

# Screening in a Matching Market

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Contract design under incomplete information is often analysed in a bilaterally monopolistic setting. If the informed party's reservation value does not depend on its private information (its type), it is a standard result that the uninformed side offers "low" types distorted contracts to reduce the information rent left to "high" types.

We challenge this result by embedding contract design in a matching market environment. We consider a market where players meet pairwise and where, in each match, either side may be chosen to make a take-it-or-leave-it offer. As frictions become sufficiently low, we find that the set of equilibria is independent of whether there is complete or incomplete information. In particular, all contracts are free of distortions.

## 1. INTRODUCTION

Consider the following canonical setting of a screening game with private values. There are two parties who can realize a gain if they conclude a contract. Utility is transferable and the payoff of one party (the agent) depends systematically on a parameter (the type). The agent has private information regarding his type. The contract specifies a transfer and additionally a variable satisfying a standard sorting condition with regards to the agent's type. Moreover, it is a standard assumption that the agent's reservation value does not depend on his type. The uninformed side (the principal) has the right to make a take-it-or-leave-it offer to the agent. The literature has developed this model both in the abstract (see Laffont and Maskin (1982) and Guesnerie and Laffont (1984) for unified treatments) and as applied to various economic problems, *e.g.* labour contracts, optimal taxation, price and quality discrimination, public goods, and regulation of monopoly. It is found that only the contract offered to the highest type maximizes joint surplus, while for lower types the sorting variable is distorted. Moreover, the participation constraint becomes only binding at the lowest type.

This paper changes three related assumptions of the standard screening model. First, we consider a matching market with many principals and agents instead of analysing an isolated pair. Secondly, we endow both sides with bargaining power. Precisely, we assume that in a given match either side is chosen with some probability to make a take-it-or-leave-it proposal. Finally, this set-up allows to endogenize the agents' reservation values. For many applications these assumptions seem to be more appropriate than those of the standard model. For instance, in a labour market it is hard to justify that the principal (employer) can act as a monopolist with all bargaining power. Instead, there should be many firms demanding labour, and an agent (prospective employee) should have the possibility to leave a match and approach a different employer. Similarly, markets for goods are typically populated by many potential sellers and buyers.<sup>1</sup>

We restrict attention to a stationary matching market. Frictions are modelled by discounting. Following Gale (1987), the primitives of our model are the constant

1. The standard assumptions seem, however, to be more appropriate to model procurement or the provision of public goods, where the government is the only relevant principal.

numbers of players arriving newly at the market fringe each period. Players on the market fringe decide whether to participate in the market or whether to realize an outside option. Our main result is that, for sufficiently low frictions, the set of equilibria is independent of whether the agents' type is their private information or not. In particular, even if agents have private information, principals propose only first-best contracts where the participation constraint binds at all types. The endogenization of reservation values in the market ensures that these contracts become incentive compatible.

The paper contributes to the recent literature on screening with type-dependent reservation values. The dependency arises in Laffont and Tirole (1990*b*) due to different initial contracts, in Champsaur and Rochet (1989) and Laffont and Tirole (1990*a*) due to alternative (bypass) products, in Biglaiser and Ma (1995) and Curien, Jullien, and Rey (1998) from the presence of a competing unregulated firm, or in Biglaiser and Mezzetti (1993) from competition between two principals with different payoff functions.<sup>2</sup>

Our model and the derived results are also reminiscent of the literature on competing mechanism design. We briefly outline the two main strands of this literature. Armstrong and Vickers (1999) and Rochet and Stole (1998) consider, amongst other things, imperfect competition between two firms in nonlinear prices. If competition is sufficiently intense, they show that equilibrium prices may reduce to simple cost-plus-fixed-fee schedules. This stands in sharp contrast to the monopolistic case, where the monopolist offers a complex menu to optimally trade-off price discrimination with the maximization of surplus. Another strand of the literature discusses competition in auction design. For the case where buyers' types are independently drawn, McAfee (1993) and Peters (1997) show that competition between auction designers erodes the value of the reserve price as an instrument to increase the auctioneer's profit. Intuitively, if buyers can choose in which mechanism to participate, setting a high reserve price reduces profits as it attracts a smaller and possibly worse subset of buyers.<sup>3</sup> One tentative conclusion of the two reviewed strands on competing mechanisms is that competition may lead to equilibrium mechanisms which are far less complex than those designed by a monopolist. Likewise, in our paper embedding contract design in a market environment ensures that uninformed principals offer non-distorted contracts as frictions become sufficiently low. In contrast to the above literature, this is, however, not a consequence of direct competition between principals. In a matching market offers are only made after the match has been concluded and are thus only addressed to the current matching partner. As argued in more detail below, this implies in particular that our results do not extend to the case where only principals make offers.

The rest of this paper is organized as follows. Section 2 presents the basic model, which is analysed in Sections 3 and 4 under both information regimes. The conclusion of Section 5 reports on extensions. Some proofs are relegated to an Appendix.

2. Related is also the literature on countervailing incentives (see Lewis and Sappington (1989*a, b*) or Bradburd and Srinagesh (1989)). Maggi and Rodriguez-Clare (1995) and Jullien (2000) have recently unified previous work on optimal contracts under countervailing incentives.

