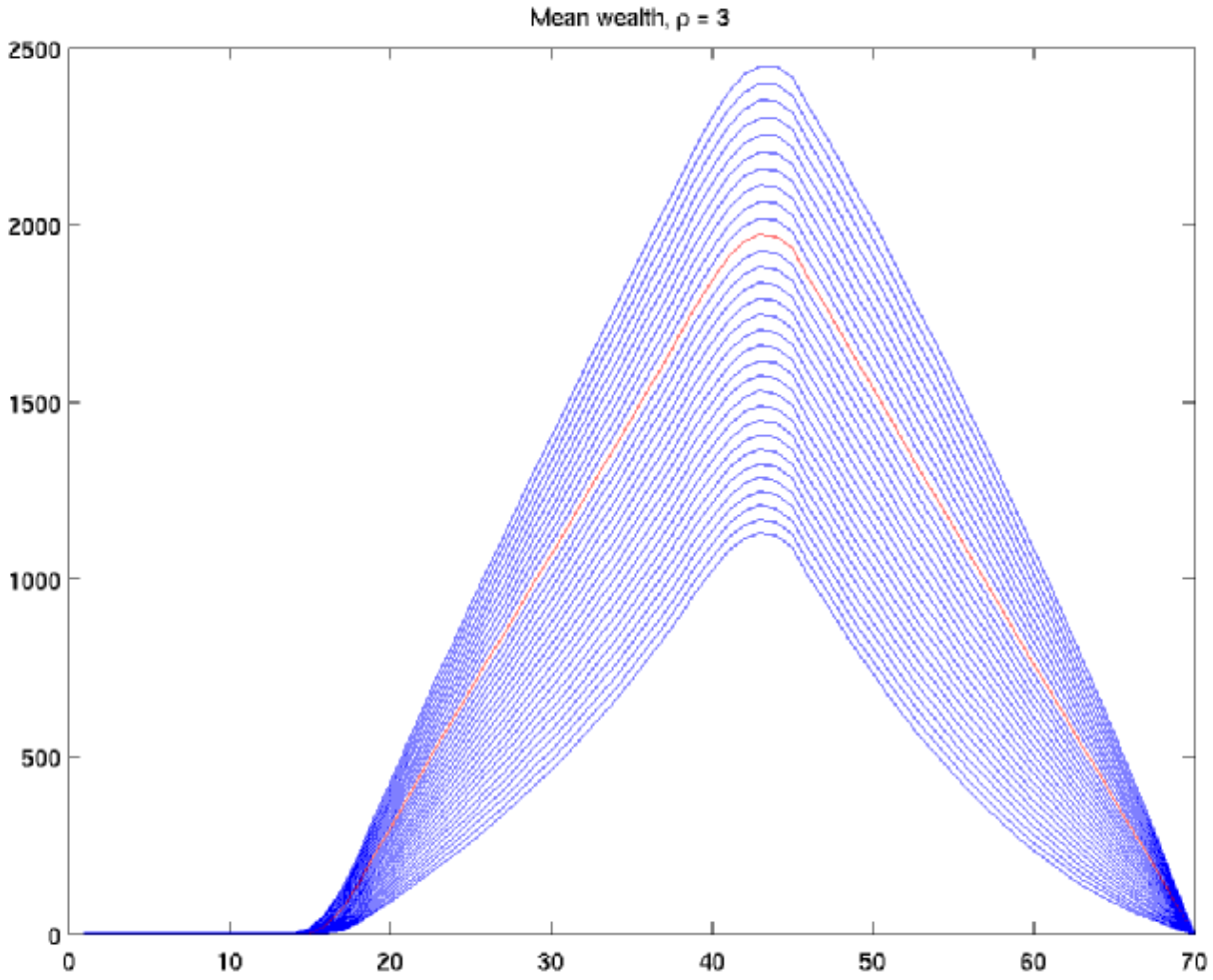
