

# Indirect versus Direct Constraints in Markets with Vertical Integration

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## Abstract

For an assessment of market power on the wholesale (or merchant) market in the presence of vertically integrated firms, we analyze the interaction of direct constraints, arising from competition on the wholesale market, and of indirect constraints, arising from substitution on the retail market. A vertically integrated firm that still participates in the merchant market exerts both direct and indirect constraints. We analyze the factors that determine the importance of indirect constraints. We find that, in contrast to a common presumption, indirect constraints are sometimes more powerful than direct constraints. We furthermore analyze the incentives of integrated firms to still participate in the merchant market, provided that this is technologically feasible.

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## I. Introduction

This paper contributes to the assessment of market power on wholesale markets in the presence of vertically integrated firms. Our analysis is motivated by several recent antitrust cases in Europe that all raised the issue of how to appropriately take into account the presence of both “direct constraints” from competition on the wholesale market and “indirect constraints” from substitution on the retail market.

In an important recent ruling, the European Court of First Instance (CFI) overruled the decision to block the proposed merger of Schneider Electric SA and Legrand SA.<sup>1</sup> While Schneider and Legrand were not vertically integrated, other firms (such as ABB and Siemens) competed only through

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<sup>1</sup> CFI Judgment of 22.10.2002 on case T-310/01: Schneider v. Commission (Application for the annulment of Commission Decision C(2001)3014 of 10.10.2001 on case COMP/M.2283—Schneider—Legrand). In July 2007, the European Union’s second-highest court ruled that Schneider is entitled to compensation.

self-supply at the downstream level. The CFI argued that by not incorporating ABB's and Siemens' market shares at the downstream market, "the Commission underestimated the economic power of the merged entity's two main competitors and correspondingly overestimated that entity's strength". Furthermore, in Europe, there is an ongoing controversy about how to take into account indirect constraints in the provision of broadband telecommunications services. Broadband services are provided over different technologies at the wholesale level (DSL, cable, fiber, etc.). National Regulatory Authorities differed substantially regarding the inclusion of technologies other than DSL, such as satellite TV networks or wireless technologies. In regulated markets, these issues are particularly important due to the direct connection between the finding of market power and the imposition of *ex ante* remedies. Such a finding of market power could, in turn, hinge crucially on whether and to what extent indirect constraints from the retail market might substitute for the absence of strong direct constraints.<sup>2</sup>

The present paper also seeks to quantify the role that indirect constraints might play, as opposed to that of direct constraints. One objective is to scrutinize the assertion that indirect constraints should typically be of only second-order importance, because their effect is "buffered" through the vertical structure. One of our findings, however, throws this into question. We find, for the tractable case with linear demand and downstream Cournot competition, that as we "remove" one competitor from the merchant market through forward integration, the loss in direct constraints is more than compensated by the additional presence of indirect constraints. Consequently, the prevailing merchant price decreases.

Another finding is that, in the presence of indirect constraints, suppliers' market power is diminished with more competition at the retail level.<sup>3</sup> In contrast, in Schneider/Legrand, the Commission argued that the high level of downstream competition (in this case, among wholesalers that were supplied by the two merging firms) increased upstream market power, because it undermined buyers' ability to exert countervailing power.

Our approach follows Salinger (1988) by supposing that interactions on the merchant market are adequately captured by presuming a "market interface" characterized by a single uniform price. This approach contrasts with a more novel contractual approach that builds on bilateral negotiations and

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<sup>2</sup> For a survey of recent European case-law concerning indirect constraints, see Ofcom, Indirect Constraints, and Captive Sales: Overview of Regulatory Practice and Competition Case Law with Regards to Indirect Constraints and Captive Sales in Market Definition and Market Power Assessment, May 2006.

<sup>3</sup> The extent of competition prevailing in the retail market has, however, no (unambiguous) impact on the wholesale price in the absence of vertically integrated firms.

speaks more to questions of the firm's ability to foreclose. We say more on this below.

Previous studies building on Salinger (1988) have focused on the incentives for vertical integration and for foreclosure. Our interest is different. We focus on the assessment of market power in the merchant market in the presence of vertically integrated firms. Here, for practical purposes, we distinguish between two different scenarios. In one scenario, an integrated firm cannot participate on the merchant market, which could be due to technological differences, as in the aforementioned case of broadband services. In the second scenario, participation is technologically feasible, though the integrated firm scales back (partially or fully) its sales on the merchant market.<sup>4</sup>

An alternative approach builds on bilateral contracting on the upstream market, which, in the case of non-observable contracts, leads to a serious problem of opportunism—as in Hart and Tirole (1990), O'Brien and Shaffer (1992) and McAfee and Schwartz (1994). In the extreme case, the upstream market structure then becomes irrelevant for final retail prices: each supplier becomes his own worst competitor. As Rey and Tirole (2007) show, vertical integration provides a commitment to hold back supply from other downstream firms (and, thus, to partially or fully foreclose the downstream market).<sup>5</sup>

The choice of a "market interface" or that of bilateral negotiations should be dictated by the respective application. A framework of bilateral negotiations can be more applicable in tight bilateral oligopolies with heterogeneous goods, possibly longer-term relationships with individually negotiated contracts, and potentially pervasive wholesale price discrimination. Instead, a market interface (i.e., with uniform and linear pricing) can be more applicable if suppliers are relatively homogeneous or if there is less scope for price discrimination as downstream firms mix and match between suppliers, often purchasing at list prices.<sup>6</sup> These conditions should be readily observable by antitrust authorities in each case.

As noted above, the primary objective of this paper is to guide the assessment of market power on *wholesale* markets. Consequently, our focus is on the price prevailing in the wholesale market. This deserves some comment. In competition cases, the assessment of market power typically precedes the overall assessment of, say, a merger in light of its impact on

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<sup>4</sup> Our analysis follows that in Gaudet and Long (1996), though, again, our focus is on the merchant market.

