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Dynamic bilateral bargaining under private information with a sequence of potential buyers

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Abstract

A seller owning a single, indivisible asset faces the random arrival of privately informed buyers, with whom he can bargain sequentially. Our key result is that despite the arrival of alternative buyers the Coase conjecture continues to hold under stationary strategies if the distribution of buyer valuations has convex support: Negotiations end almost immediately and the asset is sold almost at the minimum of the seller's own reservation value and the lowest possible valuation of a buyer. We also show existence of multiple stationary equilibria, though, in the special case where the support of buyers' valuations exhibits a sufficiently large "interior gap". Taken together, our findings thus also point to a potential pitfall when analyzing only two-type distributions in more applied work.

JEL classification: C78; D82

Keywords: Coase conjecture; Bargaining

1. Introduction

The theory of bargaining with private information has focused squarely on the case of a bilateral monopoly situation. (See, however, the contributions discussed below.) Clearly, for most applied work, though, it is more realistic to assume that either side to a transaction may have alternative options for trade. The vast literature on bargaining in search and matching markets

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takes this into account, albeit typically with the restriction to negotiations under complete information.[1]

In this paper we consider a seller with a single, indivisible asset who faces the random arrival of potential buyers, each having private information about his valuation. We restrict the trading mechanism to bilateral negotiations. Our main question is how, compared to a bilateral monopoly, the potential arrival of new buyers changes the equilibrium outcome. Our key result is that when restricting attention to stationary strategies, as the time between two consecutive offers becomes arbitrarily small the seller no longer gains from the possibility that another buyer will arrive. This result holds whenever the support of buyers' valuations is convex. In this case we find, more precisely, that the seller will almost surely trade with the first buyer at a price close to the lowest possible valuation. Though natural, the assumption that buyers' valuations have convex support is, however, also crucial. If there is a sufficiently large "gap" in the distribution's support, we find multiple stationary equilibria that support also in the limit, where the time between two consecutive offers vanishes, outcomes with substantially different payoffs for the seller.

We thus show that, given the restrictions of a convex support of valuations and stationary strategies, the Coase conjecture (Coase, 1972) survives despite the stochastic arrival of new trading partners. That is, if the time between two consecutive offers goes to zero, trade occurs almost immediately and the asset is sold almost at the minimum of the seller's own reservation value and the lowest possible valuation of a buyer.

From an applied perspective, these results should then have the following implications. First, if the assumptions of our model seem realistic in a particular setting, then we obtain the extreme result that we can, in essence, analyze a given bilateral negotiation (almost) in isolation. Under bilateral negotiations, if the seller can make new offers arbitrarily fast, then the possibility that new buyers will arrive in the future does not allow the seller to extract a higher share of the bilateral surplus. Second, given that trade takes place (almost surely) with the first buyer, who may have a very low valuation, there can be a substantial loss in gains from trade. Third, the multiplicity that arises, in particular, in the case of a two-type distribution, provided that the "gap" between the low and high valuation is sufficiently large, should warn against the potential pitfalls from making such a simplification. Our results would then suggest that if the two-type assumption is merely a simplification, then we should always pick the sequence of equilibria that supports the Coase Conjecture.

There exists a large body of literature analyzing the Coase conjecture in the bilaterally monopolistic case. The Coase conjecture was first proven in some generality by Fudenberg et al. (1985) and Gul et al. (1986). Ausubel and Deneckere (1989) consider the so-called "no-gap" case where the seller's reservation value is equal to the lowest feasible valuation of the buyer. They show that this gives rise to non-stationary equilibria that do not support the Coase conjecture.[2]

Below we relate our setting and our results to the few papers that also allow the seller to switch buyers. Most importantly, our analysis sheds new light on the results in the seminal papers of Fudenberg et al. (1987) and Samuelson (1992).[3]

[1] For an early overview see Osborne and Rubinstein (1990).

[2] Under some technical assumptions, the bargaining model is isomorphic to a model where a seller with sufficient capacity faces a continuum of buyers and cannot engage in first-degree price discrimination. McAfee and Wiseman (2003) have recently revived interest in this literature by showing that results change drastically once we allow for costs of adjusting capacity.

[3] More recently, Fuchs and Skrzypacz (2006) also consider the case with the possible arrival of new traders. In their setting, however, once new trading partners arrive, the respective "short side" switches to running an (English) auction.

The rest of this paper is organized as follows. Section 3 introduces the model. Section 4 analyzes first an example with a linear distribution of buyers' valuations, before proving our general results. Section 4 concludes with a discussion of the main assumptions and of how our results relate to the existing literature.

2. The model

We consider a monopolistic seller with a single indivisible good who faces a random inflow of potential buyers. Each buyer represents an independent draw from an interval of possible types, indexed by $q \in Q = [0, 1]$. Buyer types are uniformly distributed over Q. The buyer's type is privately known to him. It determines his valuation of the seller's good according to the nonincreasing and Lipschitz continuous function $f(q)$ satisfying $f(q) > 0$ for all $q < 1$. Hence, there exists a finite $H > 0$ such that for all $q, q' \in Q$ with $q' > q$ it holds that

$$f(q) - f(q') \leqslant H(q' - q). \tag{1}$$

To see the role of assumption (1), we must note first that Gul et al. (1986) were able to prove the Coase conjecture for the case where Lipschitz continuity is satisfied at the lowest type $q = 1$.[4] The Coase conjecture says that if the seller can make two consecutive offers arbitrarily fast, which deprives him of any commitment power, then he will end up selling almost immediately at a price that is arbitrarily close to the minimum of his own reservation value and the lowest possible valuation of the buyer. Note now that the presence of other buyers will make the seller's reservation value endogenous in our model. Clearly, he will optimally never sell to types whose valuation is below this threshold. Assuming that (1) holds for all $q \in Q$ allows us in what follows to apply the arguments of the Coase conjecture uniformly, i.e., independently of where this threshold lies.

Note also that by (1) the distribution of valuations has a convex support.[5] This will be crucial as we show below that the Coase conjecture does not hold if there exists a sufficiently large "gap" in the support.

We next normalize the distribution of valuations by setting $f(0) = 1$. As we restrict consideration to stationary strategies for buyers, it is not important whether we have $f(1) > 0$ or $f(1) = 0$. Furthermore, we denote $\underline{f} = f(1)$.

Time runs discretely with periods $n = 0, 1, \ldots$ of equal length $z > 0$. All players use the same discount factor $\delta := \mathrm{e}^{-rz}$ with $r > 0$. Each period a new buyer arrives with probability $\rho > 0$. If the seller is still haggling with an old buyer, he must decide with whom he wishes to continue negotiations. The chosen buyer stays with the seller, while the other buyer leaves immediately and cannot be re-approached later. The seller can now state a price. If the offer is accepted, the good is sold and immediately consumed. Otherwise, the stage game is repeated in the next period.

The major restriction of our setting is that the seller can communicate with only one buyer at a time. In particular, he may not set up a sales mechanism with more than one buyer, e.g., an auction.[6]

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4 This assumption has been relaxed in Ausubel and Deneckere (1989).