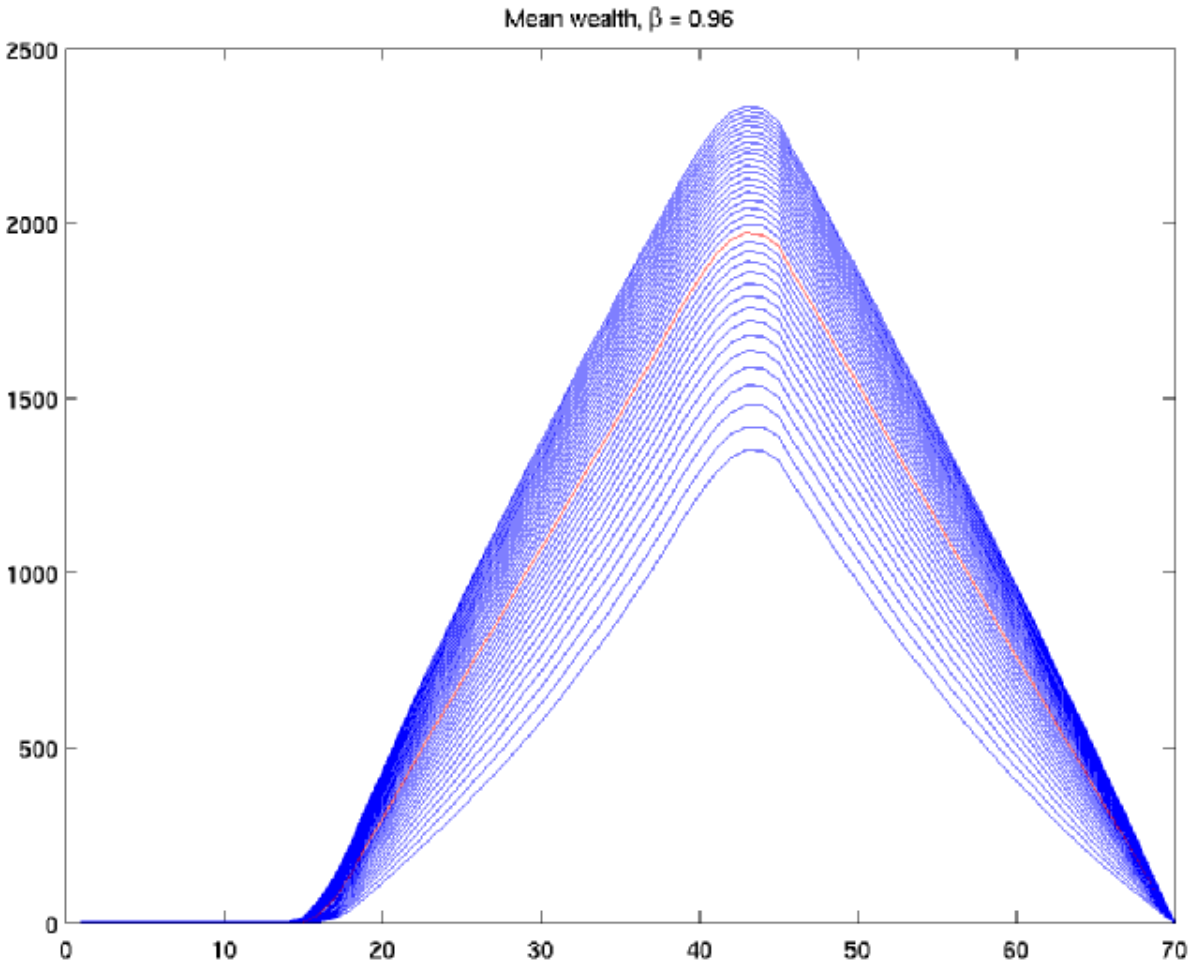


