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Competitive insurance markets under adverse selection and capacity constraints

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Abstract

Ever since the seminal work by Rothschild and Stiglitz (Q. J. Econom. 90 (1976) 629) on competitive insurance markets under adverse selection, the problem of non-existence of equilibrium in pure strategies has received much attention in the literature. We extend the original analysis by considering firms which face capacity constraints, which might be due to limited capital. We show that under mild assumptions an equilibrium exists, where every consumer obtains his appropriate Rothschild–Stiglitz contract. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Ever since the seminal work by Rothschild and Stiglitz (1976) on competitive insurance markets under adverse selection, the problem of non-existence of equilibrium in pure strategies has received much attention in the literature. The

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origin of this problem lies in the fact that only zero profit making separating contracts can constitute an equilibrium in the sense of Rothschild and Stiglitz, while in some cases a single pooling contract or a pair of cross-subsidizing contracts may be preferred by everyone and will therefore upset the Rothschild–Stiglitz equilibrium contracts.

There are many approaches to this problem in the literature. One way out of it is to allow firms to have mixed strategies (Dasgupta and Maskin, 1986); however, the economic interpretation of this modification is not clear. Another possibility is to propose different equilibrium concepts (Wilson, 1977; Miyazaki, 1977; Spence, 1978; Riley, 1979), which however lack a game-theoretic foundation. There exist a few attempts of introducing some form of dynamics in a non-cooperative model (Jaynes, 1978; Hellwig, 1987, 1988; Asheim and Nilssen, 1996).[1]

In this paper we want to add one aspect to the discussion of the non-existence problem which so far has not received any attention in the insurance literature, and which lies at the heart of the non-existence problem: If a deviating firm offers a new set of contracts, who chooses these contracts? So far it was always assumed that any new contract offer can potentially serve the whole market. Here we assume instead that firms face capacity constraints. In that case it is no longer guaranteed that a new offer may attract a fair selection of the market. Indeed, the distribution of risk types applying for a (deviating) contract at a given firm is now determined endogenously.

One reason for capacity constraints can be solvency regulation: For a given size of capital, only a finite number of risks can be added to the portfolio of the insurer, as otherwise, depending on how the solvency requirement is specified, the ratio of premium income to capital or the ratio of risk exposure to capital exceeds a given size.[2] Another argument why a firm might not serve the whole market could be the mere size of the firm, the number of employees, the size of the computer system, etc., which makes it difficult to process more than a given number of policies.

Under capacity constraints, our main result is that the Rothschild–Stiglitz (RS) contracts are equilibrium contracts, even if they do not form an equilibrium in the original game. For an illustration, consider pooling contracts which were used to destabilize the RS contracts in the original paper. If the new contract is supposed to also attract low-risk types and if the proposer intends to realize a strictly positive profit, the coverage of the low-risk type must increase compared to the RS contract. Observe now that the high-risk type's incentive

[1] Recently, an evolutionary model of the insurance market has been proposed (Ania et al., 1998). If firms copy profit making contracts and experiment with their own contracts locally, the unique long run outcome is that all firms offer the Rothschild–Stiglitz contracts.

[2] Buying reinsurance is a means to increase capacity, however at a cost. If these costs are not negligible, our result remains to hold. See Berger et al. (1992) for an account of the spillover effects from the reinsurance market to the primary insurance market in the liability insurance crisis in the mid-1980s.

compatibility constraint is binding under the RS allocation and that he benefits strictly more from an increase in the coverage (due to the single-crossing property). Hence, the high-risk type's utility will increase strictly more under the deviating contract. As a consequence, high risks are prepared to endure a more severe rationing in case a firm's capacity constraint becomes binding. This intuitive property can now be applied to make any deviating offer, even with a menu of contracts, unprofitable as it simply will not assure the firm the desired mix of types.[3]

The insurance sector is only one example of a market with adverse selection, where capacity constraints might be useful to guarantee existence of an equilibrium in pure strategies. Similar reasoning would apply to the banking market, where customers with good and bad projects demand loans (Stiglitz and Weiss, 1981; Bester, 1985). Also venture capitalists suffer from adverse selection, if entrepreneurs are of different quality. In both these markets, capacity constraints arise in the form of limited capital, which prevents a single bank/venture capitalist to serve the whole market. Also in the labor market, where workers are of either high or low productivity, adverse selection can become relevant (see e.g. Mas-Colell et al., 1995, Chapter 13; Landers et al., 1996). Here limited capacity would be the finite number of slots each individual firm has to fill.