5 Given $f(q)$, a buyer's valuation v has the cdf $G(v) = 1 - y$, where $y = \max\{q : f(q) > v\}$.

6 This assumption is also made in most matching and search models. See also Wang (1995) for a related model. For an analysis of optimal mechanisms with sequentially arriving buyers see Riley and Zeckhauser (1983), McAfee and McMillan (1988), and Ehrman and Peters (1994).

We are now more specific about the probability ρ with which a new buyer arrives in a given period. Assuming that this probability is constant over real time, we specify $\rho = 1 - e^{-\mu z}$ with $\mu > 0$. Hence, the arrival of new buyers is governed by a Poisson process with arrival rate μ.[7] To simplify the exposition of results, we further assume that a buyer arrives with probability one in the very first period $n = 0$.

We next invoke several equilibrium requirements. As they are straightforward and follow similar conditions in the literature, we omit a formal specification.[8] All buyers must apply the same strategy, which only depends on their own history. Moreover, we invoke a standard stationary requirement on buyers' strategies. Following Ausubel and Deneckere (1989), we require that a buyer's strategy is described by a *reservation price function* $P(q)$ such that a type q accepts a price p if and only if $p \leqslant P(q)$. Finally, strategies must be sequentially optimal and the seller uses Bayes' rule. For brevity we simply refer to an equilibrium satisfying these requirements as stationary.

3. Analysis

3.1. Preliminary results

It follows from standard arguments that the buyers' reservation price function $P(q)$ is non-increasing, implying that the seller's consistent beliefs are fully described by some truncation type q.[9] That is, at any given time the seller rationally expects that types in $[0, q)$ would already have accepted a previous offer while types in $[q, 1]$ would have rejected all previous offers. Moreover, a seller's continuation payoff is then only a function of this truncation. We denote the seller's continuation payoff by $R(q)$, which is evaluated at the time the seller can make a new offer. $R(q)$ is then defined by the dynamic programming equation

$$R(q) = \max_{y \in [q,1]} \frac{1}{1-q}\big[P(y)(y-q) + (1-y)\delta(1-\rho)R(y) + (1-y)\delta\rho \max\{R(0), R(y)\}\big]. \tag{2}$$

In (2) the seller essentially chooses the next truncation type $y \in [q, 1]$. Consequently, $(y-q)/(1-q)$ is the probability with which there is an agreement in the current period. If this is the case, then the good will be sold at the price of $P(q)$, i.e., at the reservation value of the next truncation type q. With the residual probability $(1-y)/(1-q)$ the buyer will reject the offer such that the game proceeds into the next round. In the latter case, payoffs are discounted by δ, and a new buyer arrives with probability ρ. If a new buyer arrives, the seller's continuation value equals $\max\{R(0), R(y)\}$. Note here again that $R(0)$ represents the continuation value when starting with a fresh buyer, for which the truncation type is still at $q = 0$, i.e., where the seller still only knows that $q \in Q$. We show below that in equilibrium the seller will always switch as $R(y) < R(0)$.

To abbreviate the exposition, we define for a given pair (δ, ρ) the parameter

$$\alpha = \frac{\delta\rho}{1-\delta(1-\rho)}. \tag{3}$$

[7] This specification of ρ assumes that in case more than one buyer arrives in the time window z only one can team up with the seller.

[8] See, for instance, the formal description in Ausubel and Deneckere (1989).

[9] If f is not everywhere strictly decreasing, we can always adequately perturb types.

For an interpretation of α, note that if the seller waited for the arrival of the next buyer and was sure to obtain the price p from him, his expected payoff would equal αp. We thus refer to $\alpha R(0)$ as the seller's *opportunity cost*. By optimality, the seller will never offer a price below $\alpha R(0)$. Note also that α is strictly increasing in both δ and ρ.

In the standard bilateral monopoly case it follows from stationarity and optimality that trade takes place with positive probability in each period until the good is sold. This result carries over to our model if there is no gap in the support of valuations. As the seller thus becomes increasingly pessimistic about the old buyer, it is intuitive that he will always switch to a newly arriving buyer.

Lemma 1. *Any stationary equilibrium exhibits the following characteristics*:

(i) *The buyer's reservation price satisfies* $P(q) \geqslant \min\{f(q), \alpha R(0)\}$ *for* $q \in Q$.
(ii) *The seller's continuation payoff* $R(q)$ *is nonincreasing, while* $R(0) > R(q)$ *holds strictly for all* $q > 0$.
(iii) *The seller always switches buyers.*

Proof. For assertion (i) note first that it is optimal for the seller not to offer the good below his opportunity costs $\alpha R(0)$. Consequently, it is also not optimal for a buyer to hold out for a lower price than $\alpha R(0)$. Consider next assertion (ii). The monotonicity of $R(q)$ follows from the monotonicity of the reservation price strategy $P(q)$. That $R(0) > 0$ can next be seen from a contradiction. If $R(0) = 0$, then the seller could profitably deviate by offering a sufficiently low but strictly positive price $\varepsilon > 0$, which given that no offer $p < \alpha R(0)$ can be rationally expected in the future would be surely accepted by all types with valuation $f(q) - \varepsilon > \delta f(q)$, which for given δ and sufficiently low ε has positive measure.

We prove next that $R(0) > R(q)$ holds strictly for all $q > 0$. We argue to a contradiction and suppose that there exists some $\bar{q} > 0$ with $R(\bar{q}) = R(0)$. If beliefs are truncated at $\bar{q}$, the optimal value y must by (2) satisfy

$$R(\bar{q}) = \frac{1}{1-\bar{q}}\big[P(y)(y-\bar{q}) + (1-y)\delta(1-\rho)R(y) + (1-y)\delta\rho R(0)\big], \tag{4}$$

where we used that $R(0) \geqslant R(y)$. Consider next the seller's program with $q = 0$. By stationarity, $R(0)$ must satisfy

$$R(0) \geqslant P(y)y + (1-y)\delta(1-\rho)R(y) + (1-y)\delta\rho R(0). \tag{5}$$

Using $R(\bar{q}) = R(0)$, we obtain from (4) and (5) the requirement

$$P(y) \leqslant \delta\big[(1-\rho)R(y) + \rho R(0)\big]. \tag{6}$$

As $\delta R(0) < R(0)$ and $R(y) \leqslant R(0)$, which holds by construction of $\bar{q}$, we obtain from (6) that $P(y) < R(0)$. Using finally $P(y) < R(0)$ together with $R(y) \leqslant R(0)$ and $R(0) = R(\bar{q})$, it follows that the right side of (4) must be strictly smaller than the left side, which yields a contradiction.

Finally, assertion (ii) implies assertion (iii) by optimality for the seller. □

By assertion (iii) of Lemma 1, we can now rewrite the seller's dynamic programming equation (2) as

$$R(q) = \max_{y\in[q,1]} \frac{1}{1-q}\big[P(y)(y-q) + (1-y)\delta(1-\rho)R(y) + (1-y)\delta\rho R(0)\big], \tag{7}$$

where we have simply substituted $R(0) = \max\{R(0), R(y)\}$.

