Consumption and Saving

Lecture 4

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General Equilibrium Heterogeneous Agent Model

Utility maximization problem:

$$\max_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E_0 u\left(C_t\right)$$

subject to

$$c_t + k_{t+1} = \left(1 + r_t\right) k_t + w_t \epsilon_t$$

and

$$k_t \geq 0.$$

Bellman Equation Formulation

Bellman equation:

$$v\left(k,\epsilon\right)=\max_{k'}\left[u\left(c\right)+\beta\sum_{\epsilon'}prob\left(\epsilon'|\epsilon\right)v\left(k',\epsilon'\right)\right]$$

subject to

$$c_t+k_{t+1}=\left(1+r_t\right)k_t+w_t\epsilon_t$$

and

$$k_t\geq 0.$$

Aggregate Economy

- Labor supply $L = \left(\text{fraction of agents in } \epsilon_{high}\right) * \epsilon_{high}$

$$+ \left(\text{fraction of agents in } \epsilon_{low}\right) * \epsilon_{low}$$

- Capital stock $K = \sum_{k}\sum_{\epsilon}\lambda\left(k,\epsilon\right)k'\left(k,\epsilon\right)$
- Aggregate Cobb-Douglas production function $Y = F\left(K,L\right)$
- From profit maximization:

$$r\left(K,L\right) = \frac{\partial F\left(K,L\right)}{\partial K}$$
$$w\left(K,L\right) = \frac{\partial F\left(K,L\right)}{\partial L}$$

Equilibrium Definition

Definition 1 *A stationary equilibrium is a decision rule* $k'(k,\epsilon)$*, stationary distribution* $\lambda(k,\epsilon)$*, aggregate quantities* K *and* L*, and aggregate prices* w *and* r*, such that*

1. *given prices* r *and* w*, the decision rule* $k'(k,\epsilon)$ *solves the maximization problem of the household,*
2. *prices are equal to marginal products (i.e. firms maximize profits),*
3. $\lambda(k,\epsilon)$ *is time-invariant,*
4. *aggregate level of* K *is implied by the households' behavior:*

$$K = \sum_{k}\sum_{\epsilon}\lambda(k,\epsilon)\,k'(k,\epsilon)\,.$$

Quantitative Results

Table 3
Main statistics of the benchmark economy and its deterministic counterpart

	D Deterministic economy	B Benchmark economy	Change $\frac{B-D}{D} * 100$
Aggregate assets	2.959	3.015	1.9%
Output	1.000	1.007	0.7%
Capital output ratio	2.959	2.994	1.2%
Interest rate	4.17%	4.02%	−3.5%
Coeff. of variation of wealth	0.0	0.748	—
Gini index of wealth	0.0	0.404	—

Diaz, Pijoan-Mas, and Rios-Rull (2003)

