

Multi-issue Bargaining with Endogenous Agenda*

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The first part of this paper shows that in a noncooperative bargaining model with alternating offers and time preferences the timing of issues (the agenda) matters even if players become arbitrarily patient. This result raises the question of which agenda should come up endogenously when agents bargain over a set of unrelated issues. It is found that simultaneous bargaining over “packages” should be a prevailing phenomenon, but we also point to the possibility of multiple equilibria involving even considerable delay. *Journal of Economic Literature* Classification Number: C78.

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1. INTRODUCTION

Envisage a situation where two parties bargain over a set of projects (issues). For each project they can determine whether it will be implemented and possibly its specific realization. Assume that projects are separable and that each subset of projects is immediately implemented after partial agreement on this set. Players have no access to any medium of exchange such as monetary side payments to facilitate bargaining. This is a realistic assumption in many institutional settings where “bribes” either are illegal or conflict with (ethical) conventions. Bargaining between political parties or between departments of an organization may satisfy this assumption. Finally, suppose that bargaining takes place in the Rubinstein (1982) extensive form with alternating offers and time preferences. Define now an agenda as an ordered partition of the set of projects such that bargaining proceeds simultaneously over projects in a given set of the partition, while bargaining on a specific set can only begin after agreement on all

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preceding sets. Hence, an agenda determines the timing of projects on the bargaining table. The first part of this paper derives equilibrium payoffs if an exogenously given agenda requires that bargaining proceed sequentially, simultaneously, or separately (in independent teams) over the set of projects. The agenda can have a marked impact on payoffs, which—in contrast to a similar result reported by Fershtman (1990)—does not vanish as players become increasingly patient. First, bargaining simultaneously over a set of projects can improve efficiency by creating trading opportunities across issues. Second, changing the agenda may have a distributive effect, and players may therefore prefer different agendas.

The impact of the agenda raises the question of which agenda should come up endogenously. It is shown in the second part that the answer depends on the types of projects and on the players' access to a randomization device (lottery). If all projects are mutually beneficial under at least one realization, the game always ends immediately with simultaneous agreement over all projects. However, if the bargaining set contains strictly controversial projects, multiple equilibria involving even considerable delay may occur if players cannot bargain over lotteries.

The role of the agenda has already been addressed in both axiomatic and strategic models of bargaining. For the limited purpose of this paper consideration is restricted to noncooperative contributions.¹ In particular, the setting is compared to Fershtman (1990) and Busch and Horstmann (1996, 1997a).² Fershtman considers alternating-offer bargaining over two linear projects (linear bargaining frontiers) by two players with time preferences and additively separable utility functions. He analyzes sequential agendas where the realization of utilities is postponed until both projects are accepted. He shows that a player prefers that the first project be least important to him but most important to the opponent. However, as bargaining frictions vanish, the impact of the agenda disappears. This is different in the setting proposed in this paper, where utilities from agreed projects are immediately realized.

A similar change to the model of Fershtman has been adopted by Busch and Horstman (1996, 1997a). As their work covers related issues, the main differences are briefly stated. Busch and Horstman (1996) consider the case with an exogenous agenda. In contrast to our analysis in Section 2, they treat only the two-project case with linear bargaining frontiers. Apart from losing generality, this restriction does not allow clear extraction of the two

¹Ponsati and Watson (1996) provide an exhaustive analysis in cooperative terms.

²An older unpublished paper on the role of the agenda in strategic bargaining is Herrero (1989). We also restrict consideration to models with complete information. Issue-by-issue negotiation is used to signal bargaining power in Bac and Raff (1996) and Busch and Horstmann (1997b, c).

basic impacts of the agenda. In another paper primarily designed to explain incomplete contracts, Busch and Horstman (1997a) partially “endogenize” a bargaining agenda in a simple setting with two linear projects by introducing a separate bargaining round over the agenda, which precedes bargaining over projects. In contrast, in Section 3 of this paper players bargain over projects without an ex ante agreed agenda. The timing of accepted projects becomes thus truly endogenous. Finally, confining the bargaining set to two linear projects does not allow the difference between bargaining with mutually beneficial and strictly controversial projects to be revealed.

The rest of this paper is organized as follows. Section 2 solves for equilibria under various exogenous agendas. Two examples illustrate how the agenda affects the bargaining outcome. Motivated by this result, Section 3 solves a model with endogenous agenda. Section 4 summarizes the results. Some proofs are relegated to an Appendix.

2. THE GAME WITH AN EXOGENOUS AGENDA

2.1. *Equilibria with Various Exogenous Agendas*

In what follows I consider bargaining games with two players called A and B. Players can realize a finite set of projects denoted by P_i with $i \in I$ where $I = \{1, \dots, |I|\}$ is an index set of finite size $|I|$. Each project P_i is characterized by a strictly decreasing, concave, and continuous function $v_i(U_i)$ which maps $U_i \in [\underline{U}_i, \bar{U}_i]$ onto $V_i \in [\underline{V}_i, \bar{V}_i]$.³ Denote the inverse of this function by $u_i(V_i)$. Throughout the paper lowercase letters u and v relate to functions, while capital letters U and V denote realizations. The respective values of U_i and V_i represent the von Neumann–Morgenstern (vNM) utilities of players A and B if a specific realization of project P_i is chosen. A full characterization of the utility functions is provided below. Players have no opportunity costs from implementing a specific project and they obtain zero future utility if they break up bargaining at any point of time.

The bargaining frontier $v_i(U_i)$ may arise in the following way. Suppose that parties bargain both over the implementation of a given project and over its specific realization with respect to the choice of a set of underlying variables. Now $v_i(U_i)$ equals the maximum utility player B can realize if he or she can freely choose these parameters under the constraint that A's utility is at least U_i . Note that we admit (degenerate) projects with $\underline{U}_i = \bar{U}_i$ and $\underline{V}_i = \bar{V}_i$. Define next two classes of projects.

³Note that monotonicity and concavity already imply continuity on $U_i \in [\underline{U}_i, \bar{U}_i]$.

DEFINITION. A project P_i is called “mutually beneficial” if there exists a realization $v_i(U_i)$ with $U_i \in [\underline{U}_i, \bar{U}_i]$ such that $U_i > 0$ and $V_i > 0$. A project is called “strictly controversial” if there is no realization $v_i(U_i)$ with $U_i \in [\underline{U}_i, \bar{U}_i]$ such that $U_i \geq 0$ and $V_i \geq 0$, while there exists a realization $v_i(U_i)$ with $U_i \in [\underline{U}_i, \bar{U}_i]$ such that $U_i > 0$ or $V_i > 0$.