3.2. Linear example

It is instructive to discuss first an example, for which we set $f(q) = 1 - q$. Before solving the model, we consider the following *auxiliary game*. Suppose that each period the game ends with probability ρ, in which case the seller realizes some exogenously specified utility $W \in [0, 1]$. We will specify a solution to the seller's respective dynamic programming equation, which we denote by $y^*(q)$. The respective prices are denoted by $p^*(q) = P(y^*(q))$. (Recall that $P(\cdot)$ represents the buyer's reservation price strategy.) Define next the parameters

$$\lambda = \sqrt{1 - \delta(1-\rho)}, \qquad (8)$$
$$\gamma = \frac{\sqrt{1-\delta(1-\rho)} - (1 - \delta(1-\rho))}{\delta(1-\rho)},$$

where $0 < \gamma < \lambda < 1$. These two parameters will be used to characterize the seller's equilibrium strategy as well as the buyer's reservation price depending on his type. We comment in this in more detail as we go along. We specify now first that the buyer's reservation price is given by

$$P(q) = \begin{cases} \alpha W + \lambda(1 - q - \alpha W) & q \in [0, 1 - \alpha W), \\ 1 - q & q \in [1 - \alpha W, 1]. \end{cases} \qquad (9)$$

That is, all types with valuation below the seller's opportunity costs αW have a reservation price $P(q)$ that is equal to their own valuation $1 - q$. For types with valuation $f(q) > \alpha W$, i.e., types with $q \in [0, 1 - \alpha W)$, their reservation value is from $\lambda < 1$ strictly lower than their valuation. Moreover, with the specified strategy this "shading" of the reservation value compared to the true valuation is a linear function of their valuation and thus their type. The extent to which such "shading" occurs depends, in turn on the parameter λ. By its definition from (8) λ is, in particular, strictly lower as the buyer becomes more patient. (Admittedly, however, as δ changes due to, say, a change in the primitives z or r, then also the other parameters in $P(q)$ such as α change as well.)

For the seller we next specify the strategy

$$y^*(q) = \begin{cases} \frac{\gamma}{\lambda} q + (1 - \alpha W)\frac{\lambda - \gamma}{\lambda} & q \in [0, 1 - \alpha W), \\ q & q \in [1 - \alpha W, 1], \end{cases}$$

which is shown in Appendix A to solve the program (7). Recall here that the truncation type function $y^*(q)$ describes how the seller works his way through the set of possible types, starting from $q = 0$. This implies a sequence of price offers

$$p^*(q) = \begin{cases} \alpha W + \gamma(1 - q - \alpha W) & q \in [0, 1 - \alpha W), \\ \alpha W & q \in [1 - \alpha W, 1], \end{cases}$$

where the initial offer is given by $p^*(0) = \alpha W + \gamma(1 - \alpha W)$.[10]

As we further show in Appendix A, with these specifications the seller's expected payoff equals

$$R(0) = \alpha W + \gamma\big(1 - \delta(1-\rho)\bar{\gamma}\big)(1 - \alpha W)^2 \frac{1}{1 - \bar{\gamma}^2}, \qquad (10)$$

[10] It may be helpful to compare the linear strategies to those obtained for a bilateral monopoly (see Stokey, 1981 or the discussion in Fudenberg and Tirole, 1992). In the bilateral monopoly case we have $P(q) = (1-q)\sqrt{1-\delta}$ and $p^*(q) = (1-q)[\sqrt{1-\delta} - (1-\delta)]/\delta$. These strategies are obtained by setting $\rho = 0$, which also implies $\alpha = 0$ and thus $\alpha W = 0$.

where we substituted $\bar{\gamma} = \gamma/\lambda$. Note here that the restriction to $W \in [0, 1]$ implies $0 \leqslant \alpha W < 1$. Clearly, $R(0)$ lies strictly above the opportunity cost αW. It is immediate to show that the difference between $R(0)$ and the opportunity cost converges to zero as the time between two consecutive offers vanishes, which is a result that we will use more generally below. Instead, we proceed immediately to solving our full model. For this recall that in the auxiliary game W denoted the (exogenous) payoff that the seller realized at the arrival of a new buyer. For our full model, we now have to substitute $W = R(0)$ into (10) and rearrange the equation to obtain

$$R(0)(1-\alpha) = \gamma \frac{(1-\delta(1-\rho)\bar{\gamma})}{1-\bar{\gamma}^2}\bigl(1-\alpha R(0)\bigr)^2. \tag{11}$$

This generates a fixed point equation for $R(0)$. The quadratic equation (11) has a unique positive solution that satisfies $R(0) < 1$. Substituting this solution into the equations used to derive $p^*(q)$ and $y^*(q)$ completes the characterization of a stationary equilibrium.

Note that this characterizes an equilibrium for any given value of z. Denote now the respective value of $R(0)$ that corresponds to some value $z > 0$ by $R_z(0)$. By changing z we thus obtain a sequence of equilibria with corresponding values $R_z(0)$. Along this sequence of equilibria it now holds that $R_z(0) \to 0$ as $z \to 0$. This is easily seen from (11) by noting that $\alpha \to \mu/(r+\mu)$, $\gamma \to 0$, and $(1-\delta(1-\rho)\bar{\gamma})/(1-\bar{\gamma}^2) \to 1/2$.

3.3. Main result

For general valuation functions $f(q)$ we obtain the following result.

Proposition 1. *Under assumption* (1) *on the valuation function $f(q)$, the Coase conjecture extends to the case where the seller faces an infinite sequence of potential new buyers. Formally, for any $\varepsilon > 0$ there exists a value $\bar{z} > 0$ such that the highest price the seller offers in any stationary equilibrium is bounded from above by $\underline{f} + \varepsilon$ if the time between two consecutive periods is not larger than $\bar{z}$.*

Proof. See Appendix B. □

The proof of Proposition 1 proceeds as in the linear example. That is, we first consider the auxiliary game with some exogenously specified payoff W, which the seller obtains if the next buyer arrives. We show that the seller's payoff converges to his opportunity cost, which in the auxiliary game is exogenous. In the second step, we then replace the exogenous payoff W with the endogenous payoff that the seller receives when a new buyer arrives, $R(0)$. Before commenting in more detail on how these two steps proceed more formally, we offer some more intuition for Proposition 1.

What is crucial for Proposition 1 to hold is that as the time between two consecutive offers shrinks, this also reduces the probability with which a buyer arrives in a given period. What is kept constant is the probability with which a buyer arrives over a given interval in *real* time. Putting it differently, while the attractiveness of the option to wait for another buyer remains unaffected, as z becomes smaller the seller's commitment power vis-a-vis any given buyer shrinks. As $z \to 0$ the seller will end up offering a price close to his opportunity cost already in the first period. As his opportunity cost reflects the value from any future negotiation, a lower initial price in any future negotiation reduces also the value of his opportunity cost, feeding in turn back into yet a lower offer in the first period and so forth. In total, as $z \to 0$ the value from waiting for another buyer and thus also the seller's initial offer converge to $\underline{f}$.

