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Alternating-offer bargaining over menus under incomplete information*

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Summary. This paper considers bargaining with one-sided private information and alternating offers where an agreement specifies both a transfer and an additional (sorting) variable. Moreover, both sides can propose menus. We show that for a subset of parameters the alternating-offer game has a unique equilibrium where efficient contracts are implemented in the first period. This stands in sharp contrast to the benchmarks of contract theory, where typically only the uninformed side proposes, and bargaining theory, where typically the agreement only specifies a transfer.

Keywords and Phrases: Screening, Bargaining.

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1 Introduction

Real-world negotiations cover often multiple issues. For instance, collective agreements between unions and firms may specify not only wages, but also working hours, overtime pay, or manning levels. This note shows that the presence of such additional issues can vastly change the equilibrium outcome if bargaining proceeds under private information.

Suppose for a start that a firm is privately informed about its profitability. Bargaining with a union over wages proceeds by alternating offers. In this case equilibria typically exhibit delay (i.e., "strike") and more profitable firms concede faster.[1]

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[1] For an overview on bargaining under private information see Kennan and Wilson (1993). Kennan and Wilson (1989) survey applications to labor models. Note also that with alternating offers considerable

Suppose now that negotiations cover also working hours. If more profitable firms derive higher profits from any additional hour of work, which seems to be a reasonable assumption, the choice of working hours can perform the role of an additional sorting variable. A first conjecture could be that the availability of an additional sorting variable increases the scope for inefficiencies, leading to equilibria where inefficient contracts are signed with delay. In contrast, we can show for a large parameter range that the game with alternating offers over menus has a unique equilibrium where an efficient agreement is reached immediately.

This outcome stands in sharp contrast to that obtained in standard contracting games where typically the uninformed side can make a take-it-or-leave-it offer (see, e.g., Laffont and Maskin, 1982). As is well known, these games of screening lead to distortions in the sorting variable. In the case with alternating offers, however, the right of the uninformed side to make counteroffers essentially creates type-dependent reservation values, which reduces the scope for profitable screening by the uninformed party.

The possibility to specify a sorting variable and offer menus is the main difference to standard models of bargaining with private information (see, e.g., Grossman and Perry, 1986). As noted above and stated more formally below, multiple equilibrium outcomes supporting long delays are typical in these models. Related to our model is Sen (2000), where negotiations also include a sorting variable. In contrast to our paper, he does not allow for menu offers by the uninformed party and uses instead refinements to prune the equilibrium set.[2]

2 The model

Two players can sign a contract $c = (x, t) \in \Re^2$. One player (the agent) has private information about his type, which is denoted by $i \in I = \{1, 2\}$. The prior probability that the agent is of type $i = 2$ is given by $\mu^0 \in (0, 1)$. Both parties have zero reservation payoff. A contract c yields the payoff $U_i(c) = V_i(x) - t$ to an agent of type i, while the uninformed side (the principal) realizes $W(c) = T(x) + t$. Denote $S_i(x) = V_i(x) + T(x)$. All functions are continuous. Moreover, we make the following assumptions.

(A.1) $S_i(x)$ is strictly concave and maximized by a finite value x_i^*, where $S_i^* = S_i(x_i^*) > 0$ for $i \in I$. Moreover, $V_2(x) > V_1(x)$ holds for all x, and $V_2(x) - V_2(x') > V_1(x) - V_1(x')$ holds for any pair $x > x'$.

By (A.1) there are gains from contracting with both types. Moreover, the possible surplus is strictly higher for $i = 2$. Finally, the last assertion represents a standard single-crossing condition.

delay can be supported even if the time between offers is short, which stands in contrast to the case where only the uninformed side proposes (see Fudenberg, Levine, and Tirole, 1985; Gul, Sonnenschein, and Wilson, 1986).

[2] Less related is Bac and Raff (1996) where the timing of issues (i.e., the agenda) becomes a signaling device. [For an analysis of agenda formation under complete information see Inderst (2000).] Our result that agreement is reached in the first period should also not be confused with a similar result in Wang (1998) and Inderst (1998) where only the uninformed party can make menu offers, but cannot commit to a final proposal.