Hence, for mutually beneficial projects there exist realizations such that both players strictly prefer implementation to reaching no agreement on this project. For a strictly controversial project such a realization does not exist. In what follows projects are either mutually beneficial or strictly controversial. Note that the definition of projects differs from the standard notion of a convex and comprehensive bargaining set confined to nonnegative pairs of utilities. The bargaining frontiers (outer contours) of these sets admit neither strictly controversial projects nor degenerate mutually beneficial projects. The latter projects become, however, particularly useful to isolate the impact of the agenda below. Moreover, the convexification of the set of feasible utilities is postponed until Section 3.2 where lotteries are introduced.

Utility functions are additively separable, and both players have the same constant and symmetric discount factor $\delta \in [0, 1)$. If a set of projects $I_A \subseteq I$ is accepted and P_i is implemented with the choice (U_i, V_i) at time t_i where $t_i \in \{0, 1, \dots, \infty\}$, then A and B derive the respective utilities $\sum_{i \in I_A} \delta^{t_i} U_i$ and $\sum_{i \in I_A} \delta^{t_i} V_i$.

Attention is now restricted to exogenous agendas which prescribe how bargaining has to proceed. We distinguish three cases.

Separate Bargaining: Time runs in discrete, equidistant periods numbered $t = 0, 1, 2, \dots$. Players announce in period $t = 0$ for each project a pair of representatives who bargain in isolated teams according to the following procedure. Players alternate in making proposals with (the representative of) player A making the first proposal at $t = 0$. A proposal specifies a pair (U_i, V_i) such that $V_i = v_i(U_i)$ with $U_i \in [\underline{U}_i, \bar{U}_i]$. After a proposal is made at time t , the responder can either accept or reject. If he or she accepts, the respective project is carried out immediately and utilities are realized. If he or she rejects, he or she becomes the proposer at time $t + 1$.

Sequential Bargaining: The agenda specifies now a partition of the set I with $|I| > 1$ into $K > 1$ nonempty subsets I_k^P with $\bigcup_{k=1}^K I_k^P = I$. Players bargain simultaneously over projects P_i with $i \in I_k^P$ for each $k \in \{1, \dots, K\}$, while bargaining on projects in I_{k+1}^P has to be preceded by agreement over projects in I_k^P . To be precise, players start to bargain on the set of projects I_1^P at $t = 0$. Offers are thus characterized by a family $\{(U_i, V_i)\}_{i \in I_1^P}$ with $U_i \in [\underline{U}_i, \bar{U}_i]$ and $V_i = v_i(U_i)$ for $i \in I_1^P$. After agreement in period \bar{t} they

realize the respective utilities and proceed to bargain over the set I_2^P in period $\bar{t} + 1$. It is assumed that player A makes the first proposal on each new set of projects.⁴ This procedure is repeated until the last set of projects I_K^P is agreed and implemented. Observe that delaying an agreement for a set of projects $I_{\bar{k}}^P$ with $\bar{k} < K$ delays also the realization of all projects belonging to sets I_k^P with $k > \bar{k}$.

Simultaneous Bargaining: Now the exogenous agenda specifies that players have to bargain over all projects simultaneously. Hence, this case represents a degenerate sequential agenda where $K = 1$.

I look now for subgame perfect equilibria (SPE) under the different agendas and show that in each case equilibrium utilities are uniquely determined. Consider first for a moment an (auxiliary) game where players bargain over any nonempty set $\hat{I} \subseteq I$ and break up after an agreement. (For $\hat{I} = I$ we obtain simultaneous bargaining as defined above.) To solve this game, one has to derive the (composite) bargaining frontier for the set \hat{I} . Define the boundary $\underline{U}(\hat{I}) = \sum_{i \in \hat{I}} \underline{U}_i$, and in analogy $\bar{U}(\hat{I})$, $\underline{V}(\hat{I})$, and $\bar{V}(\hat{I})$. Define a function $v(U, \hat{I})$ which denotes the maximum utility player B can realize under a proposal covering the set of projects \hat{I} and leaving player A with utility U . Define for player A the analogous function $u(V, \hat{I})$. It is intuitive that some characteristics of the underlying functions $v_i(U_i)$ and $u_i(V_i)$ carry over to both newly defined functions. In fact, $u(V, \hat{I})$ is concave, continuous, and strictly decreasing, while $v(U, \hat{I})$ is the inverse of $u(V, \hat{I})$. By applying the techniques of Rubinstein (1982) it can be shown that the auxiliary game ends in the first period. Though equilibrium utilities are unique, multiple SPE may exist, as pairs of utilities may arise from various realizations of the individual projects in \hat{I} . This is particularly intuitive if \hat{I} contains several projects with a linear frontier of equal slope.⁵ Moreover, as frictions vanish with $\delta \rightarrow 1$, equilibrium utilities converge to those obtained by applying the (axiomatic) Nash bargaining solution to the composite frontier.

For $\hat{I} = \{i\}$ and $\hat{I} = I$ one can immediately apply these results for the auxiliary game to the case of separate and simultaneous bargaining. In contrast, under a sequential agenda bargaining over a nonfinal set I_k^P with $k < K$ has to take additional account of players' (continuation) utilities derived from subsequent bargaining over sets I_l^P with $l \in \{k + 1, \dots, K\}$. These discounted utilities take the form of a pair of "windfall utilities"

⁴Note that this will not represent a major restriction as the rest of this paper restricts attention to the limit case $\delta \rightarrow 1$ where the first-mover advantage disappears.

⁵Note that in Rubinstein's original proof the bargaining frontier contains for each player a realization over which he or she is completely patient. With discounting this requirement is only satisfied if the frontier contains the choices $(0, V)$ and $(U, 0)$, which may not hold in our case.

(U^*, V^*) with surely $U^* \geq 0$ and $V^* \geq 0$, which are “realized” together with the implementation of the current set I_k^P . The relevant bargaining frontier is then obtained by a simple translation:

$$\begin{aligned} v^*(U, \hat{I}, U^*, V^*) &= v(U - U^*, \hat{I}) + V^* \text{ defined on} \\ &U \in [\underline{U}(\hat{I}) + U^*, \bar{U}(\hat{I}) + U^*] \\ u^*(V, \hat{I}, U^*, V^*) &= u(V - V^*, \hat{I}) + U^* \text{ defined on} \\ &V \in [\underline{V}(\hat{I}) + V^*, \bar{V}(\hat{I}) + V^*]. \end{aligned}$$

Apply now the Nash bargaining solution to the frontier $v^*(U, \hat{I}, U^*, V^*)$ by maximizing the Nash product $U \cdot v^*(U, \cdot)$ over the feasible set of utilities. Strict monotonicity and concavity ensure the existence of a unique choice of utilities denoted by $U^N(\hat{I}, U^*, V^*)$ and $V^N(\hat{I}, U^*, V^*)$. Finally, denote (for given δ) the always unique equilibrium utilities under a separate (i.e. “parallel”) agenda by (U^P, V^P) , under a simultaneous (i.e., “composite”) agenda by (U^C, V^C) , and under a sequential agenda by (U^S, V^S) . The following proposition is proved in the appendix.