We now comment in more detail on the formal steps in the proof of Proposition 1. There, we take first the auxiliary game. Using the insights from the Coase conjecture, as the time between offers z shrinks, the initial expected revenue for the seller will be close to the seller's opportunity cost. In the auxiliary game, this is equal to αW. (Of course, if $\underline{f}$ exceeds αW, then there will be convergence to $\underline{f}$ instead.) Putting it more formally, we thus show that for any $\varepsilon > 0$ we can choose the period length z sufficiently small such that the seller's payoff $R(0)$ must not exceed $\max\{\alpha W, \underline{f}\}$ by more than ε.[11] In the second step, where we endogenize W by setting $W = R(0)$, we then use that α is bounded below one as z shrinks. More precisely, we have that α converges to $\bar{\alpha} = \mu/(r + \mu) < 1$ as $z \to 0$.

What complicates the formal analysis is that in the full game the seller's opportunity cost $\alpha R(0)$ depends also on z. This does not allow to write Proposition 1 simply as a corollary to existing results in the literature.[12]

4. Concluding discussion

4.1. The role of a convex support of buyers' valuations

To derive Proposition 1 the assumption of a convex support of buyers' valuations is crucial. Without an "interior gap" in the support the seller continues to lower his price until a new seller arrives. In this section, we bring out more formally the crucial role of this assumption by allowing for a "sufficiently large" interior gap. We show that even when restricting attention to stationary strategies there exist multiple equilibria that support different limits for the seller's payoff as $z \to 0$.

As a first step in the analysis, suppose that $f(q)$ has a discontinuity at $0 < q^G < 1$ and that the seller could *commit* to only sell to types $q \in [0, q^G]$. Denote the right-side limit of $f(q)$ at q^G by $\vec{f}(q^G) = \sup\{f(q) \mid q > q^G\}$. By our previous arguments, the seller's initial offer now converges to $f(q^G)$ as $z \to 0$. From this we then obtain that the seller's expected payoff converges to

$$f\left(q^G\right)\frac{\mu q^G}{\mu q^G + r}. \tag{12}$$

To see that (12) must hold, take first again the auxiliary game, where the seller realizes W if a new buyer with type $q \leqslant q^G$ arrives. His expected payoff from waiting for a buyer with type $q \leqslant q^G$ is then

$$\frac{\delta\rho}{1 - \delta[\rho(1-q) + 1 - \rho]} W.$$

(Note here that for $q = 1$ this becomes just αW.) Substituting then $W = f(q^G)$ and taking the limit $z \to 0$ yields (12). Observe next that, as is intuitive, (12) is strictly higher the higher the valuation of the threshold type q^G, the more likely it is that a new buyer arrives in a given

[11] What is important for the formal argument is that as $f(q)$ satisfies the continuity property (1), we can derive a uniform threshold on z for any given $\varepsilon > 0$.

[12] To cope with this complication, the proof relies on a combination of arguments in Ausubel and Deneckere (1989) with those used somewhat informally in Fudenberg and Tirole (1992).

interval of time (as captured by a higher arrival rate μ), and the more patient the seller is (as captured by a lower r).

Suppose now the seller has already offered a price (close to) $f(q^G)$ and that this offer has been rejected by the old buyer as his type is $q > q^G$. If the seller could loosen his commitment for just a moment and offer a lower price, would this be optimal? Clearly, as the buyer cannot expect a lower price in the future, type q would accept if $p \leqslant f(q)$. Hence, to ensure that for $z \to 0$ it is strictly optimal for the seller *not* to offer a lower price, $\vec{f}(q^G)$ must be below his opportunity cost as derived in (12). That is, it must hold that

$$\vec{f}(q^G) < f(q^G)\frac{\mu q^G}{\mu q^G + r}.$$

This condition can be rewritten as

$$f(q^G) - \vec{f}(q^G) > f(q^G)\frac{r}{\mu q^G + r}, \tag{13}$$

implying that the gap at q^G must be sufficiently large. As is intuitive, the condition (13) is less likely to hold for high r and low μ as this makes it more costly to wait for a new buyer.

If (13) holds for some q^G, there exists a sequence of stationary equilibria where for all sufficiently low z the seller never drops his price below $f(q^G)$. The intuition is simple. If a buyer expects the seller to never price below $f(q^G)$, he will not hold out for lower prices. In turn, if the seller expects future buyers to behave in this way as well, then it is indeed optimal for him to follow this strategy. This argument, however, also makes clear that in this case there exist multiple equilibria or, more precisely, different sequences of equilibria where for $z \to 0$ initial prices either converge to $f(q^G)$ or to $\underline{f}$. The latter sequence of equilibria is supported by a different set of rational expectations, namely that the seller will lower his price below $f(q^G)$. Again, if the seller will follow this strategy when meeting a new buyer, then this lowers his opportunity cost in the current negotiation and, therefore, does not make it credible that he will sustain a high price. Consequently, for the current trading partner it is now optimal to hold out for a low price.

More formally, we have the following result.

Proposition 2. *Suppose that* (13) *holds for some* $0 < q^G < 1$. *Then there exists a sequence of stationary equilibria where the seller's payoff converges to*[13]

$$f(q^G)q^G\frac{\mu + r}{\mu q^G + r} \tag{14}$$

as $z \to 0$. *There also exists a sequence of stationary equilibria satisfying the characterization of Proposition* 1.

Proof. See Appendix C. □

4.2. Comparison with the related literature

In this section we explore several modifications of our game in order to highlight the driving forces behind Proposition 1 and discuss the relation to the extant literature, in particular Fuden-

[13] Note that (14) is easily obtained from (12) as follows. As the game starts with a new buyer, the seller realizes for $z \to 0$ with probability q^G the price $f(q^G)$ (almost) immediately. With the residual probability $1 - q^G$ the seller's continuation value for $z \to 0$ equals (12). Transforming $q^G f(q^G) + (1 - q^G) f(q^G) \frac{\mu q^G}{\mu q^G + r}$ yields (14).

berg et al. (1987), Samuelson (1992), and De Fraja and Muthoo (2000). In Fudenberg et al. (1987) a seller also has to leave his current trading partner in order to locate a new buyer. Switching is costly as the seller has to wait $d \geqslant 0$ additional periods before he can start negotiations with a new buyer. Importantly and in contrast to our main result, in their setting the frictions from search vanish as the real time to wait for another buyer, dz, converges to zero for $z \to 0$.[14] The key result in Fudenberg et al. (1987) is that such a setting generates multiple equilibria. As they argue, given that the profitability of "going outside" the current negotiations depends on how tough a seller's future trading partners are, i.e., of how much they will try to hold out for lower prices, there exists an "externality between buyers". In other words, as we also argued for the case with a gap, multiple equilibria can be sustained by different expectations on the seller's and all buyers' strategies, where a tough strategy by buyers is supported by a weak strategy for the seller and vice versa. In our setting where the (time) costs of waiting for another buyer do not decrease as z becomes smaller and as, consequently, the seller's commitment power decreases, we showed that without a gap any sequence of equilibria must satisfy, however, the Coase conjecture.

In light of the results of Fudenberg et al. (1987), where the seller must leave the current buyer in order to locate a new trading partner, it seems interesting to ask whether such a modification would also change our key results. More precisely, suppose now for a moment that the seller must actively look for a new buyer, in which case he cannot continue haggling with the old buyer. However, we assume still that search will produce a new buyer according to the Poisson process with arrival rate μ. Hence, the average real time spent on searching for a new buyer is unaffected by z.