The rest of this paper is organized as follows. Section 2 introduces the model, which is solved in Section 3. We conclude in Section 4.

2. The model

The insurance market is populated by $F = \{1, \ldots, F\}$ risk-neutral firms, each with a fixed (integer) capacity of $k_f > 0$. On the demand side there are $N = \{1, \ldots, N\}$ customers. We assume that $k_f < N$ holds for all $f \in F$ and that $\sum_{f \in F'} k_f \geq N$ holds for all sets $F' = F/\{f\}$ with $f \in F$. Hence, no single firm can serve the whole market, while all but one firm together are sufficient to serve all customers.[4] The customers face the risk of loosing a sum S. An individual may

[3] This paper is not the first to use rationing and congestion in markets with adverse selection. Congestion has also been applied as an equilibrating device by Inderst and Müller (2001) in markets for lemons where there is no contractual sorting variable. Gale (1992, 1996) considers a Walrasian approach where each contract represents a separate trading environment. In contrast to us he takes a one-shot perspective where rationed individuals cannot trade subsequently. Moreover, instead of solving a fully defined game, he imposes market clearing conditions and uses refinements to reduce the multiplicity of equilibria.

[4] This assumption is a simplification of the capacity problem due to limited capital. Given any amount of capital, k_f will in general depend on the form of the contracts offered and the types of the insured buying these contracts. However, we conjecture that making k_f an endogenous variable will not change the result. The important assumption we require is that there are enough firms to serve the least-cost separating contracts without incurring capacity constraints, while any single firm will run into severe capacity problems if it tries to serve a significant fraction of the market.

have either a high-risk probability of π_H or a low-risk probability $\pi_L < \pi_H$. The respective risk type of customer n is denoted by $t_n \in T = \{L, H\}$. All individuals have the same Von Neuman–Morgenstern utility function $U(w)$. Below we will invoke a further assumption on the severeness of the capacity constraint to support an equilibrium.

The game is modeled as follows:

Stage 0: The risk type of each individual is chosen by nature. Each person has the chance of γ_H $(1-\gamma_H)$ to be a high (low)-risk type. This draw is taken independent across individuals, so that overall the expected number of high risks is $\gamma_H N$.

Stage 1: Firm $f, f = 1, \ldots, F$, sets a menu of contracts $\{\omega_1^f, \omega_2^f, \ldots, \omega_k^f\}$ where each ω_l^f specifies a premium P_l^f and a net indemnity payment I_l^f. Each firm cannot offer more than K contracts.

Stage 2: Each customer either chooses a firm f and a contract ω_l^f or decides not to visit any firm, in which case he exits the market irrevocably.

If the number of customers choosing firm f, which we denote by n_f, does not exceed k_f, then each customer obtains his desired contract. The game ends for these customers. The expected utility, if the chosen contract ω specifies the premium P and the net indemnity I, is abbreviated by

$$U_t^{\mathrm{E}}(\omega) = (1 - \pi_t)U(w - P) + \pi_t U(w - S + I),$$

where the risk type t is either H or L. If n_f is larger than k_f, the firm runs into capacity constraints and has to apply a rationing scheme. We assume that rationing occurs randomly over all applicants.[5] Hence, each individual is rationed with probability $\rho_f = 1 - \min\{1, k_f/n_f\}$. If some customers do not obtain a contract, Stage 3 follows.

Stage 3: The rationed customer can either choose to remain uninsured and exit the market or he can visit any other firm f' which still has free capacity available and pick a contract $\omega_l^{f'}$ from the menu of respective contracts. The search for a new firm is costly. We measure these costs in utility units and assume that approaching another firm results in costs $u > 0$, which are independent of the risk type.[6] If the customer has chosen a firm f' where again demand exceeds supply, and he did not obtain a contract, Stage 3 is repeated. If all customers are either served or have exited the market, the game ends.