Figure 6. Value of the Objective Function (10), College Graduates, PSID. The minimum is .989 and 4.26.

Cagetti (2003)



this is sample wealth graph for quite extreme assumptions on life cycle income growth;
beta=0.96 is red line; other lines: beta varying from 0.92 to 0.98



this is sample wealth graph with same calibration; red line rho=3; other lines rho=1.5 to rho=5

Consumption Profile from CEX

- estimate

$$\ln \widetilde{C}_i = f_i \pi_1 + a_i \pi_2 + b_i \pi_3 + U_i \pi_4 + \mathrm{Re}\, t_i \pi_5 + \varepsilon_i$$

- construct household-level consumption that represents consumption of the observed household with "average" characteristics

$$\ln C_i = \overline{f}_t \widehat{\pi}_1 + a_i \widehat{\pi}_2 + \overline{U} \widehat{\pi}_4 + \widehat{\varepsilon}_i$$

- average these consumption data across households:

$$\ln C_a = \overline{f} \widehat{\pi}_1 + a \widehat{\pi}_2 + \overline{U} \widehat{\pi}_4$$

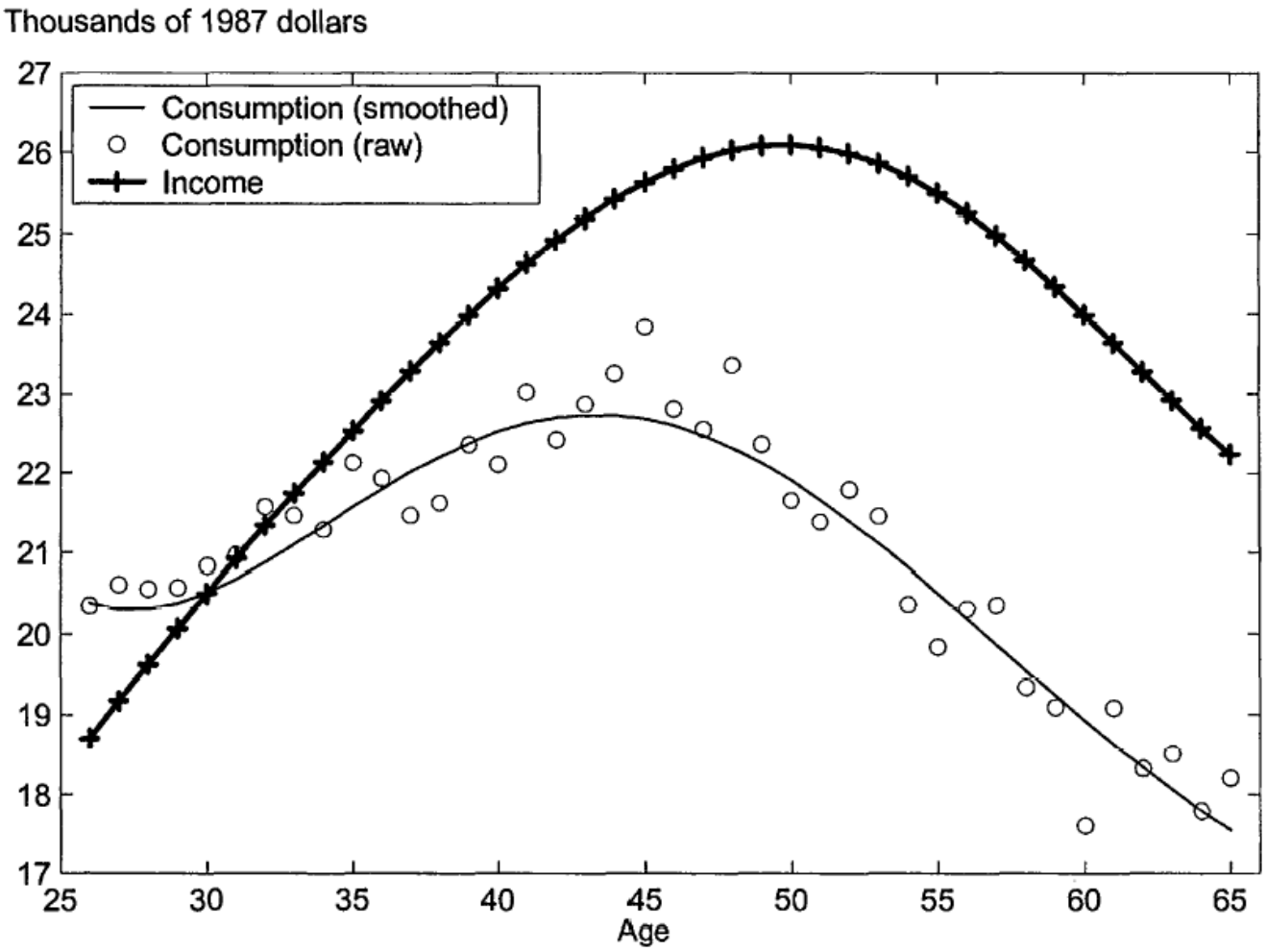


FIGURE 2.—Household consumption and income over the life cycle.

Gourinchas/Parker (2002)