Denote now $\tilde{q} = \min\{q \mid f(q) \leqslant \alpha R(0)\}$. To keep the discussion short, suppose also that $\underline{f} \leqslant \alpha R(0)$, e.g., as $\underline{f} = 0$. In our previous analysis we showed that $\tilde{q}$ converges to 1 as time passes without a new buyer arriving. Clearly, if the seller must leave the old buyer to start searching for a new buyer, he will optimally not wait infinitely long. Despite this difference, we argue next that Proposition 1 should still hold. As $z \to 0$, it becomes almost costless for the seller to stay with the current buyer just one more period in order to make yet another offer. By the arguments of the Coase conjecture, as $z \to 0$ the seller should thus trade almost instantaneously with almost all types $q < \tilde{q}$ at a price only marginally above his opportunity costs $\alpha R(0)$. But once this is established, we can rely on the arguments of Section 3, given that α remains bounded away from one.

In Samuelson (1992), a seller also has to leave the current buyer in order to find a new trading partner. Embedding bilateral negotiations in a matching model, Samuelson (1992) focuses on stationary strategies and allows for two possible types of buyers. Mirroring our "gap" result in Section 4.1, he finds that an equilibrium where the seller never switches always exists, while if the difference between types is sufficiently large there also exists an equilibrium in which the seller only trades with the high-type buyer. Our results suggest, however, that this multiplicity should vanish once we allow for the buyer's valuation to have full support (over some interval of valuations).

Finally, the model in De Fraja and Muthoo (2000) covers, as they put it, an intermediate position between the more standard one-buyer models on the one hand and the model of Fudenberg et al. (1987), as well as ours, with an infinite set of buyers on the other hand. In their model, a seller can shuttle back and forth, albeit at strictly positive switching costs, between two different buy-

[14] Based on our previous discussion we would conjecture that for $z \to 0$ the seller's expected payoff converges to $f(0) = 1$. Proving this is, however, beyond the limited scope of this short paper.

ers. Each buyer's valuation can be either low or high. They show that the Coase conjecture still holds if the costs of switching are kept fixed while the seller's commitment power erodes. This result is thus analogous to ours, though with one subtle difference. If the lowest valuation is zero, i.e., if $\underline{f}=0$, then as the supplier's opportunity cost also goes to zero so does his effective cost from waiting for another buyer, given that in our model these costs are incurred from discounting.

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Appendix A. Linear example

Here and in what follows, we use

$$\ell=\delta(1-\rho).$$

We discuss now first the auxiliary model. Optimality for buyers requires that $1-y-P(y)=\ell[1-y-p^*(y)]$, which transforms to

$$1-y-\alpha W-\lambda(1-y-\alpha W)=\ell\big[1-y-\alpha W-\gamma(1-y-\alpha W)\big].$$

This holds as $1-\lambda=\ell(1-\gamma)$. For truncations $q<1-\alpha W$ we obtain from the envelope theorem $\mathrm{d}R(q)/\mathrm{d}q=[-p^*(q)+R(q)]/(1-q)$ such that the first-order condition for $y^*(q)$ is $P(y)(1-\ell)+(y-q)\,\mathrm{d}P(y)/\mathrm{d}y-\delta\rho W=0$, where we used $p^*(q)=P(y^*(q))$. Substituting the (candidate) equilibrium strategies, we have $p^*(q)(1-\ell)-\lambda(y^*(q)-q)-\delta\rho W=0$, which transforms to

$$\big[\alpha W+\gamma(1-q-\alpha W)\big](1-\ell)-\lambda\left(\frac{\gamma}{\lambda}q+(1-\alpha W)\frac{\lambda-\gamma}{\lambda}-q\right)-\delta\rho W=0.$$

This holds as $\ell\gamma+\lambda-2\gamma=0$. For brevity we now ignore the second-order condition, which is easily checked.

To calculate the seller's expected payoff $R(0)$, we denote prices and truncations along the equilibrium path of the auxiliary game by the sequences $\{p_n\}$ and $\{q_n\}$ such that $q_0=0$, $q_n=y^*(q_{n-1})$, and $p_n=P(q_n)$. Then

$$\begin{aligned}R(0)&=\sum_{n=1}^{\infty}\ell^{n-1}\big[(q_n-q_{n-1})p_{n-1}+(1-q_n)\delta\rho W\big]\\&=\alpha W+\sum_{n=1}^{\infty}\ell^{n-1}q_n(p_{n-1}-\ell p_n)-\delta\rho W\sum_{n=1}^{\infty}\ell^{n-1}q_n,\end{aligned}\tag{15}$$

which is obtained by collecting terms for each q_n and substituting $\alpha W=\delta\rho W\sum_{n=1}^{\infty}\ell^{n-1}$ and $q_0=0$. Substitute next $p_n=\alpha W+\gamma(1-q_n-\alpha W)$. This yields

$$p_0-\ell p_1=\alpha W(1-\ell)+\gamma(1-\alpha W-q_0)-\ell\gamma(1-\alpha W-q_1).$$

Proceeding in this way for higher n, (15) transforms to

$$R(0)=\alpha W+\sum_{n=1}^{\infty} q_n\big[\gamma(1-\alpha W-q_{n-1})-\ell\gamma(1-\alpha W-q_n)\big]. \tag{16}$$

Recall next that

$$q_n=\frac{\gamma}{\lambda}q_{n-1}+(1-\alpha W)\frac{\lambda-\gamma}{\lambda} \tag{17}$$

holds by optimality. By this we have $1-\alpha W-q_n=\bar{\gamma}(1-\alpha W-q_{n-1})$, where we use $\bar{\gamma}=\gamma/\lambda$. Moreover, the recursive definition of q_n in (17) can be solved to obtain $q_n=\bar{\gamma}q_{n-1}+(1-\alpha W)(\lambda-\gamma)/\lambda$, which finally gives $q_n=(1-\alpha W)(1-\bar{\gamma}^n)$ and thus $q_n(1-\alpha W-q_{n-1})=(1-\alpha W)^2(1-\bar{\gamma}^n)\bar{\gamma}^{n-1}$. Substituting these expressions into (16), we obtain finally

$$\begin{aligned}R(0)&=\alpha W+\gamma(1-\ell\bar{\gamma})(1-\alpha W)^2\sum_{n=1}^{\infty}\bar{\gamma}^{n-1}\big(1-\bar{\gamma}^n\big)\\&=\alpha W+\gamma(1-\ell\bar{\gamma})(1-\alpha W)^2\frac{1}{1-\bar{\gamma}^2}.\end{aligned}$$

Appendix B. Proof of Proposition 1

Consider the following auxiliary game. Each period the game ends with probability ρ, in which case the seller receives the exogenously specified payoff $W\in[0,1]$. For a given equilibrium and a given W, we denote the reservation price strategy and the value function by $P(q,W)$ and $R(q,W)$, respectively. In what follows, we neglect W in the argument of P and R whenever this does not give rise to confusion. Rewriting (7), we obtain

$$R(q)=\max_{y\in[q,1]}\frac{1}{1-q}\big[P(y)(y-q)+(1-y)\delta(1-\rho)R(y)+(1-y)\delta\rho W\big]. \tag{18}$$

Let $T(q)$ denote the argmax correspondence for (18) with $t(q)=\inf\{T(q)\}$ and $S(q)=P(t(q))$. Without loss of generality assume that P is left-continuous. From the arguments of Ausubel and Deneckere (1989) we know that the seller may randomize only in the first period and offers thereafter prices $S(q)$. Optimality for buyers then implies $f(q)-P(q)=\delta(1-\rho)[f(q)-S(q)]$. Define next the "unconditional value function" $R^U(q)=(1-q)R(q)$ and for all values $x\in[0,1]$ the function $q^D(x)=\max\{q\colon\ f(q)\geqslant x\}$. The following lemma extends a result from Ausubel and Deneckere (1989) to the auxiliary game.