Equilibrium Asset Distribution

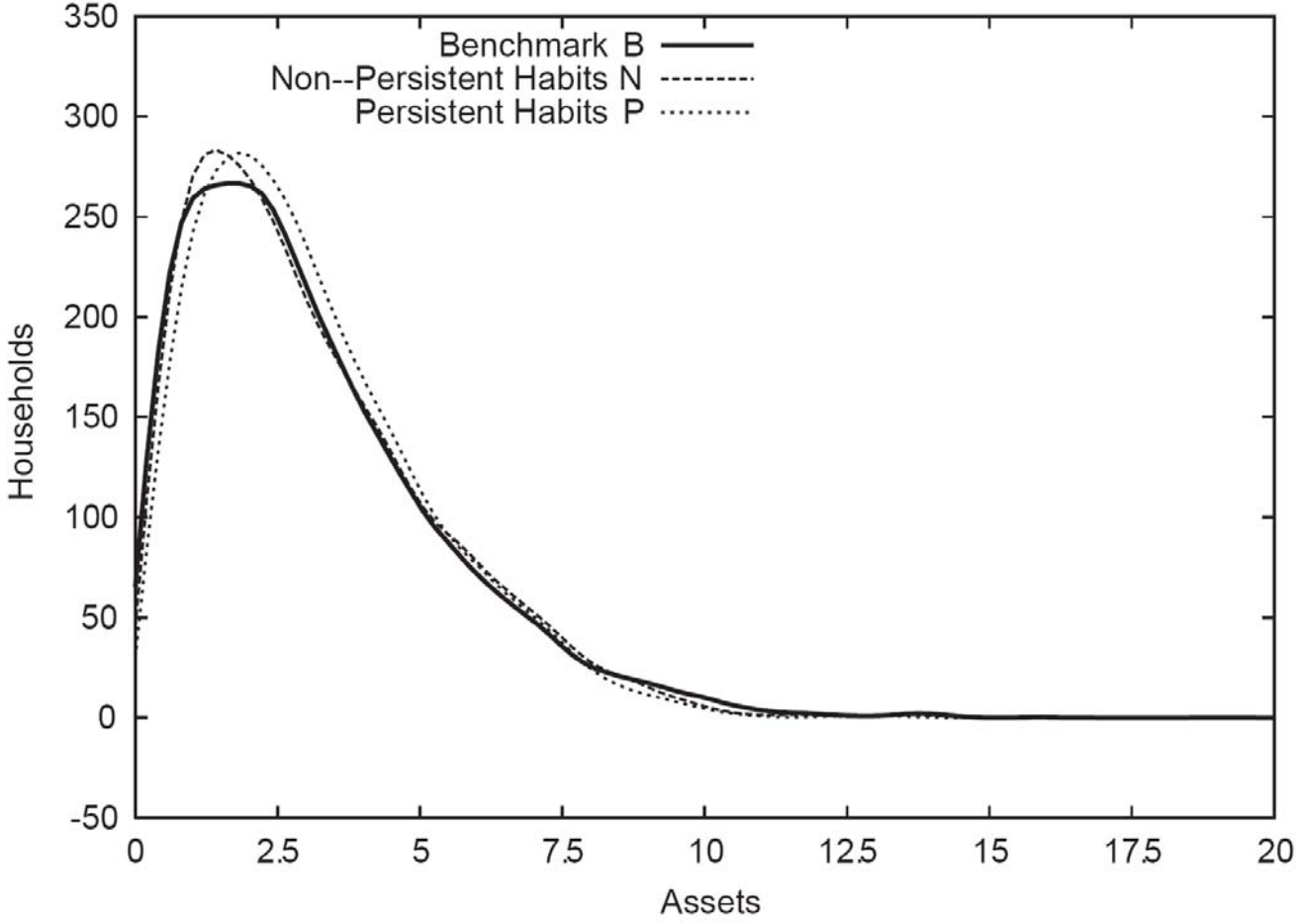

Fig. 2. Histogram for assets: unadjusted economies $\sigma = 2$, $\beta = 0.96$. General equilibrium.

Diaz, Pijoan-Mas, and Rios-Rull (2003)

Krusell and Smith (1998)

- idiosyncratic shocks $\epsilon \in \{0; 1\}$
- aggregate shocks $z \in \left\{z_g; z_b\right\}$
- aggregate production function: $F = zK^{\alpha}L^{1-\alpha}$
- both shocks are correlated: $u_g < u_b$
- joint transition probability $\pi_{ss'\epsilon\epsilon'}$
- impose

$$-\pi_{ss'00} + \pi_{ss'01} = \pi_{ss'10} + \pi_{ss'11} = \pi_{ss'}$$

$$-u_s \frac{\pi_{ss'00}}{\pi_{ss'}} + (1 - u_s) \frac{\pi_{ss'10}}{\pi_{ss'}} = u_{s'}$$

Krusell and Smith (cont.)