PROPOSITION 1. *The following results hold with exogenous agendas:*

(i) *Separate bargaining: The game has a unique SPE and ends in the first period. Equilibrium utilities for $\delta \rightarrow 1$ are uniquely characterized by $U^P = \sum_{i=1}^{|I|} U^N(\{i\}, 0, 0)$ and $V^P = \sum_{i=1}^{|I|} V^N(\{i\}, 0, 0)$.*

(ii) *Simultaneous bargaining: The game ends in the first period, there exists at least one SPE, and all SPE yield the same utilities. Equilibrium utilities for $\delta \rightarrow 1$ are uniquely characterized by $U^C = U^N(I, 0, 0)$ and $V^C = V^N(I, 0, 0)$.*

(iii) *Sequential bargaining: The set of SPE is not empty. In each SPE the set of projects I_k^P is accepted as soon as possible, i.e., in $t = k - 1$. Equilibrium utilities are unique and for $\delta \rightarrow 1$ derived by recursively applying the Nash bargaining solution.⁶*

2.2. Isolating the Effects of Exogenous Agendas

This section uses two simple examples to isolate two effects of the agenda. The first effect is purely distributive and illustrated by combining a linear and a degenerate project. By combining two linear projects with different slopes the second example shows how simultaneous bargaining

⁶Define for the last bundle I_k^P the utility $U_k^S = U^N(I_k^P, 0, 0)$ and for all other bundles with $k \in \{1, \dots, K - 1\}$ recursively $U_k^S = U^N(I_k^P, U_{k+1}^S, V_{k+1}^S)$. Defining V_k^S similarly, one obtains $U^S = U_1^S$ and $V^S = V_1^S$.

improves efficiency. To highlight the difference from Fershtman (1990), attention is restricted to the case where $\delta \rightarrow 1$.

Assume first that $|I| = 2$ with a degenerate P_1 represented by a single realization with $U_1 = a$ and $U_2 = b$ where $a, b > 0$. P_2 is linear and characterized by $V_2 = g - rU_2$ with $g, r > 0$, $U_2 \in [0, \frac{g}{r}]$, and $V_2 \in [0, g]$. As $(\frac{g}{2r}, \frac{g}{2})$ maximizes the Nash product for P_2 , Proposition 1 yields $U^P = a + \frac{g}{2r}$ and $V^P = b + \frac{g}{2}$ for separate bargaining. For simultaneous bargaining the composite bargaining frontier becomes $v(U, I) = g + b - r(U - a)$ for $U \in [a, a + \frac{g}{r}]$. Maximizing $U \cdot v(U, I)$ yields from Proposition 1

$$(U^C, V^C) = \begin{cases} (a, b + g) & \text{if } ar > b + r \\ \left(\frac{g + b + ar}{2r}, \frac{g + b + ar}{2} \right) & \text{if } b + r \geq ar \geq b - r \\ \left(a + \frac{g}{r}, b \right) & \text{if } ar < b - r \end{cases} \quad (1)$$

Utilities under separate and simultaneous bargaining are only equal for $r = \frac{b}{a}$. For $r > \frac{b}{a}$ player B is strictly better off under simultaneous bargaining, whereas for $r < \frac{b}{a}$ the reverse result holds. Recall that $\frac{b}{a}$ is the ratio of players' utilities in P_1 , while r is the ratio if players bargain separately over P_2 . If $r > \frac{b}{a}$ holds, P_1 can be considered to be more important to player A than to player B if one compares the fixed payoff ratio in P_1 to the slope in P_2 . By linking P_1 to P_2 player A's loss from delay is thus relatively more increased than that of player B compared to separate bargaining over P_2 . This increases B's equilibrium payoff realized from project P_2 . Figure 1 illustrates an extreme example for this case.

With a sequential agenda there are two cases with either $I_1^P = \{1\}$ or $I_1^P = \{2\}$. In case bargaining proceeds first over the degenerate project P_1 , it is immediate that the sequential case equals that with a separate agenda. And backward calculation shows for $I_1^P = \{2\}$ that payoffs are the same as under simultaneous bargaining. This confirms the above intuition for the difference between separate and simultaneous bargaining. As delaying P_2

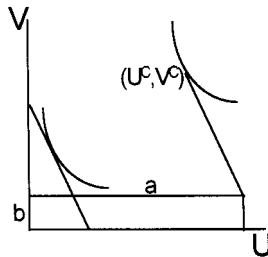


FIGURE 1

also delays P_1 , a player gains bargaining power over the issue P_2 if P_1 is relatively more important to the other party.

To illustrate the impact on bargaining efficiency, consider next the pair of linear projects

$$P_1: v_1(U_1) = g - rU_1 \quad \text{for } U_1 \in \left[0, \frac{g}{r}\right],$$

$$P_2: v_2(U_2) = h - sU_2 \quad \text{for } U_2 \in \left[0, \frac{h}{s}\right].$$

With separate bargaining the players' utilities (U^P, V^P) equal $(\frac{g}{2r}, \frac{h}{2s}, \frac{g}{2} + \frac{h}{2})$. The composite frontier $v(U, I)$ now maximizes $(g + h - rV_1 - sV_2)$ for $U \in [0, \frac{g}{r} + \frac{h}{s}]$ subject to the constraints $U_1 + U_2 \geq U$, $U_1 \in [0, \frac{g}{r}]$, and $U_2 \in [0, \frac{h}{s}]$. For $r = s$, $v(U, I)$ is a straight line connecting the points $(0, g + h)$ and $(\frac{g}{r} + \frac{h}{s}, 0)$. If player B has to make a concession to player A, he is indifferent to whether the additional utility comes from P_1 or P_2 . This becomes different if the slopes of the two frontiers differ. With $r > s$ player B prefers to make a concession on P_2 . With $r \geq s$ the frontier is characterized by

$$v(U, I) = \begin{cases} g + h - sU & \text{if } 0 \leq U < \frac{h}{s} \\ g - r(U - \frac{h}{s}) & \text{if } \frac{h}{s} \leq U \leq \frac{g}{r} + \frac{h}{s} \end{cases}$$