Lemma B.1. *For $q<q^D(\alpha W)$ it holds that $R^U(q)$ is decreasing and Lipschitz continuous, satisfying $0<R^U(q')-R^U(q'')\leqslant q''-q'$ for all $q'<q''\leqslant q^D(\alpha W)$.*

Proof. The strict monotonicity follows from the monotonicity of $R(\cdot)$ (Lemma 1). Define next $y_0=t(q')$, $y_1=t(t(q'))$ and continue up to some value $y_{\bar{n}}$ such that $y_{\bar{n}}\geqslant q''$ and $y_{\bar{n}-1}<q''$. Note for this that for all $q<q^D(\alpha W)$, i.e., for all q with $f(q)>\alpha W$, we use that $t(q)>q$. This in turn follows as, by an argument analogous to that in Lemma 1, we have for all these types that $R(q)>\alpha W$.

Observe next that

$$R^U(q')=p_0(y_0-q')+\sum_{n=1}^{\bar{n}}\ell^n p_n(y_n-y_{n-1})+\ell^{\bar{n}+1}R^U(y_{\bar{n}})+\sum_{n=0}^{\bar{n}}\delta\rho W\ell^n(1-y_n),$$

where the sequence of p_n represents the respective prices. Optimality for the seller with beliefs truncated at q'' implies

$$\begin{aligned} R^U(q'') \geqslant{}& \ell^{\bar{n}} p_{\bar{n}}(y_{\bar{n}} - q'') + \ell^{\bar{n}+1} R^U(y_{\bar{n}}) + \delta\rho W \ell^{\bar{n}}(1 - y_{\bar{n}}) \\ &+ \delta\rho W(1 - q'') \sum_{n=1}^{\bar{n}-1} \ell^{n-1}. \end{aligned} \tag{19}$$

Note that we use for this inequality that the seller could set a price of 1, which clearly no buyer would accept, for $\bar{n}$ periods and then continue with a price of $p_{\bar{n}}$. From (19) it then follows that

$$\begin{aligned} &R^U(q') - R^U(q'') \\ &\quad\leqslant p_0(y_0 - q') + \sum_{n=1}^{\bar{n}-1} p_n(y_n - y_{n-1})\ell^n + p_{\bar{n}}(q'' - y_{\bar{n}-1})\ell^{\bar{n}} + \delta\rho W \sum_{n=0}^{\bar{n}} (q'' - y_n)\ell^n. \end{aligned}$$

Substituting $1 \geqslant p_n \geqslant \alpha W$ and $\alpha = \delta\rho/(1-\ell)$, this term bounded from above by $q'' - q'$, which proves the claim. □

We prove now a uniform version of the Coase conjecture for the auxiliary game. We turn to the question of existence of a stationary equilibrium below.

Proposition B.1. *Consider the auxiliary game. Then for any $\varepsilon > 0$ there exists $\bar{z} > 0$ such that for all $z \leqslant \bar{z}$, all $W \in [0, 1]$, and any stationary equilibrium supported by a pair (P, R) it holds that $R(0) < \max\{\alpha W, \underline{f}\} + \varepsilon$ and $S(0) < \max\{\alpha W, \underline{f}\} + \varepsilon$.*

Proof. Note first that it is sufficient to prove the final assertion, namely that $S(0) < \max\{\alpha W, \underline{f}\} + \varepsilon$. (The boundary on the expected payoff follows immediately as the price schedule is nonincreasing in any equilibrium.) We argue to a contradiction and suppose that there exists a value $\varepsilon > 0$, sequences $\{z_k\}_{k=1}^{\infty}$ with $z^k \to 0$ and $\{W_k\}_{k=1}^{\infty}$ with $W_k \in [0, 1]$, and a sequence of stationary equilibria described by $\{(R_k, P_k)\}_{k=1}^{\infty}$ (respectively, $\{(R_k^U, P_k)\}_{k=1}^{\infty}$) such that $S_k(0) \geqslant \max\{\alpha_k W_k, \underline{f}\} + \varepsilon$.

Recall now the definition that $\alpha_k = \delta_k\rho_k/(1 - \delta_k(1 - \rho_k))$, where $\delta_k = \mathrm{e}^{-rz_k}$ and $\rho_k = 1 - \mathrm{e}^{-\mu z_k}$, and denote $c_k = \max\{\alpha_k W_k, \underline{f}\}$. (Recall that if $c_k = \alpha_k W_k$, then this represents the seller's opportunity costs from selling to the given buyer instead of waiting until W is realized in the auxiliary game.) Without loss of generality we suppose that $\{W_k\}_{k=1}^{\infty}$ converges to some value W, implying that $\{c_k\}_{k=1}^{\infty}$ converges to some value c, $\{R_k^U\}_{k=1}^{\infty}$ converges uniformly to some continuous function R^U, and $\{P_k\}_{k=1}^{\infty}$ converges pointwise for all rationals to some nonincreasing and left continuous P. This can be assured by taking successive subsequences, while using from Lemma B.1 that $\{R_k^U\}_{k=1}^{\infty}$ is an equicontinuous family and applying a diagonal argument for $\{P_k\}_{k=1}^{\infty}$. Define next a sequence $\{q_k^D\}_{k=1}^{\infty}$ with $q_k^D = q^D(c_k)$ and a value $q^D = q^D(c)$.

Recall that any stationary equilibrium has sales with positive probability in each period as long as the truncation type q has a valuation above $\alpha_k W_k$. Hence, $S_k(0) \geqslant c_k + \varepsilon$ implies $P_k(0) \geqslant c_k + \varepsilon$ such that $q = 0$ will indeed accept in the first period. Denote next

$$\bar{y} = \inf\{r\colon\ P(r) < c + \varepsilon/3\} \tag{20}$$

and observe that $0 \leqslant \bar{y} < q^D \leqslant 1$. (Formally, this follows from Lipschitz continuity of the valuation function and monotonicity of P_k.) We show that for all sufficiently high k and thus sufficiently short period lengths the seller has an incentive to deviate from the supposed equilibrium path by speeding up the sale.

We first suppose that $R^U(\bar{y}) > (1-\bar{y})c$. In this case, the argument is completely analogous to that in Ausubel and Deneckere (1989) and thus omitted. (Intuitively, $R^U(\bar{y}) > (1-\bar{y})c$ implies that the seller's expected payoff from $\bar{y}$ onwards will remain bounded away from this opportunity cost even as z vanishes, which makes it strictly profitable to speed up the sale.)