We consider a bargaining game with alternating offers set in discrete time. In even periods, starting with $n = 0$, the principal can propose a menu of deterministic contracts, from which, after acceptance, the agent is free to choose. After rejection, the game moves on and the agent is chosen in an odd period to make a proposal. The agent can also propose a menu, from which, after acceptance, he is again free to choose his preferred contract. By allowing the agent to propose a menu, we follow Maskin and Tirole (1990, 1992). Denote the duration of a single period by $\Delta > 0$. The agent (principal) discounts future payoffs by the factor $\delta_A = e^{-r_A \Delta}$ ($\delta_P = e^{-r_P \Delta}$) with $r_A > 0$ ($r_P > 0$).

We consider perfect Bayesian equilibria, requiring that strategies are sequentially optimal and beliefs are consistently updated.[3] In particular, we will not impose any further restrictions or refinements on beliefs to obtain our uniqueness result.

In the introduction we discussed one application of our model, in which a union bargains with a firm over wages and working hours. In the case of union-firm bargaining $V_i(\cdot)$ represent the firm's profits, while t equals the wages paid to workers. The sorting variable x may represent a measure of output, working hours, or additional (overtime) shifts. Alternatively, we may suppose that a firm bargains with a single worker and that this time it is the worker who possesses private information. The worker is privately informed about his ability, which affects his (dis-)utility from working. In this case the sorting variable x may be a measure of output, while $-t$ denotes the wage. As a final application, suppose the agent is a franchisee who is privately informed about the costs from running the local outlet. In this case x may represent the specified delivery of inputs by the principal, while t covers both the franchise fee and the price paid for the inputs.

All these applications share one common feature, i.e., that the principal and the agent interact bilaterally and have both the authority to change the contractual terms. This would be different in some alternative applications of contract theory where, for instance, an insurance broker meeting a potential customer may have only limited or even no power to change the company's contracts. In such environments it is typically competition between principals and not bargaining with agents that (mainly) shapes the features of equilibrium contracts.

3 Analysis

3.1 Three benchmarks

If the agent's type was common knowledge, we know from Rubinstein (1982) that there is a unique (subgame perfect) equilibrium, in which the game ends in the first period with the implementation of a contract $c_i^{R,P}$. This contract specifies the respective first-best choice $x_i^{R,P} = x_i^*$, while the transfer $t_i^{R,P}$ is chosen to split the surplus such that the principal realizes the fraction $(1 - \delta_A)/(1 - \delta_A \delta_P)$, i.e., we obtain $t_i^{R,P} = S_i^*(1 - \delta_A)/(1 - \delta_A \delta_P) - T(x_i^*)$. Moreover, if players have deviated and it is the agent's turn to make a proposal, the (continuation) game has

[3] For a formal description of strategies and equilibrium conditions in a related model with alternating offers of transfers see Ausubel and Deneckere (1992).

again a unique equilibrium leading to an immediate agreement. The implemented contract is denoted by $c_i^{R,A}$. It specifies again the first-best choice $x_i^{R,A} = x_i^*$, while the agent receives now the fraction $(1-\delta_P)/(1-\delta_A\delta_P)$ of the total surplus, i.e., we obtain $t_i^{R,A} = S_i^*\delta_P(1-\delta_A)/(1-\delta_A\delta_P) - T(x_i^*)$.

Return now to the case with private information and consider first a game of screening where the principal can commit to a take-it-or-leave-it offer in $n = 0$. Under (A.1) it is a standard result that he will offer only the high type an efficient contract. In contrast, the low type will either not receive an acceptable offer, which is the case if μ^0 is sufficiently high, or his contract will be distorted as the respective value of x is lower than the first-best choice x_1^*.

As a final benchmark, suppose that bargaining proceeds by alternating offers under private information, while now the choice of x is (physically) fixed. For instance, take $x = 0$ and assume $T(0) = 0$. Trade with the low type is now strictly profitable if $V_1(0) > 0$. Recall also that (A.1) implies $V_2(0) > V_1(0)$. For $\Delta \to 0$ we can support for the low type's expected equilibrium transfer, which we denote by t_1, the following outcomes (see Grossman and Perry, 1986):

$$\frac{r_P}{r_P + r_A} V_1(0) \le t_1 \le max \left\{ V_1(0), \frac{r_P}{r_P + r_A} V_2(0) \right\}.$$

Moreover, some equilibria involve considerable delay.