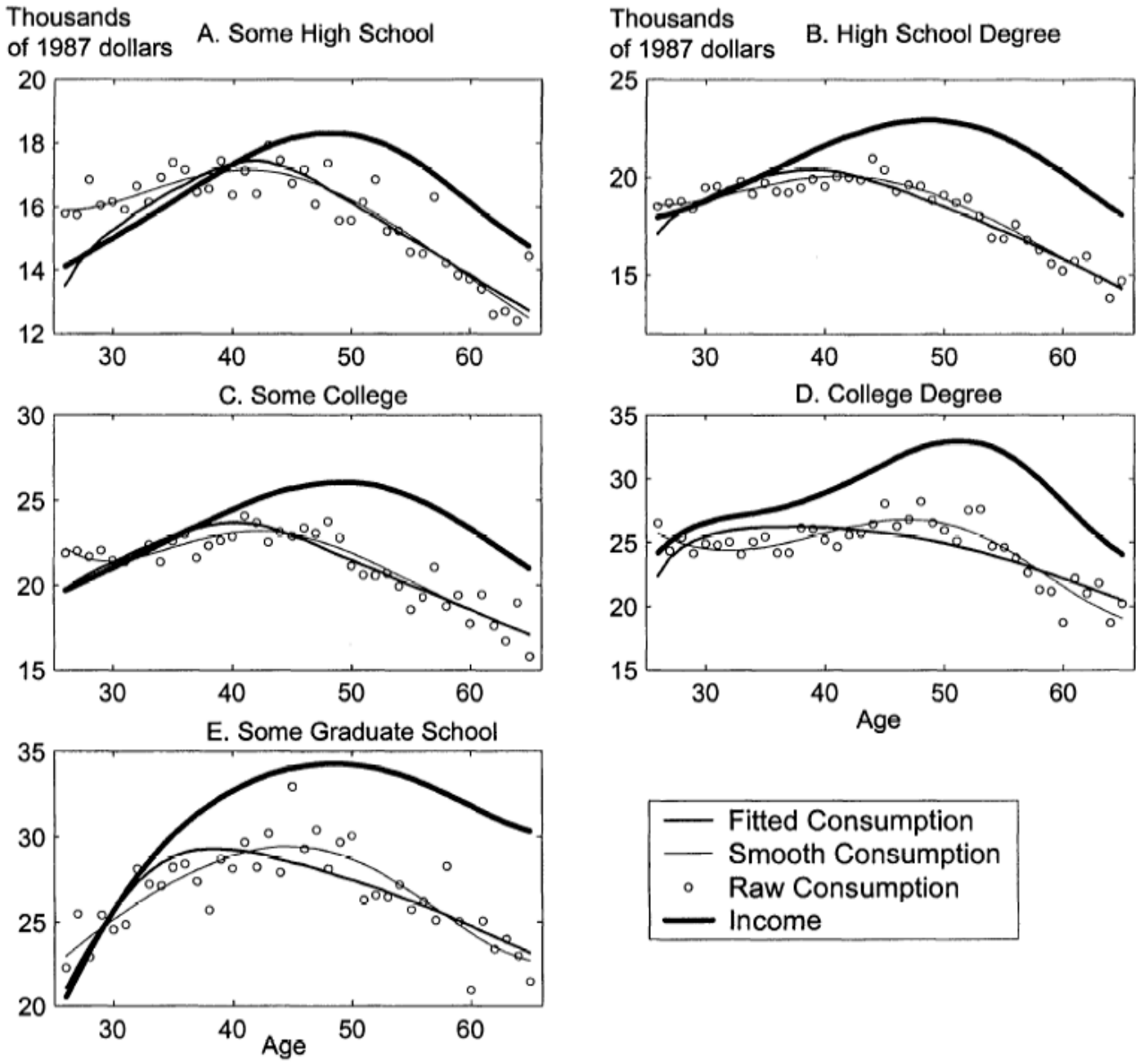


FIGURE 3.—Household consumption and income over the life cycle, by education group.

G/P(2002)

TABLE III

STRUCTURAL ESTIMATION RESULTS

MSM Estimation	Robust Weighting	Optimal Weighting
Discount Factor (β)	0.9598	0.9569
S.E.(A)	(0.0101)	
S.E.(B)	(0.0179)	(0.0150)
Discount Rate ($\beta^{-1}-1$)(%)	4.188	4.507
S.E.(A)	(1.098)	
S.E.(B)	(1.949)	(1.641)
Risk Aversion (ρ)	0.5140	1.3969
S.E.(A)	(0.1690)	
S.E.(B)	(0.1707)	(0.1137)
Retirement Rule:		
γ_0	0.0015	$5.68\,10^{-6}$
S.E.(A)	(3.84)	
S.E.(B)	(3.85)	(16.49)
γ_1	0.0710	0.0613
S.E.(A)	(0.1215)	
S.E.(B)	(0.1244)	(0.0511)
χ^2(A)	175.25	
χ^2(B)	174.10	185.67

Note: MSM estimation for entire group. Standard errors calculated without (A) and with (B) correction for first stage estimation. Cell size is 36,691 households. The last row reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71. Efficient estimates are calculated with a weighting matrix $\widehat{\Omega}$ computed from the robust estimates.