3. Recently, Peters (2001) has considered the case where buyers have correlated private information. In contrast to the complex equilibrium mechanisms offered by a monopolist (see Cremer and McLean (1988), McAfee and Reny (1992)), he derives an equilibrium where sellers hold simple second-price auctions where reserve prices are set equal to their costs

2. THE MODEL

2.1. *Players and payoffs*

We consider a market with two types of players called principals and agents. A principal and an agent can generate a surplus if they conclude a contract. The contract specifies two real variables: a monetary transfer  $t$  and an additional variable  $x$ . Below we offer various interpretations for  $x$ . We summarize a contract by  $c = (x, t)$  with  $c \in C = \mathfrak{R}^2$ . There are two types of agents indexed by  $i \in I = \{1, 2\}$ . An agent of type  $i$  realizes the utility  $V_i(c) = v_i(x) + t$  from a contract  $c$ , while the principal's utility is given by  $U(c) = u(x) - t$ . Note that payoffs exhibit private values as a principal's payoff is independent of the agent's type. Furthermore, utilities are transferable. Below we will discuss both the case where the agent's type is known and the case where it is the agent's private information. The sum of payoffs is  $w_i(x) = v_i(x) + u(x)$ . In what follows, we assume that  $v_i(\cdot)$  and  $u(\cdot)$  are twice continuously differentiable. We invoke the following additional assumptions:

$$(A.1) \quad d^2w_i/dx^2 < 0 \text{ is bounded away from zero; } v_2(x) > v_1(x), dv_2/dx > dv_1/dx, \text{ and } d^2v_2/dx^2 \geq d^2v_1/dx^2.$$

As  $v_i$  increases with the agent's type  $i$ , we refer to  $i = 2$  as the high-type agent. The assumption  $dv_2/dx > dv_1/dx$  is known as a single-crossing condition. Note that  $dv_i/dx$  represents the marginal rate of substitution between the "sorting" variable  $x$  and the transfer  $t$  for type  $i$ . Hence, the condition  $dv_2/dx > dv_1/dx$  asserts that the agent's type affects the marginal rate of substitution in a systematic way. The further assumptions in (A.1) are standard to ensure existence and uniqueness of optimal contracts. Denote the unique value of  $x$  maximizing  $w_i(x)$  by  $x_i^*$  and the realized surplus by  $w_i^* = w_i(x_i^*)$ .

The framework allows for various interpretations. A partnership may consist of an employer and an employee, who negotiate over the agent's verifiable effort  $x$  and the wage  $t$ . The employer realizes the revenue  $u(x)$ , while the employee derives the (dis)utility  $v_i(x)$  from providing the corresponding level of effort. As the disutility of working is smaller for a high-type agent, the corresponding surplus realized under employment  $w_i(x)$  is larger. Alternatively, the principal may be the owner of a resource used by an entrepreneur. In this case  $-t$  represents a (usage) fee paid by the agent, and  $x$  becomes a measure of output produced with the transferred resource. The agent is privately informed about his productivity. Finally, consider a buyer-seller relationship as in Mussa and Rosen (1978) where  $x$  specifies the quality of traded goods, while  $-t$  denotes the price paid by the agent, *i.e.* the buyer. Costs of production are now denoted by  $u(x)$ . High-type agents, who may have a higher income, have a higher absolute valuation for the good and additionally a higher marginal valuation for an increase in quality.<sup>4</sup>

2.2. *Matching*

We consider a matching market with endogenous entry.<sup>5</sup> Time runs discretely and the market operates for an infinite number of periods. All players discount future utilities by a constant factor  $\delta \in (0, 1)$ . We frequently refer to the degree of impatience, *i.e.* to the value of  $\delta$ , as the level of frictions. The primitives of the model are the time invariant measures of

4. If  $x$  denotes quantity, it is only reasonable to allow nonnegative values. In what follows, this could give rise to a corner solution in the (screening) programme.

5. We follow Peters (1992) and Gale (1987). In contrast, Rubinstein and Wolinsky (1985) take the stocks of players in the market as the primitives and adjust entry flows to ensure stationarity. See Osborne and Rubinstein (1992, Chapter 7) on this distinction.

players appearing newly on the market fringe each period. Below we provide some examples in detail. We denote the measure of newly arriving principals by  $n^P > 0$  and that of agents of type  $i$  by  $n_i^A > 0$ . These measures are aggregated by  $\mathbf{n} = (n^P, n_2^A, n_1^A)$ . Principals realize the utility  $U^0 > 0$  from their outside option, while agents of both types derive the utility  $V^0 > 0$  if they decide not to enter the market. Observe that the existence of valuable options makes waiting on the market fringe costly. This will ensure that neither the market fringe nor the matching market are flooded with passive players over time. We invoke the following assumptions.

$$(A.2) \quad n_2^A < n^P.$$

$$(A.3) \quad w_1^* > V^0 + U^0.$$

These assumptions ensure that for sufficiently low frictions both types of agents will enter the market. From (A.2) the fraction

$$\mu^0 = \frac{n_2^A}{n_2^A + \max[n_1^A, n^P - n_2^A]},$$

is strictly between zero and one. Note that  $\mu^0 = n_2^A/(n_2^A + n_1^A)$  holds if the measure of agents does not exceed that of principals. Otherwise, it holds that  $\mu^0 = n_2^A/n^P$ . Hence,  $\mu^0$  denotes the fraction of high-type agents among the set of “relevant” agents.