3.2 Result

Return now to the model of Section 2 with alternating offers of menus. In what follows, we make the following parameter restriction.

(A.2) $r_P(S_2^* - S_1^*)/(r_A + r_P) > V_2(x_1^*) - V_1(x_1^*)$.

This assumption has the following important implication. Given (A.2) it holds for all sufficiently low values of Δ that the pair of contracts $(c_1^{R,P}, c_2^{R,P})$ is incentive compatible. Precisely, if the time between periods becomes sufficiently short, (A.2) implies for $i = 2$ that[4]

$$U_2(c_2^{R,P}) \ge U_2(c_1^{R,P}). \quad (1)$$

Observe also that for $i = 1$ the respective inequality $U_1(c_1^{R,P}) \ge U_1(c_2^{R,P})$ is always satisfied, i.e., irrespective of whether (A.2) holds or not. Moreover, it is easily checked that (1) implies incentive compatibility also for the menu $(c_1^{R,A}, c_2^{R,A})$, i.e., for the family of offers proposed by the agent in the alternating-offer game under complete information.

Assumption (A.2) ensures that we can derive clear-cut results in what follows. The requirement clearly holds for given payoff functions if r_P/r_A is sufficiently large. Moreover, it is more easily satisfied if the difference in types becomes sufficiently large. To formalize this, denote for a moment an agent's type by the real variable θ_i and the respective payoff and surplus function by $V(x, \theta)$ and $S(x, \theta)$,

[4] To see that this holds, substitute $\delta_A = e^{-r_A\Delta}$ and $\delta_P = e^{-r_P\Delta}$ and observe that $(1-\delta_A)/(1-\delta_A\delta_P)$ converges to $r_A/(r_A + r_B)$.

respectively. Given differentiability, (A.1) implies then $V_\theta > 0$ and $V_{\theta x} > 0$. To see that (A.2) is more easily satisfied if the difference between types $\theta_2 - \theta_1$ is large, note that $[V(x_1^*, \theta_2) - V(x_1^*, \theta_1)]/[S(x_2^*, \theta_2) - S(x_1^*, \theta_1)]$ strictly decreases as we decrease θ_1.[5] Finally, our main result is not restricted to the case of two types. Building on our argument in the proof we conjecture that it can be extended to any finite distribution as long as a condition equivalent to (A.2) holds. Precisely, it must hold for any adjacent pair of types $j > i$ with $i, j \in I$ that $r_P(S_j^* - S_i^*)/(r_A + r_P) > V_j(x_i^*) - V_i(x_i^*)$.[6]

Proposition. *Under (A.1)-(A.2) the following result holds if the period length Δ is sufficiently small. The game with alternating offers has a unique equilibrium outcome where the agent of type i accepts the contract $c_i^{R,P}$ in the first period.*

The Proposition is proved in Section 3.3. If (A.2) applies and if the time between offers is sufficiently short, implying that (1) is satisfied, the outcome of the game with alternating offer does not depend on whether the agent's type is his private information or not. In particular, all implemented contracts are first-best. This stands in sharp contrast to the benchmark of the screening model discussed briefly in Section 3.1. We like to interpret this difference to the screening game in the following way. By allowing alternating offers, we essentially create type-dependent reservation values. Indeed, one of the first steps in the proof is to show that (in any equilibrium) the payoff of type i is bounded from below by $U_i(c_i^{R,P})$. From the principal's perspective in $n = 0$, the high type has then a higher "reservation value". For the parameters considered in the Proposition this difference is now sufficiently large to make the menu $(c_1^{R,P}, c_2^{R,P})$ incentive compatible. This, however, eliminates any incentives for the principal to offer distorted contracts. In fact, in contrast to the one-shot screening game, the principal no longer faces a trade-off between maximizing surplus and reducing the information rent left to $i = 2$.

