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Capacity constrained firms in (labor) markets with adverse selection

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Summary. We discuss a competitive (labor) market where firms face capacity constraints and individuals differ according to their productivity. Firms offer two-dimensional contracts like wage and task level. Then workers choose firms and contracts. Workers might be rationed if the number of applicants exceeds the capacity of the firm.

We show that under reasonable assumptions on the distribution of capacity an equilibrium in pure strategies (by the firms) exists. This result stands in contrast to the case of unlimited capacity. The utility level is uniquely determined in equilibrium. No rationing occurs in equilibrium, but it does off the equilibrium path.

Keywords and Phrases: Adverse selection, Capacity constraints, Labor markets, Competitive equilibrium.

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1 Introduction

It has long been noted that firms which face capacity constraints compete differently than those with unlimited capacity (Edgeworth, 1897). Capacity constraints change the structure in the market in two fundamental respects. First, by undercutting a rival, a firm will not necessarily serve a larger fraction of the market, as it might have filled up its capacity already. Second, by increasing the price,

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a firm will not loose all customers, as all the other firms might not have enough capacity to serve these customers. Most of the literature in this area however concentrates on competition under symmetric information (Allen and Hellwig, 1986, 1989; Kreps and Scheinkmann, 1983).

With asymmetric information, it is usually assumed that firms are able to serve the whole market (see e.g., Mas-Colell et al., 1995, Chapter 13).[1] In this paper we analyze competition under asymmetric information where firms face capacity constraints, i.e., job opportunities or vacancies are (in the short run) limited. If a firm offers a very attractive contract, it may end up with a queue of applicants exceeding its demand. We depart from previous analysis in several respects: First, the strategic interaction in this market is modelled explicitly. The equilibrium concept we use is the standard subgame perfect equilibrium. Second, the demand game, where customers choose firms and contracts is also modelled explicitly. Here we differ from the standard analysis in the Bertrand model (see e.g., Allen and Hellwig, 1986, 1989), where a residual demand function is exogenously imposed, rather than endogenously derived. Finally, with adverse selection capacity constraints have an additional effect on the strategic interaction in a competitive market: If a firm undercuts its rivals, it will not necessarily attract the mix of types which it would if capacity were unlimited. Workers anticipate that there might be rationing which might affect differently the expected utility of different types.

The model is such that firms offer a menu of contracts at Stage 1, while workers, who are either of high or low ability, choose a firm and a contract at Stage 2. To abstract from issues of coordination failures at the stage where workers apply to the various firms, we consider a market with a continuum of workers. If the queue of applicants at any single firm exceeds the capacity of this firm, positions get filled randomly and workers face a loss from being turned down. Precisely, we specify that workers who have failed to receive a job in the first round stay unemployed. This set of assumptions will allow us to endogenize the distribution of types attracted by a deviating offer.

With this model we derive the following results:

(i) For any set of contracts offered by the firms, an equilibrium in the subsequent demand game exists. All equilibria lead to a unique utility level for the two types.

(ii) If market capacity exceeds market demand, the only possible equilibrium allocation where firms use pure strategies is where all customers obtain their least cost separating contracts almost surely. Hence, firms make zero profits with each type, the contract for the low-ability type is efficient, and the low-ability type is just indifferent between his contract and the contract for the high-ability type. There is no rationing in equilibrium.

[1] An exception is Gale (1992, 1996) where there is a continuum of uninformed agents with a single capacity (or trading opportunity). Instead of solving for a market game, Gale takes a Walrasian approach where each contract essentially constitutes a single market which has to be cleared in equilibrium. He employs refinements to restrict the set of equilibrium allocations.

(iii) If capacity is sufficiently dispersed among firms, an equilibrium in pure strategies exists.

Result (i) is new, as to our knowledge demand games with different types have not been solved so far for capacity constrained firms. Result (ii) is the same as in standard models with unlimited capacity. However, the proof of this result differs from usual analysis, as the queue of applicants to out-of-equilibrium offers has to be defined with care. Result (iii) stands in contract to standard analysis (following Rothschild-Stiglitz (RS), 1976): While there an equilibrium in pure strategies ceases to exist if the distribution of types is sufficiently skewed to the high-ability type, we obtain that for any distribution of types an equilibrium exists if firms' capacity is sufficiently low. Crucial for this result is that even if in equilibrium no rationing occurs, out of equilibrium a firm might attract more applicants than it is able to serve. The distribution of types queueing at such a firm will be determined endogenously, and, as we will show, will make any deviating offer unattractive.

Related strands of literature

There are alternative proposals to overcome the equilibrium existence problem (in pure strategies)[2,3].

The first attempt was the introduction of different equilibrium concepts, which however lack a game-theoretic foundation (Wilson, 1977; Miyazaki, 1977; Spence, 1978; Riley, 1979). Subsequently, structural extensions like withdrawal of contracts were added to the basic model. In contrast, we stay with the most simple model proposed by RS. As a consequence, our result is identical to theirs if a pure strategy equilibrium exists; however, limited capacities bring additional existence in other cases. Thus the basic insight gained by RS remains: Firms offer least-cost separating contracts, which yield zero-profit and prevent any form of further cream-skimming.

If firms can withdraw contracts after they observe the choices made by workers (Hellwig, 1987), under appropriate belief refinements a pooling contract will be the outcome. In this case cream-skimming, the attraction of the desired type, is avoided, because if firms observe such a contract, they will refuse to accept applicants for their initial contracts. In contrast, in our setup firms have no further choice of action once the contract menu is offered.

[2] Dasgupta and Maskin (1986) have shown that an equilibrium exists in mixed strategies.

[3] We will exclusively focus on the literature modelling contract design strategically. (See, for instance, Gale (1996) on the distinction between a non-cooperative (or strategic) and a cooperative (or axiomatic) approach.) In particular, this excludes the following two strands. First, we do not discuss Walrasian (or competitive) approaches following the pioneering work of Prescott and Townsend (1984a, b). [Recall, however, our brief discussion of Gale (1992, 1996) above.] We refer the reader to Bisin and Gottardi (1999) for a recent overview. Second, we ignore extensions of cooperative solution concepts to private information environments (see, for instance, Kahn and Mookherjee, 1995, for some references).

Another strand of literature deals with contractual games of signalling, where the informed party makes a contract offer, which the uninformed party can accept or reject.[4] One of the major problems of this literature is the multiplicity of equilibria, which has lead to an extensive discussion of refinements. One refinement concept is the intuitive criterion, where beliefs are made about who is most likely to deviate (Cho and Kreps, 1987).With the intuitive criterion some equilibria are not stable as a deviation is to be expected to come from the high ability type. In contrast, in our work an equilibrium is established, because any deviation is more likely to attract the low-ability type.[5]

As mentioned above, rationing (on- and off-equilibrium) is a well-known phenomenon in the industrial organization literature. As in our paper, rationing introduces a new dimension: the trade-off between price and trading probability (for an overview see Peters, 1998). Typically, on-equilibrium rationing arises as either aggregate capacity is too low or as players suffer from a coordination failure. For coordination failure to arise, it is, however, essential that the number of buyers and sellers is not too different. For instance, there is a continuum of both buyers and sellers in the one-stage trading environment of Peters (1997a) or in the search market model of Moen (1997). If the population is heterogeneous as e.g., buyers differ in their valuations, different types may face different prices and trading probabilities in equilibrium.[6] In contrast, in our model (with asymmetric information) there will be no rationing in equilibrium as a finite number of firms face a continuum of workers, while additionally aggregate capacity is more than sufficient to serve the market.

2 The model

2.1 The economy

We consider a market for labor. The demand side consists out of a finite number F of identical firms indexed by $f \in F = \{1, ..., F\}$. Each firm can employ at most the measure $l > 0$ of workers.[7] To characterize the supply side, let $I = [0, 1]$ be the set of workers. There are two possible types of workers, who have either low or high ability. We denote the low-ability type by L and the high-ability type by H, where $T = \{L, H\}$. The ability of a worker is his private information.

[4] Observe that, in contrast to the Spence (1973) model, both the signaling (or sorting) variable and the transfer are contractually specified. For a general analysis see, for instance, Kreps and Sobel (1994).

[5] The authors have recently proposed new alternatives to model markets with adverse selection. Ania et al. (1998) employ an evolutionary approach, where firms copy profit making contracts and experiment with their own contracts. In Inderst (1997) contract design is embedded in a matching or search market environment.

[6] To be more precise, in Moen (1997) firms with a higher productivity offer a higher wage to reduce the expected time it takes to fill a vacancy. (Inderst (2000) has extended this approach to bilateral heterogeneity.) In Peters (1997b) sellers with a higher reservation value offer auctions with a higher reserve price and attract bidders with a lower probabiltiy.

[7] Though we assume that firms have symmetric capacities, it is straightforward to extend our analysis to the asymmetric case.

Initially, for each worker it represents a random draw with $Pr(t = H) = \pi$, where $0 < \pi < 1$. In what follows, we will be concerned with a game where nature has already chosen a worker's type. We denote the type of worker $i \in I$ by $\tau(i) \in T$. To ensure that capacities are indeed limited and that there is competition for workers, we invoke the following assumption on capacities.

(A.0) $l < 1,\ l(F - 1) \geq 1$.