- distribution Γ over capital and employment status evolves according to H:

$$\Gamma' = H\left(\Gamma, z, z'\right)$$

- prices equal marginal products

$$w\left(K, L, z\right) = \left(1-\alpha\right) z \left(\frac{K}{L}\right)^{\alpha}$$

$$r\left(K, L, z\right) = \alpha z \left(\frac{K}{L}\right)^{\alpha-1}$$

Bellman Equation

$$v\left(k,\epsilon;\Gamma,z\right)=\max_{k'}\left\{U\left(c\right)+\beta E\left[v\left(k',\epsilon';\Gamma',z'\right)|z,\epsilon\right]\right\}$$

subject to

$$\begin{aligned}c+k' &= r\left(K,L,z\right)k+w\left(K,L,z\right)\epsilon+\left(1-\delta\right)k\\ \Gamma' &= H\left(\Gamma,z,z'\right)\\ k' &\geq 0\end{aligned}$$

- denote policy function as $f:k'=f\left(k,\epsilon;\Gamma,z\right)$

Definition Equilibrium

Definition 2 *An equilibrium is a law of motion H, a decision rule $f(k, \epsilon; \Gamma, z)$, and aggregate prices w and r, such that*

1. *the decision rule f solves the maximization problem of the household,*
2. *prices are equal to marginal products,*
3. *H is generated by f.*

Calibration

- quarterly periods
- $\beta = 0.99$, depreciation rate $\gamma = 0.025$, capital share $\alpha = 0.36$
- log utility
- states and transition probabilities:
  - $z_g = 1.01$, $z_b = 0.99$
  - unemployment rates $u_g = 0.04$, $u_b = 0.1$
  - average duration of good and bad state is eight quarters
  - average duration of unemployment spell is 1.5 quarters in good times, and 2.5 quarters in bad times

Results

Law of motion

- in good times:

$$\log K' = 0.095 + 0.962 \log K$$
$$R^2 = .999998,\ \widehat{\sigma} = 0.0028\%$$

- in bad times:

$$\log K' = 0.085 + 0.965 \log K$$
$$R^2 = .999998,\ \widehat{\sigma} = 0.0036\%$$

TABLE 2

DISTRIBUTIONS OF EARNINGS AND WEALTH IN THE U.S. ECONOMY (%)

	Quintile					Top Groups (Percentile)		
Gini	First	Second	Third	Fourth	Fifth	90th–95th	95th–99th	99th–100th
	A. Distribution of Earnings							
.63	−.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
	B. Distribution of Wealth							
.78	−.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55

Castaneda, Diaz-Gimenez and Rios-Rull (2003)

TABLE 1

Distribution of Wealth: Models and Data

Model	Percentage of Wealth Held by Top					Fraction with Wealth < 0	Gini Coefficient
	1%	5%	10%	20%	30%		
Benchmark model	3	11	19	35	46	0	.25
Stochastic-β model	24	55	73	88	92	11	.82
Data	30	51	64	79	88	11	.79

Krusell/Smith (1998)