The fact that each type $i \in I$ must realize at least the respective payoff $U_i\,(c_i^{R,P})$ also eliminates the multiplicity of equilibrium outcomes observed in the bargaining game over transfers (see also Sect. 3.1). Under (A.2) alternating offers over menus pin down equilibrium payoffs and rule out delay.

3.3 Proof

We restrict ourselves to sufficiently low values of Δ such that (1) holds by (A.2). The proof proceeds now as follows. We first derive in a series of lemmas boundaries for the payoffs of both players. Using the lower boundaries for the agent's payoff, we solve next an auxiliary problem in which the principal can make a take-it-or-leave-it offer, which must, however, respect the derived lower boundaries for the agent's payoff. Given these type-dependent boundaries we find a unique solution to

[5] Using the envelope theorem, the sign of the derivative is given by $[V(x_1^*, \theta_2) - V(x_1^*, \theta_1)]V_\theta(x_1^*, \theta_1) - [S(x_2^*, \theta_2) - S(x_1^*, \theta_1)]V_\theta(x_1^*, \theta_1)$, which from $V_\theta > 0$ and from $S(x_2^*, \theta_2) - S(x_1^*, \theta_1) > V(x_1^*, \theta_2) - V(x_1^*, \theta_1)$ is strictly negative.

[6] Admittedly, the condition becomes more and more difficult to satisfy if the density of the type space increases.

the principal's auxiliary problem, in which the menu $(c_1^{R,P}, c_2^{R,P})$ is implemented in $n = 0$. As the principal's payoff in the bargaining game is surely not higher than his payoff under the auxiliary problem, we can finally prove the Proposition.

We start by deriving boundaries for the principal's payoff. The following result follows by standard arguments (see Grossman and Perry, 1986).

Lemma 1. *The principal accepts any offer which, given his beliefs, realizes not less than* $W(c_2^{R,A})$.

Proof. Let $\bar{W}$ denote the highest payoff (i.e., the supremum of all such utilities) which the principal can realize in any equilibrium of some continuation game starting in an even period. By the agent's individual rationality constraint, the supremum exists. If the assertion does not hold, i.e., if it is optimal for the principal to reject an offer yielding at least $W(c_2^{R,A})$, $\bar{W}$ must satisfy $\bar{W} = W(c_2^{R,P}) + b$, where $b > 0$ holds. (Observe that $W(c_2^{R,A}) = \delta_P W(c_2^{R,P})$.) For convenience we assume that the supremum $\bar{W}$ is attained. This implies that at least one type i realizes in the respective continuation game a payoff not exceeding $S_i^* - \bar{W}$.[7] Observe also that $S_i^* - \bar{W} \geq 0$ follows from individual rationality. We show that type i can profitably deviate by rejecting and offering in the next period a single contract $c = (x, t)$ such that

$$x = x_i^*,$$
$$t = \delta_P \bar{W} + b\frac{1 - \delta_A \delta_P}{2\delta_A} - T(x_i^*).$$

By $W(c) > \delta_P \bar{W}$ and the construction of $\bar{W}$ the principal will not reject. It remains to show that this strategy is strictly profitable to type i. To see this, note that the difference $\delta_A U_i(c) - \left[S_i^* - \bar{W}\right]$ transforms to

$$\delta_A \left[S_i^* - W(c_2^{R,A}) - \delta_P b - b\frac{1 - \delta_A \delta_P}{2\delta_A}\right] - \left[S_i^* - W(c_2^{R,P}) - b\right]$$
$$= \delta_A \left[S_i^* - W(c_2^{R,A})\right] - \left[S_i^* - W(c_2^{R,P})\right] + b\frac{1 - \delta_A \delta_P}{2},$$

which is strictly positive regardless of the choice of $i \in I$.[8] □

We derive next a lower boundary for the principal's payoff. For this we need first the following intuitive auxiliary result.

Lemma 2. *Consider any odd period* n. *If the principal's beliefs specify* $\mu = Pr(i = 2) = 1$, *it holds in any equilibrium of the continuation game that the payoff of the high type is bounded from above by* $U_2(c_2^{R,A})$. *If beliefs specify* $\mu = 0$, *the low type's payoff is bounded from above by* $U_1(c_1^{R,A})$.