Maximizing $U \cdot v(U, I)$ yields finally

$$(U^C, V^C) = \begin{cases} \left(\frac{g+h}{2s}, \frac{g+h}{2}\right) & \text{if } \frac{g}{h} \leq 1 \\ \left(\frac{h}{s}, g\right) & \text{if } 1 < \frac{g}{h} < \frac{r}{s} \\ \left(\frac{g}{2r} + \frac{h}{2s}, \frac{g}{2} + \frac{hr}{2s}\right) & \text{if } \frac{g}{h} \geq \frac{r}{s} \end{cases}$$

Compare now separate with simultaneous bargaining. If efficiency gains are not possible when $r = s$, both agendas obtain the same outcome. On the other side, for $r > s$ both players strictly prefer simultaneous bargaining in case $1 < \frac{g}{h} < \frac{r}{s}$. This case is illustrated in Fig. 2.

Equilibrium utilities are realized in this case by choosing $U_1 = 0$ and $V_1 = g$ in P_1 , while P_2 is realized with $U_2 = \frac{h}{s}$ and $V_2 = 0$. Note that the "win-win" situation arises only if $g > h$ and $\frac{g}{s} < \frac{h}{s}$ hold simultaneously. If players could choose a single project on which they would have discretionary power, they would in this case select different projects.

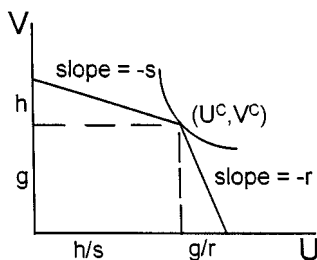


FIGURE 2

3. ENDOGENOUS AGENDA

3.1. *The Case of Mutually Beneficial Projects*

The examples in the preceding section show that the agenda influences the realized utilities even if consideration is restricted to only mutually beneficial projects. Moreover, the distributive effect creates conflicting preferences over the choice of the agenda. It is therefore a natural question which agenda should arise endogenously. Can multiple equilibria arise as players can successfully commit to their most preferred timing of projects? Or does subgame perfection reduce the set of equilibria as in the standard Rubinstein game? To answer this question, proposals are no longer restricted to fixed (sub)sets of projects. Instead, a proposer can freely choose the subset of projects for which he or she makes an offer. As this is a novel feature in the literature, the game is now stated in detail. Again the two players alternate in making proposals with player A starting in $t = 0$. Let h_t denote the history in period t and h'_t the history including the offer of period t . H_t and H'_t denote the respective sets of histories. Moreover, denote the set of remaining (undecided) projects at time t by I'_t with $I'_0 = I$. The strategy of player A (B) is a possibly infinite sequence of functions $\{f_t\}_{t=0}^{\infty}$ ($\{g_t\}_{t=0}^{\infty}$) mapping each time the respective history into the strategy space. In even periods t player A proposes for a (possibly empty) set of projects $I_t \subseteq I'_t$ utilities $\{(U_i^t, V_i^t)\}_{i \in I_t}$ with $U_i^t \in [\underline{U}_i, \bar{U}_i]$ and $V_i^t = v_i(U_i^t)$ for each $i \in I_t$. Acceptance leads to $I'_{t+1} = I'_t \setminus I_t$ while otherwise $I'_{t+1} = I'_t$. And in odd periods f_t maps the history H'_t into the dichotomous action space $\{Y, N\}$.⁷ Player B's strategies are defined analogously. With only mutually beneficial projects the option to break up bargaining before all projects are implemented can again be neglected without loss of generality.

It is found that equilibria with an endogenous agenda basically mirror the case with an exogenous simultaneous agenda.

⁷Though we restrict the notation to pure strategies, Proposition 2 still applies if mixed strategies are allowed.

PROPOSITION 2. *In any SPE with an endogenous agenda and only mutually beneficial projects, the game ends in the first period where all projects are simultaneously accepted. The set of SPE is not empty and equilibrium utilities are uniquely determined. Moreover, utilities are equal to the case with a simultaneous exogenous agenda.*

The proof in the Appendix proceeds in analogy to that of Shaked and Sutton (1984) for Rubinstein's model. At the heart of the proof is the argument that no player can commit to follow a specific nonsimultaneous agenda as the other player could profitably deviate by an acceptable counteroffer. The underlying idea becomes particularly clear in an alternative setting where bargaining over projects is preceded by a first round in which players agree on the agenda.⁸ In this case it is obvious that only utilities which may be realized by a simultaneous agreement constitute the relevant bargaining frontier. To see this, note that for each feasible value of U (V) the respective value $v(U, I)$ ($u(V, I)$) is strictly larger than any utility of which player B (A) could ensure himself or herself under a sequential agenda while leaving the other player with utility U (V). This follows as a simultaneous offer both saves time and makes full use of all valuable trading opportunities across issues. If lotteries were admitted both over the realizations of projects and over the choice of an agenda, the simultaneous frontier would indeed form the outer contour over all realizable utilities. For $\delta \rightarrow 1$ Proposition 2 therefore tells us just the usual story to apply the Nash bargaining solution to the relevant frontier.

Casual observation indeed reveals that parties often prefer to create "packages" of seemingly unrelated issues instead of bargaining issue-by-issue. Proposition 2 provides a formal underpinning for this observation. Though each player may prefer a different agenda, players cannot successfully commit to their optimal choice.

3.2. *The Case with Strictly Controversial Projects*

Extend now the range of admissible projects to strictly controversial projects for which there is no realization yielding nonnegative utilities for both players. Such projects would not be adopted if players bargained separately over all issues. While the restriction to mutually beneficial projects was indeed only for convenience in Section 3, it was crucial for the derivation of Proposition 2. In fact, it turns out that with strictly controversial projects an analogous result holds only if a randomization device is introduced. Otherwise, multiple equilibria may exist with even considerable delay.

⁸I thank L. A. Busch for helpful suggestions on the interpretation of Proposition 2.

I personally feel that agreements on lotteries should rarely occur in the institutional settings mentioned in the Introduction. This view finds support in the literature where nonconvex bargaining problems are increasingly studied. (See Conley and Wilkie (1996) for a summary.) As the hypothesis of vNM preferences is not rejected in most of these contributions, nonconvexities are indeed only explainable by a restricted access to lotteries.