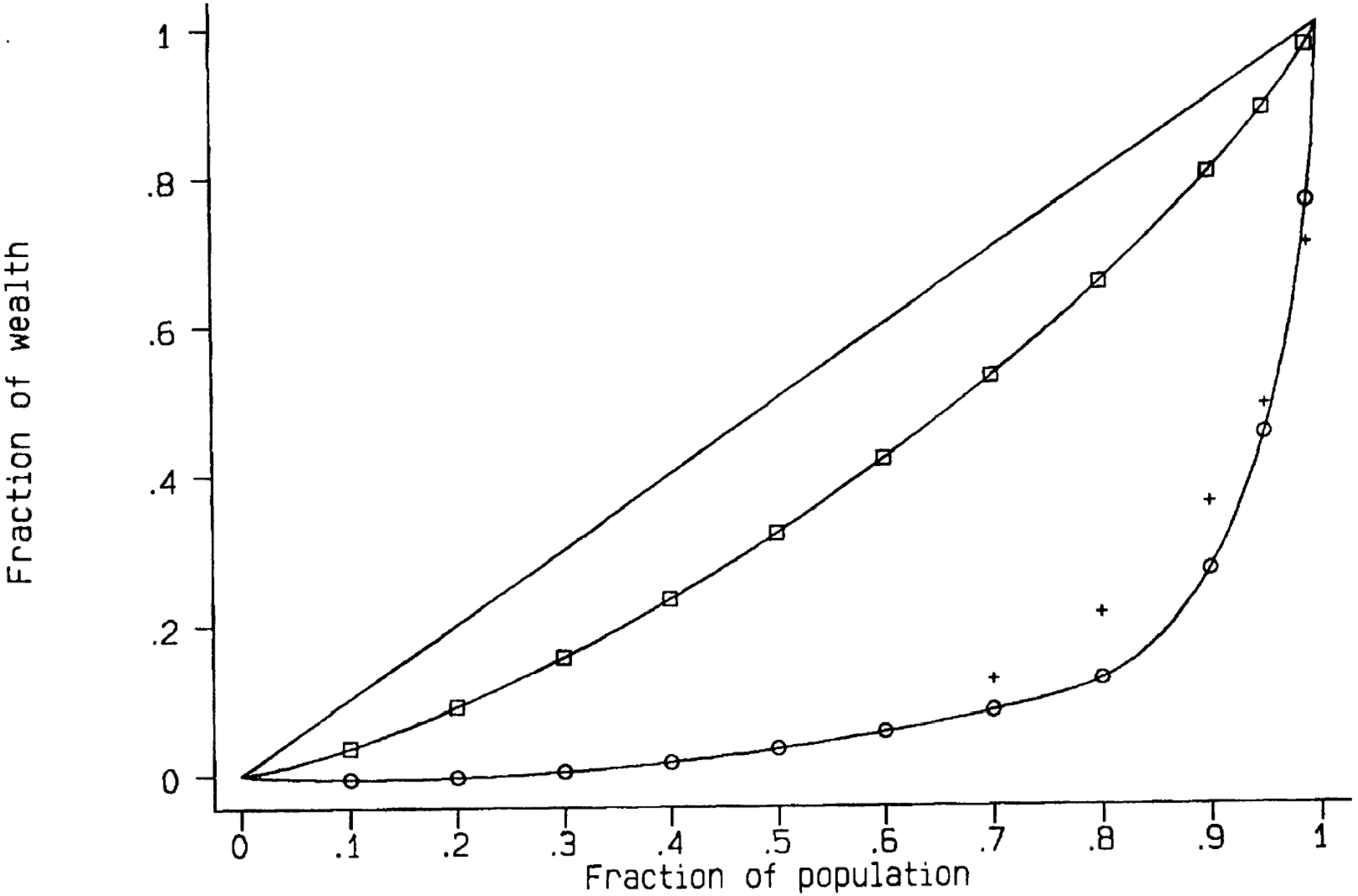

FIG. 3.—Lorenz curves for wealth holdings (+ refers to the data, □ to the benchmark model, and ○ to the stochastic-β model).

TABLE 2

AGGREGATE TIME SERIES

Model	Mean(k_t)	Corr(c_t, y_t)	Standard Deviation (i_t)	Corr(y_t, y_{t-4})
Benchmark:				
Complete markets	11.54	.691	.031	.486
Incomplete markets	11.61	.701	.030	.481
$\sigma = 5$:				
Complete markets	11.55	.725	.034	.551
Incomplete markets	12.32	.741	.033	.524
Real business cycle:				
Complete markets	11.56	.639	.027	.342
Incomplete markets	11.58	.669	.027	.339
Stochastic-β:				
Incomplete markets	11.78	.825	.027	.459

TABLE 7

DISTRIBUTIONS OF EARNINGS AND OF WEALTH IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES (%)