[5] This rationing scheme can be motivated by assuming that customers visit the insurer one after the other in a random order. Additionally, it can be shown that our results still apply if we allow firms to offer more complicated mechanisms, which e.g., give different contracts a different priority when the firm must ration. Indeed, this follows immediately from incentive compatibility. Allowing for such mechanisms on a general level would, however, severely complicate the notation without adding insights.

[6] Note that we assume that the first visit is free. Our results continue to hold if the costs of a first visit do not exceed the difference between the utility derived by the low-risk type under his RS contract and his utility without insurance.

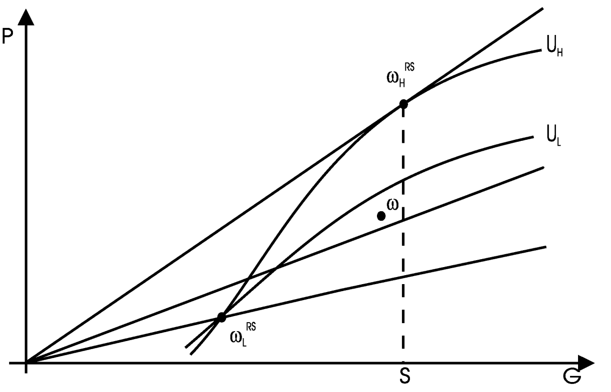

Fig. 1. Rothschild–Stiglitz contracts.

3. RS contracts as equilibrium contracts

Let us first recall the pair of RS contracts, which are the least-cost separating contracts. They are uniquely derived by the following conditions.

The RS contract ω_H^{RS} for the high-risk type specifies full coverage with $I_H^{RS} = S - P_H^{RS}$, while the premium is determined by the zero-profit condition for the risk-neutral insurer as $P_H^{RS} = \pi_H S$. We denote the realized (expected) utility by $U_H^{RS} = U_H^E(\omega_H^{RS})$.

The RS contract ω_L^{RS} for the low-risk type is chosen to maximize $U_L^E(\omega)$ subject to the firms' participation constraint $(1 - \pi_L)P \geq I\pi_L$ and the high-risk type's incentive compatibility constraint $U_H^{RS} \geq U_H^E(\omega)$. It can be shown that both constraints become binding, while the contract provides less than full coverage with $I_L^{RS} < S - P_L^{RS}$. We denote the utility by $U_L^{RS} = U_L^E(\omega_L^{RS})$. Denote $U_t^0 = U_t^E(0,0)$ for the expected utility without insurance coverage. It is clear that U_L^{RS} exceeds the expected utility from staying uninsured U_L^0.

The contracts are shown in Fig. 1.

On the two axes are the premium and the gross indemnity $G = I + P$. Full insurance is obtained at the vertical line which is given by $G = S$. The three straight lines denote the zero-profit lines, if only high risks (the top line), only low risks (the bottom line), or a mixture of the two risks buy such a contract. Note that with the pooling line as drawn, a contract like ω would be preferred by everyone and would be strictly profitable if it can assure a fair selection.

Throughout this paper we focus on (subgame-perfect) equilibria where firms play pure strategies in Stage 1. As the number of firms is finite, the market will always clear after a finite number of repetitions of Stage 3. Hence, once contracts are in place, we face a finite (continuation) game, which therefore has always an equilibrium in (possibly mixed) strategies.

As indicated in the introduction, the novel feature of our approach to markets with adverse selection is that we abandon the particular specification in the Rothschild–Stiglitz environment that a deviator can either serve the whole market or can always assure himself a fair selection of risks. For our result to hold we require that the capacity problem is sufficiently severe. This contains three elements:

First, the search costs u should not be negligible. Suppose otherwise: For search costs of zero there are no direct costs of being rationed by a firm which makes a deviating offer. Thus again, everyone might try to obtain such a contract which in turn makes the distribution of risks equal to the distribution in society.

Second, search costs u should not be too high. Otherwise, consumers only have one possibility to search around. If they do not receive a contract at their first choice, they prefer to stay uninsured. As high risks suffer more from being uninsured, by offering a contract which is going to be rationed firms might deter high risks from choosing this contract.