<sup>5</sup> De Fontenay and Gans (2005) have extended this to upstream competition. Gans (2007) uses such a framework to derive concentration measures for markets with multiple vertical segments, though he also explores uniform pricing.

<sup>6</sup> In addition, parallel trade clearly rules out, or at least limits, the scope for individual discounts, as well as for non-linear pricing.

final retail prices and, thus, welfare and consumer surplus, provided the respective standard is applied. Our analysis seeks to inform, in particular, the first step of an assessment of market power. Moreover, as noted above, in regulated industries, the imposition of *ex ante* remedies is typically closely tied to the finding of market power or dominance in the respective market, which is, the wholesale market in our case.

The rest of the paper is organized as follows. Section II analyzes a benchmark case with linear demand. The analysis is generalized in Section III, which also allows us to obtain new implications. Section IV discusses both the practical relevance of our analysis as well as some of the underlying modeling assumptions. Section V concludes.

## II. Linear Demand

### *Benchmark Case*

Throughout this paper, we employ a two-stage Cournot approach, as pioneered in Salinger (1988). In the benchmark case, which we analyze in this section, there will not be a vertically integrated firm. As is standard, our procedure will be, first, to solve for the equilibrium on the downstream market, thereby obtaining a derived demand function. We will subsequently use this to characterize the equilibrium on the upstream (supply) market.

At the downstream level,  $N \geq 2$  firms serve final consumers. Downstream firms use a homogeneous upstream good in fixed proportions. We normalize the ratio of input to output to be one. In this section, to obtain an explicit characterization, we specify a linear demand system with symmetric product differentiation. Thus, let the inverse demand for the generic firm  $n$ , given its own output  $q_n$  and output  $q_i$  of each of its downstream rivals, be given by

$$p_n = \alpha - q_n - \gamma \sum_{i \neq n} q_i, \quad \text{with } 0 \leq \gamma \leq 1,$$

whenever this is positive. The parameter  $0 \leq \gamma \leq 1$  describes the degree of homogeneity. When  $\gamma = 1$ , goods are homogeneous, and when  $\gamma = 0$ , we have independent demand. Each downstream firm  $n$  has a constant marginal cost  $c_n$ , where we simplify the analysis by setting  $c_n = c$ .

There are  $M \geq 2$  independent and, for the time being, non-integrated suppliers that produce a homogeneous good and compete in quantities. Each upstream firm  $m$  has a constant marginal cost  $\bar{c}_m$ , where we again simplify the analysis by setting  $\bar{c}_m = \bar{c}$ . The resulting uniform price at the upstream market is denoted by  $\bar{p}$ . Note that we refer to variables of the upstream market by using an upper bar; that is, by writing  $\bar{c}$  or  $\bar{p}$ , respectively.

As each downstream firm maximizes its profits  $\pi_n := (p_n - \bar{p} - c)q_n$ , we have, without vertical integration, the symmetric quantities

$$q_n = \frac{\alpha - \bar{p} - c}{2 + \gamma(N - 1)}.$$

The (inverse) derived demand is thus given from aggregation by

$$\bar{P}(\bar{q}) := \alpha - c - \frac{2 + \gamma(N - 1)}{N} \bar{q},$$

where  $\bar{q}$  is the total quantity supplied to and purchased from the upstream market. (Because the ratio of input to output is exactly one, we have  $\bar{q} = \sum_{i=1}^M \bar{q}_i = \sum_{i=1}^N q_i$ .) In light of our subsequent results, it is interesting to note that the elasticity of derived demand, which is given by

$$\bar{\varepsilon} := - \frac{\bar{p}}{\bar{q}} \frac{1}{d\bar{P}/d\bar{q}} = \frac{\bar{p}}{\alpha - c - \bar{p}},$$

is independent of the parameters that presently characterize the degree of downstream competition:  $N$  and  $\gamma$ . Intuitively, as we will see next, this implies that the equilibrium wholesale price will be independent of downstream competition.

Upstream firms maximize  $\bar{\pi}_m := (\bar{P}(\bar{q}) - \bar{c})\bar{q}_m$ . In equilibrium, we have that

$$\bar{q}_m = \frac{(\alpha - c - \bar{c})N}{(1 + M)(2 + \gamma(N - 1))},$$

such that, after substitution, the equilibrium wholesale price is given by

$$\bar{p}_{NVI} = \bar{c} + \frac{\alpha - \bar{c} - c}{M + 1}. \tag{1}$$

The subscript *NVI* denotes the fact that this is the equilibrium price without vertical integration.

### Indirect Constraints under Captive Sale

The main change to the preceding benchmark analysis is that one firm, namely  $n = m = 1$ , is now vertically integrated. In this section, we stipulate that the integrated firm does not participate in the merchant market, either through sales or through purchases. As noted in the Introduction, the fact that all of the production of  $m = 1$  is self-supplied (or “captive”) could be due to technological constraints (e.g. cable operators cannot offer a wholesale product but compete at the retail level in broadband markets).

The differences from the preceding benchmark analysis are thus two-fold. There are now only  $M - 1$  suppliers to the merchant market, while on the downstream level, the integrated firm operates at a (gross) marginal cost of  $\bar{c} + c$  (instead of  $\bar{p} + c$ ). The resulting equilibrium merchant price

under vertical integration (and captive sales), which we denote by  $\bar{p}_{VI}$ , is given by

$$\bar{p}_{VI} = \bar{c} + \frac{(\alpha - c - \bar{c})(2 - \gamma)}{2M} \quad (2)$$

and compares as follows with the price  $\bar{p}_{NVI}$ , which prevailed without vertical integration.