We consider an anonymous market with random matching.<sup>6</sup> Suppose that the measure of principals is given by  $s^P$  and that of agents of type  $i$  by  $s_i^A$ . If the market is active, it holds that  $s^P > 0$  and  $s^A = s_1^A + s_2^A > 0$ . Denote the relation of principals to agents by  $\theta = s^P/s^A$ . As it is common in the labour literature, we refer to  $\theta$  as the tightness of the market. We assume that a principal’s probability to be matched with an agent depends only on the tightness  $\theta$ . It is denoted by the function  $m^P(\theta)$ . Similarly, an agent’s matching probability is denoted by  $m^A(\theta)$ . Intuitively,  $m^P(\theta)$  should be nonincreasing and  $m^A(\theta)$  should be nondecreasing. Furthermore, we specify that matching probabilities are continuous and satisfy  $m^A(0) = 0$ ,  $m^P(0) = 1$ ,  $\lim_{\theta \rightarrow \infty} m^P(\theta) = 0$ , and  $\lim_{\theta \rightarrow \infty} m^A(\theta) = 1$ .<sup>7</sup> To illustrate the assumptions on the matching technology, we provide some examples.

*Standard (linear) matching technology:* Suppose that all players are randomly matched each period. As a consequence, any player is matched with a principal with probability  $m = s^P/(s^A + s^P)$ , which transforms to  $m = \theta/(1 + \theta)$ , while with probability  $1 - m$  he is matched with an agent. Similarly, an agent finds a principal with probability  $1 - m$ . As a consequence, in a given period a principal (agent) finds a trading partner with probability  $1 - m(m)$ .<sup>8</sup>

*Classical matching technology:* Suppose that the short side in the market is matched with probability one, while players on the long side are (randomly) rationed. Hence, agents are matched with probability  $\min\{1, \theta\}$ , while principals are matched with probability  $\min\{1, 1/\theta\}$ .

6. Anonymity implies that players cannot observe the previous history of their trading partners.

7. Hence, our main assumption is that the matching technology is homogenous of degree one. Observe that this implies  $m^A(\theta) = \theta m^P(\theta)$  so that  $m^P(\theta) - m^A(\theta)$  is strictly decreasing. This restriction is, however, only made for notational convenience as it allows us to represent matching probabilities by a single variable  $\theta$ .

8. This technology is very prominent and applied, for instance, in Binmore and Herrero (1988, Assumption 2) or Gale (1987).

*Exponential matching technology:* Principals are matched with probability  $(1 - e^{-\theta})/\theta$ , while agents are matched with probability  $1 - e^{-\theta}$ . For a strategic foundation of this technology we refer the reader to Peters (1991).

We conclude the description of the market environment with some applications. Suppose first that agents are workers and principals are firms with a single vacancy. Each period a new cohort of prospective employees arrives together with firms who have separated from retired workers. Firms can decide to search for a new worker in the matching market or to disinvest their capital. Workers have the option to enter the matching market in order to search for a (skilled) job or to take up an outside option such as unskilled work. In what follows, we will discuss the three cases where  $n^P > n_1^A + n_2^A$ ,  $n^P < n_1^A + n_2^A$ , and  $n^P = n_1^A + n_2^A$ . We show that by stationarity of the market the longer side (in terms of potential entrants) must realize exactly the utility derived from its outside option. Hence, for  $n^P > n_1^A + n_2^A$  the firms' payoff is equal to  $U^0$ . In this case our results would be unchanged if we assumed instead that the entry of firms is determined by a zero-profit condition, as it is standard in the labour literature.

If we assume that principals are the sellers of a good, our assumptions on the matching market are analogous to those in Gale (1987). The novelty of our model is that a contract specifies both the price and the quality of the exchanged good.

### 2.3. Bargaining

In a given period a player can bargain only with his current matching partner. The bargaining procedure is unchanged in all periods. First, one of the parties in a match is selected randomly to propose a mechanism described below. We assume that the principal is selected with probability  $b \in (0, 1)$  and the agent with probability  $1 - b$ .<sup>9</sup> The other party can only accept or reject this proposal. We refer to  $b$  as a measure of the principals' bargaining power. If the mechanism leads to an agreement, the contract is implemented and both parties leave the market. Note that an accepted contract is not open to renegotiations. If the mechanism does not lead to an accepted proposal, the match dissolves and both players re-enter the market.

If the principal is chosen to make a proposal, we specify that he can offer a menu of deterministic contracts. If the agent accepts, he is free to implement any of the offers in the menu.<sup>10</sup> Recall that we consider both the (benchmark) case where the agent's type is complete information and the case where the agent is privately informed about his type. Borrowing from the terminology of information economics, we refer to the case where principals make offers as the screening game.

If the agent is chosen to make an offer, we specify that he can propose a single deterministic contract. Recall that the agent's type does not directly enter the principal's

9. Random choice is applied in Rubinstein and Wolinsky (1985), Gale (1987), and Binmore and Herrero (1988). Alternatively, offers can be restricted to one party (Samuelson (1992), Fudenberg, Levine and Tirole (1987)), or both sides may simultaneously announce a proposal (Wolinsky (1990), Serrano and Yosha (1992)).