Third, the capacity of a single firm must be sufficiently low compared to the economy. If not, a firm would by offering a deviating contract attract maybe not all of the population, but nearly all. This would make the risk distribution more and more favorable.

We next provide a formalization of these assumptions. Assumption A.1 assures that rationed players prefer to newly approach an insurer to sign their respective RS contract instead of staying uninsured.

Assumption A.1.

$$U_t^{\mathrm{RS}} - u > U_t^0 \quad \text{for } t \in \{L, H\}. \tag{1}$$

We next derive a combined requirement which puts a lower bound on the search costs u, while assuring that capacity is sufficiently dispersed. Let $k^{\mathrm{M}} = \max_{f \in F} k_f$ be the maximum capacity of a single firm. Recall next that at Stage 0 the type of an individual is determined randomly. Therefore, the true distribution of types in the population is unknown to all market participants.[7] Suppose that individual n expects that all high-risk individuals choose to visit a particular firm f with the maximum capacity k^{M}, while all low-risk individuals pick different firms. This allows us to calculate an expected rationing probability for individual n if he chooses firm f as well. We denote this probability by ρ^{M}. [8] Let U_H^{P} be the expected utility of a high risk under his preferred contract with

[7] By the law of large numbers this uncertainty vanishes as the number of individuals increases.

[8] Formally, suppose that there are N_H high risks visiting firm f in addition to individual i. This gives rise to the rationing probability $\rho^{\mathrm{M}}(N_H) = 1 - \min\{1, k^{\mathrm{M}}/(N_H + 1)\}$. The probability that there are exactly m high risks in the population is equal to $\Pr(N_H = m) = \binom{N-1}{m}\gamma_H^m(1-\gamma_H)^{N-1-m}$, such that $\rho^{\mathrm{M}} = \sum_{m=0}^{N-1} \Pr(N_H = m)\rho^{\mathrm{M}}(N_H)$.

the low-risk type fair premium, i.e. the utility from a contract ω satisfying $(1-\pi_L)P = \pi_L I$, where I is chosen to maximize $U_H^{\mathrm{E}}(\omega)$.

Assumption A.2.

$$(1-\rho^{\mathrm{M}})U_H^{\mathrm{P}} + \rho^{\mathrm{M}}(U_H^{\mathrm{RS}} - u) < U_H^{\mathrm{RS}}. \tag{2}$$

Assumption A.2 implies that if individual n is a high risk, he is better off buying his RS contract than queuing at a firm together with all other high risks for the insurance contract at the low-risk premium, and in case he is unlucky in the draw, buying the RS contract in the next round. That is, the expected rationing at one single firm is sufficiently severe if all high risks are expected to turn up.

Note that Assumption A.2 holds for a given level of u if the number of customers N is sufficiently large and if the capacity is sufficiently dispersed among firms. To see this, consider a sequence of economies where the expected fraction of high-risk types $0 < \gamma_H < 1$ is kept fixed together with the maximum capacity k^{M} of a single insurer. If the size of the economy N increases, ρ^{M} will converge to 1, so that (2) is satisfied for any positive costs $u > 0$. Thus a limit result holds: For any type distribution and capacity per firm, if the economy is replicated sufficiently often, Assumption A.2 will hold.

We can now prove our main result.

Proposition. If Assumptions A.1 and A.2 hold, there exists an equilibrium where any customer $n \in N$ obtains his respective RS contract in Stage 2.

Proof. We claim that one possible equilibrium strategy of firms is to offer the two RS contracts each. To show this, we first need to discuss the equilibrium strategy of the customers. Let h be a search history. A strategy of a customer is a map from history h into its action space, consisting out of the choice to visit some firm and pick a particular contract or to exit the market.