**Proposition 1.** *With linear demand and differentiated products, the merchant price is strictly lower in the presence of indirect constraints (given that firms  $n = m = 1$  are vertically integrated) if and only if products are not too differentiated. That is,  $\bar{p}_{VI} < \bar{p}_{NVI}$  holds if and only if  $\gamma > \gamma^* = 2/(M + 1)$ .*

*Proof:* In the downstream market, the integrated firm,  $n = 1$ , maximizes  $(p - \bar{c} - c)q_1$ , while for the vertically separated firms,  $n \geq 2$ , the first-order conditions are as in the case without vertical integration. The downstream equilibrium is then characterized by

$$q_1 = \frac{(\alpha - c - \bar{c})(2 - \gamma) + \gamma(N - 1)(\bar{p} - \bar{c})}{(2 - \gamma)(2 + \gamma(N - 1))},$$

$$q_n = \frac{(\alpha - c)(2 - \gamma) - 2\bar{p} + \gamma\bar{c}}{(2 - \gamma)(2 + \gamma(N - 1))} \quad \text{for } n \geq 2.$$

Aggregating over the  $n \geq 2$  firms gives rise to the (inverse) derived demand

$$\bar{P}(\bar{q}) = \frac{1}{2} \left[ (\alpha - c)(2 - \gamma) + \gamma\bar{c} - \bar{q} \frac{(2 - \gamma)(2 + \gamma(N - 1))}{N - 1} \right], \quad (3)$$

where now  $\bar{q} = \sum_{i=2}^M \bar{q}_i = \sum_{i=2}^N q_i$ . Next, a non-integrated upstream firm  $m \geq 2$  maximizes  $(\bar{p} - \bar{c})\bar{q}_m$ , such that we have from the respective first-order condition the unique symmetric equilibrium

$$\bar{q}_m = \frac{(\alpha - c - \bar{c})(N - 1)}{(2 + \gamma(N - 1))M}.$$

After substitution, this yields expression (2) for the equilibrium input price with vertical integration. Comparing, finally,  $\bar{p}_{VI}$  from (2) and  $\bar{p}_{NVI}$  from (1), we have that  $\bar{p}_{VI} < \bar{p}_{NVI}$  in case

$$\frac{1}{M + 1} > \frac{2 - \gamma}{2M},$$

which, after solving for  $\gamma$ , obtains the asserted condition  $\gamma > \gamma^*$ . ■

Note that  $\gamma^*$  decreases with  $M$ . When there are more independent upstream firms, the loss in direct constraint due to vertical integration weighs less.

It is worthwhile to recall that in the absence of a vertically integrated firm, the degree of differentiation  $\gamma$  has no impact on the elasticity of derived demand, and therefore it also has no impact on the equilibrium wholesale price. In contrast, the indirect constraints that are exerted by a backwards integrated firm become stronger when products are less differentiated at the retail level (higher  $\gamma$ ). This results in a more elastic derived demand and, consequently, in a lower equilibrium merchant price. More formally, we have from (3) that the elasticity of derived demand equals

$$\bar{\epsilon} = \frac{2\bar{p}}{(2 - \gamma)(\alpha - c - \bar{p}) - \gamma(\bar{p} - \bar{c})}$$

That indirect constraints become stronger as products are less differentiated is reflected in the fact that the merchant price after vertical integration is lower if and only if  $\gamma$  is sufficiently high. In particular, in the case of homogeneous goods ( $\gamma = 1$ ), indirect constraints are always sufficiently strong to more than compensate for the loss of direct constraints, which results as the number of suppliers to the wholesale market shrinks from  $M$  to  $M - 1$ .

Finally, Proposition 1 shows that the common presumption that indirect constraints are of only second-order importance as they are buffered through the additional vertical layer is flawed. In fact, Proposition 1 offers a formalized “experiment” that trades off direct for indirect constraints, showing that the loss of direct constraints due to vertical integration weighs more than the addition of indirect constraints only if downstream competition is weak.

In the rest of this section, we provide some robustness checks for Proposition 1 by showing that the result does not depend qualitatively on the timing of moves.<sup>7</sup>

**Proposition 2.** *When the backwards integrated firm can act as a Stackelberg leader on the downstream market, indirect constraints are still stronger; that is, the merchant price is strictly lower under vertical integration, when  $\gamma > \gamma^{**}$ , with  $\gamma^{**} > \gamma^*$ .*

*Proof:* Taking the quantity  $q_1$  of the integrated firm as given, the downstream equilibrium is characterized by

$$q_n = \frac{\alpha - c - \bar{p} - \gamma q_1}{2 + \gamma(N - 2)} \quad \text{for } n \geq 2.$$

<sup>7</sup> We thank a referee for suggesting this extension. It follows arguments in the literature that vertical integration can provide a commitment device to “move first” at the price-setting stage.

Aggregation gives rise to the (inverse) derived demand on the wholesale market

$$\bar{P}(\bar{q}) = (\alpha - c) - \frac{(2 + \gamma(N - 2))\bar{q} + \gamma(N - 1)q_1}{N - 1},$$

where, again,  $\bar{q} = \sum_{i=2}^M \bar{q}_i = \sum_{i=2}^N q_i$ . It also proves useful to derive from this the elasticity of derived demand

$$\bar{\varepsilon} = \frac{\bar{p}}{\alpha - c - \bar{p} - \gamma q_1}, \tag{4}$$

which is increasing in  $q_1$ . Next, upstream firm  $m \geq 2$  chooses  $\bar{q}_m$  to maximize  $(\bar{P}(\bar{q}) - \bar{c})\bar{q}_m$  at the same time that the integrated firm chooses  $q_1$  to maximize  $(p_1 - \bar{c} - c)q_1$ . In equilibrium, we have

$$\begin{aligned} \bar{q}_1 = q_1 &= \frac{(\alpha - c - \bar{c})(2M - \gamma(1 + M - N))}{4M + 2\gamma M(N - 2) - \gamma^2(M - 1)(N - 1)}, \\ \bar{q}_m &= \frac{(\alpha - c - \bar{c})(N - 1)(2 - \gamma)}{4M + 2\gamma M(N - 2) - \gamma^2(M - 1)(N - 1)}, \end{aligned}$$

for the supply of the  $m \geq 2$  non-integrated firms. After substitution, the equilibrium merchant price is

$$\bar{p}_{VI}^{Stack} = \bar{c} + \frac{(\alpha - c - \bar{c})(2 + \gamma(N - 2))}{4M + 2\gamma M(N - 2) - \gamma^2(M - 1)(N - 1)}. \tag{5}$$