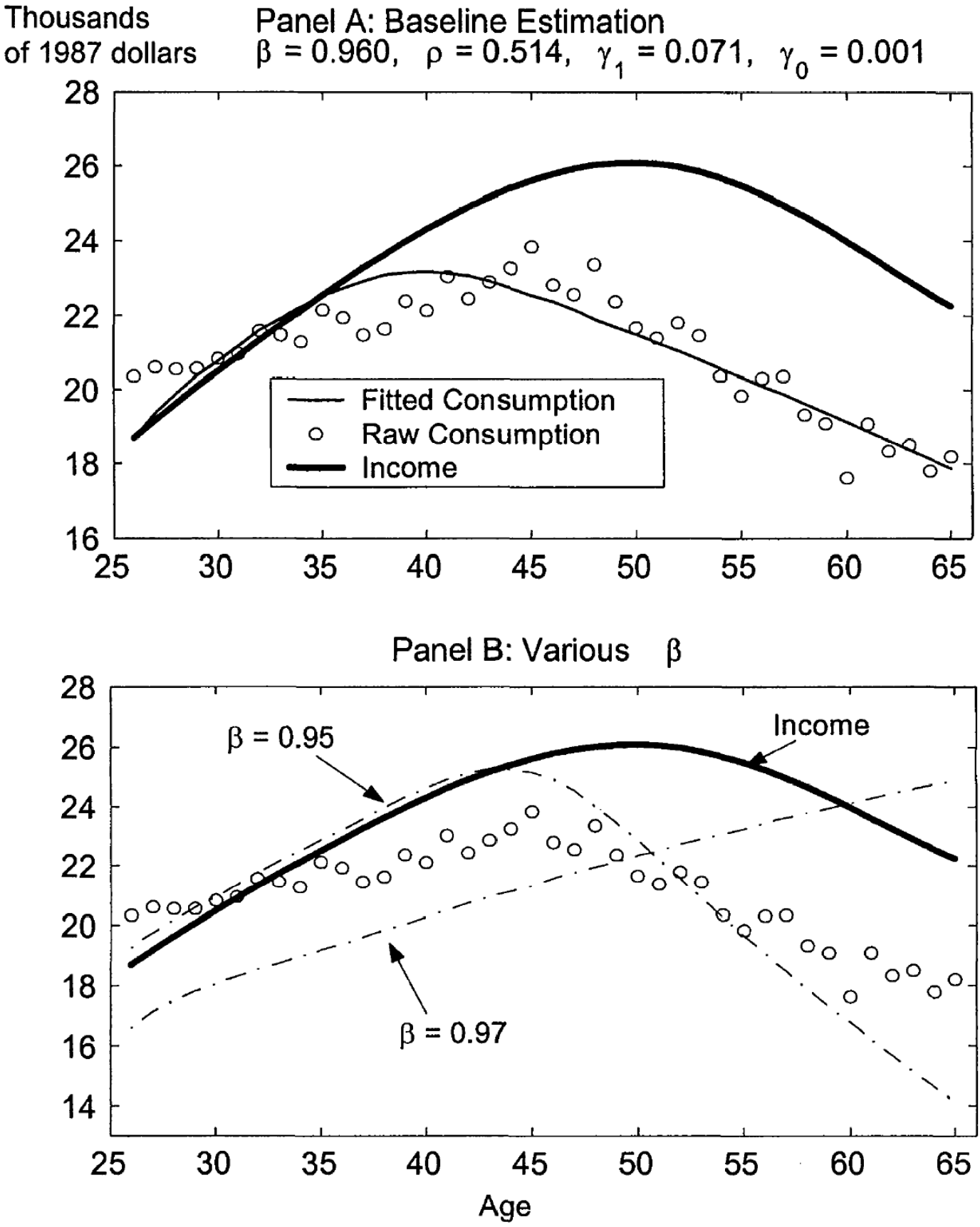


FIGURE 5.—The fitted consumption profile.