Consider the case where all firms have offered the two RS contracts. After any search history h, let $m(h)$ be an ordered set of sellers who still have unused capacity $k_f(h)$. Let $n(h)$ be an ordered set of buyers who have not yet found contracts. Define $f(i,h)$ as the smallest index $f \in \{1,\dots,m(h)\}$ satisfying $\sum_{f'=1}^{f} k_{f'}(h) \geq i$. The equilibrium strategy of customer $i \in \{1,\dots,n(h)\}$ is to choose firm $f(i,h)$ and to pick the RS contract according to his type. By doing so the customers always solve their coordination problem and in equilibrium, they are all served in Stage 2. No customer has an incentive to deviate, as all are served with the best possible contract on offer and no rationing occurs. Moreover, all firms make zero profit with these contracts. To support the resulting allocation as an equilibrium, it remains to specify strategies if a single firm deviates to a different menu of contracts in order to show that there are no profitable deviations.[9]

Consider now the case where all firms offer the two RS contracts apart from firm $\bar{f}$, whose offer if given by the contract menu $\{\bar{\omega}_1^{\bar{f}}, \bar{\omega}_2^{\bar{f}}, \dots, \bar{\omega}_k^{\bar{f}}\}$.[10]

Rationing, if it occurs, is the same for all customers of a single firm. Therefore it suffices to concentrate on the two contracts which are optimal for each type. Let $\bar{\omega}_H$ be the contract out of $\{\bar{\omega}_1^{\bar{f}}, \bar{\omega}_2^{\bar{f}}, \dots, \bar{\omega}_k^{\bar{f}}\}$ which the high-risk type prefers, and define $\bar{\omega}_L$ for the low-risk type analogously. Both contracts may well be the same. We consider four cases.

Case 1: We begin with the most interesting case, where $U_L^{\mathrm{E}}(\bar{\omega}_L) \geq U_L^{\mathrm{RS}}$ and $(1-\pi_L)\bar{P}_L - \pi_L \bar{I}_L > 0$. The deviating offer is supposed to attract (some) low-risk types, with whom a profit can be made. As can be seen in Fig. 1, this can only hold if this offer involves more coverage $\bar{I}_L > I_L^{\mathrm{RS}}$ and also a higher premium $\bar{P}_L > P_L^{\mathrm{RS}}$ than the RS contract for the low-risk type. Due to incentive compatibility it follows that $U_H^{\mathrm{E}}(\bar{\omega}_H) > U_H^{\mathrm{RS}}$. We claim that in this case there exists an (continuation) equilibrium where only high-risk types turn up at the deviator.

We construct the continuation equilibrium such that all customers visit $\bar{f}$ in Stage 2 with a probability ϕ if and only if they are of the high-risk type. With probability $1-\phi$ a high-risk type turns to a firm where he obtains his RS contract with probability 1. Low-risk types choose a firm other than $\bar{f}$ with probability 1 in order to buy their RS contract. Formally, the equilibrium strategies are as follows: After any search history h, let $m'(h)$ be an ordered set of *non-deviating* sellers who still have unused capacity $k_f(h)$. Let $n(h)$ be an ordered set of buyers who have not yet found contracts. Define $f'(i,h)$ as the smallest index f satisfying $\sum_{f'=1}^{f} k_{f'}(h) \geq i$. In Stage 2, the strategy of customer i is to go to firm $f'(i,h)$ and choose his RS contract, if he is of the low-risk type. If he is of the high-risk type, his strategy is to go to firm $f'(i,h)$ with probability $(1-\phi)$ and to firm $\bar{f}$ with probability ϕ and choose the appropriate contracts. For any further stage, and after any search history, if firm $\bar{f}$ has no free capacity, customer i will go to firm $f'(i,h)$ and choose the RS contract according to his type. If firm $\bar{f}$ still has free capacity, take any continuation equilibrium from there on, which we know does exist.[11]

We will now show that the above mentioned ϕ is uniquely determined. For a given ϕ calculate the expected rationing probability $\rho(\phi)$ for an individual who

[9] We know that a continuation equilibrium exists for every contract offers. Therefore, we can take any of these continuation equilibria if more than one firm offer contracts different from the two RS contracts.

[10] In the standard discussion by Rothschild and Stiglitz, a pooling contract as shown in Fig. 1 was the only possible profitable deviation being analyzed. However, in our model, we must consider deviations with a menu of contracts.