10. It can be shown that making a take-it-or-leave-it offer is indeed the optimal mechanism (see Inderst (1998)). Moreover, the restriction to deterministic offers is without loss of generality. Indeed, we show below that with private information only the incentive compatibility constraint of the high type may become binding. As shown by Maskin and Riley (1984), the assumption  $d^2v_2/dx^2 \geq d^2v_1/dx^2$  in (A.1) together with the concavity of  $w_i$  ensure that only deterministic offers are optimal.

utility function (private values). Hence, even if the agent's type is his private information, his offer does not convey any relevant information. This would be different under common values.<sup>11</sup>

Observe that our specification of the bargaining game has two main properties: Both sides can make a proposal with some probability, and the respective proposer can commit to a take-it-or-leave-it offer. Below we comment on relaxing these assumptions.

#### 2.4. Equilibrium requirements

We impose next a series of equilibrium requirements. First, we require that players choose symmetric, stationary, and sequentially optimal strategies in the bargaining games.<sup>12</sup> Additionally, for notational convenience we restrict attention to pure strategies. Denote next by  $\emptyset$  the case where a match is dissolved unsuccessfully and extend  $C^0 = C \cup \{\emptyset\}$ . In equilibrium a match with an agent of type  $i$  leads to a choice  $c_i^A \in C^0$  if the agent makes the proposal and to a choice  $c_i^P \in C^0$  if it is the principal's turn. We summarize these contracts by  $\mathbf{c} = (c_1^A, c_1^P, c_2^A, c_2^P)$ . We further restrict attention to a stationary matching environment where stocks denoted by  $\mathbf{s} = (s^P, s_2^A, s_1^A)$  and entries denoted by  $\mathbf{e} = (e^P, e_2^A, e_1^A)$  are time independent. We denote the distribution of types by  $\mu = s_2^A/s^A$ , where  $s^A = s_1^A + s_2^A$ . Recall also that the relation of principals to agents is denoted by the tightness  $\theta = s^P/s^A$ .

Finally, we require that the entry decision is made optimally. To formalize this condition, we must first determine the utilities realized in the market. These are called the players' reservation values and are denoted by  $V_i^R$  for agents and by  $U^R$  for principals. If  $c_i^A$  and  $c_i^P$  are the outcomes of optimal strategies played in a match, the reservation value of agent  $i$  is given by

$$V_i^R = \delta[m^A(\theta)(bV_i(c_i^P) + (1-b)V_i(c_i^A)) + (1-m^A(\theta))V_i^R], \quad (1)$$

while the reservation value of a principal is defined by

$$\begin{aligned} U^R &= \delta m^P(\theta)\mu(bU(c_2^P) + (1-b)U(c_2^A)) \\ &+ \delta m^P(\theta)(1-\mu)(bU(c_1^P) + (1-b)U(c_1^A)) + \delta(1-m^P(\theta))U^R. \end{aligned} \quad (2)$$

For an illustration, take the case of  $V_i^R$ . The expected payoff from a match equals  $bV_i(c_i^P) + (1-b)V_i(c_i^A)$ . With probability  $1-m^A(\theta)$  the agent is not matched with a principal and has to wait for the next round. By stationarity, the expected utility from waiting another round is again his reservation value. The optimality requirement for the agents' entry decision can now be formalized as follows

$$e_i^A = \begin{cases} 0, & \text{if } V_i^R < V^0, \\ \in [0, n_i^A], & \text{if } V_i^R = V^0, \\ n_i^A, & \text{if } V_i^R > V^0. \end{cases}$$

11. As shown by Maskin and Tirole (1990), the quasilinearity of payoffs ensures that the offer of a single contract is optimal under private values. (This would be different under common values, see Maskin and Tirole (1992).)

12. With a continuum of players the symmetry assumption is not restrictive. We should note that stationarity of strategies is not already implied by assuming that the market is stationary. As shown in Gale (1987), this would be the case if contracts only specified prices. In our setting, however, principals may become indifferent between offering various menus if agents have private information, implying that they may randomize differently in each period.

Analogously, the requirement for principals becomes

$$e^P = \begin{cases} 0, & \text{if } U^R < U^0, \\ \in [0, n^P], & \text{if } U^R = U^0, \\ n^P, & \text{if } U^R > U^0. \end{cases}$$

In summary, a market equilibrium is described by a profile  $\psi = (c, s, e)$  of contracts, stocks, and entries such that the market is stationary, while players choose sequentially optimal, symmetric, stationary, and pure strategies.<sup>13</sup>

Observe that we have suppressed the exogenous parameters  $(U^0, V^0, b, \mathbf{n}, \delta)$  in the description of a market equilibrium. In what follows, we frequently keep the parameters  $(U^0, V^0, b, \mathbf{n})$  fixed while changing the discount factor  $\delta$ , which represents frictions. For given  $\delta$  we denote the set of equilibria by  $\Psi_\delta$  if the agents' type is observable and by  $\bar{\Psi}_\delta$  if the agents' type is their private information.