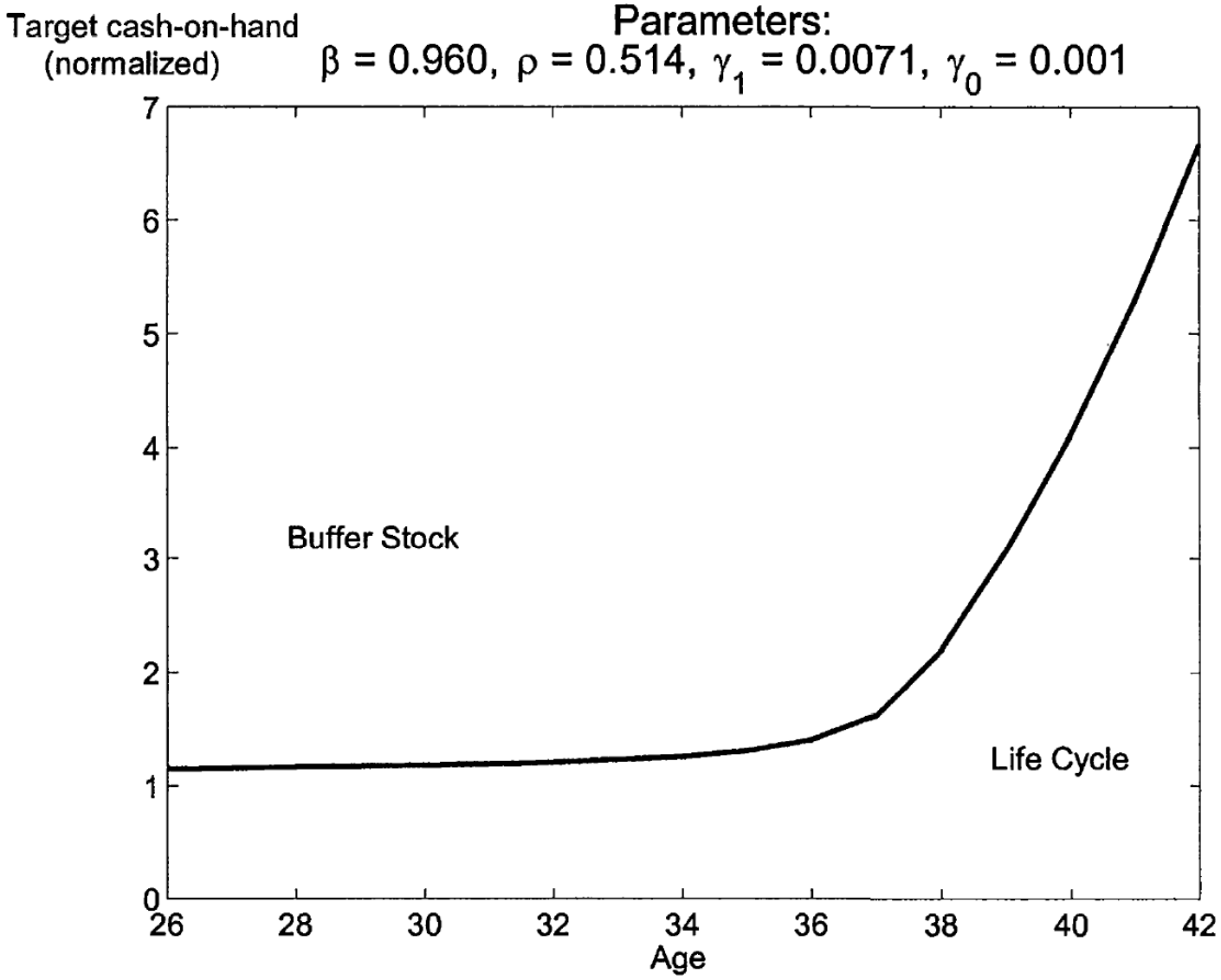


FIGURE 6.—Normalized target cash-on-hand by age.

G/P (2002)