[11] For the equilibrium analysis this continuation equilibrium is irrelevant. Either all customers are served in Stage 2, or they are rationed, in which case firm $\bar{f}$ has no free capacity.

contemplates choosing firm $\bar{f}$

$$\rho(\phi) = 1 - \sum_{m=0}^{N-1} \binom{N-1}{m} (\phi\gamma_H)^m (1-\phi\gamma_H)^{N-1-m} \min\{1, k_{\bar{f}}/(m+1)\}.$$

Note that $\rho(\phi)$ is strictly increasing in ϕ.

Define by $U_t^{\mathrm{D}}(\bar{\omega}_t, \rho) = (1-\rho)U_t(\bar{\omega}_t) + \rho(U_t^{\mathrm{RS}} - u)$ the expected utility of a risk type $t \in \{L, H\}$ who chooses the deviating firm, and, if rationed, buys the RS contract in the next round. There exists a unique $0 < \phi < 1$ satisfying

$$U_H^{\mathrm{D}}(\bar{\omega}_H, \rho(\phi)) = U_H^{\mathrm{RS}}. \tag{3}$$

$U_H^{\mathrm{D}}(\bar{\omega}_H, \rho)$ is strictly decreasing and continuous in ρ, while $\rho(\phi)$ is strictly increasing and continuous in ϕ. Moreover, it holds that $U_H^{\mathrm{D}}(\bar{\omega}_H, \rho(0)) > U_H^{\mathrm{RS}}$, while $U_H^{\mathrm{D}}(\bar{\omega}_H, \rho(1)) < U_H^{\mathrm{RS}}$. The latter assertion follows directly from Assumption A.2, by noting that $\rho(1) = \rho^{\mathrm{M}}$, which was defined before invoking Assumption A.2.

It now remains to show that, given the uniquely chosen $0 < \phi < 1$ satisfying (3), it is indeed optimal for any low-risk type not to visit $\bar{f}$. Recall first that by Assumption A.1 and our specification of continuation strategies, an individual implements his RS contract at Stage 3 if he was rationed when visiting $\bar{f}$ at Stage 2. It therefore remains to show that $U_L^{\mathrm{D}}(\bar{\omega}_L, \rho(\phi)) \leq U_L^{\mathrm{RS}}$. This again holds if $U_H^{\mathrm{E}}(\bar{\omega}_H) - U_H^{\mathrm{RS}}$ is not smaller than $U_L^{\mathrm{E}}(\bar{\omega}_L) - U_L^{\mathrm{RS}}$, which after substitution of $U_H^{\mathrm{E}}(\omega_L^{\mathrm{RS}}) = U_H^{\mathrm{RS}}$ and $U_H^{\mathrm{E}}(\bar{\omega}_H) \geq U_H^{\mathrm{E}}(\bar{\omega}_L)$ holds if

$$U_L^{\mathrm{E}}(\bar{\omega}_L) - U_L^{\mathrm{E}}(\omega_L^{\mathrm{RS}}) \leq U_H^{\mathrm{E}}(\bar{\omega}_L) - U_H^{\mathrm{E}}(\omega_L^{\mathrm{RS}}). \tag{4}$$

But this must be satisfied even strictly due to the single-crossing property, $\bar{I}_L > I_L^{\mathrm{RS}}$, and $\bar{P}_L > P_L^{\mathrm{RS}}$. [12]

Case 2: $U_L^{\mathrm{E}}(\bar{\omega}_L) \leq U_L^{\mathrm{RS}}$ *and* $U_H^{\mathrm{E}}(\bar{\omega}_H) \leq U_H^{\mathrm{RS}}$. We specify the strategy of the customers as follows: Define $m'(h)$, $n(h)$, and $f'(i,h)$ as before. After any search history customer i will go to firm $f'(i,h)$ and choose the RS contract according to his type. Recall that $N-1$ firms are sufficient to serve the whole market. Therefore with this strategy, all customers obtain their RS contract and no rationing occurs.