[7] To see this, recall that S_i^* is the maximum surplus that can be realized with type i under a single deterministic contract. By concavity of $S_i(x)$, the gains from trade cannot increase under randomization, i.e., if players mix along the equilibrium path.

[8] To see this, note that $\delta_A \left[S_i^* - W(c_2^{R,A})\right] - \left[S_i^* - W(c_2^{R,P})\right]$ is equal to zero for $i = 2$, while for $i = 1$ it is equal to $(S_2^* - S_1^*)(1 - \delta_A) > 0$.

Proof. Take the case where $\mu = 1$. Denote by $\bar{U}_2 = U_2(c_2^{R,A}) + b$ the supremum for the payoff which the high type can realize in any equilibrium of a continuation game starting in an odd period. Suppose that $b > 0$. The principal can now successfully deviate by rejecting and offering $c = (x_2^*, t)$, where t satisfies $V_2(x_2^*) - t = \delta_A \bar{U}_2 + b(1 - \delta_A \delta_B)/(2\delta_P)$. By construction of $\bar{U}_2$, the offer is accepted by the high type, while it holds that $\delta_P W(c) > S_2^* - \bar{U}_2$. The same argument applies to $\mu = 0$. □

Lemma 3. *Take any odd period. Then in any equilibrium of the continuation game the payoff of the principal is bounded from below by $W(c_1^{R,A})$.*

Proof. Denote the infimum for the principal's payoff starting from an odd period by $\underline{W}$ and suppose that $\underline{W} = W(c_1^{R,A}) - b$, where $b > 0$. We show that the principal can profitably deviate by rejecting the agent's offer and proposing a menu (c_1, c_2). We next specify the respective contracts c_i. Choose $x_i = x_i^*$, while the transfer t_1 is determined by

$$t_1 = V_1(x_1^*) - \delta_A \left[S_1^* - W(c_1^{R,A}) + b \right] - b \frac{1 - \delta_A \delta_P}{2\delta_P}.$$

Note that this implies $U_1(c_1) > \delta_A (S_1^* - \underline{W})$. Moreover, the difference $\delta_P \ (T(x_1^*) + t_1) - (W(c_1^{R,A}) - b)$ transforms to

$$\delta_P \left[(S_1^* - W(c_1^{R,P})) - \delta_A (S_1^* - W(c_1^{R,A})) \right] + b \frac{1 - \delta_A \delta_P}{2},$$

which by construction of $c_1^{R,P}$ and $c_1^{R,A}$ is equal to $b(1 - \delta_A \delta_P)/2 > 0$. It remains to specify the transfer t_2. We choose t_2 to satisfy

$$\delta_P (T(x_2^*) + t_2) - (W(c_1^{R,A}) - b) = b \frac{1 - \delta_A \delta_P}{2}.$$

Note next that $V_2(x_2^*) - t_2 > \delta_A \left[S_2^* - \underline{W} \right]$. By substitution this holds if

$$\delta_P \left[(S_2^* - W(c_1^{R,P})) - \delta_A (S_2^* - W(c_1^{R,A})) \right] + b \frac{1 - \delta_A \delta_P}{2} > 0,$$

which is satisfied as the first term transforms to $\delta_P (S_2^* - S_1^*)(1 - \delta_A) > 0$. Observe further that the menu is (strictly) incentive compatible. To prove this for $i = 2$, we can use $T(x_2^*) + t_2 = T(x_1^*) + t_1$ to transform the high type's incentive compatibility constraint $V_2(x_2^*) - t_2 > V_2(x_1^*) - t_1$ to $S_2^* > S_2(x_1^*)$. In analogy, the low type's incentive compatibility constraint $V_1(x_1^*) - t_1 > V_1(x_2^*) - t_2$ transforms to $S_1^* > S_1(x_2^*)$.