Suppose first that lotteries are not feasible and that I consists of three degenerate projects with unique realizations (U_i, V_i) for P_i where $i \in \{1, 2, 3\}$. Moreover, suppose P_1 is mutually beneficial with $U_1 > 0$ and $V_1 > 0$, while P_2 and P_3 are strictly controversial with $U_i > 0$ and $V_i < 0$ for $i \in \{1, 2\}$. Player A's utilities satisfy

$$\begin{aligned} \delta(U_1 + U_i) &> U_1 \quad \text{for } i \in \{2, 3\}, \\ \delta(U_1 + U_2 + U_3) &> U_1 + U_i \quad \text{for } i \in \{2, 3\}, \end{aligned} \quad (2)$$

while it holds for player B that

$$\begin{aligned} V_1 + V_2 + V_3 &> 0, \\ \delta V_1 &> V_1 + V_i \quad \text{for } i \in \{2, 3\}, \\ \delta(V_1 + V_i) &> V_1 + V_2 + V_3 \quad \text{for } i \in \{2, 3\}. \end{aligned} \quad (3)$$

As long as $V_1 + V_2 + V_3 > 0$ holds, these assumptions are surely satisfied if δ is sufficiently close to 1. With only degenerate projects the strategies of a proposer can be reduced to the proposed set I_t . Note that no further agreement is reached after time t if $\{1\} \notin I'_t$. To avoid ambiguities assume that the game terminates if $\{1\} \notin I'_t$. Consider now for a given $\hat{I} \subseteq I$ the following strategies depending only on the set of remaining projects I'_t : At even t A proposes $I_t = \hat{I} \cap I'_t$, while B accepts I_t if $\{1\} \in I_t$ and $I_t \subseteq \hat{I}$. At odd t B proposes $I_t = \hat{I} \cap I'_t$, while A accepts I_t either if $\{1\} \notin I_t$ or if $\hat{I} \cap I'_t \subseteq I_t$ in case $\{1\} \in I_t$.

Given (2) and (3) these strategies support an equilibrium for

$$\hat{I} \in \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}.$$

These equilibria can now be used to construct equilibria involving considerable delay. To give an example, suppose that until a given period \bar{t} player A proposes $I_t = \{1, 2, 3\}$ while player B rejects and proposes $I_t = \{1\}$, which is again rejected by A. If either player deviates to a different offer in period $t \leq \bar{t}$, that player is "punished" by implementing in $t + 1$ the most unfavorable (continuation) equilibrium involving immediate acceptance of $I_t = \{1\}$ if player A deviates and acceptance of $I_t = \{1, 2, 3\}$ if B deviates. At time $\bar{t} + 1$ play switches to an equilibrium with an intermediate offer ($I_{\bar{t}+1} = \{1, 2\}$ or $I_{\bar{t}+1} = \{1, 3\}$), which is immediately accepted. Agreement

in period \bar{t} is indeed an equilibrium if \bar{t} is sufficiently small and δ sufficiently high. The maximum delay which can be supported depends on the skew between the maximum and the minimum payoffs for each player in the set of equilibria used to punish and reward players.⁹ The example gives rise to the following proposition.

PROPOSITION 3. *If the set of projects contains strictly controversial projects, bargaining with endogenous agenda and without a randomization device can lead to multiple equilibria involving even considerable delay.*

The existence of multiple equilibria is not surprising given the results of van Damme *et al.* (1990) on bargaining with a nonconnected frontier arising from a smallest unit of money. The set of equilibria with immediate agreement can from (2) and (3) be sustained simply due to the physical impossibility of proposing a counteroffer which makes oneself better off while making the other party at least indifferent to realizing his or her original offer with one period delay. Recall that constructing such counteroffers is at the heart of the proof of uniqueness in Rubinstein (1982).

To finally introduce lotteries, assume that players propose additionally for each project in I_t a number $p_i^t \in [0, 1]$ which specifies the “success” probability after implementation of P_i . Players realize the specified utilities if the realization is successful, while otherwise they derive zero utility from this project. Assume also that expected utilities from a set of accepted projects I_A where P_i with $i \in I_A$ is implemented in t_i with realization (U_i, V_i) and success probability p_i equal $\sum_{i \in I_A} p_i \delta^{t_i} U_i$ and $\sum_{i \in I_A} p_i \delta^{t_i} V_i$. With this enlarged strategy space it can now be proved that in any SPE of the game with an endogenous agenda all projects accepted with strictly positive probability are indeed accepted in the first period, and that equilibrium utilities are identical to those with an exogenous simultaneous agenda. Moreover, all projects which are not accepted (or implemented with zero success probability) are strictly controversial, and exactly one player obtains strictly negative utility for any realization of any of these projects. Hence, this set does not allow any further mutually beneficial agreement. As the proof is virtually identical to that of Proposition 2, the result is stated as a corollary.¹⁰

COROLLARY. *In any SPE of the game with an endogenous agenda and a randomization device, all projects implemented with positive probability are*

⁹Exact conditions are stated for a similar result in Fernandez and Glaser (1991). Note that the use of multiple (quasi-)stationary equilibria to construct equilibria with long delay is not new to the literature. A summary of such approaches is provided by Avery and Zemsky (1994).

¹⁰To be precise, Proposition 2 is a special case of this assertion where only mutually beneficial projects are admitted. Note also that the corollary uses implicitly the (straightforward) generalization of Proposition 1 to the case with strictly controversial projects and a randomization device.

accepted in the first period. The set of SPE is not empty, and equilibrium utilities are uniquely determined. Utilities are equal to the case with an exogenous simultaneous agenda. Moreover, the set of projects which are not accepted or which are accepted with zero probability does not allow any further mutually beneficial agreement.

4. CONCLUSION

The paper presents a strategic model of multiissue bargaining with alternating offers and time preferences which allows the impact of the agenda to be analyzed. It is shown that the choice of the agenda may have a distributive effect and may also change the efficiency of bargaining. In contrast to a previous result of Fershtman (1990) for a similar setting, these effects persist if players become increasingly patient. As players may have conflicting preferences over various agendas, the question arises which agenda is chosen endogenously. It is found that players bargain simultaneously over all issues (projects) if these are mutually beneficial. If the bargaining set contains strictly controversial projects, an analogous result holds only if players have access to a randomization device. With strictly controversial projects and without lotteries there might be multiple equilibria involving even considerable delay.