### 3. THE BENCHMARK WITH COMPLETE INFORMATION

Throughout the paper we restrict consideration to the case where  $\delta$  is sufficiently high. By (A.3) this will ensure that agents of both types enter the market. This constitutes the interesting case for the comparison of the two information regimes.

Under complete information we derive a unique equilibrium where all matches are successful and all contracts specify the first-best value of  $x$ . Consider first the bargaining game between an agent of type  $i$  and a principal. If the match is successful, the contract must by optimality implement the first-best value of  $x$ , while the utility of the responding party must be equal to its reservation utility. Hence, if the match is successful, we obtain  $x_i^A = x_i^P = x_i^*$ ,  $t_i^A = u(x_i^*) - U^R$ , and  $t_i^P = V_i^R - v_i(x_i^*)$ . Clearly, if the aggregate surplus  $w_i^*$  exceeds the sum of reservation values  $V_i^R + U^R$ , the match must be successful. It is thus intuitive that all matches are successful. (This and the following arguments are formalized in the proof of Proposition 1 below.)

We can next substitute the derived equilibrium contracts into (1)–(2) to solve for the reservation values. For this purpose it is convenient to define

$$d^A = \frac{\delta m^A(\theta)(1 - b)}{1 - \delta[1 - m^A(\theta)(1 - b)]},$$

$$d^P = \frac{\delta m^P(\theta)b}{1 - \delta[1 - m^P(\theta)b]}.$$

For instance, an agent's reservation value becomes now  $V_i^R = d^A(w_i^* - U^R)$ . Observe also that  $d^A$  is increasing in both  $\theta$  and  $\delta$ , while  $d^P$  decreases with  $\theta$  and increases with  $\delta$ .

13. Incidentally, for the description of an equilibrium the pair  $(\mathbf{c}, \mathbf{s})$  would be sufficient, as entries are already determined by the choice of  $(\mathbf{c}, \mathbf{s})$  and the stationarity requirement. Note also that we only consider equilibria where the market opens up. In particular, this rules out the case where the market remains closed due to a coordination failure. Existence of an equilibrium where the market is active is proved below for all sufficiently high  $\delta$ .

Substitution yields next

$$\begin{aligned}
 U^R &= \frac{d^P(1-d^A)}{1-d^A d^P} [(1-\mu)w_1^* + \mu w_2^*] \\
 &= \frac{\delta m^P(\theta)b}{1-\delta[1-m^P(\theta)b-m^A(\theta)(1-b)]} [(1-\mu)w_1^* + \mu w_2^*]. \quad (3)
 \end{aligned}$$

The principals' reservation value is continuous and strictly increasing in  $\theta$ . If we plough this expression back into  $V_i^R = d^A(w_i^* - U^R)$ , we obtain likewise that an agent's reservation value decreases with  $\theta$ . This is intuitive as increasing the ratio of principals to agents should favour the latter side. Moreover, for given choice of  $\theta$  both  $U^R$  and  $V_i^R$  are increasing in  $\delta$ .

Consider next the choice of stocks and entry flows, which determine the distribution of types  $\mu$  and the relation of principals to agents in the market. Take first the case  $n^P > n_1^A + n_2^A$ , where principals outnumber agents on the market fringe. To ensure stationarity, not all principals may enter the market in equilibrium, implying  $U^R = U^0$ . Recall now from (A.3) that the first-best surplus  $w_i^*$  exceeds the sum  $U^0 + V^0$  for both types of the agent. This implies for sufficiently low frictions that the equation  $U^R = U^0$  has a (unique) solution  $\theta$ , where it holds that  $V_i^R > V^0$ . Hence, by optimality all agents enter, while the flow of new principals into the market satisfies  $e^P = n_1^A + n_2^A$  to ensure stationarity. Finally, stocks in the market are given by  $s_i^A = e_i^A/m^A(\theta)$  and  $s^P = e^P/m^P(\theta)$ . We have thus derived for the case  $n^P > n_1^A + n_2^A$  the full specification of the unique equilibrium if frictions are sufficiently low.

We can proceed analogously for the opposite case where  $n^P < n_1^A + n_2^A$ . Given (A.2), the tightness  $\theta$  is now determined by the requirement  $V_1^R = V^0$ , *i.e.* low types must be indifferent between entering the market or taking up their outside option. Finally, for the case  $n^P = n_1^A + n_2^A$  we obtain multiple equilibria where all players enter and reservation values satisfy  $V_i^R \geq V^0$  for  $i \in I$  and  $U^R \geq U^0$ . These results are formalized in Proposition 1.