Offering (c_1, c_2) is by construction strictly profitable for the principal if it is accepted by both types of the agent. By Lemma 2 there is no equilibrium of the continuation game where only one type rejects with positive probability. It remains to discuss the case where both types reject with positive probability. We show that this cannot occur. By construction of $\underline{W}$ there is at least one type i whose continuation payoff in $n + 2$ is bounded from above by $S_i^* - \underline{W}$. Hence, for this type it holds that $V_i(x_i^*) - t_i > \delta_A (S_i^* - \underline{W})$, which implies that he must accept by optimality. The constructed deviation is thus profitable for the principal, which concludes the proof. □

We now proceed to derive boundaries for the payoff of the agent. Using Lemma 1 and the possibility to offer menus, we can derive the following lower boundary for both types.

Lemma 4. *Take any odd period. Then for any equilibrium of the continuation game the payoff of an agent of type i is bounded from below by $U_i(c_i^{R,A})$.*

Proof. For $i = 2$ the assertion follows immediately from Lemma 1. Consider next $i = 1$. In analogy to Lemma 1, let $\bar{W}_1$ denote the highest payoff which the principal can realize with $i = 1$ in any equilibrium of a continuation game starting from an even period. We show that $\bar{W}_1 = W(c_1^{R,P})$. This will allow us to extend the assertion to $i = 1$. Suppose this does not hold and denote $\bar{W}_1 = W(c_1^{R,P}) + b$, where $b > 0$. We assume again that the supremum is attained at some period n. By construction the payoff for $i = 1$ in the continuation game is bounded from above by $S_1^* - \bar{W}_1 \geq 0$. We show that type $i = 1$ gains from rejecting and proposing in $n+1$ the contract c_1, where $x_1 = x_1^*$ and t_1 is defined by

$$t_1 = \delta_P \bar{W}_1 + b\frac{1 - \delta_A \delta_P}{2\delta_A} - T(x_1^*).$$

Note that $\delta_A U_1(c_1) > S_1^* - \bar{W}_1$. We next distinguish between two cases, a) and b), to show that the principal must accept a menu containing c_1. Case a) is characterized by $W(c_1) > W(c_2^{R,A})$, which by Lemma 1 implies acceptance of the single offer c_1.

In case b) it holds that $W(c_1) \leq W(c_2^{R,A})$. The agent offers now an additional contract c_2, where $x_2 = x_2^*$ and $t_2 = t_2^{R,A} + b(1 - \delta_A \delta_P)/(4\delta_A)$. Note first that the menu (c_1, c_2) satisfies downwards incentive compatibility as $U_2(c_2^{R,A}) - b(1 - \delta_A \delta_P)/(4\delta_A)$ exceeds $V_2(x_1^*) - V_1(x_1^*) + U_1(c_1)$. Moreover, it is (strictly) upwards incentive compatible if

$$U_1(c_1) = S_1^* - W(c_1) > V_1(x_2^*) - V_2(x_2^*) + S_2^* - W(c_2^{R,A}) - b\frac{1 - \delta_A \delta_P}{4\delta_A},$$

which holds if $W(c_1) < W(c_2^{R,A}) + S_1^* - S_1(x_2^*)$. This is satisfied in case b) by assumption. Hence, in case b) the principal realizes more than $\delta_P W(c_2^{R,P})$ if he faces a high type and more than $\delta_P \bar{W}_1$ if he faces a low type. Regardless of his beliefs acceptance is thus optimal as he could never realize more with $i = 2$ by Lemma 1 or more with $i = 1$ by construction of $\bar{W}_1$. We have thus proved that $\bar{W}_1 \leq W(c_1^{R,P})$.

Consider now a menu (c_1, c_2) specifying $x_i = x_i^*$ and $t_i = t_i^{R,A} + \varepsilon$, where $\varepsilon > 0$. This offer is by assumption incentive compatible. Moreover, by our previous results it is surely accepted by the principal. As ε can be chosen arbitrarily small, the assertion in Lemma 4 extends to $i = 1$. □

Finally, we derive an upper boundary for the agent's payoff. Note that the lower boundary for the principal's payoff in Lemma 3 still leaves much room for the payoff of a particular type $i \in I$. We are now mainly concerned with the payoff of the low type, for whom we derive an upper boundary in two steps.