Comparing  $\bar{p}_{VI}^{Stack}$  from (5) and  $\bar{p}_{NVI}$  from (1), we have that  $\bar{p}_{NVI} > \bar{p}_{VI}^{Stack}$  holds if

$$\gamma > \gamma^{**} = \frac{3 + M - N - \sqrt{M^2 - 2M(N - 1) + N^2 + 2N - 3}}{3 + M - 2N}.$$

As  $\gamma^{**}$  strictly increases in  $N$ , and  $\gamma^{**} = 2/(M + 1)$  for  $N = 1$  while  $\gamma^{**} \rightarrow 1$  for  $N \rightarrow \infty$ , we also have that  $\gamma^* < \gamma^{**} < 1$ . ■

Note that a Stackelberg leader typically produces more than in the comparable simultaneous Cournot game and thereby displaces the quantity of followers. In the presently considered game with a vertical structure, however, there is a second effect at work. The more the integrated firm produces and thus displaces downstream rivals, the less elastic the derived demand on the wholesale market becomes (as evidenced by expression (4) in the preceding proof). This in turn implies, for given quantities, a lower price prevailing at the merchant market and thus more competitive conditions for the integrated firm’s downstream rivals. This novel feedback mechanism dampens the integrated firm’s (i.e., the Stackelberg leader’s) incentives to expand its output.<sup>8</sup> Still, Proposition 2 shows the robustness of our

<sup>8</sup> Despite the novel effect arising from indirect constraints, it is still the case that the quantity supplied by the vertically integrated firm under Stackelberg leadership is higher than the quantity supplied under simultaneous Cournot competition. This can be readily established by comparing  $q_1$  as expressed in the respective proofs of Proposition 2 and Proposition 1.

previous results. The merchant-market price is lower as indirect constraints “replace” direct constraints following vertical integration if and only if the downstream market is sufficiently competitive (in terms of product differentiation).

*Participation in the Merchant Market*

We have assumed so far that the integrated firm does not participate in the merchant market because its captive supply cannot be used by other downstream firms for technological reasons. This section considers the alternative case, in which this is technologically feasible.

**Lemma 1.** *With linear demand and differentiated products, there exists a threshold  $\bar{\gamma}$  such that the integrated firm strictly prefers to sell on the merchant market if  $\gamma < \bar{\gamma}$  and strictly prefers to buy on the merchant market if  $\gamma > \bar{\gamma}$ .*

*Proof:* Note that the integrated firm can now participate in the merchant market with quantity  $\bar{q}^1$ . Following by now standard calculations, we have in equilibrium that the integrated firm chooses

$$\bar{q}^1 = \frac{2(\alpha - c - \bar{c})(N - 1)[2 - \gamma(2 + M - N) - \gamma^2(N - 1)]}{[2 + \gamma(N - 1)][4(1 + M) + 2\gamma(1 + M)(N - 2) + \gamma^2(M + 2)(N - 1)]},$$

while all non-integrated suppliers  $m \geq 2$  choose

$$\bar{q}^m = \frac{(\alpha - c - \bar{c})(N - 1)[4 + \gamma(2 - \gamma)(N - 1)]}{[2 + \gamma(N - 1)][4(1 + M) + 2\gamma(1 + M)(N - 2) + \gamma^2(M + 2)(N - 1)]}.$$

From the condition that  $\bar{q}^1 > 0$ , we obtain a threshold

$$\bar{\gamma} := \frac{N - M - 2 + \sqrt{8(N - 1) + (N - M - 2)^2}}{2(N - 1)},$$

such that for  $\gamma < \bar{\gamma}$ , the vertically integrated firm still wants to sell on the upstream market with  $\bar{q}^1 > 0$ . Note that as  $(\partial\bar{\gamma}/\partial M) < 0$ ,  $\lim_{M \rightarrow \infty} \bar{\gamma} = 0$ ,  $(\partial\bar{\gamma}/\partial N) > 0$  and  $\lim_{N \rightarrow \infty} \bar{\gamma} = 1$ , it is always the case that there is an interior solution as  $0 < \bar{\gamma} < 1$ . ■

When products are completely independent such that  $\gamma = 0$ , the integrated firm surely gains from participating in the upstream market, as this has no effects on its retail profits. Instead, when products are fairly homogeneous (high  $\gamma$ ), there are no incentives to sell. For instance, we obtain for  $N = M = 3$  the threshold  $\bar{\gamma} \cong 0.618$ .

When participation is feasible, a lower value of  $\gamma$  is more generally associated with higher incentives to continue to participate on the merchant market. Once this is taken into account, it holds that regardless of the

degree of differentiation at the retail level, the loss of direct constraints due to vertical integration is always more than compensated for by the additional indirect constraints.