In the second case, we assume that $R^U(\bar{y}) = (1-\bar{y})c$. Here, we proceed as follows. We use $R^U(\bar{y}) = (1-\bar{y})c$ to show first that over two consecutive periods the seller would strictly prefer to sell faster if the probability of trade was sufficiently small in the first period of these two periods. We then use convergence to $R^U(\bar{y}) = (1-\bar{y})c$ to show that the probability of trade must indeed be so small for high k as, otherwise, the seller would realize a strictly higher payoff.

Formally, for the second case we construct a sequence of rationals $\{\bar{y}_k\}_{k=1}^{\infty}$ converging to $\bar{y}$ and find for any $\varepsilon_1 > 0$ an integer $\bar{k}(\varepsilon_1)$ such that for all $k > \bar{k}(\varepsilon_1)$ it holds that $P_k(\bar{y}_k) < c_k + \frac{\varepsilon}{2}$ and $R_k^U(\bar{y}_k) < (1-\bar{y}_k)c_k + \varepsilon_1$. This follows from the construction of $\bar{y}$, the convergence of P_k on all rationals, the uniform convergence of R_k^U, the convergence of c_k, the equicontinuity of all R_k^U, and the assumption of Case 2 that $R^u(\bar{y}) = (1-\bar{y})c$.

Now suppose the seller deviates and offers a first-period price of $c_k + \varepsilon/2$. As by assumption $S_k(0) \geqslant c_k + \varepsilon$, implying $P_k(0) > c_k + \varepsilon/2$, optimality for a buyer of type $q = 0$ requires that the price does not drop below $c_k + \varepsilon/4$ before some real time t, where t must satisfy

$$1 - c_k - \frac{\varepsilon}{2} \geqslant \mathrm{e}^{-t(r+\mu)}\left(1 - c_k - \frac{\varepsilon}{4}\right).$$

We argue now to a contradiction by proving that the seller strictly prefers to sell faster. The proof follows an argument in Fudenberg and Tirole (1992).

Choose m_k as the smallest integer satisfying $m_k \geqslant t/z_k$. For simplicity we assume equality. We denote the respective offers by $p_{n,k}$ and the set of buyer types accepting in n by $[q_{n,k}, q_{n+1,k}]$. Note that by construction it holds that $p_0 = c_k + \varepsilon/2$. Choose z_k sufficiently low to ensure $t > z_k$. Simple calculations show that there must exist two consecutive periods $n_k + 1$ and $n_k + 2$ with $0 \leqslant n_k \leqslant m_k - 2$ such that

$$\max\{q_{n_k+1,k} - q_{n_k,k}, q_{n_k+2,k} - q_{n_k+1,k}\} \leqslant 2\frac{z_k}{t - z_k}(q_{m_k,k} - q_{1,k}). \tag{21}$$

$R_k^U(q_{n_k,k})$ can now be explicitly written as

$$\begin{aligned} &P_k(q_{n_k+1,,k})[q_{n_k+1,k} - q_{n_k}] + \ell_k[q_{n_k+2,k} - q_{n_k+1,k}]P_k(q_{n_k+2,k}) \\ &\quad + \ell_k^2[1 - q_{n_k+2,k}]R_k(q_{n_k+2,k}) + W_k\delta_k\rho_k\big[(1 - q_{n_k+1,k}) + \ell_k(1 - q_{n_k+2,k})\big]. \end{aligned} \tag{22}$$

Suppose the seller would offer p_{n_k+1} already at n_k, which yields from stationarity the (unconditional) payoff

$$P_k(q_{n_k+2,k})[q_{n_k+2,k} - q_{n_k,k}] + \ell_k[1 - q_{n_k+2,k}]R_k(q_{n_k+2,k}) + W_k\delta_k\rho_k[1 - q_{n_k+2,k}]. \tag{23}$$

By optimality it must hold that (22) $\geqslant$ (23), which by $P_k(q_{n_k+1,k}) = f(q_{n_k+1,k})(1-\ell_k) + \ell_k P_k(q_{n_k+2,k})$ and after dividing by $1 - \ell_k$ transforms to the requirement[15]

$$\begin{aligned} &f(q_{n_k+1,k})[q_{n_k+1,k} - q_{n_k,k}] - P_k(q_{n_k+2,k})[q_{n_k+2,k} - q_{n_k,k}] + c_k[q_{n_k+2,k} - q_{n_k+1,k}] \\ &\quad \geqslant \ell_k(1 - q_{n_k+2,k})\big[R_k(q_{n_k+2,k}) - c_k\big]. \end{aligned}$$

[15] We use here that in an equilibrium the seller will not randomize in the continuation game.

For this inequality to be satisfied, it must finally hold by $P_k(q_{n_k+2,k}) \geqslant c_k$ and by $f(q_{n_k+1,k}) - P_k(q_{n_k+2,k}) \leqslant 1$ that

$$q_{n_k+1,k} - q_{n_k,k} \geqslant \ell_k(1 - q_{n_k+2,k})\big[R_k(q_{n_k+2,k}) - c_k\big]. \tag{24}$$

We now contradict (24) for high k. For this we derive first a lower boundary for $R_k(q_{n_k+2,k})$, where we can use from Lemma 1 that $R_k(q_{n_k+2,k}) \geqslant R_k(q_{m_k,k})$ due to $q_{n_k+2,k} \leqslant q_{m_k,k}$. Suppose now that in period m_k the seller chooses to sell to types $(q_{m_k,k}, q_k]$ by offering some price $p \leqslant p_{m_k-1,k}$, where $p_{m_k-1,k} \geqslant c_k + \varepsilon/4$. To determine for given q_k a lower boundary for the respective price $p = p(q_k)$, note first that the seller will never charge a price below c_k. We thus have from optimality for the buyer that

$$p(q_k) - c_k \geqslant (1 - \ell_k)\big[f(q_k) - c_k\big]. \tag{25}$$

Note next that with $f(q_{m_k}) \geqslant c_k + \varepsilon/4$ and with $f(q_{m_k} + \Delta) \geqslant c_k + \varepsilon/4 - H\Delta$ for all $\Delta \geqslant 0$ due to (1), we have for $\Delta = \varepsilon/(8H)$ that $f(q_{m_k} + \Delta) \geqslant c_k + \varepsilon/8$. Using first that

$$R_k(q_{n_k+2,k}) \geqslant R(q_{m_k,k}) \geqslant c_k + \frac{q_k - q_{m_k,k}}{1 - q_{m_k,k}}\big[p(q_k) - c_k\big],$$

we can next set $q_k - q_{m_k,k} = \Delta$ and use also (25) to transform this further into

$$\begin{aligned} R_k(q_{n_k+2,k}) &\geqslant c_k + \frac{\Delta}{1 - q_{m_k,k}}(1 - \ell_k)\big[f(q_k) - c_k\big], \\ &\geqslant c_k + \frac{1}{1 - q_{m_k,k}}\frac{\varepsilon}{8H}(1 - \ell_k)\frac{\varepsilon}{8}. \end{aligned}$$

With this inequality at hands, (24) holds only if

$$q_{n_k+1,k} - q_{n_k,k} \geqslant \ell_k \frac{1 - q_{n_k+2,k}}{1 - q_{m_k,k}}(1 - \ell_k)\frac{\varepsilon}{8H}\frac{\varepsilon}{8}.$$

By $1 - q_{n_k+2,k} \geqslant 1 - q_{m_k,k}$ and after substitution of (21) this yields then the requirement

$$2\frac{z_k}{t - z_k}(q_{m_k,k} - q_{1,k})\frac{1}{1 - \ell_k} \geqslant \ell_k \frac{\varepsilon}{8H}\frac{\varepsilon}{8}. \tag{26}$$

Summing up, we have shown so far that to contradict (24) for high k it is sufficient to contradict (26). We do so by providing an upper boundary for the interval $q_{m_k,k} - q_{1,k}$.