In words, any individual firm cannot serve the whole market and is also dispensable to ensure full employment. Firms may employ workers by signing a contract specifying two variables, a wage w and an additional sorting variable y, which may represent a nonnegative level of training or hours worked. Contracts $c = (w, y)$ belong thus to $C = \mathbb{R} \times \mathbb{R}_0^+$. Given a contract c, a worker of type t realizes the utility $u_t(c)$, while a firm realizes $v_t(c)$. Note that the firm's payoff depends on the worker's type (common values).

We apply the following specification which is frequently used in the labor or managerial context, namely that the utility function of the worker is quasi-linear in the two variables:

(A.1) $u_t(c) = \beta(w) + \eta_t(y)$.

Indeed, it is common in application on managerial or labor contracts to assume $u_t(c) = w + \eta_t(y)$. This assumption guarantees that the transfer impact is type independent, and, as we will see, this excludes the use of rationing as an efficient screening instrument. Note that our results will also go through for more general specifications of the utility function as long as the properties derived below are satisfied. In other words: As long as rationing is not an efficient device to separate types, our results will hold.

The following assumptions on the utility functions are standard:

(A.2) $u_t(\cdot)$ *and* $\mu v_H(\cdot) + (1 - \mu)v_L(\cdot)$ *are quasiconcave (one strictly), monotonic (one strictly), continuous and differentiable in* c, *for all* $0 \leq \mu \leq 1$. *Also, when* $u_t(c)$ *and* $v_t(c)$ *are both strictly monotonic in* c, *we assume that* $sign \nabla_c u_t(c) = -sign \nabla_c v_{t'}(c)$ *for* $t, t' = L, H$.
(A.3) $0 > \eta_H' > \eta_L'$.
(A.4) $v_H(c) > v_L(c)$ *for all* $c \in C$.

(A.2) ensures the existence and uniqueness of solutions to the considered programs. (A.3) describes the impact of the sorting variable. An increase in y, which for example means a higher level of training, is worse for the L type than for the H type. (A.4) indicates common values: Firms prefer to employ high-ability types.

In our context it is moreover reasonable to specify that the reservation value without contracting is equivalent to the payoff under a contract specifying $w = 0$ and $y = 0$, i.e., no transfer and, for instance, zero hours of work or no training. For brevity we will frequently denote this by the null contract $\emptyset$. Below we will specify an example in detail.

Define next the program to maximize $u_t(c)$ subject to $v_t(c) \geq 0$. A solution exists and is unique due to (A.2). We denote it by c_t^*. To ensure that there are gains from employing either type, we assume that $v_L(c_L^*) > 0$. Additionally, we make the following assumption.

(A.5) $\quad u_L(c_L^*) < u_L(c_H^*)$.

Hence, by (A.5) the pair of first-best contracts (for workers) is *not* incentive compatible. Consider next the program to maximize $u_H(c)$ subject to $v_H(c) \geq 0$ and the low type's incentive compatibility constraint $u_L(c) \leq u_L(c_L^*)$. By (A.2) the solution is again unique and denoted by c_H^{RS}. Denote additionally $c_L^{RS} = c_L^*$. We call these contracts the Rothschild-Stiglitz (or RS) contracts. The following result is standard under (A.1)-(A.5).

Lemma 0. *Given (A.1)-(A.5), the uniquely determined family of RS contracts* $\left\{c_t^{RS}\right\}_{t\in T}$ *satisfies:* $u(c_L^{RS}) = u_L(c_H^{RS})$, $v_t(c_t^{RS}) = 0$ *for* $t \in T$, $y_H^{RS} > y_L^{RS}$, *and* $y_H^{RS} > y_H^*$.

Intuitively, the sorting condition (A.3) together with (A.5) ensure that the variable y specified in the RS contract for the high type must be distorted upwards.

Recall that we specified $u_t(c) = \beta(w) + \eta_t(y)$, where by (A.3) $0 > \eta_H'(y) > \eta_L'(y)$. This has two major implications. First, by $u_t(\emptyset) = 0$ it implies that $u_H(c) > u_L(c)$ if $y > 0$. In words, our assumptions invoked so far imply that high-type workers receiving a contract are always not worse off in *absolute* terms (compared to their reservation value of *not* working) than low-type workers. In the labor context this seems to be a reasonable assumption.[8] Second, we obtain for all contracts $c = (w, y)$ with $y < y_H^{RS}$ and $u_H(c) \geq u_H(c_H^{RS})$ that $u_H(c)/u_H(c_H^{RS}) < u_L(c)/u_L(c_H^{RS})$.[9] As this relationship will become key when supporting an equilibrium where firms play pure strategies, we briefly comment on it. Take any contract c which specifies a smaller level of the sorting variable y than the RS contract of the high type and which ensures that the high type is not made worse off. When comparing the payoff of each type under this contract with the payoff under his respective RS contract, we find that the low type is relatively better off under the new contract. Given the additive separability of payoffs in (A.1), this is ensured by the sorting condition (A.3).

We conclude the specification of the economy with an example:

Let $u_t(c) = w - y/\lambda_t^W$, where y specifies hours of work or training, while the parameters $\lambda_H^W > \lambda_L^W > 0$ denote the efficiency of the two types. Firms' payoff is given by $v_t(c) = \lambda_t^F - w$, where $\lambda_H^F > \lambda_L^F > 0$ denote the productivity parameters of the two types. In this example y is a completely unproductive signal, as it

[8] This would change if we introduced type-dependent reservation values. This could arise from two plausible reasons. First, wages in a second market could depend on the worker's (productivity) type t. Second, if we allowed for repeated interaction instead of considering a one-shot game, workers who failed to receive a contract previously could still try to obtain a contract from another firm, which itself might depend on the types. We will return to these issues in the conclusion.

[9] Proof: $u_H(c)/u_H(c_H^{RS}) - 1 = [\beta(w) - \beta(w_H^{RS}) + \eta_H(y) - \eta_H(y_H^{RS})]/[\beta(w_H^{RS}) + \eta_H(y_H^{RS})]$. This expression is smaller than $u_L(c)/u_L(c_H^{RS}) - 1 = [\beta(w) - \beta(w_H^{RS}) + \eta_L(y) - \eta_L(y_H^{RS})]/[\beta(w_H^{RS}) + \eta_L(y_H^{RS})]$, because due to Assumption (A.3) the denominator is smaller and the numerator is larger in the latter case, as long as $y < y_H^{RS}$.

does not enter the profit of the firm. If we were to maximize workers payoff subject only to the participation constraint of firms (under complete information), the respective contracts would specify $y = 0$ and $w_t = \lambda_t^F$. For the low type this (first-best) contract $(\lambda_L^F, 0)$ is identical to his RS contract, while the RS contract of the more productive type is given by $(\lambda_H^F, \lambda_L^W(\lambda_H^F - \lambda_L^F))$. Note that if $\pi > 1 - \lambda_L^W/\lambda_H^W$, i.e., if there are sufficiently many of the high-ability type, then the contract $(\lambda_P^F, 0)$, where $\lambda_P^F = \pi\lambda_H^F + (1 - \pi)\lambda_L^F$ is the average productivity, will be preferred by both types. In this case an equilibrium in pure strategies will not exist if capacity is unlimited. As we will see below, if capacity is sufficiently constrained, even in this case an equilibrium in pure strategies exists.

2.2 The game

We consider the following two-stage game.

Stage 1: Firms $f \in F$ offer a menu of contracts.

Stage 2: Workers choose either to take up their outside option (of staying unemployed) or they apply to a single firm. If the measure of applicants at a given firm f exceeds l, the firm randomly picks workers to fill all vacancies. If a worker i applies to f and is accepted, he is free to choose any of the contracts in the firm's menu. Otherwise, workers stay unemployed.

We comment on both stages in turn. A firm's menu may contain an arbitrary number, say K, of possibly different deterministic contracts. To simplify the notation we will, however, introduce the following restriction. We specify that a firm f offers at most two different contracts. One of these contracts, which we denote by c_L^f, is chosen by an applicant of type $t = L$ if this applicant is accepted, while the other contract, which we denote by c_H^f, is picked by type $t = H$. (Note that we do not exclude the case where $c_L^f = c_H^f$.) Thus, a firm's action space is given by $C \times C$.

We turn next to workers' application strategies. Workers share the common action space $\Phi = F \cup \{0\}$, where $\psi = 0$ denotes the choice to stay unemployed and receive the reservation value of zero. A (pure) strategy profile for workers is a measurable function from $I \times T$ to Φ. It is denoted by $a(i, t)$, where $a(i, \tau(i)) \in \Phi$ is the strategy of workers with index $i \in I$ and type $\tau(i) \in T$. We denote the space of all strategy profiles by A. As types are independently drawn, there is no aggregate uncertainty in the economy. Denote the measure of workers of type t choosing strategy ψ under profile $a \in A$ by $m_t^\psi(a)$. We aggregate $m^\psi(a) = m_L^\psi(a) + m_H^\psi(a)$. For $\psi = f \in F$, the measure $m^\psi(a)$ represents the queue of applicants at firm f. The probability of an individual worker to get accepted at f, which is one minus the rationing probability, is then determined by the function

$$\rho(m^\psi(a)) = \min\left\{1, \frac{l}{m^\psi(a)}\right\}.$$

To simplify the analysis, we assume in addition that applying at a firm $f \in F$ comes at the constant costs γ, which are expressed in utility terms. We specify that $\gamma > 0$ becomes arbitrarily small. The introduction of application costs allows to rule out the case where, in equilibrium, workers of some type t are indifferent between various degrees of congestion or rationing at a firm f in case the respective menu specifies $u_t(c_t^f) = 0$. Though our arguments would still hold for $\gamma = 0$, we can rule out some tedious case distinctions by choosing $\gamma > 0$.