**Proposition 1.** *For all sufficiently high  $\delta$  the set of equilibria under complete information  $\Psi_\delta$  is non-empty. All equilibria exhibit the following characteristics:*

- (i) *All players on the “short” side enter, i.e.  $n^P < n_2^A + n_1^A$  implies  $e^P = n^P$ ,  $e_2^A = n_2^A$ ,  $e_1^A = n^P - n_2^A$ ;  $n^P > n_2^A + n_1^A$  implies  $e^P = n^P - n_1^A - n_2^A$ ,  $e_2^A = n_2^A$ ,  $e_1^A = n_1^A$ ;  $n^P = n_2^A + n_1^A$  implies  $\mathbf{e} = \mathbf{n}$ .*
- (ii) *All matches are successful and implement  $x_i^A = x_i^P = x_i^*$ ,  $t_i^A = u(x_i^*) - U^R$ , and  $t_i^P = V_i^R - v_i(x_i^*)$ . Stocks are given by  $s^P = e^P/m^P(\theta)$  and  $s_i^A = e_i^A/m^A(\theta)$ .*
- (iii) *If  $n^P > n_2^A + n_1^A$ ,  $\theta$  is uniquely defined by the requirement  $U^R = U^0$ .*
- (iv) *If  $n^P < n_2^A + n_1^A$ ,  $\theta$  is uniquely defined by the requirement  $V_1^R = V^0$ .*
- (v) *If  $n^P = n_2^A + n_1^A$ , there exist  $0 < \underline{\theta} < \bar{\theta}$  such that  $\psi \in \Psi_\delta$  if and only if  $\theta \in [\underline{\theta}, \bar{\theta}]$ .*

*Proof.* See Appendix A. ||

In what follows, we restrict attention to sufficiently high values of  $\delta$  such that Proposition 1 applies. Proposition 1 implies the following Corollary.

**Corollary.** *As  $\delta$  approaches one, the difference between the high type's and the low type's reservation values converges to  $w_2^* - w_1^*$ .*

*Proof.* We first restate the claim more formally. We claim that for each  $\varepsilon > 0$  there exists  $\hat{\delta} < 1$  such that for all  $\delta > \hat{\delta}$  and  $\psi \in \Psi_\delta$  it holds that  $V_2^R - V_1^R \in [w_2^* - w_1^* - \varepsilon, w_2^* - w_1^* + \varepsilon]$ .<sup>14</sup> Observe next that  $V_2^R - V_1^R = d^A(w_2^* - w_1^*)$  holds by Proposition 1. Moreover, by the proof of Proposition 1 any equilibrium satisfies  $\theta \geq \underline{\theta}$ , where  $\underline{\theta}$  is uniquely determined by the requirement  $V_1^R = V^0$ . Denote by  $\underline{\theta}_\delta$  the value of  $\underline{\theta}$  for the respective choice of  $\delta$ . Given  $\delta$  and  $\underline{\theta}_\delta$ , calculate next the respective values of  $d^A$  and  $d^P$ , which we denote by  $d_\delta^A$  and  $d_\delta^P$ . To complete the proof, it remains to show that  $\lim_{\delta \rightarrow 1} d_\delta^A = 1$ . We argue to a contradiction, which by  $d_\delta^A \in [0, 1]$  implies existence of a sequence  $\{\delta_n\}$  where  $\delta_n \rightarrow 1$  and  $d_{\delta_n}^A \rightarrow d_1^A < 1$ . By the definition of reservation values and by  $w_2^* > w_1^*$ , the low type's reservation value is bounded from above by  $[d_{\delta_n}^A(1 - d_{\delta_n}^P)/(1 - d_{\delta_n}^A d_{\delta_n}^P)]w_1^*$ . By  $\delta_n \rightarrow 1$ ,  $d_{\delta_n}^A \rightarrow d_1^A$ , and  $d_{\delta_n}^P \rightarrow 1$ , this boundary converges to zero, which by  $V^0 > 0$  contradicts the definition of  $\underline{\theta}_\delta$ .  $\parallel$

As frictions vanish, the difference between the high type's payoff and that of the low type converges to the difference in first-best surplus  $w_2^* - w_1^*$ . As an immediate consequence, for  $\delta \rightarrow 1$  principals become indifferent between contracting with low or high types. In Gale (1987), where contracts specify only the transfer between buyers and sellers of a homogenous good, this has been termed the “law of one price”.<sup>15</sup>

#### 4. INCOMPLETE INFORMATION

We now introduce private information about the agent's type. It is a standard result that this favours the high-type agent. He must be induced to choose “his” contract instead of mimicking the low type. As a consequence, in a bilaterally monopolistic setting with type-independent reservation values, the principal distorts the low type's contract to reduce the high type's information rent. In contrast, we find in the matching market environment that for sufficiently low frictions all contracts are free of distortions. More precisely, we show that the set of equilibria with incomplete information is equal to that with complete information derived in Proposition 1.

We proceed in two steps. First, Section 4.1 sets up the programme to derive the optimal screening contracts  $c_i^P$ . In a second step, in Sections 4.2–4.3 we embed the screening game in the market environment where reservation values are endogenized.

##### 4.1. Analysis of the screening game

Suppose a principal makes an offer to an agent whose type is his private information. As previously, we denote beliefs by  $\mu$  and the agent's type-dependent reservation value by  $V_i^R$ . Suppose further that the principal's menu must contain an acceptable offer  $c_i^P \in C$  for both types. Hence, the offer maximizes

$$\mu(u(x_2^P) - t_2^P) + (1 - \mu)(u(x_1^P) - t_1^P),$$

subject to the individual rationality and incentive compatibility constraints for both types  $i \in I$

$$V_i(c_i^P) \geq V_i^R, \tag{IR}_i$$

$$V_i(c_i^P) \geq V_i(c_j^P), \quad j \neq i. \tag{IC}_i$$

14. Observe that for  $n^P = n_1^A + n_2^A$ , where we find multiple equilibria for sufficiently high values of  $\delta$ , this formulation implies uniform convergence along any sequence of equilibria where frictions vanish.