Case 3: $U_L^{\mathrm{E}}(\bar{\omega}_L) \geq U_L^{\mathrm{RS}}$ *and* $(1-\pi_L)\bar{P}_L - \pi_L \bar{I}_L < 0$. We consider two subcases. In the first subcase $\bar{\omega}_H$ is such that $(1-\pi_H)\bar{P}_H - \pi_H \bar{I}_H < 0$. Here we can again take any continuation equilibrium. It is sufficient to notice that if the deviating firm serves a customer, it will make a loss.

In the second case $(1-\pi_H)\bar{P}_H - \pi_H \bar{I}_H > 0$ holds. This implies that the high-risk type prefers his RS contract to $\bar{\omega}_H$. In this case we specify the strategies

[12] Note that inequality (4) is equivalent to $(\pi_H - \pi_L)[U(w - \bar{P}_L) - U(w - P_L^{\mathrm{RS}})] + (\pi_L - \pi_H)[U(w - S + \bar{I}_L) - U(w - S + I_L^{\mathrm{RS}})] \leq 0$.

of the customers similar to case 1, but now the low-risk type chooses firm $\bar{f}$ with some probability ϕ at Stage 2. By doing so, only low-risk types will go to the deviating firm which makes a loss.

Case 4: $U_L^{\mathrm{E}}(\bar{\omega}_L) \leq U_L^{\mathrm{RS}}$ *and* $U_H^{\mathrm{E}}(\bar{\omega}_H) > U_H^{\mathrm{RS}}$. It is clear that $(1-\pi_H)\bar{P}_H - \pi_H \bar{I}_H < 0$ must hold. The equilibrium strategy in this case can be defined as in case 1, so that only high-risk types go to the deviating firm.

In all four cases the deviating firm either attracts no customer, or customers with whom it makes a loss. Thus deviating is not profitable. □

The proposition has a simple intuition which comes out most clearly if we suppose that a deviating offer intends to attract a mixed set of types. To realize a profit with low-risk types, the offer must specify a higher coverage than the RS contract designated for these types. By the single-crossing property, high-risk types gain more under the new offer than low-risk types. They are thus prepared to accept a higher (expected) rationing probability than low-risk types who would not apply for the deviating offer at this level of congestion.

Standard analysis of competitive insurance markets under adverse selection has shown that if the ratio of high-risk to low-risk types is small, no equilibrium in pure strategies exists. Contrary to this result, the proposition and the remarks we made before on Assumption A.2 imply that for *any* distribution of risk types we can find an economy in which capacity is sufficiently dispersed among firms such that an equilibrium where all insurers offer the same contracts exists.

In a previous version of this paper we have shown that the analogue of the proposition holds if firms can potentially serve all customers, but face the risk of bankruptcy. If acquisition of further contracts increases the bankruptcy risk, then more customers will make each policy less attractive to a potential insured.[13] This in effect restricts the number of applicants for any contract offer. Therefore also in this setup one can devise equilibrium strategies such that only high risks turn up at a deviating firm.

4. Conclusion

We showed how an existence result in pure strategies for firms can be obtained in an insurance market if there is limited capacity to write contracts, which moreover is sufficiently dispersed among the competing firms. A family of (least-cost separating) Rothschild–Stiglitz contracts cannot be destabilized. Any deviating contract which is designed to make positive profits with low-risk customers will be (relatively) more attractive to high-risk types. As customers take into account capacity constraints and the expected degree of congestion

[13] A more detailed model of insurers facing bankruptcy is given in Rees et al. (1999).

prevailing at the deviating firm, the offer will fail to attract a sufficient number of low-risk types to become profitable.

The derived equilibrium is, however, not unique under capacity constraints. In particular, the possibility of coordination failure among customers allows to support equilibria where firms make positive profit even though each individual firm is dispensable. Under complete information this has been shown by Peters (1984). Inderst (1999) discusses (under complete information) two ways to make prices converge to the unique 'competitive' outcome. First, if the number of buyers increases while the ratio of capacity to buyers and the number of firms stay constant, coordination is facilitated as buyers predict more accurately the congestion prevailing at an individual seller. Second, coordination failure becomes less serious if the costs of visiting another seller decrease. It remains to be seen how these arguments can be exploited to restrict the set of equilibria for the insurance model analyzed in this paper.[14]

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