APPENDIX

Proof of Proposition 1. Define first formally for $U \in [\underline{U}(\hat{I}), \bar{U}(\hat{I})]$ the function $v(U, \hat{I})$ by the program

$$v(U, \hat{I}) = \max_{\{U_i\}_{i \in \hat{I}}} \sum_{i \in \hat{I}} v_i(U_i) \quad (4)$$

with the constraints

$$\begin{aligned} \sum_{i \in \hat{I}} U_i &\geq U, \\ U_i &\in [\underline{U}_i, \bar{U}_i] \quad \forall i \in \hat{I}. \end{aligned} \quad (5)$$

As the objective function is surely continuous and the domain of $\{U_i\}_{i \in \hat{I}}$ compact, $v(U, \hat{I})$ is well defined. Define $u(V, \hat{I})$ analogously.

STEP 1. For all nonempty $\hat{I} \subseteq I$ and all $U^* \geq 0, V^* \geq 0$ it holds that:

- (i) $v^*(U, \hat{I}, U^*, V^*)$ and $u^*(V, \hat{I}, U^*, V^*)$ are continuous, strictly decreasing, and concave in U and V .
- (ii) $u^*(v^*(U, \hat{I}, U^*, V^*), \hat{I}, U^*, V^*) = U$.

Proof. As $u^*(\cdot)$ and $v^*(\cdot)$ are translations of $u(\cdot)$ and $v(\cdot)$, I focus on the case $U^* = 0$ and $V^* = 0$. The proof of assertion (i) is restricted to $v(U, \hat{I})$ as it is analogous for $u(V, \hat{I})$. Select for $v(\hat{U}, \hat{I})$ a member of the (possibly not single-valued) set of maximizing project realizations (the argmax-correspondence) and denote it by $\{\hat{U}_i\}_{i \in \hat{I}}$ ($v(\hat{U}, \hat{I}) = \sum_{i \in \hat{I}} v_i(\hat{U}_i)$). For a feasible $\check{U} < \hat{U}$ the vector $\{\check{U}_i\}_{i \in \hat{I}}$ would also satisfy the constraints (5) if A's utility was \check{U} . This yields $v(\check{U}, \hat{I}) \leq v(\hat{U}, \hat{I})$, and strict inequality follows from the strict monotonicity of all functions $v_i(U_i)$ and as the constraint (5) is surely binding. It is shown next that for any $\lambda \in [0, 1]$ and an arbitrary pair (\check{U}, \hat{U}) :

$$v(\lambda\check{U} + (1-\lambda)\hat{U}, \hat{I}) \geq \lambda v(\check{U}, \hat{I}) + (1-\lambda)v(\hat{U}, \hat{I}). \quad (6)$$

Abbreviate $\tilde{U} = \lambda\check{U} + (1-\lambda)\hat{U}$ and select again $\{\hat{U}_i\}_{i \in \hat{I}}$ from the argmax-correspondence for $v(\hat{U}, \hat{I})$, $\{\check{U}_i\}_{i \in \hat{I}}$ for $v(\check{U}, \hat{I})$, and $\{\tilde{U}_i\}_{i \in \hat{I}}$ for $v(\tilde{U}, \hat{I})$. Replace $\{\tilde{U}_i\}_{i \in \hat{I}}$ by the feasible linear combination $\{\lambda\check{U}_i + (1-\lambda)\hat{U}_i\}_{i \in \hat{I}}$. Constraints (5) are still satisfied, while the concavity of $v_i(\cdot)$ yields (6) from

$$\begin{aligned} \sum_{i \in \hat{I}} V_i[\lambda\check{U}_i + (1-\lambda)\hat{U}_i] &\geq \sum_{i \in \hat{I}} [\lambda v_i(\check{U}_i) + (1-\lambda)v_i(\hat{U}_i)] \\ &= \lambda v(\check{U}, \hat{I}) + (1-\lambda)v(\hat{U}, \hat{I}). \end{aligned}$$

Continuity on $U \in [\underline{U}(\hat{I}), \overline{U}(\hat{I})]$ follows from the two previous results, while it can be derived by an ε -argument for $U = \overline{U}(\hat{I})$. To prove assertion (ii), define $V = v(U, \hat{I})$ and $\tilde{U} = u(V, \hat{I})$ and claim that $\tilde{U} > U$. If \tilde{U} is realized by choosing project realizations $\{\tilde{V}_i\}_{i \in \hat{I}}$ ($\sum_{i \in \hat{I}} \tilde{V}_i \geq V$, $\sum_{i \in \hat{I}} u_i(\tilde{V}_i) = \tilde{U}$), then from $\tilde{U} > U$ there exists a project $\hat{i} \in \hat{I}$ where $\tilde{V}_{\hat{i}} < \overline{V}_{\hat{i}}$. Therefore, from continuity of $u_i(\cdot)$ there exists a value $\varepsilon > 0$ such that $\sum_{i \in \hat{I} \setminus \{\hat{i}\}} u_i(\tilde{V}_i) + u_{\hat{i}}(\tilde{V}_{\hat{i}} + \varepsilon) \geq U$, which from $\sum_{i \in \hat{I}} \tilde{V}_i + \varepsilon > V$ contradicts the construction of $V = v(U, \hat{I})$. The claim $\tilde{U} < U$ can be similarly contradicted. ■

I now derive equilibrium strategies and utilities if players bargain with alternating offers over a nonempty set of mutually beneficial projects $\hat{I} \subseteq I$ and with windfall utilities (U^*, V^*) . Call this (auxiliary) game the “ \hat{I} -simultaneous” game. With $l = (\delta, \hat{I}, U^*, V^*)$ define $N(l) = (U^A(l), V^A(l), U^B(l), V^B(l))$ by:

$$\begin{aligned} V^A(l) &= \max \left[V^*, \underline{V}(\hat{I}) + V^*, \delta v^*(\max[U^*, \underline{U}(\hat{I}) + U^*, \delta U^A(l)], \hat{I}) \right] \\ V^A(l) &= v^*(U^A(l), \hat{I}) \\ U^B(l) &= \max \left[U^*, \underline{U}(\hat{I}) + U^*, \delta u^*(\max[V^*, \underline{V}(\hat{I}) + V^*, \delta V^B(l)], \hat{I}) \right] \\ U^B(l) &= u^*(V^B(l), \hat{I}). \end{aligned} \quad (7)$$

Below the superscripts used in (7) denote whether a respective pair of utilities is proposed by A or B in the set of SPE.

STEP 2. $N(l)$ defined in (7) is uniquely determined.