Regarding the equilibrium concept, we look for a subgame-perfect equilibrium in which firms choose pure strategies. At $n = 2$, where workers apply for jobs, this implies the following condition.[10] Given a family of offers $\{c^f\}_{f\in F}$ where $c^f = (c_L^f, c_H^f)$, the strategy profile $a \in A$ satisfies the following requirements for almost all $i \in I$:

i) If $a(i,t) = f \in F$ with $t = \tau(i)$, then

$$u_t(c_t^f)\rho(m^f(a)) - \gamma \geq \max\left\{0, \max_{f'\in F} u_t(c_t^{f'})\rho(m^{f'}(a)) - \gamma\right\}$$

ii) If $a(i,t) = 0$ with $t = \tau(i)$, then

$$\max_{f\in F} u_t(c_t^f)\rho(m^f(a)) - \gamma \leq 0.$$

For $n = 1$, where firms offer contracts, we require that firms maximize their aggregate payoffs. Denote for a given choice of $\{c^f\}_{f\in F}$ and a strategy profile $a \in A$ the payoff of firm f by $V^f(\{c^f\}_{f\in F}, a)$. Formally, payoffs are defined by

$$V^f(\{c^f\}_{f\in F}, a) = \min\left\{1, \frac{l}{m^f(a)}\right\}\left[m_H^f(a)\upsilon_H(c_H^f) + m_L^f(a)\upsilon_L(c_L^f)\right].$$

3 Equilibria of the demand game

We turn first to an analysis of the demand game at $n = 2$. Our benchmark case is the standard textbook analysis where firms have infinite capacity (see e.g., Mas-Colell et al., 1995, Chapter 13) or at least enough capacity to serve the whole market each, i.e., $l \geq 1$. In that case the demand game is easy to solve: Every type just chooses the best available contract. This implies that a) an equilibrium of the demand game always exists, and b) that each person of the same type obtains the same utility level.

The same implications a) and b) hold if capacity at each firm is smaller than one. An equilibrium exists for the final stage and, with a continuum of workers, almost all workers i with type $\tau(i) = t$ will realize the same (expected) utility. We call this utility level the equilibrium utility of type t. Given some offers $\{c^f\}$, we will show that this type-dependent utility is uniquely determined in equilibrium.

[10] Note that we suppress the requirement that an accepted worker chooses optimally between the contracts in the menu.

We denote it by $U_t(\{c^f\})$. Moreover, $U_t(\{c^f\})$ changes continuously with the firms' offers.

However, in contrast to the benchmark case, the proof of these results is more elaborate. In particular, as firms are not able to serve the whole market, it is in general not an equilibrium strategy just to choose the firm with the best available contract.

Proposition 1. *Given any family of offers $\{c^f\}$, there exists at Stage 2 an equilibrium profile $a \in A$ for workers. Moreover, any equilibrium profile gives rise to the same type-dependent utilities $U_t(\{c^f\})$, which are continuous in contracts c_t^f.*

The proof is relegated to Appendix 1. We briefly comment on the finding that equilibrium utilities (for the continuation game at Stage 2) are uniquely determined. Suppose equilibrium utilities were not unique. Then there would exist two different equilibrium profiles a and $\overline{a}$ such that some type t would realize more utility under a than under $\overline{a}$. This would imply that for all firms f with $m_t^f(a) > 0$ it holds that $m^f(\overline{a}) > m^f(a)$, i.e., the rationing probability has increased at those firms. As the mass of type t is restricted to be either π or $1-\pi$, this implies that also type $t' \neq t$ must be in the queue of applicants at those firms. From this it follows that t' will also be better off with equilibrium profile a. This in turn implies that rationing at any active firm under $\overline{a}$ will be more severe than under a, which, however, cannot be true as the total number of workers is fixed.

Before deriving the set of equilibria for the full game, we introduce some additional notation. Given $\{c^f\}$, denote the (by Proposition 1 non-empty) set of equilibrium strategy profiles at Stage 2 by $\alpha(\{c^f\}) \subset A$. Given a family of offers $\{c^f\}$, a profile $a \in \alpha(\{c^f\})$ gives rise to an *allocation* of contracts to workers (and types). Recall that we have defined $c = \emptyset$ if a worker stays unemployed. Denote next $C_0 = C \cup \{\emptyset\}$. Formally, we define an allocation as a mapping of T into the set of distribution functions over C_0.[11] In what follows, we are interested in a particular allocation which specifies for almost all workers their respective RS contract with probability one.[12]

4 Equilibria of the game

We now turn to the equilibrium of the overall game. Recall that we look for an equilibrium where firms use pure strategies. We will derive two results: First, we show that the only possible candidate for an equilibrium allocation is the RS allocation, as defined above. In a second step we derive a necessary and sufficient condition for the existence of such an equilibrium. We find existence if aggregate capacity is sufficiently dispersed among firms.

[11] Hence, we abstract from perturbations of individual workers.

[12] Formally, a profile a implements the RS allocation if for almost all $i \in I$ and $t = \tau(i)$ it holds that $a(i,t) = f \in F$, $c^f_{a(i,t)} = c_t^{RS}$, and $m^f(a) \leq l$.

4.1 Characterization of the equilibrium

Take first again the benchmark case of unlimited capacity. In this case the only possible equilibrium allocation is the RS allocation. We obtain the same result under capacity constraints. So before proceeding, it is worth spending some remarks on how the analysis here differs from the benchmark case. Concerning the derivation of the RS allocation as the only possible allocation if firms use pure strategies, the standard reasoning goes that for any other set of contracts, a firm will either undercut its rivals, which comes with a larger market share and therefore larger profits, or it will offer a contract which attracts the high-ability types only. These arguments do not apply here, mainly for two reasons:
First, with limited capacities, the applications at some firm f may not jump by a positive measure if the firm slightly "undercuts" the offer of some other firm f'. This may be due to the fact that f already enjoys a supply of labor exceeding its capacity.
Second, the distribution of types at a deviating offer is endogenously determined and depends on how the equilibrium utilities (after reallocation of workers) adjust. Therefore, it is not clear that a new contract which is designed to attract one particular type will indeed be chosen by this type, as the rationing at this firm might make it more profitable for the other type to queue at this firm.

The next proposition shows that any equilibrium allocation in pure strategies is the RS allocation.

Proposition 2. *A family of offers* $\left\{c^f\right\}$ *can only be supported by an equilibrium where firms choose pure strategies if all resulting strategy profiles for workers* $a \in \alpha(\left\{c^f\right\})$ *implement the RS allocation.*

The proof of Proposition 2 is relegated to Appendix 2. It proceeds in two steps. We first show that firms cannot realize profits with any contract accepted by some positive mass of workers in equilibrium. This result can then be used to show that each type must indeed implement his RS contract without rationing. The argument for the first step is somewhat involved given that firms have limited capacities. As a consequence, a deviating offer has a non-marginal impact on workers' payoffs and their distribution among firms. Hence, when constructing a deviating offer which intends to attract a particular type of workers from another firm (realizing positive profits), we have to ensure that the deviator gets the "right" distribution of types.

Proposition 2 characterizes all equilibria where firms choose pure strategies. Although the proof differs substantially, the final result is essentially the same as in the standard analysis with unlimited capacities. The major difference comes now with regards to existence.

4.2 Existence of equilibrium

Suppose all firms offer the RS menu of contracts. With *unlimited capacities* or with the exogenous specification that any new offer attracts a fair distribution of types, a firm f can profitably deviate if there exists a pair c_L, c_H satisfying the following conditions:[13]

$$\begin{aligned} u_t(c_t) &\geq u_t(c_t^{RS}) \text{ for t} \in \text{T}, \\ u_L(c_L) &\geq u_L(c_H), \\ \pi v_H(c_H) + (1-\pi)v_L(c_L) &> 0. \end{aligned}$$

Given the characterization of RS contracts in Lemma 0, the upwards distortion in y_H^{RS} implied by (A.5) ensures that such a profitable deviation exists in the standard analysis if π becomes sufficiently large (see also end of Section 2.1).

Return now to the case with *limited capacities*. We first go through a series of arguments to outline the intuition of the condition for the existence of an equilibrium in pure strategies. First, any profitable deviation can only be done with type H. Second, by construction of c_H^{RS} and Lemma 0, any such contract satisfying $u_H(c) \geq u_H(c_H^{RS})$ and $v_H(c) > 0$ must also satisfy $y < y_H^{RS}$ and $w < w_H^{RS}$. In words, the (upwards) distortion in the high type's contract must be reduced. Third, take now any such deviation c and consider some type t who may choose between receiving his RS contract c_t^{RS} for sure or implementing c with some probability ρ. In the latter case, his expected utility is equal to $\rho u_t(c)$. We can now calculate for any c satisfying $u_H(c) \geq u_H^{RS}$ and $v_H(c) > 0$ the threshold values

$$\rho_t(c) = \frac{u_t(c_H^{RS})}{u_t(c)},$$

at which type t is just indifferent between the two options. Recall now that $u_L(c_H^{RS}) = u_L(c_L^{RS})$. By (A.3) and our specification of utility functions it holds that

$$\rho_L(c) < \rho_H(c) \leq 1.$$

In words, the L type is willing to endure more rationing at any contract which makes the H type better off. Basically, given the additive separability in (A.1), this result is driven by the sorting condition in (A.3).