15. This was generalized by Mortensen and Wright (1997).

It is useful to rewrite  $IC_i$  as

$$V_i(c_i^P) - V_j(c_j^P) \geq v_i(x_j^P) - v_j(x_j^P),$$

which already implies  $x_1^P \leq x_2^P$  by the sorting condition in (A.1). The solution to this programme depends crucially on the difference between reservation values  $V_2^R - V_1^R$ . For the purpose of our matching market model, we can restrict the range of relevant values. Indeed, we show below that the difference is always nonnegative and not larger than  $w_2^* - w_1^*$ . Moreover, we can restrict attention to values  $\mu \in (0, 1)$ .

Suppose first that  $V_2^R - V_1^R = 0$ . This represents the standard case where reservation values do not depend on the agent's type. It is well known that in this case only the incentive compatibility constraint of the high type  $IC_2$  and the participation constraint of the low type  $IR_1$  bind. Substituting the binding constraints, we obtain the transfers  $t_1^P = V_1^R - v_1(x_1^P)$  and  $t_2^P = t_1^P + v_2(x_1^P) - v_2(x_2^P)$ . From the first-order conditions, we obtain next  $x_2^P = x_2^*$ , while  $x_1^P$  is uniquely determined by the requirement

$$\frac{dw_1(x)}{dx} = \mu \frac{dw_2(x)}{dx}. \quad (4)$$

Note that (4) has a unique solution by (A.1), which satisfies  $x_1^P < x_1^*$ . For what follows it is convenient to denote the value for  $x_1^P$  derived in this case by  $x_1^S$ . We also refer to the case where only  $IR_1$  and  $IC_2$  bind as case (C1).

It is intuitive that (C1) still applies if  $V_2^R - V_1^R$  remains small. (This claim and the following arguments are made formal in the proof of Lemma 1 below.) However, if the difference becomes sufficiently large, the (neglected) participation constraint for  $i = 2$  ceases to hold. In this case the three constraints  $IR_1$ ,  $IR_2$  and  $IC_2$  become binding. Substitution yields the transfers  $t_i^P = V_i^R - v_i(x_i^P)$  for  $i \in I$ . By the binding constraint  $IC_2$ , we obtain  $x_1^P$  from

$$v_2(x_1^P) - v_1(x_1^P) = V_2^R - V_1^R. \quad (5)$$

In what follows, it will be ensured that (5) has a solution whenever this case applies. Finally, the first-order condition yields  $x_2^P = x_2^*$ . We refer to this solution candidate as case (C2).

If the difference  $V_2^R - V_1^R$  increases even further, the incentive compatibility constraint for the high type no longer binds. Indeed, the set of (first-best) contracts offered under complete information, where  $x_i^P = x_i^*$  and  $t_i^P = V_i^R - v_i(x_i^P)$ , becomes now incentive compatible. Hence, only the two participation constraints  $IR_i$  bind in this case, which is referred to as (C3).

To determine when the three cases apply, we make use of the two thresholds

$$\check{\Delta} = v_2(x_1^S) - v_1(x_1^S),$$

$$\hat{\Delta} = v_2(x_1^*) - v_1(x_1^*).$$

Recall that  $x_1^S$  was determined in (C1). By  $x_1^S < x_1^*$  and (A.1) it follows that  $0 < \check{\Delta} < \hat{\Delta} < w_2^* - w_1^*$ .

**Lemma 1.** *The screening programme has a unique solution, which depends on  $V_2^R - V_1^R$  as follows*

- (C1) *applies if  $0 \leq V_2^R - V_1^R \leq \check{\Delta}$ .*
- (C2) *applies if  $\check{\Delta} < V_2^R - V_1^R < \hat{\Delta}$ .*
- (C3) *applies if  $\hat{\Delta} \leq V_2^R - V_1^R \leq w_2^* - w_1^*$ .*

*Proof.* See Appendix B. ||

Observe that the high type’s contract always specifies the first-best value  $x_2^P = x_2^*$ . In contrast, the level of (downwards) distortion in  $x_1^P$  depends on the difference  $V_2^R - V_1^R$ . Indeed, inspection of the solution candidates for (C1)–(C3) reveals that  $x_1^P$  is continuous and monotonic in  $V_2^R - V_1^R$ . It is equal to  $x_1^S$  if (C1) applies. For  $\check{\Delta} < V_2^R - V_1^R < \hat{\Delta}$ , where (C2) applies, it is strictly increasing in  $V_2^R - V_1^R$ . Finally, for  $V_2^R - V_1^R \geq \hat{\Delta}$  it is equal to the first-best value  $x_1^*$ . In the last case both contracts are free of distortions.

Lemma 1 has a straightforward intuition. The principals’ optimal menu trades off the maximization of surplus with the minimization of information rent left to the high type. Observe that for  $V_2^R = V_1^R$ , where (C1) applies, this information rent is equal to  $v_2(x_1^S) - v_1(x_1^S)$ . As the difference between reservation values increases, the importance of rent extraction decreases and vanishes completely for  $V_2^R - V_1^R \geq \hat{\Delta}$ , where the pair of first-best contracts  $(x_i^P, V_i^R - v_i(x_i^*))$  becomes incentive compatible.