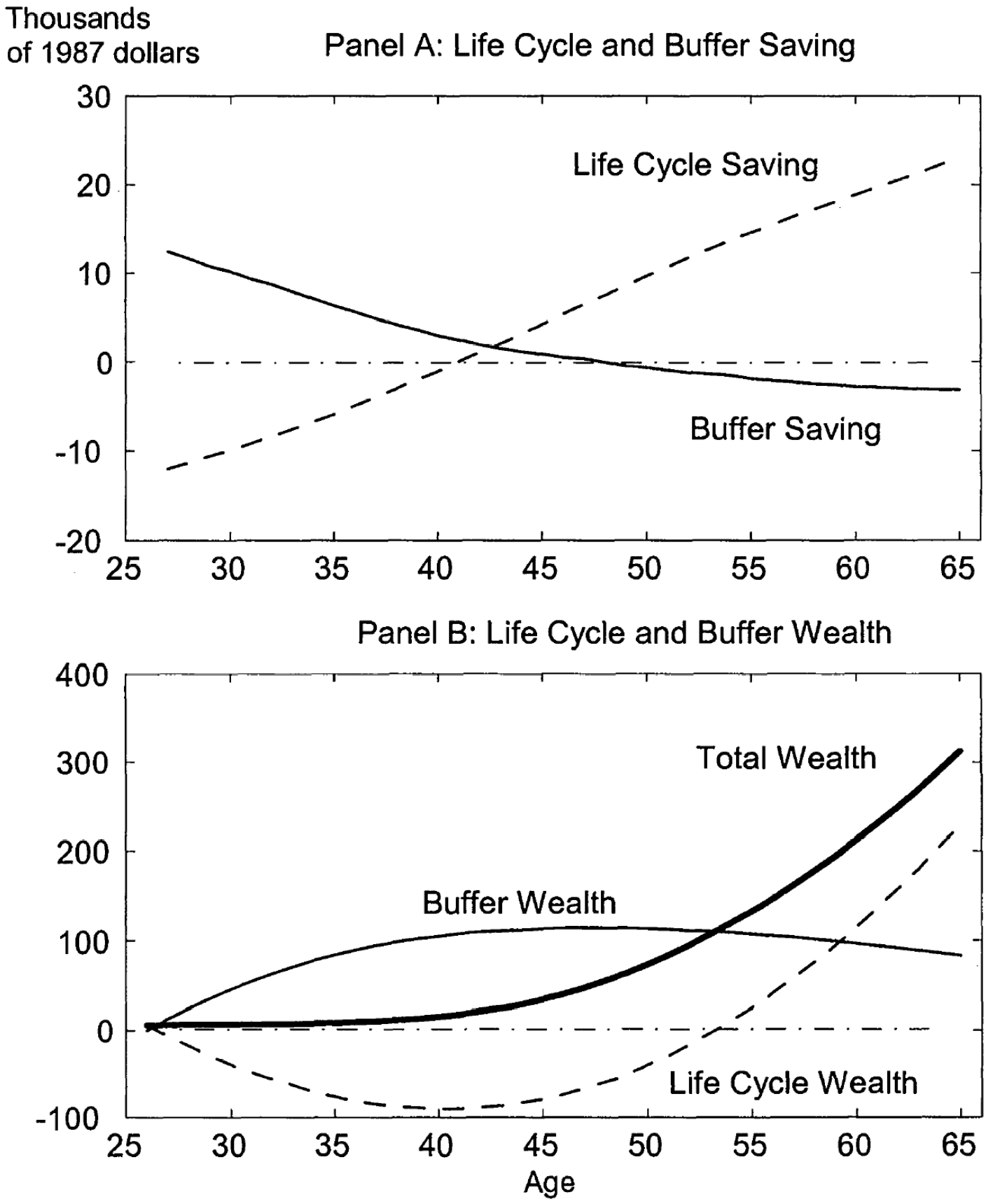


FIGURE 7.—The role of risk in saving and wealth accumulation.