Lemma 5. *Consider any odd period n. If the low type's payoff in the continuation game exceeds $U_1(c_1^{R,A})$, the payoff of the high type cannot exceed $U_2(c_2^{R,A})$.*

Proof. We argue by contradiction and denote the supremum for the high type's payoff after any such history by $\bar{U}_2 = U_2(c_2^{R,A}) + b$, where $b > 0$. We start by deriving an upper boundary for $\bar{U}_2$. By Lemma 3 it holds that $\bar{U}_2 \leq S_2^* - W(c_1^{R,A})$, as otherwise the principal's payoff in the continuation game would surely be less than $W(c_1^{R,A})$. (Recall that the low type's payoff exceeds $U_1(c_1^{R,A})$ by assumption.) Moreover, the principal's payoff (in the considered continuation game) is bounded from above by $\mu(S_2^* - \bar{U}_2) + (1-\mu)W(c_1^{R,A})$ for given beliefs μ.

We show now again that the principal can improve by rejecting in the considered odd period n and offering in $n+1$ a menu (c_1, c_2), where $x_i = x_i^*$, $t_1 = t_1^{R,P} + \varepsilon$, and t_2 satisfies

$$V_2(x_2^*) - t_2 = \delta_A \bar{U}_2 + \eta b \frac{1-\delta_A\delta_P}{2\delta_P}.$$

The values $\varepsilon > 0$ and $\eta > 0$ are chosen arbitrarily small. Note that the menu is (strictly) incentive compatible for sufficiently small ε and η as $U_2(c_2^{R,A}) < \bar{U}_2 \leq S_2^* - W(c_1^{R,A})$. Moreover, in case of acceptance the principal is strictly better off. This follows from $\delta_P W(c_2) > S_2^* - \bar{U}_2$ and as ε can be chosen arbitrarily small. It remains to show that the menu (c_1, c_2) is indeed accepted by both types. Note first that under this menu each type $i \in I$ realizes strictly more than $\delta_A U_i(c_i^{R,A})$ respectively. Consider now acceptance by $i = 1$. Obviously, he only rejects with positive probability if, after rejection, the payoff in the continuation game exceeds $\delta_A U_1(c_1^{R,A})$. By construction of $\bar{U}_2$ and c_2 the high type must then accept for sure. By Lemma 2 this implies, however, that the low type must also accept for sure as otherwise his payoff is bounded from above by $\delta_A U_1(c_1^{R,A})$. Having proved acceptance by $i = 1$, this ensures by Lemma 2 that also the high type accepts. □

Lemma 6. *Consider any odd period n. Then for any equilibrium of the continuation game the low type's payoff is bounded from above by $U_1(c_1^{R,A})$.*

Proof. By Lemma 2 we only have to consider the case where the principal's beliefs specify $\mu > 0$. We argue again by contradiction. Denote the supremum of the low type's payoff after any such history by $\bar{U}_1 = U_1(c_1^{R,A}) + b$ and suppose that $b > 0$. By Lemma 4 the high type's payoff in any equilibrium of the continuation game is not below $U_2(c_2^{R,A})$. As a consequence, the principal's payoff, which we denote by W, satisfies $W \leq (1-\mu)W(c_1^{R,A}) + \mu W(c_2^{R,A}) - b'$ for some $b' > 0$.

We show again that the principal is better off by rejecting the agent's offer and proposing a menu (c_1, c_2), where we specify $x_i = x_i^*$ and $t_i = t_i^{R,P} - b'$ for $i \in I$. If this menu is accepted the principal clearly realizes more than W/δ_P. It thus remains to prove that the offer is indeed accepted by both types. Suppose the low type rejects with positive probability. This is only the case if, after rejection, his payoff in the continuation game exceeds $\delta_A U_1(c_1^{R,A})$. But in this case we know from Lemma 5 that the high type will not realize more than $U_2(c_2^{R,A})$ and must therefore accept the offer. By Lemma 2 this implies that also the low type must accept for sure. Finally, since we have shown that the low type accepts, the high type must accept by Lemma 2. □

We can now use Lemma 6 to determine a lower boundary for the principal's payoff in the bargaining game.