Proof. Focus again on $U^* = V^* = 0$. As the argument is symmetric, the assertion holds if

$$v(U, \hat{I}) - \max\left[\underline{V}(\hat{I}), \delta v([\underline{U}(\hat{I}), \delta U], \hat{I})\right] \quad (8)$$

becomes zero for a unique value of $U \in [\underline{U}(\hat{I}), \overline{U}(\hat{I})]$. Note first that with $U = \underline{U}(\hat{I})$ (8) is strictly positive, while it is nonpositive at $U = \overline{U}(\hat{I})$. As the expression is continuous from Step 1, it becomes zero for at least one choice of U . Uniqueness follows if (8) is strictly monotone decreasing on $U \in [\underline{U}(\hat{I}), \overline{U}(\hat{I})]$. For $U \in [\underline{U}(\hat{I}), \hat{U}]$ with $\hat{U} = \min[\underline{U}(\hat{I})/\delta, \overline{U}(\hat{I})]$ (8) reduces to $v(U, \hat{I}) - \max[\underline{V}(\hat{I}), \delta \underline{V}(\hat{I})]$, which is strictly decreasing from the respective characteristic of $v(U, \hat{I})$. Hence, for $\underline{U}(\hat{I})/\delta \geq \overline{U}(\hat{I})$ the proof is concluded. Otherwise, define $\tilde{V} = \min[\underline{V}(\hat{I})/\delta, \overline{V}(\hat{I})]$ and $\tilde{U} = u(\tilde{V}, \hat{I})$ with $u(\tilde{V}, \hat{I}) = v^{-1}(\tilde{V}, \hat{I})$ from Step 1. Distinguish now between two cases. For $\tilde{U} \leq \hat{U}$ (8) reduces for all $U \in [\hat{U}, \overline{U}(\hat{I})]$ to $v(U, \hat{I}) - \delta v(\delta U, \hat{I})$. For $\tilde{U} > \hat{U}$ this transformation holds only for $U \in [\hat{U}, \tilde{U}]$, while for $U \in [\tilde{U}, \overline{U}(\hat{I})]$ (8) equals $v(U, \hat{I}) - \underline{V}(\hat{I})$. In the latter case the monotonicity is again immediate, and it thus remains to prove that $v(U, \hat{I}) - \delta v(\delta U, \hat{I})$ is strictly monotone decreasing. This is now implied by the concavity of $v(U, \hat{I})$. ■

Define the set of “quasi-stationary” SPE as follows: In even periods A makes a proposal which realizes utility $U^A(l)$ for A and $V^A(l)$ for B. Player B rejects a proposal if and only if it realizes less utility than $V^A(\delta, \hat{I})$. In odd periods B makes a proposal realizing $U^B(l)$ for A and $V^B(l)$ for B, which A rejects if and only if he realizes less utility than $U^B(l)$.

STEP 3. Any SPE of an \hat{I} -simultaneous game is quasi-stationary. Hence, the game ends in the first period and players realize the utilities $(U^A(l), V^A(l))$. The set of SPE is moreover nonempty.

Proof. With Steps 1 and 2 the proof is a straightforward extension of Theorem 3.4 in Osborne and Rubinstein (1990) to the case of this paper where the relevant bargaining frontier may not contain zero-utility points for both players. ■

STEP 4. For $\delta \rightarrow 1$ $U^A(l)$ and $U^B(l)$ converge to a limit $U^N(\hat{I}, U^*, V^*)$, while $V^A(l)$ and $V^B(l)$ converge to a limit $V^N(\hat{I}, U^*, V^*)$ such that $(U^N(\hat{I}, U^*, V^*), V^N(\hat{I}, U^*, V^*))$ maximizes the product $U \cdot V$ subject to the constraint $V = v^*(l)$.

Proof. Convergence can be easily deduced from monotonicity following from (7). From Binmore *et al.* (1986) (BRW) the second assertion is immediate if $(U^N(\hat{I}, U^*, V^*), V^N(\hat{I}, U^*, V^*))$ lies in the (adequately defined) interior of the (graph of the) frontier. Otherwise, the result follows in complete analogy to BRW by considering the (from the strict monotonicity of $u(I)$ surely existing) derivative w.r.t. V at the respective (boundary) pair $(\underline{U}(\hat{I}) + U^*, \bar{V}(\hat{I}) + V^*)$ or $(\bar{U}(\hat{I}) + U^*, \underline{V}(\hat{I}) + V^*)$. ■

The proof of assertions (i) and (ii) for a separate and simultaneous agenda follows from Steps 3 and 4 where \hat{I} represents either a single project or the whole set I . Backward induction from I_K^P to I_1^P proves assertion (iii).

Proof of Proposition 2. Define first the set of simultaneous-proposal SPE (SPSPE): At even t with nonempty I_t^r A proposes $(I_t^r, \{U_i^t, V_i^t\}_{i \in I_t^r})$ which realizes the utilities $(U^A(\delta, I_t^r), V^A(\delta, I_t^r))$.¹¹ Player B accepts any proposal $(I_t, \{U_i^t, V_i^t\}_{i \in I_t})$ with $I_t \subseteq I_t^r$ which satisfies

$$\sum_{i \in I_t} V_i^t + \delta V^B(\delta, I_t \setminus I_t^r) \geq V^A(\delta, I_t^r). \quad (9)$$

At odd t with nonempty I_t^r B makes a proposal $(I_t^r, \{U_i^t, V_i^t\}_{i \in I_t^r})$ which realizes the utilities $(U^B(\delta, I_t^r), V^B(\delta, I_t^r))$. Player A accepts any proposal $(I_t, \{U_i^t, V_i^t\}_{i \in I_t})$ with $I_t \subseteq I_t^r$ which satisfies

$$\sum_{i \in I_t} U_i^t + \delta U^A(\delta, I_t \setminus I_t^r) \geq U^B(\delta, I_t^r). \quad (10)$$

The proof proceeds in four steps. The first step proves that the set of SPSPE is nonempty. Steps 2–4 derive suprema and infima for both players' (possibly discounted) utilities in any equilibrium and show that they are equal to the utilities defined by $N(\delta, I)$. This allows us finally to prove that Step 1 indeed characterizes the complete set of SPE.