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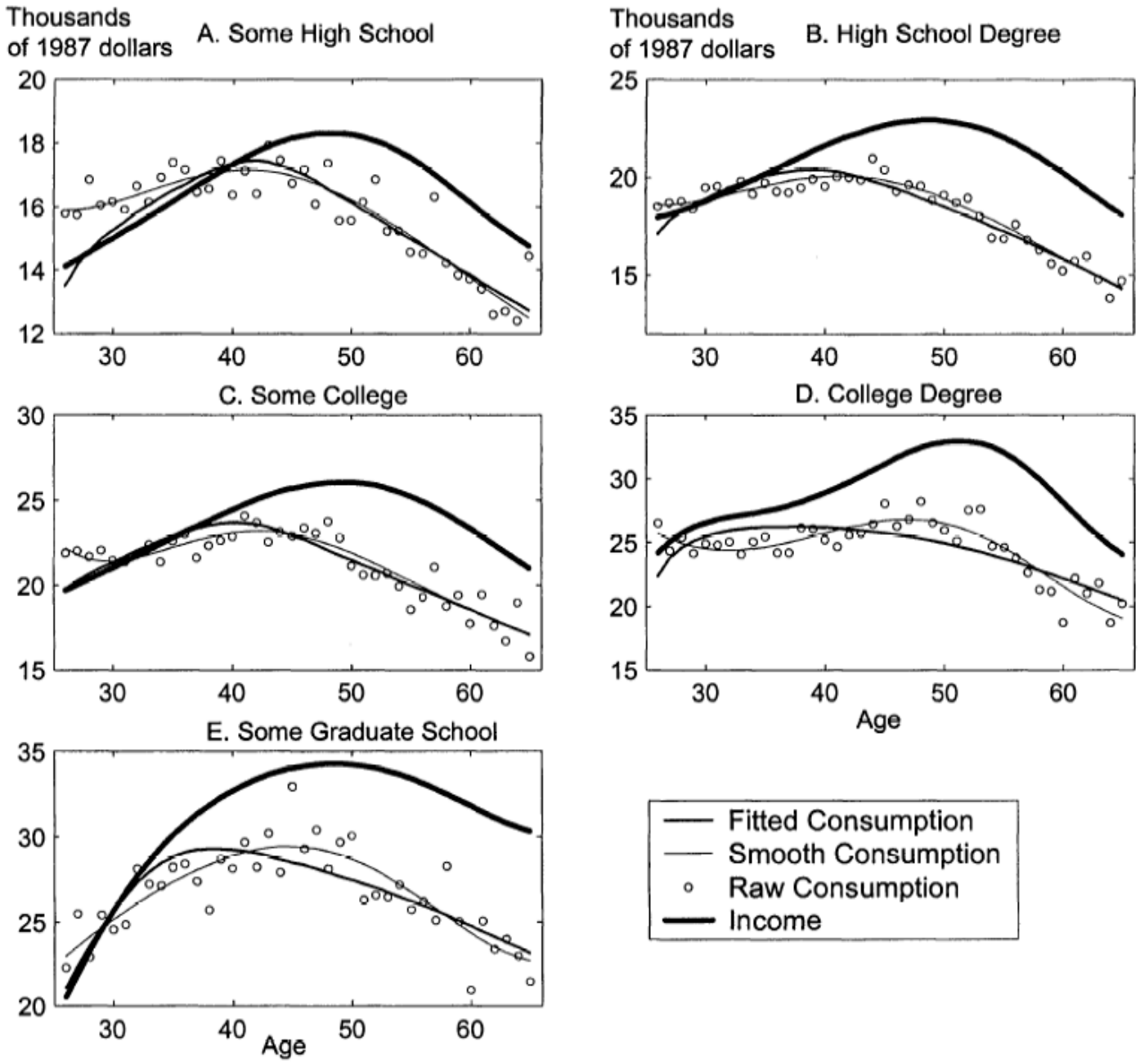
TABLE IV

ESTIMATES FROM THE STOCHASTIC MODEL BY EDUCATION AND OCCUPATION

Group	Discount Factor (β)	Discount Rate ($\beta^{-1}-1$)	Risk Aversion (ρ)	Retirement Rule γ_0	Retirement Rule γ_1	χ^2	N
Education							
Some High School	0.962	3.94	0.282	0.209	0.072	53.60	4,270
	(0.082)	(8.92)	(1.481)	(5.04)	(2.360)		
High School Graduate	0.949	5.30	0.869	$3.79\,10^{-3}$	0.059	59.12	12,445
	(0.015)	(1.64)	(0.220)	(20.05)	(0.049)		
Some College	0.960	4.15	0.394	0.351	0.043	84.21	9,653
	(0.159)	(17.29)	(2.344)	(4.095)	(3.156)		
College Graduate	0.930	7.48	2.290	$1.55\,10^{-8}$	0.049	111.70	6,350
	(0.060)	(6.97)	(0.423)	(54.60)	(0.075)		
Graduate School	0.944	5.93	1.694	$1.06\,10^{-7}$	0.057	87.26	5,973
	(0.087)	(9.77)	(0.843)	(18.23)	(0.076)		
Occupation							
Managerial and Prof.	0.946	5.71	1.672	$5.20\,10^{-8}$	0.050	115.62	12,693
	(0.060)	(6.69)	(0.524)	(22.78)	(0.067)		
Tech., Sales, Admin.	0.953	4.90	1.059	$2.13\,10^{-7}$	0.049	64.02	6,548
	(0.037)	(4.11)	(0.339)	(39.42)	(0.064)		
Precision Prod., Craft	0.953	4.97	0.990	0.003	0.054	52.86	4,469
	(0.333)	(36.77)	(3.895)	(18.49)	(0.997)		
Operators, Laborers	0.953	4.90	0.867	$3.14\,10^{-6}$	0.049	57.58	6,063
	(0.489)	(53.80)	(4.846)	(1365.28)	(2.35)		

Note: MSM estimation in levels. Standard errors calculated with correction for first stage estimation. The next to last column reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71.



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