This feature can now be used in the following way. A potentially profitable deviation (which must make the H type better off) *first* attracts the measure $(1-\pi)$ of L types *before* a positive mass of high-ability workers might turn up. We now proceed as follows. First, we formalize a necessary and sufficient condition for existence. Second, we provide a verbal restatement of this condition. Third, we show how this condition can be endogenized by ensuring that aggregate capacity is sufficiently dispersed. Finally, we provide an example.

The above arguments allow us to provide an immediate *sufficient* condition for existence. Denote $\overline{u}_H = u_H(c_H^{RS})/\rho_L$ where $\rho_L = \min\{1, l/(1-\pi)\}$. Observe that when applying to the deviating firm f, ρ_L is the maximum probability with which a H type receives a contract. Denote next by $\overline{v}_H$ the maximum utility which can be obtained by maximizing $v_H(c)$ subject to $c \in C$ and $u_H(c) \geq \overline{u}_H$.

[13] In short, the RS contracts do not constitute an equilibrium if this allocation is not interim efficient in the sense of Holmström and Myerson (1983).

Obviously, if all other firms offer the RS menu of contracts and if $\overline{v}_H < 0$, a single firm cannot profitably deviate as it simply fails to attract a positive measure of high-type workers.

To derive a condition which is both *sufficient* and *necessary* for existence, define for all $u \geq u_H(c_H^{RS})/\rho_L$ the function

$$\mu(u) = \max\left\{1 - (1-\pi)\max\left\{\frac{u_H(c_H^{RS})}{lu}, 1\right\}, 0\right\}.$$

For any $u \geq u_H^{RS}/\rho_L$ choose next $c_L, c_H \in C$ to maximize $\mu(u)v_H(c_H) + (1 - \mu(u))v_L(c_L)$ subject to $u_H(c_H) \geq u$ and $u_L(c_L) \geq u_L(c_H)$. By (A.2) the program has a solution. Denote the realized expected payoff by $\overline{v}(u)$. In a final step choose $u \geq u_H(c_H^{RS})/\rho_L$ to maximize $\overline{v}(u)$ and denote the realized payoff by $\overline{\overline{v}}$.[14] The following assumption is now necessary and sufficient for existence.

(A.6) $\overline{\overline{v}} \leq 0.$

In words, (A.6) implies the following condition. Suppose workers could always obtain their respective RS contract for sure. (Recall that one firm is always dispensable to ensure full employment by (A.0).) By the previous arguments, any (deviating) contract which intends to attract high types must attract (almost) *all* low types. The respective distribution of types is then given by $\mu(u)$, i.e., $\mu(u)$ is the expected proportion of high-ability type a deviating firm will finally end up with. The payoff $\overline{\overline{v}}$ is then the maximum payoff the deviating firm can ensure itself.

Note next that (A.6) can be easily endogenized by using the primitives of the model. Precisely, fix a value of π. We can then choose for any fixed *aggregate* capacity $\overline{l} = Fl > 1$ a sufficiently large threshold $\overline{F}$, such that (A.6) is satisfied for all $F > \overline{F}$ and respective individual capacities $l = \overline{l}/F$. In words, (A.6) is surely satisfied if capacity becomes sufficiently dispersed among a finite number of firms.

Proposition 3. *An equilibrium where firms play pure strategies exists if and only if (A.6) is satisfied, which holds if capacity is sufficiently dispersed among firms.*

The proof of Proposition 3 is relegated to Appendix 3. The proof is done in two steps. First it is shown that (A.6) implies existence of an equilibrium. This should be clear by the preceeding arguments. In a second step we prove that an equilibrium where firms choose pure strategies does not exist if (A.6) does not hold. By Proposition 2 we know that any equilibrium where firms play pure strategies must implement the RS allocation. Starting from this observation, we can construct a deviating offer and show that the respective firm will realize a strictly positive profit.

[14] Existence follows as $\overline{v}(u)$ is continuous, while $u \to \infty$ is surely not optimal by (A.2).

An example

As an illustration of the effect of capacity constraints consider again the Spence model we discussed in Section 2.1. We restrict attention to possible deviations where a single contract is offered to both types. As argued above, if firms have unlimited capacity, a profitable deviating pooling contract exists if and only if $\pi > 1 - \frac{\lambda_L^W}{\lambda_H^W}$. Now consider firms with limited capacity. The best deviating pooling contract such a firm could offer will specify $y = 0$ and w such that

$$\rho w = u_H^{RS}, \tag{1}$$

where ρ is the probability that someone who queues at this firm is served. The wage w must be chosen large enough such that high types have an incentive to queue at this firm. Otherwise no profitable deviation can take place due to the construction of the RS contracts. As all L types will also queue at this firm, the expected number of high types which the firm will finally employ is given by $1 - (1 - \pi)\rho/l$. Then the profit of the deviating firm is

$$(1 - (1 - \pi)\rho/l)(\lambda_H^F - w) + (1 - \pi)\rho/l(\lambda_L^F - w).$$

Maximizing this expression with regard to ρ and w by using the above constraint (1) yields the maximum deviating profit of

$$\lambda_H^F - 2\sqrt{(1 - \pi)(\lambda_H^F - \lambda_L^F)u_H^{RS}/l}.$$

It can easily be seen that if the individual capacity l is small enough, this expression is negative.

5 Conclusion

We have analyzed a competitive (labor) market where the number of vacancies at any firm is strictly limited. Although in equilibrium rationing will not occur if overall capacity is sufficiently large, the possibility of *rationing out-of-equilibrium* allows to stabilize the Rothschild-Stiglitz (RS) outcome. This result is in contrast to the standard analysis where capacity is assumed to be unlimited.

There are several limits to our analysis. First, the specification of utility functions was made to rule out efficient rationing. As an example consider the case were reservation utilities are type-dependent, e.g., a high-ability worker has a more valuable outside option. In that case it might be efficient to ration the contract for the high-ability type, as this makes it less attractive for the low type to mimic the high type. Our conjecture is that if the market opens up for only one period an efficient level of rationing will be established. This brings us to the second shortcoming of the present analysis: If we additionally leave the one-shot framework, then previous arguments do not apply. More precisely, assume that visiting another firm is costly. Moreover, these costs are sufficiently low such that a worker might indeed approach another firm instead of exiting the market. Then it is not at all obvious that rationing, even if it is efficient, can prevail in equilibrium. In Inderst and Wambach (2001) we have modelled a multi-stage game with a *finite* number of agents in the explicit context of an insurance market, where incidentally reservation values are type-dependent. We show existence of an equilibrium in pure strategies, where no rationing occurs.[15] Based on this result we conjecture that with a continuum of (informed) agents the following results will still hold under the assumptions on payoffs made in this paper: There will be no rationing, only RS contracts will be implemented; and our existence result will carry over.[16] Finally, though our analysis is restricted to only two types for the informed agent, it can in principle be extended to larger type spaces. It is well-known that in the RS model conditions of existence of equilibrium become more severe the more types are added. Similarly, we expect that our sufficient and necessary conditions on the dispersion on rationing become more demanding, once the type space is enlarged.

Appendix 1: Proof of Proposition 1

We first prove existence. To do so, we define a program which derives for given contracts a distribution of types over firms and the outside option of no work. The respective measures are chosen such that, given the choices for the other type, the maximal utility which some type can obtain is *minimized*. This will assure that almost all individuals of the same type obtain the same utility. Below we will use this distribution to construct an equilibrium of the game in Stage 2.

Program $P(\{c^f\})$. A pair of families $\left\{M_H^\psi\right\}_{\psi\in\Psi}$ and $\left\{M_L^\psi\right\}_{\psi\in\Psi}$ solves $P(\{c^f\})$ if the following two requirements are satisfied:

i) Given $\left\{M_H^\psi\right\}_{\psi\in\Psi}$, $\left\{M_L^\psi\right\}_{\psi\in\Psi}$ is chosen to minimize

$$\max\left\{u_L(c_L^f)\rho(M_H^f + M_L^f) - \gamma \mid f \in F\right\}$$

subject to $\sum_{\psi=0}^F M_L^\psi = 1 - \pi$ and $u_L(c_L^f)\rho(M_H^f + M_L^f) - \gamma \geq 0$ if $f \in F$ and $M_L^f > 0$.

ii) Given $\left\{M_L^\psi\right\}_{\psi\in\Psi}$, $\left\{M_H^\psi\right\}_{\psi\in\Psi}$ is chosen to minimize

$$\max\left\{u_H(c_H^f)\rho(M_H^f + M_L^f) - \gamma \mid f \in F\right\}$$

subject to $\sum_{\psi=0}^F M_H^\psi = \pi$ and $u_H(c_H^f)\rho(M_H^f + M_L^f) - \gamma \geq 0$ for $f \in F$ and $M_H^f > 0$.

We prove next the following claim.

[15] With a finite number of agents we do not obtain uniqueness as in this paper.

[16] This conjecture holds only for the case where frictions, i.e., the costs of being rationed, are modeled by (type-independent) additive search or waiting costs. In case of discounting, Inderst and Müller (2000) analyze a search market environment with a continuum of informed and uninformed agents where both the contractual sorting variable and delay are used for efficient separation.