STEP 1. *The set of SPSPE is nonempty.*

Proof. The proof proceeds by induction on the number of projects in I_t^r after an arbitrary history H_t or H_t' . For $|I_t^r| = 1$ Proposition 1 shows that the SPSPE strategies form indeed a continuation equilibrium (which is unique in this case). Assume now that the assertion holds for any strict subset of a set of remaining projects I_t^r with $|I_t^r| > 1$. More precisely, assume that for any $\bar{t} > t$ with $I_{\bar{t}}^r \subset I_t^r$ there is a continuation equilibrium belonging to the set of SPSPE. Consider only the strategies of player A as the argument

¹¹With $U^* = 0$ and $V^* = 0$ in (7) the zero vector $(0, 0)$ is now suppressed in all expressions.

is symmetric. A profitable deviating proposal for A ($I_t, \{U_i^t\}_{i \in I_t}, \{V_i^t\}_{i \in I_t}$) must satisfy¹²

$$\begin{aligned} \sum_{i \in I_t} U_i^t + \delta U^B(\delta, I_t \setminus I_t) &> U^A(\delta, I_t^t), \\ \sum_{i \in I_t} V_i^t + \delta V^B(\delta, I_t \setminus I_t) &\geq V^A(\delta, I_t^t), \end{aligned} \quad (11)$$

$$\begin{aligned} U_i^t &\in [\underline{U}_i, \bar{U}_i] \quad \text{and} \quad V_i^t = v_i(U_i^t) \quad \text{for } i \in I_t, \\ I_t &\subseteq I_t^t. \end{aligned}$$

The first two lines use for $I_t \subset I_t^t$ the inductive assumption and players' supposed equilibrium strategies. It follows from the definition of $N(\delta, I_t^t)$, the construction of $u(V, \hat{I})$, and the loss incurred from potential delay that a deviation satisfying (11) is not feasible. ■

STEP 2. *Suppose I_t^t is nonempty for even t . Then $U^A(\delta, I_t^t)$ is the supremum of the discounted utility player A can realize in any continuation equilibrium. The analogous result holds for player B and odd t with the supremum $V^B(\delta, I_t^t)$*

Proof. For even t and given I_t^t denote the supremum of player A's possibly discounted utility in any continuation equilibrium by $\bar{U}^A(I_t^t)$. As $\bar{U}^A(I_t^t)$ is surely bounded below by zero and bounded above by $\sum_{i \in I_t^t} \bar{U}_i$, the supremum indeed exists. By arguing to a contradiction it is proved that $\bar{U}^A(I_t^t) = U^A(\delta, I_t^t)$ where from Step 1 $U^A(\delta, I_t^t)$ is indeed realized in a continuation equilibrium (SPSPE). Suppose therefore $\bar{U}^A(I_t^t) > U^A(\delta, I_t^t)$ holds in a specific continuation equilibrium.¹³ Denote B's utility in this equilibrium by $\hat{V}^A(I_t^t)$ and define $\bar{V}^A(I_t^t) = v(\bar{U}^A(I_t^t), I_t^t)$. As projects are possibly delayed on the equilibrium path, it holds that $\hat{V}^A(I_t^t) \leq \bar{V}^A(I_t^t)$ as surely $\hat{V}^A(I_t^t) \geq 0$. Construct now a deviation for Player B from the supposed equilibrium path in which he or she offers in $t+1$ a proposal over all remaining projects such that players realize the utilities (\check{U}, \check{V}) with

$$\begin{aligned} \check{U} &> \delta \bar{U}^A(I_t^t), \\ \delta \check{V} &> \bar{V}^A(I_t^t). \end{aligned} \quad (12)$$

From the construction of the supremum, player A must accept. It therefore remains to show that for any $\bar{U}^A(I_t^t) > U^A(\delta, I_t^t)$ there exists a proposal realizing a pair (\check{U}, \check{V}) which satisfies (12). As in the proof of Rubinstein (1982), this is now an immediate consequence of the construction of

¹²This uses $U^B(\delta, \hat{I}) = V^B(\delta, \hat{I}) = 0$ if \hat{I} is empty.

¹³Though there might be no equilibrium in which the supremum is indeed realized, one can choose by the nature of the supremum an equilibrium path of offers in which the utility becomes arbitrarily close to $\bar{U}^A(I_t)$.

$N(\delta, I_t^r)$ in (7) and of the continuity of the frontier $v(\cdot, I_t^r)$. The analogous argument applies now to the supremum for player B. ■

STEP 3. *Suppose I_t^r is nonempty for even t . Then $U^A(\delta, I_t^r)$ is the infimum of the discounted utility player A realizes in any continuation equilibrium. The analogous result holds for player B and odd t with the infimum $V^B(\delta, I_t^r)$.*

Proof. Take first the claim for player A and denote the infimum by $\underline{U}^A(I_t^r)$, which again surely exists. Suppose $\underline{U}^A(I_t^r) < U^A(\delta, I_t^r)$ holds in a continuation equilibrium. From Step 2 player B cannot realize a higher utility than $V^B(\delta, I_t^r)$ in any continuation equilibrium after rejection. As a consequence, player A can strictly improve by making a proposal over all issues realizing (\check{U}, \check{V}) with

$$\begin{aligned}\check{U} &> \underline{U}^A(I_t^r) \\ \check{V} &> \delta V^B(\delta, I_t^r).\end{aligned}\tag{13}$$

Again from (7) such a pair surely exists for all cases $\underline{U}^A(I_t^r) < U^A(\delta, I_t^r)$. Finally, from Step 1 $\underline{U}^A(I_t^r) = U^A(\delta, I_t^r)$ is indeed realized. The same argument now applies to player B. ■

STEP 4. *Suppose I_t^r is nonempty for even t . Then $V^A(\delta, I_t^r)$ is the infimum and the supremum of the discounted utility player B realizes in any continuation equilibrium. The analogous result holds for player A with $U^B(\delta, I_t^r)$.*

Proof. From Step 3 B's utility in any continuation equilibrium after rejection is bounded below by $V^B(\delta, I_t^r)$. By construction of (7) he must therefore realize at least $V^A(\delta, I_t^r)$ in any continuation equilibrium starting in even t . On the other side, he cannot obtain more than this value from Step 3 and the construction of $U^B(\delta, I_t^r)$ in (7). The same argument now applies to odd periods.

To finish the proof, Steps 2–4 show that for any period t with a nonempty set I_t^r any continuation equilibrium must end the game in t with a joint proposal realizing the utilities specified by $N(\delta, I_t^r)$. To see this, note first that from Steps 2–4 the (possibly discounted) utilities realized in continuation equilibria for even t must be exactly $U^A(\delta, I_t^r)$ for player A and $V^A(\delta, I_t^r)$ for player B as the respective suprema and infima coincide for each player. With costly delay, $V^A(\delta, I_t^r) = v(U^A(\delta, I_t^r), I_t^r)$, and the strict monotonicity of $v(\cdot, I_t^r)$, it is immediate that only a simultaneous proposal is feasible. The same argument applies to odd periods. Therefore, the set of SPE is completely characterized by that of SPSPE in Step 1. ■

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