**Proposition 3.** *With linear demand and differentiated products, the equilibrium wholesale price is lower, and for  $\gamma > 0$  strictly lower, under vertical integration, in case it is technologically feasible for the integrated firm to continue to participate in the merchant market. That is, indirect constraints always more than compensate for the loss of direct constraints in the merchant market.*

*Proof:* After substitution from the proof of Lemma 1, the equilibrium input price under continued participation becomes

$$\bar{p}_{VI} = \bar{c} + \frac{(\alpha - \bar{c} - c)(2 - \gamma)(\gamma^2(N - 2) - 2\gamma(N - 1) - 4)}{2\gamma^2(N - 2)(2 + M) - 4\gamma(N - 2)(M + 1) - 8(M + 1)}.$$

Recall from (1) that  $\bar{p}_{NVI}$  is independent of  $\gamma$ . The difference  $\bar{p}_{VI} - \bar{p}_{NVI}$ , which equals

$$-(\alpha - \bar{c} - c)\gamma^2 \times \frac{(N - 1)(2M - \gamma(M + 1))}{2(1 + M)(-\gamma^2(N - 1)(2 + M) + 2\gamma(N - 2)(M + 1) + 4(M + 1))},$$

is, then, indeed strictly negative for every  $0 < \gamma \leq 1$ . ■

### III. General Analysis

#### *Captive Sales*

Suppose that the downstream market is characterized by some inverse demand  $P(q)$ . First, take the case without vertical integration. Aggregating over the first-order conditions of all downstream firms under Cournot competition, while making use of the fact that the aggregate input volume  $\bar{q}$  equals the aggregate sales volume  $q$ , we obtain the (inverse) derived demand for the input

$$\bar{P}(\bar{q}) = P(\bar{q}) - c + \frac{\bar{q}}{N} P'(\bar{q}).$$

Note that, as  $N \rightarrow \infty$ , the inverse derived demand converges to  $P(\bar{q}) - c$ , which is just the inverse final demand adjusted for downstream firms' own (symmetric) marginal costs.

In the upstream market, we now introduce, by assumption symmetric and constant, “the conjectural-variation” parameter  $\lambda := \sum_{i \neq m} (\partial \bar{q}_i / \partial \bar{q}_m)$ . Here, our choice of the conjectural-variations approach clearly deserves comment. We are not agnostic to the conceptual shortcomings of this approach. Moreover, all our results, with the obvious exception of those where we conduct a comparative analysis in  $\lambda$ , clearly also hold when choosing the “Cournot conjectures” with  $\lambda = 0$ . We want, however, to allow for more general values of  $\lambda$ , as this approach is still prominent in parts of the empirical literature.<sup>9</sup> More importantly for the objective of the present paper, it is still frequently used for applied work undertaken by and for antitrust authorities. Because one of our objectives is to inform the discussion on the different role and importance of direct and indirect constraints, it seems appropriate to allow for changes in the “mode” of upstream (direct) competition along this dimension. From this perspective, lower values of  $\lambda$  then correspond to more intense upstream competition and thus stronger direct constraints (the limiting case  $\lambda \rightarrow -1$  corresponds to “Bertrand conjectures”).

Next, take the case in which firm  $n = m = 1$  is vertically integrated. Note that in this case, total downstream sales  $q = \sum_{n=1}^N q_n$  are strictly higher than the total quantity of the input sold in the merchant market  $\bar{q} = \sum_{m=2}^M \bar{q}_m$ . The following lemma characterizes the derived-demand function.

**Lemma 2.** *With a vertically integrated firm, where  $\bar{q} = q - q_1$ , we have the (inverse) derived demand*

$$\bar{P}(\bar{q}) = [P(q) - c] + \frac{\bar{q}}{N - 1} P'(q), \tag{6}$$

which satisfies

$$\bar{P}'(\bar{q}) = \frac{P'(q)}{N - 1} \frac{(N + 1)P'(q) + qP''(q)}{2P'(q) + q_1P''(q)}. \tag{7}$$

*Proof:* Aggregating over the non-integrated firms’ first-order conditions to have

$$\sum_{i=2}^N [P(q) - \bar{p} - c + q_i P'(q)] = 0$$

and making use of the fact that  $\bar{q} = q - q_1$ , we obtain (6). Next, from total differentiation of the first-order condition for  $n = 1$ , namely

$$P(q) - c + P'(q)q_1 - \bar{c} = 0,$$

<sup>9</sup> Cf. Bresnahan (1989). (See, however, Corts, 1999, for a critique.)

and of the previously derived first-order conditions for all firms  $n \geq 2$ , we have that

$$\begin{pmatrix} \frac{N}{N-1}P'(q) + \frac{\bar{q}}{N-1}P''(q) & P'(q) + \frac{\bar{q}}{N-1}P''(q) \\ P'(q) + q_1P''(q) & 2P'(q) + q_1P''(q) \end{pmatrix} \begin{pmatrix} d\bar{q} \\ dq_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} d\bar{p},$$

which gives (7) from Cramer’s rule. ■

It is useful to express the equilibrium characterization for the wholesale market in terms of the respective elasticity. With vertical integration and the respective elasticity of derived demand  $\bar{\epsilon}_{VI}$ , the equilibrium wholesale price then satisfies the standard Lerner condition

$$\frac{\bar{p}_{VI} - \bar{c}}{\bar{p}_{VI}} = \frac{1}{M-1} \frac{1}{\bar{\epsilon}_{VI}} (1 + \lambda), \tag{8}$$

while, likewise, we obtain for the case without vertical integration that

$$\frac{\bar{p}_{NVI} - \bar{c}}{\bar{p}_{NVI}} = \frac{1}{M} \frac{1}{\bar{\epsilon}_{NVI}} (1 + \lambda). \tag{9}$$

A comparison of expressions (8) and (9) generalizes our previous insights on linear demand. As the number of upstream firms that compete directly in the merchant market decreases from  $M$  to  $M - 1$  due to vertical integration, the prevailing direct constraints are reduced and thus the wholesale price is pushed up. On the other hand (as we show next more formally), derived demand becomes more elastic under vertical integration, testifying to the presence of indirect constraints.