Claim 1. *$P(\{c^f\})$ has a solution. Moreover, a profile $a \in A$ constitutes an equilibrium at Stage 2 if and only if the respective measures $m_t^{\psi}(a)$ solve $P(\{c^f\})$.*

Proof. The program $P(\{c^f\})$ uniquely defines a pair of type-dependent utilities U_t. If $\left\{M_H^{\psi}\right\}_{\psi\in\Psi}$ and $\left\{M_L^{\psi}\right\}_{\psi\in\Psi}$ solve $P(\{c^f\})$,we define $U_t = u_t(c_t^f)\rho(M_H^f + M_L^f) - \gamma$ if $f \in F$ and $M_t^f > 0$, while we set $U_t = 0$ if $M_t^0 > 0$.

Let us now restate the program $P(\{c^f\})$. Given $M_H = \left\{M_H^{\psi}\right\}_{\psi\in\Psi}$ and $M_L = \left\{M_L^{\psi}\right\}_{\psi\in\Psi}$, the two requirements i) and ii) define a non-empty correspondence $\sum(M_L, M_H)$ such that $(M_H^*, M_L^*) \in \sum(M_L, M_H)$ if $M_L^* = \left\{M_L^{\psi,*}\right\}_{\psi\in\Psi}$ satisfies i) and if $M_H^* = \left\{M_H^{\psi,*}\right\}_{\psi\in\Psi}$ satisfies ii). By the continuity of $\rho(\cdot)$, the respective objective functions in i) and ii) and the (compact) constraint sets change continuously in (M_L, M_H). Hence, by the maximum theorem $\sum(\cdot)$ is upper-semincontinuous. Note that $\sum(\cdot)$ is convex. To see this, observe that $(M_H^*, M_L^*) \in \sum(M_L, M_H)$ implies $M_L^{f,*} = 0$ if $u_L(c_L^f) \leq 0$ and $M_H^{f,*} = 0$ if $u_H(c_H^f) \leq 0$. As the requirements i) and ii) define a unique utility level for either type, we may therefore only shift masses across firms where there is *no* rationing. Finally, note that $\sum(\cdot)$ maps the compact space

$$\{(M_L, M_H) \mid \sum_{\psi=0}^{F} M_L^{\psi} = 1 - \pi, \sum_{\psi=0}^{F} M_H^{\psi} = \pi\}$$

into itself. By the properties of $\sum(\cdot)$ and Kakutani's fixed-point theorem we can thus conclude that there exists some $(M_L, M_H) \in \sum(M_L, M_H)$, i.e., a solution to $P(\{c^f\})$.

We prove next that a profile a constitutes an equilibrium at Stage 2 if and only if it holds that $(m_L(a), m_H(a)) \in \sum(m_L(a), m_H(a))$, where $m_t(a) = \left\{m_t^{\psi}(a)\right\}_{\psi\in\Psi}$. For the "*if*" direction, consider some $(M_L, M_H) \in \sum(M_L, M_H)$, which define a pair of type-specific utilities denoted by $U_t \geq 0$. It is straightforward to derive $a \in A$ with $m_t^{\psi}(a) = M_t^{\psi}$. By construction of $P(\{c^f\})$ there exists no $f \in F$ such that $u_t(c_t^f)\rho(M_L^f + M_H^f) - \gamma > U_t$, while $f \in F$ and $M_t^f > 0$ imply $u_t(c_t^f)\rho(M_L^f + M_H^f) - \gamma \geq 0$. This proves that a constitutes an equilibrium profile.

For the "*only if*" direction, consider some supposed equilibrium profile $a \in A$. We show that the choices $m_t^{\psi}(a)$ must satisfy the respective requirements of $P(\cdot)$, i.e., i) for L and ii) for H. We argue to a contradiction. If $u_t(c_t^f)\rho(m^f(a)) - \gamma < 0$ holds for some $f \in F$ with $m_t^f(a) > 0$, by continuity of ρ a positive measure of workers of type t could profitably deviate from a. If there exists $m_t^f(a) > 0$, while $\max\left\{u_t(c_t^f)\rho(m^f(a)) - \gamma \mid f \in F\right\}$ is not minimized, this implies existence of $f', f'' \in F$ such that $u_t(c_t^{f'})\rho(m^{f'}(a)) - \gamma$ exceeds $u_t(c_t^{f''})\rho(m^{f''}(a)) - \gamma \geq 0$, which cannot be optimal. This concludes the proof of Claim 1. **QED (Claim 1)**

Having established existence of an equilibrium, we show next that equilibrium utilities are uniquely determined for both types.

Claim 2. *Any equilibrium profile (at Stage 2) gives rise to the same type-dependent utilities $U_t(\{c^f\})$.*

Proof. The proof is by contradiction. Suppose thus that there exist different equilibrium profiles a and $\overline{a}$ under which some type $t \in T$ realizes different payoffs denoted by $U_t > \overline{U}_t$. We will show that this implies that also the other type $t' \neq t$ will be better off with equilibrium profile a. This in turn implies that rationing at any active firm under $\overline{a}$ will be more severe than under a, which, however, violates the condition that measures have to add up to one.

Formally, if type t strictly prefers strategy profile a to $\overline{a}$, this implies that $U_t > 0$, $m_t^0(a) = 0$, and the existence of a $f \in F$ with $m_t^f(a) > 0$, $u_t(c_t^f) > 0$. Denote $F_t = \left\{f \in F \mid m_t^f(a) > 0\right\}$. $U_t > \overline{U}_t$ implies for $f \in F_t$ that $m^f(\overline{a}) > m^f(a)$ and $m^f(\overline{a}) > l$. As $\sum_{f\in F_t} m_t^f(a) = 1 - \pi \geq \sum_{f\in F_t} m_t^f(\overline{a})$, this requires $\sum_{f\in F_t} m_{t'}^f(\overline{a}) > \sum_{f\in F_t} m_{t'}^f(a)$ for $t' \neq t$. By $\gamma > 0$ this implies $\overline{U}_{t'} < U_{t'}$, which by $\overline{U}_{t'} \geq 0$ holds only if $m^f(\overline{a}) > m^f(a)$ and $m^f(\overline{a}) > l$ for all $f \in F_{t'}$, where $F_{t'} = \left\{f \in F \mid m_{t'}^f(a) > 0\right\}$. As additionally $\sum_{f\in F_{t'}} m_{t'}^f(a) = \pi$, the derived requirements on the masses at F_t and $F_{t'}$ in $\overline{a}$ cannot be jointly satisfied simultaneously. This completes the proof of Claim 2. **QED (Claim 2)**

It remains to prove continuity of $U_t(\{c^f\})$. By the arguments used in Claim 1, it is immediate that, for given contracts $\{c^f\}$, the fixed-points defined by $P(\{c^f\})$ constitute a convex set, while the correspondence is upper-semicontinuous in the contracts $\{c^f\}$. (This uses that u_t is continuous.) As by Claims 1-2 all solutions to $P(\{c^f\})$ realize the same utilities equal to $U_t(\{c^f\})$, continuity of these utilities is immediate from the continuity of ρ and u_t.

Appendix 2: Proof of Proposition 2

We abbreviate an equilibrium of the total game by a tuple $\omega = (\{c^f\}, a)$, where $a \in \alpha(\{c^f\})$ (by sequential optimality). The set of equilibria is denoted by Ω. Recall that we denoted the firms' payoffs by $V^f(\omega)$.

The proof proceeds by a series of steps. In Claim 1 we show that any firm f where there is no rationing may not realize positive profits. Though this uses a standard "undercutting" argument, the proof is considerably complicated by the necessity to ensure incentive compatibility in the presence of possible rationing *after* a deviation has occurred. In Claim 2 we extend the argument to the case where there is rationing at f. In Claims 3-4 we show that these results imply that (almost) all workers of type t must realize c_t^{RS} with probability one.

Claim 1. *If $\omega \in \Omega$, then there is no $f \in F$ such that $0 < m^f(a) \leq l$, $m_t^f(a) > 0$, and $v_t(c_t^f) > 0$ for some type t. Hence, a non-rationing firm may not realize positive profits with any contract.*

Proof. We argue by contradiction and assume that such a firm exists. Precisely, consider some f, $\omega \in \Omega$, and a type t such that $v_t(c_t^f) > 0$. Furthermore, as $m^f(a) > 0$, we can find by (A.1) some firm f' with $m^{f'}(a) < l$. We will show that f' can profitably deviate. In what follows, we restrict attention to the case where $t = H$.[17]

We further distinguish between three different cases depending on the supposed equilibrium demand at f' under a.

Case 1: Suppose $m_L^{f'}(a) = 0$.

Hence, we assume that in the supposed equilibrium the potential deviator f' does not attract a positive measure of type L. This assumption will allow us to basically neglect the type L. Suppose first that

$$v_t(c_H^f) > v_t(c_H^{f'}), \tag{2}$$

i.e., that the contract designed for H by firm f is more attractive to firms than that offered by f'. We will use c_H^f to construct a profitable deviation denoted by $\overline{c}$. Observe next that $m^f(a) \leq l$ implies $U_H(\{c^f\}) = u_H(c_H^f) - \gamma$ and (using incentive compatibility) $U_L(\{c^f\}) \geq u_L(c_L^f) - \gamma$. (Recall that equilibrium utilities are uniquely determined by Proposition 1.) Hence, by continuity, (A.3), and $v_H(c_H^f) > 0$, there exists $\overline{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \overline{\varepsilon}]$ we can construct a contract $\overline{c}$ satisfying

$$\begin{aligned} v_H(\overline{c}) &> v_H(c_H^f) - \varepsilon, \\ u_H(\overline{c}) &> u_H(c_H^f) + \varepsilon, \\ u_L(\overline{c}) - \gamma &< U_L(\{c^f\}). \end{aligned} \tag{3}$$

For notational convenience we do not index $\overline{c}$ by ε. Fix now some ε and suppose that f' offers a respective contract $\overline{c}$. Denote the new family of contracts by $\{\overline{c}^f\}$ and denote $\overline{U}_t = U_t(\{\overline{c}^f\})$. Pick some equilibrium profile (at Stage 2) $\overline{a} \in \alpha(\{\overline{c}^f\})$.