		Quintile					Top Groups (Percentile)		
Economy	Gini	First	Second	Third	Fourth	Fifth	90th–95th	95th–99th	99th–100th
				A. Distributions of Earnings					
United States	.63	−.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Benchmark	.63	.00	3.74	14.59	15.99	65.68	15.15	17.65	14.93
				B. Distributions of Wealth					
United States	.78	−.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Benchmark	.79	.21	1.21	1.93	14.68	81.97	16.97	18.21	29.85

Castaneda, Diaz-Gimenez and Rios-Rull (2003)

TABLE 4

TRANSITION PROBABILITIES OF THE PROCESS ON THE ENDOWMENT OF EFFICIENCY LABOR UNITS FOR WORKING-AGE HOUSEHOLDS THAT REMAIN AT WORKING AGE ONE PERIOD LATER, $\mathbf{\Gamma}_{\varepsilon\varepsilon}$ (%)

	TO s'			
FROM s	$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	96.24	1.14	.39	.006
$s = 2$	3.07	94.33	.37	.000
$s = 3$	1.50	.43	95.82	.020
$s = 4$	10.66	.49	6.11	80.51

TABLE 5

RELATIVE ENDOWMENTS OF EFFICIENCY LABOR UNITS, $e(s)$, AND THE STATIONARY DISTRIBUTION OF WORKING-AGE HOUSEHOLDS, $\boldsymbol{\gamma}_{\varepsilon}^{*}$

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$e(s)$	1.00	3.15	9.78	1,061.00
$\boldsymbol{\gamma}_{\varepsilon}^{*}$ (%)	61.11	22.35	16.50	.0389

TABLE 8

DISTRIBUTIONS OF CONSUMPTION IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES (%)

		Quintile					Top Groups (Percentile)		
Economy	Gini	First	Second	Third	Fourth	Fifth	90th–95th	95th–99th	99th–100th
United States:									
Nondurables	.32	6.87	12.27	17.27	23.33	40.27	9.71	10.30	4.83
Nondurables+*	.30	7.19	12.96	17.80	23.77	38.28	9.43	9.69	3.77
Benchmark:									
Wealthiest 1% excluded	.40	5.23	12.96	13.55	20.41	47.85	12.77	14.89	3.83
Entire sample	.46	4.68	11.58	12.07	18.68	52.99	12.82	13.45	11.94

* Includes imputed services of consumer durables.

TABLE 9

EARNINGS AND WEALTH PERSISTENCE IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES: FRACTIONS OF HOUSEHOLDS THAT REMAIN IN THE SAME QUINTILE AFTER FIVE YEARS

	Quintile				
Economy	First	Second	Third	Fourth	Fifth
	A. Earnings Persistence				
United States	.86	.41	.47	.46	.66
Benchmark	.76	.55	.65	.80	.80
	B. Wealth Persistence				
United States	.67	.47	.45	.50	.71
Benchmark	.81	.80	.80	.75	.89

TABLE 1

DISTRIBUTIONS OF EARNINGS AND OF WEALTH IN THE UNITED STATES AND IN SELECTED MODEL ECONOMIES

	Gini	Bottom 40%	Top 5%	Top 1%
		A. U.S. Economy		
Earnings	.63	3.2	31.2	14.8
Wealth	.78	1.7	54.0	29.6
		B. Aiyagari (1994)		
Earnings	.10	32.5	7.5	6.8
Wealth	.38	14.9	13.1	3.2
		C. Castañeda et al. (1998)		
Earnings	.30	20.6	10.1	2.0
Wealth	.13	32.0	7.9	1.7
		D. Quadrini (1998)		
Earnings	…	…	…	…
Wealth	.74	…	45.8	24.9
		E. Krusell and Smith (1998)		
Earnings	…	…	…	…
Wealth	.82	…	55.0	24.0
		F. Huggett (1996)		
Earnings	.42	9.8	22.6	13.6
Wealth	.74	.0	33.8	11.1
		G. De Nardi (1999)		
Earnings	…	…	…	…
Wealth	.61	1.0	38.0	15.0