By construction of the sequence $\{\bar{y}_k\}_{k=0}^{\infty}$ we have $R_k^U(\bar{y}_k) < (1 - \bar{y}_k)c_k + \varepsilon_1$ and $P_k(\bar{y}_k) < c_k + \varepsilon/2$ for all $k > \bar{k}(\varepsilon_1)$. Using the monotonicity of the value function from Lemma 1 and $\bar{y}_k \leqslant q_{1,k}$ from $p_{0,k} = c_k + \varepsilon/2$, it holds that

$$R_k(q_{1,k}) \leqslant R_k(\bar{y}_k) \leqslant \frac{1}{1 - \bar{y}_k}\big[(1 - \bar{y}_k)c_k + \varepsilon_1\big],$$

and thus that

$$R_k^U(q_{1,k}) \leqslant (1 - q_{1,k})c_k + \varepsilon_1 \frac{1 - q_{1,k}}{1 - \bar{y}_k}.$$

This in turn implies

$$R_k^U(q_{1,k}) \leqslant (1 - q_{1,k})c_k + \varepsilon_1 \tag{27}$$

due to $1 - q_{1,k} \leqslant 1 - \bar{y}_k$. But by construction, i.e., as prices along the considered sequence were not below $c_k + \varepsilon/4$, we also have that

$$R_k^U(q_{1,k}) \geqslant c_k(1 - q_{1,k}) + \ell_k^{m_k} \frac{\varepsilon}{4}(q_{m_k,k} - q_{1,k}). \tag{28}$$

Together with $\ell_k^{m_k} \geqslant e^{-t(r+\mu)}$, the two conditions (27) and (28) can only be jointly satisfied if it holds that

$$c_k(1 - q_{1,k}) + \mathrm{e}^{-t(r+\mu)} \frac{\varepsilon}{4}(q_{m_k,k} - q_{1,k}) \leqslant (1 - q_{1,k})c_k + \varepsilon_1$$

and if thus

$$q_{m_k,k} - q_{1,k} \leqslant \frac{4\varepsilon_1}{\varepsilon \mathrm{e}^{-t(r+\mu)}}. \tag{29}$$

Hence, while the right side of (26) is bounded away from zero, we must have from (29) that the left side converges to zero for $k \to \infty$, which leads to a contradiction. More precisely, substitution yields

$$2\frac{1}{t - z_k} \frac{4\varepsilon_1}{\varepsilon \mathrm{e}^{-t(r+\mu)}} \frac{z_k}{1 - \mathrm{e}^{-z_k(r+\mu)}} \geqslant \mathrm{e}^{-z_k(r+\mu)} \frac{\varepsilon}{8H} \frac{\varepsilon}{8},$$

where $z_k/(1 - \mathrm{e}^{-z_k(r+\mu)})$ converges to $1/(r + \mu)$. We can then choose ε_1 sufficiently low and $k > \bar{k}(\varepsilon_1)$.

This completes the proof of Proposition B.1. □

Before applying Proposition B.1, we turn to existence. Once more we can extend arguments from Ausubel and Deneckere (1989) to ensure existence of a stationary equilibrium. The proof is again omitted.

Proposition B.2. *Consider the auxiliary game. Then for any z and $W' < W''$ there exist stationary equilibria, supported by $(P(\cdot, W'), R(\cdot, W'))$ and $(P(\cdot, W''), R(\cdot, W''))$ respectively, such that $R(0, W'') \geqslant R(0, W')$.*

Observe next that for any $z > 0$ and any equilibrium it holds at $W = 0$ that $R(0, 0) > 0$, while we obtain for $W = 1$ that $R(0, 1) < 1$.

We now return to the original game where W is no longer exogenous. Substitute $W = R(0)$. Given the results for the auxiliary game at $W = 0$ and $W = 1$ and Proposition B.2, we obtain by Tarski's fixed point theorem a value $W \in (0, 1)$ satisfying $R(0, W) = W$. Finally, as Proposition B.1 holds uniformly for all possible values $W \in [0, 1]$ and as any equilibrium payoff of the original game $R(0)$ is restricted to this set, we have the following implication: For any $\varepsilon' > 0$ there exists a value $\bar{z}(\varepsilon') > 0$ such that for all $z < \bar{z}'(\varepsilon')$ it holds in any stationary equilibrium that $R(0) < \max\{\alpha R(0), \underline{f}\} + \varepsilon'$. Proposition 1 follows now immediately after observing that α converges to $\bar{\alpha} = \mu/(r + \mu) < 1$ as $z \to 0$.

Appendix C. Proof of Proposition 2

The proof proceeds in complete analogy to that of Proposition 1. Denote by Δ the difference $f(q^G)\mu q^G/[\mu q^G + r] - \overrightarrow{f}(q^G)$ and take first the auxiliary game analyzed in Proposition B.1. Restrict now consideration to values $W \in [\underline{W}, 1]$, where

$$\underline{W} = \left[\overrightarrow{f}\left(q^G\right) + \Delta/2\right] \frac{\mu q^G + r}{\mu q^G},$$

and define $\rho^G = \rho q^G$. Denote also $\alpha^G = \delta\rho^G/[1-\delta(1-\rho^G)]$ such that the seller's opportunity cost equals $\alpha^G W$. Then Proposition B.1 can be applied after exchanging ρ for ρ^G. For all $W \geqslant \underline{W}$ and sufficiently small z it then holds that $\alpha^G W > \overrightarrow{f}(q^G)$, implying that the seller will not sell to types $q > q^G$. Moreover, the Coase conjecture applies uniformly for all $W \in [\underline{W}, 1]$ and the first offer converges for $z \to 0$ to $\max\{\alpha^G W, f(q^G)\}$. Setting $W = R(0)$ as in Appendix B concludes the proof.

Turning to the second assertion, we can again apply the proof of Proposition 1, albeit with some care. To derive a uniform version of the Coase conjecture in Proposition B.1, we used that condition (1) holds for *all* values of q. While this is now no longer the case due to the interior gap, it still holds at $q = 1$. This implies that the Lipschitz condition (with a possibly higher value H) holds over a sufficiently small interval $[\bar{q}, 1]$, where $\bar{q} < 1$. To apply the analysis of Proposition B.1, we next restrict the feasible values for W to some set $W \in [0, \overline{W}]$ such that $q^D(\bar{\alpha}W) > \bar{q}$. (Recall that over segments of continuity of $f(q)$ we defined $q^D(x) = \max\{q\colon f(q) \geqslant x\}$.) With this restriction on W the result in Proposition B.1 again applies uniformly. Again, the proof is concluded by using $W = R(0)$.

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