We analyze next how the demand at the deviator f' changes under $\overline{a}$. We show first that $m_L^{f'}(\overline{a}) = 0$, i.e., that the deviator attracts no low-type workers. This is intuitive as $\overline{c}$ was constructed to be more favorable to high-type workers than the previous offer at f', while low-type workers would be worse off. To make this formal, we argue to a contradiction. If $m_L^{f'}(\overline{a}) > 0$, this implies by (3) that $0 \leq \overline{U}_L < U_L(\{c^f\})$. As a consequence, the set $F(L) = \left\{f'' \in F \mid m_L^{f''}(a) > 0\right\}$ is non-empty and for all $f'' \in F(L)$ it holds that $m^{f''}(\overline{a}) > m^{f''}(a)$ and $m^{f''}(\overline{a}) > l$. This requires $m_H^{f''}(\overline{a}) > 0$ for some $f'' \in F(L)$, implying $0 \leq \overline{U}_H < U_H(\{c^f\})$. As previously for L, this implies for H that $m^{f''}(\overline{a}) > m^{f''}(a)$ and $m^{f''}(\overline{a}) > l$ for all f'' in the non-empty set $F(H) = \left\{f'' \in F \mid m_H^{f''}(a) > 0\right\}$. As $m_t^0(a) = 0$ holds by $U_t(\{c^f\}) > 0$, it cannot hold that the measures of workers increase at both $F(L)$ and $F(H)$, which yields a contradiction.

[17] The argument for $t = L$ is simpler due to (A.4) (common values).

Having shown that no low-type workers apply at f', we turn next to high-type workers. We argue that $\varepsilon > 0$ implies

$$m_H^{f'}(\overline{a}) \geq \min\left\{l, m_H^{f'}(a) + m_H^{f}(a)\right\}, \tag{4}$$

such that applications of high types at f' jump. We argue to a contradiction. Recall first that $m_L^{f'}(\overline{a}) = 0$. Hence, if (4) does not hold, it follows that $\overline{U}_H > U_H(\{c^f\})$ such that $m_H^{f''}(\overline{a}) = 0$ for *all* f'' satisfying $m^{f''}(a) \leq l$. To ensure $\overline{U}_H > U_H(\{c^f\})$ this implies existence of some f'' with $m_H^{f''}(\overline{a}) > 0$, $m^{f''}(a) > m^{f''}(\overline{a})$, and $m^{f''}(a) > l$. As the measure of high types has to add up to π this can only hold if $m_L^{f''}(\overline{a}) < m_L^{f''}(a)$ is satisfied at some f''. The latter implication requires $\overline{U}_L > U_L(\{c^f\})$. It is now immediate that $\overline{U}_H > U_H(\{c^f\})$ and $\overline{U}_L > U_L(\{c^f\})$ cannot hold simultaneously if (4) is not satisfied. (As we have already argued repeatedly by now, this is simply due to the fact that the measure of applying workers does not decrease.)

We are now in a position to conclude the argument for Claim 1. As (4) holds for any $\varepsilon > 0$ and as the arguments apply to any $\overline{a} \in \alpha(\{\overline{c}^f\})$, we can conclude by (3) that for sufficiently low ε the deviation is indeed strictly profitable for f' (in any equilibrium of the continuation game).

To conclude the proof for Case 1, recall that we have assumed $v_H(c_H^f) > v_H(c_H^{f'})$ in (2). Suppose now that $v_H(c_H^f) \leq v_H(c_H^{f'})$. For $m^{f'}(a) = 0$ we can apply the previous argument. If $m^{f'}(a) > 0$, the argument is analogous with the difference that $\overline{c}$ is now constructed from $c_H^{f'}$ in (3).

Case 2: Suppose $m_L^{f'}(a) > 0$ and $v_L(c_L^{f'}) \leq 0$.

We are now brief as the argument is analogous to that in Case 1. We may again focus on the case where $v_H(c_H^f) > v_H(c_H^{f'})$. In difference to Case 1, the deviation consists now of a pair of incentive compatible contracts $(\overline{c}_H, \overline{c}_L)$ constructed from $(c_H^f, c_L^{f'})$. Observe first that both the pair $c_H^f, c_L^{f'}$ and the pair $c_L^f, c_H^{f'}$ are incentive compatible as there is no rationing at f and f' in a. In analogy to (3), we can use the assumptions on the utility functions to ensure (for low ε) existence of a pair of contracts $(\overline{c}_H, \overline{c}_L)$ satisfying

$$\begin{aligned} v_H(\overline{c}_H) &> v_H(c_H^f) - \varepsilon, \\ v_L(\overline{c}_L) &\geq v_L(c_L^{f'}), \\ u_H(\overline{c}_H) &> u_H(c_H^f) + \varepsilon, \\ u_L(\overline{c}_L) &< u_L(c_L^{f'}). \end{aligned}$$

Instead of proving $m_L^{f'}(\overline{a}) = 0$ as in Claim 1, we can show by the same arguments that $m_L^{f'}(\overline{a}) \leq m_L^{f'}(a)$. Moreover, instead of (4), we can show next that

$$m_H^{f'}(\overline{a}) \geq \min\left\{l - m_L^{f'}(\overline{a}), m_H^{f'}(a) + m_H^{f}(a)\right\}, \tag{5}$$

which by $m_L^{f'}(\overline{a}) \leq m_L^{f'}(a)$ and $m^{f'}(a) < l$ implies again a "jump" in the supply of (profitable) workers of type H.

Case 3: Suppose $m_L^{f'}(a) > 0$ *and* $v_L(c_L^{f'}) > 0$.

In contrast to Cases 1-2, it is now no longer sufficient to argue that f' can attract more high-type workers, as this might imply that it looses (disproportionally many) low-type workers with whom it also makes a profit. We focus on the case where $0 < v_L(c_L^{f'}) < v_H(c_H^{f'})$. The analysis of the case with $v_L(c_L^{f'}) \geq v_H(c_H^{f'})$ is analogous. Moreover, we assume that $v_H(c_H^{f'}) \geq v_H(c_H^{f})$. As argued in Cases 1-2, the argument for $v_H(c_H^{f'}) < v_H(c_H^{f})$ is analogous.

We first construct a possible deviation $(\overline{c}_H, \overline{c}_L)$. By continuity, (A.3), and incentive compatibility of the original contracts, we can find for any sufficiently small $\varepsilon > 0$ a threshold $\overline{\varepsilon}'(\varepsilon) > 0$, such that for all $0 \leq \varepsilon' < \overline{\varepsilon}'$ there exists some incentive compatible menu $(\overline{c}_H(\varepsilon, \varepsilon'), \overline{c}_L(\varepsilon, \varepsilon'))$ satisfying

$$\begin{aligned} v_H(\overline{c}_H(\varepsilon,\varepsilon')) &> v_H(c_H^{f'}) - \varepsilon, \\ u_H(\overline{c}_H(\varepsilon,\varepsilon')) &> u_H(c_H^{f}) + \varepsilon, \\ v_L(\overline{c}_L(\varepsilon,\varepsilon')) &\geq v_L(c_L^{f'}) - \varepsilon', \\ u_L(c_L^{f'}) + \varepsilon'/2 &\leq u_L(\overline{c}_L(\varepsilon,\varepsilon')) \leq u_L(c_L^{f'}) + \varepsilon'. \end{aligned} \tag{6}$$

In particular, the payoff of high types increases by at least ε under the new contracts, while the payoff for low types increases by at least $\varepsilon'/2$ but by not more than ε'. By choosing the ratio of ε' to ε we will control the distribution of types applying at f'. For completeness we further specify that $\overline{c}_L(\varepsilon, 0) = c_L^{f'}$ for any ε. Given contracts $(\overline{c}_H(\varepsilon, \varepsilon'), \overline{c}_L(\varepsilon, \varepsilon'))$, we denote the workers' equilibrium utilities by $\overline{U}_t(\varepsilon, \varepsilon')$. Recall from Proposition 1 that equilibrium utilities are continuous in the utilities derived from the offered contracts. This implies by (6) that for given ε, it holds that $\overline{U}_t(\varepsilon, \varepsilon') \to \overline{U}_t(\varepsilon, 0)$ for $\varepsilon' \to 0$. This observation will be crucial below. Along such a sequence, an argument as in Case 1 implies by (6) that

$$\begin{aligned} \overline{U}_H(\varepsilon,\varepsilon') &\geq U_H(\{c^f\}), \\ \overline{U}_L(\varepsilon,0) &\geq u_L(c_L^{f'}) - \gamma. \end{aligned} \tag{7}$$

Finally, denote for given $(\varepsilon, \varepsilon')$ the set of equilibria (for the continuation game at Stage 2) by $\alpha(\varepsilon, \varepsilon')$. We analyze next the size and the distribution of demand attracted by f'. Most importantly, by choosing ε' sufficiently small for some fixed choice of ε, the share of high types is not worse than under a (in case rationing occurs at f' under $\overline{a}$).