We invoke the standard condition for Cournot games (in strategic complements):

$$P'(q) + qP''(q) < 0. \tag{10}$$

**Proposition 4.** *Comparing the cases with and without vertical integration, we have the following results:*

- (i) *When evaluated at the same level of final demand  $q$ , derived demand on the merchant market,  $\bar{q}$ , is more responsive in the presence of indirect constraints; that is, it holds that*

$$\left| \frac{d\bar{q}}{d\bar{p}_{VI}} \right| > \left| \frac{d\bar{q}}{d\bar{p}_{NVI}} \right|. \tag{11}$$

- (ii) *Suppose that, in addition, derived demand is concave. Then,  $N \leq M$  is a sufficient condition for the equilibrium merchant price to be strictly lower under vertical integration despite the loss of direct constraints.*

*Proof:* Without vertical integration, we have that

$$\frac{d\bar{p}_{NVI}}{d\bar{q}} = \frac{1}{N}[(N + 1)P'(q) + qP''(q)]. \tag{12}$$

For (11) to hold, using (7) and (12) we must have that

$$\frac{1}{N} > \frac{P'}{2P' + q_1P''} \frac{1}{N - 1},$$

which further transforms to

$$P'(q) + \frac{N - 1}{N - 2}q_1P''(q) < 0. \tag{13}$$

To see that condition (13) holds, note first that it is always satisfied in case  $P''(q) \leq 0$  holds everywhere. Take the case in which  $P''(q) > 0$ . Then, condition (13) is implied by condition (10) after noting that clearly

$$\frac{N - 1}{N - 2}q_1 < q.$$

We turn next to the second assertion. First, we analyze when, if evaluated at the same price  $\bar{p}_{VI} = \bar{p}_{NVI} = \bar{p}$ , we have that

$$\left| \frac{d\bar{p}_{VI}}{d\bar{q}} \right| \leq \left| \frac{d\bar{p}_{NVI}}{d\bar{q}} \right|. \tag{14}$$

Note that for given  $q$ , we already know that

$$\left| \frac{d\bar{p}_{VI}}{d\bar{q}} \right| < \left| \frac{d\bar{p}_{NVI}}{d\bar{q}} \right|$$

and thus that  $\bar{p}_{NVI} > \bar{p}_{VI}$ . Holding  $\bar{p}_{VI} = \bar{p}$  fixed, now we have to increase  $\bar{q}$ , thereby lowering  $\bar{p}_{NVI}$ , in order to compare the derivatives at the same value for  $\bar{p}$ . The inequality surely still holds if  $d\bar{p}_{NVI}/d\bar{q}$  is everywhere non-increasing in  $\bar{p}$ ; that is, if the function  $\bar{P}_{NVI}(\bar{q})$  is concave. With (14) and as the share of the  $N - 1$  non-integrated downstream firms in the final market is smaller than  $(N - 1)/N$ , assertion (ii) follows from a simple comparison of the two Lerner indices (8) and (9). ■

By assertion (i), under vertical integration, derived demand becomes more responsive to a change in the merchant price. This is intuitive. In contrast to all other downstream firms, the backwards integrated firm  $n = 1$  is not affected by a change in the merchant price. Consequently, as the merchant price increases, the integrated firm’s share of the downstream market increases at the expense of all other firms  $n > 1$ . In fact, this mechanism is the very essence of the substitution that underlies the indirect constraints.

For a final observation, recall that with linear demand and “Cournot conjectures” ( $\lambda = 0$ ), we found for homogeneous downstream products ( $\gamma = 1$ ) that vertical integration lowers the wholesale price since indirect constraints

are sufficiently strong. As we intensify upstream competition by assuming conjectures  $\lambda < 0$  (down to “Bertrand conjectures”  $\lambda = -1$ ), it is intuitive that the loss of direct constraints weighs in even less, implying that  $\bar{p}_{NVI} > \bar{p}_{VI}$  is still true.<sup>10</sup>

### Participation

To analyze the integrated firm’s incentives to continue to participate in the merchant market, provided that this is technologically feasible, we proceed as follows. Starting from the equilibrium *without* participation (see above), we analyze the effect of a marginal deviation by firm  $m = 1$  to supply:  $\bar{q}_1 > 0$ .<sup>11</sup> Whether this is profitable depends on a trade-off between realizing profits on the upstream market and, via reducing  $\bar{p}$  and thereby increasing competition among downstream firms, losing profits on the retail market. At  $\bar{q}_1 = 0$  the marginal profits from selling at the upstream markets are just  $\bar{p} - \bar{c}$ , and therefore a (marginal) participation on the merchant market is thus profitable if

$$\bar{p} - \bar{c} > - \frac{d\bar{q}}{d\bar{q}_1} \frac{d\bar{p}}{d\bar{q}} \frac{d}{d\bar{p}} [q_1 [P(q) - \bar{c} - c]], \quad (15)$$

where the RHS of (15) intuitively “walks through” the mechanism by which  $\bar{q}_1 > 0$  affects the profits realized with the firm’s own downstream operations,  $q_1 [P(q) - \bar{c} - c]$ .

**Proposition 5.** *If this is technologically feasible, then for the vertically integrated firm, it is optimal to (at least marginally) sell into the merchant market whenever (15) holds, which simplifies to*

$$\frac{\bar{p} - \bar{c}}{1 + \lambda} > p - \bar{c} - c. \quad (16)$$

*Proof:* To transform (15), note first that  $d\bar{p}/d\bar{q}_1 = 1$ . Next, using the first-order condition for  $q_1$ , namely  $P - \bar{c} - c + P'(q)q_1 = 0$ , we have

$$\frac{d}{d\bar{p}} [q_1 [P(q) - \bar{c} - c]] = -q_1 P' \left( -\frac{d\bar{q}}{d\bar{p}} \right),$$

<sup>10</sup> Formally, we then obtain

$$\bar{p}_{NVI} - \bar{p}_{VI} = (\alpha - c - \bar{c}) \frac{1 + \lambda}{2} \frac{M + \lambda - 1}{(M + \lambda)(M + \lambda + 1)},$$

which is indeed strictly positive for all  $\lambda \in (-1, 0]$ .