Lemma 7. *The principal's payoff in the bargaining game is bounded from below by* $\sum_{i \in I} \mu_i^0 W(c_i^{R,P})$.

Proof. The assertion follows immediately from the observation that for any $\varepsilon > 0$ the principal can end the game in $n = 0$ by offering a menu (c_1, c_2) satisfying $x_i = x_i^*$ and $t_i = t_i^{R,P} - \varepsilon$. To see that this holds, note that the low type must accept by Lemma 6. If the low type accepts with probability one, the high type must also accept by Lemma 2. As $\varepsilon > 0$ can be chosen arbitrarily small, $\mu^0 W(c_2^{R,P}) + (1 - \mu^0) W(c_1^{R,P})$ indeed constitutes a lower boundary for the principal's payoff. □

Recall now from Lemma 4 that in any odd period the payoff of an agent of type i is bounded from below by $U_i(c_i^{R,A})$. As the agent can always simply reject an offer, his payoff in an even period must therefore be bounded from below by $U_i(c_i^{R,P})$. This holds in particular for $n = 0$ where the game starts. Hence, combining Lemma 4 with Lemma 7, the payoffs obtained by implementing the contracts $(c_1^{R,P}, c_2^{R,P})$ represent lower boundaries for the equilibrium payoffs of the principal and both types of the agent. As these contracts implement the first-best value of x for both types and as delay is costly, it follows that implementing $(c_1^{R,P}, c_2^{R,P})$ in $n = 0$ must be the unique equilibrium outcome. (Otherwise, total expected surplus must decrease and there is no party who will bear the costs.)

It remains to show existence of an equilibrium supporting the characterized outcome. We specify the following strategies. In any even period, irrespective of the previous history, the principal offers the menu $(c_1^{R,P}, c_2^{R,P})$, while type i accepts any offer which leaves him at least the payoff $U_i(c_i^{R,P})$. In any odd period, irrespective of the previous history, the agent offers the menu $(c_1^{R,A}, c_2^{R,A})$. The principal's response in an odd period depends only on his beliefs μ. He will accept any menu (c_1, c_2) under which, given the agent's subsequent optimal choice from the menu, he will not realize less than

$$W^E(\mu) := \mu W(c_2^{R,A}) + (1 - \mu) W(c_1^{R,A}).$$

To fully characterize an equilibrium we must next specify the principal's out-of-equilibrium beliefs. We specify that, following a rejection of his own offer, the principal does not change his beliefs. Suppose now the agent deviates and offers a menu different from $(c_1^{R,A}, c_2^{R,A})$. If the deviating menu contains a contract c satisfying $U_1(c) > U_1(c_1^{R,A})$, we specify $\mu = 0$. If this is not the case, we specify $\mu = 1$. This completes the specification of strategies and (out-of-equilibrium) beliefs.

The specified set of strategies clearly supports the outcome in the Proposition. It remains to prove that these strategies are optimal. Given the agent's response the principal's strategy is indeed optimal. Moreover, the agent's response to an offer is optimal given the strategies specified for the continuation game following a rejection. It therefore remains to consider the agent's proposal. We use our specification of out-of-equilibrium beliefs to show that it is unprofitable for an agent to offer a different menu than $(c_1^{R,A}, c_2^{R,A})$. By the stationarity of the specified strategies the deviation can only be profitable if it leads to immediate acceptance. For the low type we consider thus the case where the menu contains a contract c satisfying $U_1(c) > U_1(c_1^{R,A})$. As this implies $W_1(c) < W(c_1^{R,A})$ due to $x_1^{R,A} = x_1^*$ and as we have specified for this case that the principal's beliefs are given by $\mu = 0$, the principal's expected payoff is below $W^E(\mu)$. He will therefore reject, which completes the argument for the low type. Regarding a deviation of the high type, note that we can already exclude menus that contain a contract satisfying $U_1(c) > U_1(c_1^{R,A})$ as we have shown that the principal will reject. As we have also specified that the principal's beliefs satisfy $\mu = 1$ for all other deviations, we can again argue that the principal must reject if the menu contains additionally a contract c satisfying $U_2(c) > U_2(c_2^{R,A})$. This rules out profitable deviations for the high type.

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