Auxiliary assertion: *There exists* $\overline{\varepsilon}''(\varepsilon) > 0$ *such that for all* $0 < \varepsilon' < \overline{\varepsilon}''(\varepsilon)$ *and all* $\overline{a} \in \alpha(\varepsilon, \varepsilon')$ *it holds that:*

i) $m^{f'}(\overline{a}) - m^{f'}(a) \geq \min\left\{l - m^{f'}(a), m_H^{f}(a)\right\}$

ii) $m_H^{f'}(\overline{a})/m^{f'}(\overline{a}) > m_H^{f'}(a)/m^{f'}(a)$ *if* $m^{f'}(a) > l$

iii) $m_H^{f'}(\overline{a}) - m_H^{f'}(a) \geq min\left\{l - m^{f'}(a), m_H^{f}(a)\right\}$ *if* $m^{f'}(a) \leq l$

Proof of the assertion. The proof of assertion i) is omitted as it is analogous to that in Cases 1 and 2. It implies that the aggregate measure of applications "jumps" for all $\varepsilon > 0$, $\varepsilon' > 0$.

In contrast, assertions ii) and iii) deal with the distribution of new applications. We distinguish between two subcases.

Subcase 1: $\overline{U}_H(\varepsilon, 0) < u_H(\overline{c}_H(\varepsilon, 0)) - \gamma$. This implies $m^{f'}(\overline{a}) > l$ for *all* $\overline{a} \in \alpha(\varepsilon, 0)$ and thus $m_L^{f'}(\overline{a}) = 0$ due to (7). By convergence of $U_H(\varepsilon, \varepsilon')$, i.e., by $\overline{U}_t(\varepsilon, \varepsilon') \to \overline{U}_t(\varepsilon, 0)$ for $\varepsilon' \to 0$, there must be rationing at f' in any $\overline{a} \in \alpha(\varepsilon, \varepsilon')$ if $\varepsilon' > 0$ remains sufficiently low, while at the same time it holds that $m_L^{f'}(\overline{a}) = 0$, which proves assertion ii).

Subcase 2: $\overline{U}_H(\varepsilon, 0) = u_H(\overline{c}_H(\varepsilon, 0)) - \underline{\gamma}$. By (7) this implies $m^{f'}(\overline{a}) \leq l$ for *all* $\overline{a} \in \alpha(\varepsilon, 0)$. If additionally $\overline{U}_L(\varepsilon, 0) > \overline{U}_L(\{c^f\})$, this must additionally imply by (7) that $m_L^{f'}(\overline{a}) = 0$. We can then apply the same argument as in Subcase 1. Assume therefore that $\overline{U}_L(\varepsilon, 0) = U_L(\{c^f\})$. Denote next for given $(\varepsilon, \varepsilon')$ (and respective contracts as constructed in (6)) the *unique* equilibrium probability of being served at f' by $\rho_{f'}(\varepsilon, \varepsilon')$. (Uniqueness is implied by the uniqueness of equilibrium utilities.) Moreover, convergence of equilibrium utilities implies convergence of $\rho_{f'}(\varepsilon, \varepsilon')$ to one as $\varepsilon' \to 0$. We show next that for low ε' and respective $\overline{a} \in \alpha(\varepsilon, \varepsilon')$ it holds that

$$m_H^{f'}(\overline{a}) - m_H^{f'}(a) \geq min\left\{l - m^{f'}(a), m_H^{f}(a)\right\} \tag{8}$$

To prove (8), observe first that by construction (and as $\varepsilon > 0$) $\rho u_H(\overline{c}_H(\varepsilon, \varepsilon')) > u_H(c_H^f)$ holds if ρ is close to one. As this is true for $\varepsilon' = 0$, we can find a sufficiently low ε' such that the inequality holds for $\rho_{f'}(\varepsilon, \varepsilon')$. As a consequence, workers of type H do not apply at firms f'' with $m^{f''}(a) \leq l$ and they do not choose the outside option. In other words, they only select firms where there was rationing for profile a. But this implies that the rationing probability at all these firms must drop. By the convergence of $\overline{U}_L(\varepsilon, \varepsilon')$, we can then conclude that this cannot be due to the case that low types no longer turn to these firms, which finally yields a contradiction as the total measure of high types applying to firms does not decrease. Hence, we have proved that (8) holds for sufficiently low ε'.

It is now immediate that the continuity of $\rho_{f'}(\varepsilon, \varepsilon')$, $\rho_{f'}(\varepsilon, 0) = 1$, and (8) jointly imply assertions ii) and iii) for Subcase 2. This completes the proof of the auxiliary assertion. **QED (Auxiliary assertion)**

We are now in a position to complete the proof for Case 3. Observe first that for some choice of $(\varepsilon, \varepsilon')$ and respective contracts defined in (6), the payoff of f' in some $\overline{a} \in \alpha(\varepsilon, \varepsilon')$ is given by

$$\left[v_H(\overline{c}_H(\varepsilon, \varepsilon'))m_H^{f'}(\overline{a}) + v_L(\overline{c}_L(\varepsilon, \varepsilon'))m_L^{f'}(\overline{a})\right] min\left\{1, l/m^{f'}(\overline{a})\right\}.$$

By the auxiliary assertion and the construction in (6) we can now choose $\varepsilon > 0$ sufficiently small and, respectively, some $0 < \varepsilon' < \overline{\varepsilon}''(\varepsilon)$ such that the deviation is strictly profitable for f' in *any* equilibrium of the continuation game. This concludes the argument for Case 3.

Summing up, as Cases 1-3 exhaust all possibilities, this completes the proof of Claim 1. **QED (Claim 1)**

While Claim 1 proves that a firm f cannot make positive profits with any contract if there is no rationing at f, we next extend this assertion to the case where there is rationing at f.

Claim 2. *If $\omega \in \Omega$, then there is no $f \in F$ such that $m^f(a) > l$, $m_t^f(a) > 0$, and $v_t(c_t^f) > 0$ for some type t. Hence, a rationing firm may not realize positive profits with any contract.*

Proof. The argument is again by contradiction. Hence, assume that there exists a type t with $v_t(c_t^f) > 0$ and $m_t^f(a) > 0$. We restrict again attention to the case where $t = H$ as the argument for $t = L$ is similar (but simpler due to common values). We distinguish between two cases, depending this time on the demand at other firms.

Case 1. Suppose that there exists $f' \in F \setminus \{f\}$ such that $m^{f'}(a) = 0$.

Note first that this implies $V^{f'}(\omega) = 0$, i.e., f' makes zero profits. We show that f' can profitably deviate. The argument is now abbreviated as it is somewhat analogous to that in Claim 1. We claim that for sufficiently small $\varepsilon > 0$ there exists $\overline{c}$ such that

$$\begin{aligned} v_H(\overline{c}) &> 0, \\ u_H(\overline{c}) - \gamma &> U_H(\{c^f\}) + \varepsilon, \\ u_L(\overline{c}) - \gamma &< U_L(\{c^f\}) - \varepsilon. \end{aligned} \tag{9}$$

To see this, observe first that $U_H(\{c^f\}) = \rho(m^f(a))u_H(c_H^f)$, while incentive compatibility for low-type workers also implies $U_L(\{c^f\}) \geq \rho(m^f(a))u_L(c_L^f)$. Moreover, recall that $u_t(c) = \beta(w) + \eta_t(y)$ implies $u_H(c) \geq u_L(c)$, such that high types do not gain less in *absolute* terms if ρ, i.e., the probability of receiving a contract, is increased. This observation together with incentive compatibility of the initial offers (c_L^f, c_H^f) and (A.3) enables the construction in (9).

If f' offers $\overline{c}$ (for some $\varepsilon > 0$), denote again the resulting family of contracts by $\{\overline{c}^f\}$ and the resulting equilibrium utilities by $\overline{U}_t$. Observe next that $\overline{a} \in \alpha(\{\overline{c}^f\})$ implies $m_L^{f'}(\overline{a}) = 0$ and $m_H^{f'}(\overline{a}) > 0$ for *any* $\varepsilon > 0$. (The proof is analogous to that in Claim 1 (Case 1) where a corresponding result was shown to hold.) By choosing ε sufficiently small, the deviation is therefore strictly profitable for any equilibrium of the continuation game.

Case 2. Suppose that there exists no $f' \in F \setminus \{f\}$ such that $m^{f'}(a) = 0$.

The argument is now analogous to that of Case 1 after making the following observation. By Claim 1 any firm f' with $l \geq m^{f'}(a) > 0$ realizes zero profits with *any* contract accepted by some type. Hence, we obtain $v_H(c_H^{f'}) = 0$ in case

$m_H^{f'}(a) > 0$ and also $v_L(c_L^{f'}) = 0$ in case $m_L^{f'}(a) > 0$. As a consequence, when constructing a deviating menu which intends to attract high-ability types, the firm f' is indifferent towards losing low-ability types. We can then proceed by combining the argument for Case 1 with that used in Claim 1 (for Case 2).

This completes the proof of Claim 2. **QED (Claim 2)**

By Claims 1–2 firms must realize zero profits with all contracts implemented in equilibrium. We show next that this implies that both types receive their respective RS contract with probability one. We turn first to low-ability workers.