<sup>11</sup> The effect on profits yields a sufficient condition for participation. To obtain a necessary and sufficient condition for  $\bar{q}_1 > 0$  to hold in equilibrium, we must appeal also to quasi-concavity of the profit function.

where we can further substitute  $P'(q)q_1 = -(P(q) - \bar{c} - c)$ . With these transformations, (15) becomes

$$\frac{\bar{p} - \bar{c}}{1 + \lambda} > (P(q) - \bar{c} - c) \frac{d\bar{p}}{d\bar{q}} \frac{d\bar{q}}{d\bar{p}},$$

finally yielding condition (16). ■

An immediate implication from (16) is that with upstream Cournot conjectures,  $\lambda = 0$ , the integrated firm will not want to sell on the merchant market. (Formally, (16) then transforms to  $p < \bar{p} + c$ , which clearly does not hold.) On the other hand, we can show that, at least if  $N$  is not too small, as the upstream market becomes sufficiently competitive, the integrated firm strictly prefers to sell a positive quantity on the merchant market.

**Corollary 1.** *While for  $\lambda = 0$  the converse of (16) holds strictly, we have, at least for sufficiently high  $N$ , that (16) holds as upstream competition becomes sufficiently intense with  $\lambda \rightarrow -1$ .*

*Proof:* Note, first, that the RHS of (16) converges to  $P(q^*) - \bar{c} - c > 0$  as  $\lambda \rightarrow -1$ , where  $q^*$  is the Cournot equilibrium quantity if  $N$  firms have symmetric marginal costs  $c + \bar{p}$ . Using l'Hôpital's rule, the LHS of (16) converges to  $d\bar{p}/d\lambda|_{\lambda=-1}$ . Next using the first-order condition on the upstream market,

$$\bar{p} - \bar{c} + \frac{d\bar{P}}{d\bar{q}} \frac{1}{M-1} (1 + \lambda) = 0,$$

we obtain in the limit

$$\left. \frac{d\bar{p}}{d\lambda} \right|_{\lambda=-1} = - \frac{d\bar{P}}{d\bar{q}} \frac{1}{M-1}.$$

Note that given  $\bar{p} = \bar{c}$  at  $\lambda = -1$ , all downstream firms will have symmetric quantities such that  $q_1 = q/N$ . Using (7) for  $d\bar{P}/d\bar{q}$  and noting that from the first-order condition of downstream firms, it holds that  $p - \bar{c} - c = -(1/N)/P'(q)$ , (16) becomes

$$\frac{N}{N-1} \frac{(N+1)P'(q) + qP''(q)}{2P'(q) + qP''(q)/N} > M-1. \tag{17}$$

As  $q$  converges to  $q^*$ , which for  $N \rightarrow \infty$  further converges to a finite value  $q^{**} > 0$  satisfying  $P(q^{**}) = \bar{c} + c$ , we clearly have that the LHS of (17) is strictly larger than  $M - 1$  for all sufficiently large values of  $N$ . ■

The result in Corollary 1 is somewhat surprising. According to Corollary 1, the incentives of the integrated firm to continue to participate in the merchant market are sufficiently high when upstream competition is sufficiently intense, in which case the upstream margin,  $\bar{p} - \bar{c}$ , should

clearly be very low. To obtain more intuition, take the extreme cases with conjectures  $\lambda = -1$  and  $\lambda = 0$ . The case with  $\lambda = 0$  corresponds to the standard case of quantity competition (Cournot conjectures), implying that any deviation to a higher level of a firm's own supply will be in addition to all the existing supply of other upstream firms. In contrast, for  $\lambda < 0$  (and thus, in particular, for the case with standard price competition, where  $\lambda = -1$ ), there is a replacement effect. As firm  $m = 1$  increases its quantity by  $\Delta\bar{q}$ , total supply will increase by strictly less than  $\Delta\bar{q}$ . Putting it somewhat differently, for  $\lambda < 0$ , the participation of  $m = 1$  will then not only take away market share from other suppliers, but it will also replace some of the overall quantity sold by other suppliers and thus will ultimately have a lower impact on the retail market.

#### IV. Discussion

##### *Assessing Direct and Indirect Constraints in Competition Policy*

As noted in the Introduction, our main deviation from previous contributions is the focus on the market power assessment for the merchant market. Our preceding analysis shows that, in contrast to a common presumption, indirect constraints that work through downstream competition can be very powerful, in particular when downstream competition is intense. In the Introduction, we argued that this finding—that is, the comparative analysis in downstream competition—contrasts with an argument that the European Commission used in the landmark Schneider/Legrand case.

The case in which a vertically integrated firm exerts *only* indirect constraints on the merchant market is clearly extreme. As discussed for the case of broadband services in the Introduction, such a restriction to “captive sales” can sometimes follow from technological constraints. Even then, though, technological change might ease up these constraints, at least in the medium run. Our analysis of an integrated firm's incentives to continue to participate in the merchant market provides some guidelines on when a vertically integrated firm can be expected to exert not only indirect, but also direct constraints on the merchant market.

In this section, we take some further steps in applying our analysis to guide competition policy. To assess market power, antitrust agencies have to rely frequently, at least at some initial stage, on relatively crude though readily available indicators. Market shares undoubtedly constitute one of the most important of these indicators. For an assessment of market power on the wholesale market, the presence here of an integrated firm with captive sales creates some obvious additional complications. Calculating the wholesale market shares of each of the remaining  $M - 1$  independent suppliers could be seriously misleading, as it overstates market power through

neglecting indirect constraints. In fact, note that the vertical integration of firms  $n = m = 1$  would mechanically increase the market share of each independent upstream firm from  $1/M$  to  $1/(M - 1)$ , though we know that the resulting wholesale price might be lower. To capture these indirect constraints, it has been suggested, instead, to fully incorporate captive sales of integrated firms in any (provisional) market share analysis for the wholesale market. Applied to our setting, where the volume of inputs equals that of final sales, market shares would then be based on final sales,  $q$ , instead of only on sales in the merchant market,  $\bar{q}$ .