Claim 3. *If $\omega \in \Omega$, then almost all $i \in I$ with $\tau(i) = L$ select $a(i,\tau(i)) \in F$ with $c^f_{a(i,\tau(i))} = c_L^{RS}$ and $m^f(a) \leq l$. In words, (almost) all low-ability workers must implement their RS contract with probability one.*

Proof. Consider some $\{c^f\}$ and corresponding $a \in \alpha\left(\{c^f\}\right)$. If $\omega \in \Omega$, Claims 1-2 and the construction of c_L^{RS} imply $U_L(\{c^f\}) \leq u_L(c_L^{RS}) - \gamma$. We show next that this must hold with equality. Suppose that it is not the case, i.e., that $U_L(\{c^f\}) < u_L(c_L^{RS}) - \gamma$. We can then construct some $\overline{c} \in C$ such that $v_L(\overline{c}) > 0$ and $u_L(\overline{c}) - \gamma > U_L(\{c^f\})$. Applying by now standard arguments, $\overline{c}$ will constitute a profitable deviation for some firm f' (in *any* continuation equilibrium at Stage 2). (Precisely, if f' offers $\overline{c}$, any subsequent equilibrium $\overline{a}$ must satisfy $m^{f'}(\overline{a}) > 0$, which by $v_L(\overline{c}) > 0$ and (A.3) makes the deviation strictly profitable.) Hence, we have shown that $U_L(\{c^f\}) = u_L(c_L^{RS}) - \gamma$. By Claims 1-2, however, this implies that (almost) all L workers receive c_L^{RS} with probability one. **QED (Claim 3)**

We turn next to high-ability workers.

Claim 4. *If $\omega \in \Omega$, then almost all $i \in I$ with $\tau(i) = H$ select $a(i,\tau(i)) \in F$ with $c^f_{a(i,\tau(i))} = c_H^{RS}$ and $m^f(a) \leq l$. In words, (almost) all high-ability workers must implement their RS contract with probability one.*

Proof. We show first that $\omega \in \Omega$ implies that there is no rationing of H types, i.e., that $m_H^f(a) > 0$ implies $m^f(a) \leq l$ for all $f \in F$. We argue by contradiction and assume $m_H^f(a) > 0$ and $m^f(a) > l$. Recall first that by Claim 3 $U_L(\{c^f\}) = u_L(c_L^{RS}) - \gamma$, while Claim 3 also implies $m_L^f(a) = 0$ as otherwise low types would be rationed. Moreover, as f realizes zero profits it must hold that $v_H(c_H^f) = 0$. As in Claim 2, we can now construct from c_H^f a contract $\overline{c}$ such that $v_H(\overline{c}) > \varepsilon$, $u_L(\overline{c}) < u_L(c_L^{RS})$, and $u_H(\overline{c}) - \gamma > U_H(\{c^f\}) + \varepsilon$ for small ε. We consider next how applications change if some firm offers $\overline{c}$. Given $m^f(a) > l$ and no rationing of type L workers, we can by (A.0) find some f' with $m^{f'}(a) < l$ such that $F^L = \left\{f'' \in F \backslash f' \mid m_L^{f''}(a) > 0\right\}$ satisfies $l\left|F^L\right| \geq 1 - \pi$. (In words, after taking out f', there are still enough firms left who can serve low types with c_L^{RS} without rationing.) If f' offers $\overline{c}$, we can apply the arguments of Claim 1 to show that in any continuation equilibrium $\overline{a}$ it holds that $m_L^{f'}(\overline{a}) = 0$ and $m_H^{f'}(\overline{a}) > 0$. As this is profitable for f', we obtain a contradiction. Hence, $\omega \in \Omega$ and $m_H^f(a) > 0$ must imply $m^f(a) \leq l$.

By construction of c_H^{RS} and (A.5), this implies that $U_H(\{c^f\}) > u_H(c_H^{RS}) - \gamma$ must by incentive compatibility contradict $U_L(\{c^f\}) = u_L(c_L^{RS}) - \gamma$, which holds by Claim 3. Suppose next that $U_H(\{c^f\}) < u_H(c_H^{RS}) - \gamma$. In this case we can again apply by now standard arguments to construct a profitable deviation. The assertion follows now as $U_H(\{c^f\}) = u_H(c_H^{RS}) - \gamma$ and as workers of type H are not rationed. **QED (Claim 4)**

Claims 3–4 finally establish Proposition 2.

Appendix 3: Proof of Proposition 3

We first show that (A.6) implies existence of an equilibrium. Suppose that all firms offer the RS menu of contracts, i.e., $c_t^f = c_t^{RS}$ for $t \in T$, $f \in F$. (It is then immediate that any $a \in \alpha(\{c^f\})$ must implement the RS allocation.) Suppose now that some firm $\overline{f}$ deviates, which yields a new family of contracts $\{\overline{c}^{\overline{f}}\}$. By Proposition 1, $\alpha(\{\overline{c}^{\overline{f}}\})$ is not empty. To realize positive profits, the menu of $\overline{f}$ must specify $c = \overline{c}_H^{\overline{f}}$ with $y < y_H^{RS}$. To establish optimality of the original strategies (for firms), it is sufficient to construct one $a \in \alpha(\{\overline{c}^{\overline{f}}\})$ such that $V^{\overline{f}}(\{\overline{c}^{\overline{f}}\}, a) \leq 0$. By the arguments preceding Proposition 3 this is ensured by construction of $\overline{\overline{v}}$ and (A.6).

We show next that there exists no equilibrium if (A.6) does not hold. From Proposition 2 we know that, given an equilibrium family of contracts $\{c^f\}$, any $a \in \alpha(\{c^f\})$ must implement the RS allocation. This implies that we can pick a firm $\overline{f}$ such that $F_H = \{f \in F \backslash \overline{f} \mid c_H^f = c_H^{RS}\}$ satisfies $|F_H|\, l \geq \pi$, while $F_L = \{f \in F \backslash \overline{f} \mid c_L^f = c_L^{RS}\}$ satisfies $|F_L|\, l \geq 1 - \pi - l$. Moreover, there exists no contract c contained in $\{c^f\}$ such that either $u_L(c) > u(c_L^{RS})$ or $u_H(c) > u(c_H^{RS})$. Recall now the discussion preceding Proposition 3. Denote a pair of contracts $(\overline{\overline{c}}_L, \overline{\overline{c}}_H)$ realizing the expected utility $\overline{\overline{u}}$ defined by $\overline{\overline{v}} = \overline{v}(\overline{\overline{u}})$. (Observe that by construction it holds that $u_H(\overline{\overline{c}}_H) = \overline{\overline{u}}$.) By construction, if firm $\overline{f}$ deviates and offers $(\overline{\overline{c}}_L, \overline{\overline{c}}_H)$, it strictly gains if the realized distribution is given by $\mu(\overline{\overline{u}})$. (Recall that $\mu(\overline{\overline{u}})$ was calculated by assuming that (almost) all low-type workers (i.e., the measure $1 - \pi$) apply while high types apply additionally until $\mu(\overline{\overline{u}})$ is realized.).

Recall next that we can restrict consideration to offers such that $\overline{\overline{u}} = u_H(c_H^{RS})/\rho(m^{\overline{f}}(a))$ implies $u_L(\overline{\overline{c}}_L)\rho(m^{\overline{f}}(a)) \geq u_L(c_L^{RS})$, while surely $u_L(\overline{\overline{c}}_L) > u_L(c_L^{RS})$. Moreover, by assumption firms $f \neq \overline{f}$ do not offer a contract c realizing either $u_L(c) > u_L(c_L^{RS})$ or $u_H(c) > u_H(c_H^{RS})$. As a consequence, $a \in \alpha(\{\overline{c}^{\overline{f}}\})$ implies $m_L^{\overline{f}}(a) \geq min\,\{l, 1 - \pi\}$ such that, by the construction of F_L and F_H any worker may obtain his RS contract at some other firm with probability one. In case of $\overline{\overline{u}} = u_H(\overline{\overline{c}}_H) > u_H^{RS}$ the previous arguments imply that any $a \in \alpha(\{\overline{c}^{\overline{f}}\})$ satisfies $\rho(m^{\overline{f}}(a)) > 1$, and $m_L^{\overline{f}}(a) = 1 - \pi$, while $m_H^{\overline{f}}(a) = \pi$ holds for $\overline{\overline{u}}\rho(1) \geq u_H^{RS}$ and $m_H^{\overline{f}}(a)$ solves $\overline{\overline{u}} = u_H(c_H^{RS})/\rho(1 - \pi + m_H^{\overline{f}}(a))$ if $\overline{\overline{u}}\rho(1) < u_H^{RS}$. As a consequence, $V^{\overline{f}}(\{\overline{c}^{\overline{f}}\}, a) = \overline{\overline{v}} > 0$ holds for *any* $a \in \alpha(\{\overline{c}^{\overline{f}}\})$ implying that the deviation is always profitable.

It thus remains to consider the case where $\overline{\overline{u}} = u_H(\overline{\overline{c}}_H) = u_H^{RS}$. This case is most easily dealt with by considering for some arbitrarily low $\varepsilon > 0$ an alternative pair of incentive compatible contracts $(\overline{\overline{c}}'_L, \overline{\overline{c}}'_H)$ realizing $u_H(\overline{\overline{c}}'_H) > u_H^{RS}$ and $v_t(\overline{\overline{c}}'_t) \geq v_t(\overline{\overline{c}}_t) - \varepsilon$ for $t \in T$. As $\overline{\overline{v}} > 0$ holds strictly, the previous argument carries over to the newly constructed deviation. **QED**

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