In the rest of this section, we use the preceding analysis to evaluate this proposal. For this purpose, we return to the linear case. In addition to allowing, once again, for differentiated products, we also omit the parameter  $\lambda$  from the general analysis to vary the mode of upstream competition.

**Proposition 6.** *In the presence of indirect constraints through the vertically integrated firm  $n = m = 1$  and with linear demand and differentiated products, the retail market share of any of the remaining  $M - 1$  independent suppliers is strictly lower if:*

- (i) *the retail market is more competitive (as  $\gamma$  is higher);*
- (ii) *the wholesale market is less competitive (as  $\lambda$  is higher).*

*Proof:* The analysis is the same as in Proposition 1, with the difference that we also bring in the conjectural variation parameter at the upstream level. For suppliers that are not forward integrated, we have

$$\bar{q}_m = \frac{(\alpha - c - \bar{c})(N - 1)}{(2 + \gamma(N - 1))(M + \lambda)}.$$

After substitution, the equilibrium merchant price now equals

$$\bar{p}_{VI} = \bar{c} + \frac{(\alpha - c - \bar{c})(2 - \gamma)(1 + \lambda)}{2(M + \lambda)},$$

which generalizes (1). This can be substituted back into the downstream quantities to calculate the market share of the vertically integrated firm  $s_1 := q_1/q$ , with  $q = \sum_{i=1}^N q_i$ :

$$s_1 = \frac{1}{N} + \frac{(1 + \lambda)(N - 1)(2 + \gamma(N - 1))}{N[2(1 + \lambda + (M - 1)N) + \gamma(1 + \lambda)(N - 1)]}. \tag{18}$$

For the (indirect) retail market share of any of the other independent suppliers, we then have  $(1 - s_1)/(M - 1)$ . Thus, assertions (i) and (ii) follow from the differentiation of (18), which yields  $\partial s_1 / \partial \gamma > 0$  and  $\partial s_1 / \partial \lambda > 0$ . ■

To be more specific, the insights of Proposition 6 could be applied to the assessment of a merger between two or more independent suppliers,

as well as to the assessment of whether one or a few independent suppliers have (jointly) substantial market power (i.e., a dominant position). Including captive sales, a high market share of the respective independent suppliers may thus be a sign of either weak indirect constraints or strong direct constraints. Without a more detailed analysis of market conditions at the upstream and downstream levels, a market share analysis that merely includes captive sales may not be informative about the overall strength of the competitive constraints on the merchant market.

### *Modeling the Merchant Market*

As noted in the Introduction, our approach to modeling the merchant market follows an older tradition in the literature, using a two-stage Cournot game. A central tenet of this approach is that transactions in the wholesale (or merchant) market are undertaken at a uniform price, precluding price discrimination (in terms of both quantity discounts for any given buyer and of varying terms for different buyers).

A more novel influential approach (see Hart and Tirole, 1990) has stressed, instead, precisely the role of individually negotiated contracts, which undermine even a monopolistic supplier's ability to sustain high wholesale prices. In this literature, vertical integration is thus typically looked upon negatively, as it enhances the supplier's commitment power, leading to higher wholesale and retail prices (provided that the integrated supplier still sells to other downstream firms). Crucially, though we are not aware of a formal analysis of this, the commitment power that a vertically integrated firm obtains does *not* affect the behavior of other suppliers. Each non-integrated supplier is, at least if the opportunism problem is sufficiently severe, still his own worst competitor. Hence, the presence of neither a vertically integrated firm nor other, independent, upstream suppliers—that is, neither the presence of indirect nor direct constraints—affects the terms at which a given supplier sells. For all practical purposes, these conclusions appear to be too strong.

We see two alternatives for making further progress in modeling merchant markets, in particular in the presence of integrated firms. First, building on the “contractual approach”, one could use a less extreme variant of the opportunism problem. Rey and Vergé (2004) have laid some foundations for the analysis with downstream price competition in differentiated products and with alternative beliefs. Such a model would need to be extended to upstream competition in heterogeneous goods or with convex costs (respectively, capacity constraints). Alternatively, one may want to stick to a characterization of the merchant market in terms of a uniform wholesale price. As noted in the Introduction, such a choice may be warranted in the absence of price discrimination, when there are many parties on either

side of the market or, in particular, when there is scope for parallel trade to undermine price discrimination.<sup>12</sup>

## **V. Conclusion**

This paper focuses on the assessment of market power on the wholesale (or merchant) market in the presence of a vertically integrated firm. In particular, we analyze how direct constraints due to competition on the wholesale market interact with indirect constraints arising from substitution on the retail market.

Our analysis is particularly relevant to assessing market power in settings where direct constraints are weak and where, consequently, the overall finding of market power (or dominance) could hinge crucially on the strength of indirect constraints. As noted in the Introduction, the role of indirect constraints was important in a number of recent cases, both in antitrust (merger) cases and for regulation, such as in markets for electronic communications. Practical considerations in these cases also justify the focus on the wholesale (or merchant) market alone, instead of conducting an analysis in terms of retail prices and resulting consumer surplus.

The presence of a vertically integrated firm crucially alters the role that retail competition plays for the assessment of market power on the wholesale market. If a vertically integrated firm exerts indirect constraints, then these are stronger (in the sense of inducing a more elastic derived demand) as retail competition becomes more intense. In contrast, if all downstream firms purchase from the same merchant market, then the degree of retail competition should not be expected to have an unambiguous effect on the prevailing wholesale price. In particular, in case of high downstream competition, we show that indirect constraints can be very strong and thus are not necessarily “buffered” through the vertical structure. In addition, a vertically integrated firm may still have incentives to participate in the merchant market, thus also providing direct constraints. Overall, we showed that the strength of direct vs. indirect constraints and their likely relevance for an assessment of market power on the merchant market hinges crucially on the degree of competition on both the upstream and the downstream level.

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<sup>12</sup> In ongoing research, we are currently developing a model that allows for upstream price competition in differentiated products.

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