We conclude this section with a final comment on the considered (screening) programme. Recall that we have restricted the principal to menus which specify an acceptable contract for both types. In what follows, this restriction is without loss of generality as contracting with both types will indeed be optimal for high values of  $\delta$ .

#### 4.2. Analysis of the matching market: preliminary results

We now return to the market environment. We again restrict attention to the case where frictions become small. It is intuitive that, in analogy to the case with complete information, all players of the short side (in terms of potential entrants) must enter the market as  $\delta$  becomes sufficiently high. Moreover, we can prove that the difference in reservation values  $V_2^R - V_1^R$  must always satisfy the following two conditions. First, it is strictly positive, which follows from (A.1) for  $\delta > 0$ . Second, it is bounded from above by the difference in the levels of first-best surplus  $w_2^* - w_1^*$ .

**Lemma 2.** *For all sufficiently high  $\delta$  all equilibria under incomplete information  $\psi \in \bar{\Psi}_\delta$  satisfy the following conditions:*

- (i) *All players of the “short” side enter, i.e.  $n^P < n_2^A + n_1^A$  implies  $e^P = n^P$ ,  $e_2^A = n_2^A$ ,  $e_1^A = n^P - n_2^A$ ;  $n^P > n_2^A + n_1^A$  implies  $e^P = n^P - n_1^A - n_2^A$ ,  $e_2^A = n_2^A$ ,  $e_1^A = n_1^A$ ;  $n^P = n_2^A + n_1^A$  implies  $\mathbf{e} = \mathbf{n}$ .*
- (ii) *It holds that  $0 < V_2^R - V_1^R < w_2^* - w_1^*$ .*

*Proof.* See Appendix C. ||

In what follows, we restrict consideration to sufficiently high values of  $\delta$  where Lemma 2 applies.

Suppose now that in equilibrium all matches are successful. We prove in Section 4.3 that this must indeed be the case if  $\delta$  is sufficiently high. Indeed, our main result will be that

for low frictions the set of equilibria is independent of the information regime. If all matches are successful, optimality implies that the contracts  $c_i^A$  offered by agents are uniquely determined by  $x_i^A = x_i^*$  and  $t_i^A = u(x_i^*) - U^R$ . Consider next principals' offers. By Lemma 2 we know that in any equilibrium the difference in reservation values satisfies  $0 < V_2^R - V_1^R < w_2^* - w_1^*$ , which allows to conclude by Lemma 1 that only cases (C1)–(C3) may apply. In the rest of this section we study how, given some fixed choice of  $\theta$ , the principals' offer must change in  $\delta$  along a sequence of candidate equilibria.

Consider thus a tightness  $\theta > 0$ . Define the threshold  $\check{\delta}$  which uniquely solves

$$\frac{\check{\delta}m^A(\theta)(1-b)}{1-\check{\delta}[1-m^A(\theta)(1-b)]}[w_2^* - w_1^*] = \check{\Delta}, \quad (6)$$

and the threshold  $\hat{\delta}$  which uniquely solves

$$\frac{\hat{\delta}m^A(\theta)(1-b)}{1-\hat{\delta}[1-m^A(\theta)(1-b)]}[w_2^* - w_1^*] = \hat{\Delta}. \quad (7)$$

Observe that  $0 < \check{\delta} < \hat{\delta} < 1$ . Suppose next that for given  $\delta$  and  $\theta$  there exists an equilibrium where all matches are successful. The choice of  $\delta$  determines the solution to the principals' screening programme as follows.

**Lemma 3.** *If for given  $\delta$  and  $\theta$  there exists an equilibrium under incomplete information  $\psi \in \check{\Psi}_\delta$  where all matches are successful, then the principals' offer is determined by (C1) for  $\delta \leq \check{\delta}$ , by (C2) for  $\check{\delta} < \delta < \hat{\delta}$ , and by (C3) for  $\delta \geq \hat{\delta}$ .*

*Proof.* By combining assertion (ii) in Lemma 2 with Lemma 1, we can restrict consideration to the solutions specified by (C1), (C2) and (C3). Given the specification of offers  $c_i^A$  and the respective choices of  $c_i^P$ , we obtain for (C2) and (C3) the difference  $V_2^R - V_1^R = d^A(w_2^* - w_1^*)$ . For fixed  $\theta$  the difference  $V_2^R - V_1^R$  therefore exceeds  $\check{\Delta}$  if and only if  $\delta > \check{\delta}$ , while it exceeds  $\hat{\Delta}$  if and only if  $\delta > \hat{\delta}$ . For (C1) we obtain  $V_1^R = d^A(w_1^* - U^R)$ , while

$$V_2^R = \delta(1 - m^A(\theta))V_2^R + \delta m^A(\theta)[(1-b)(w_2^* - U^R) + b(V_1^R + v_2(x_1^S) - v_1(x_1^S))].$$

This yields for (C1)

$$(V_2^R - V_1^R)[1 - \delta(1 - m^A(\theta))] = \delta m^A(\theta)[(1-b)(w_2^* - w_1^*) + b(v_2(x_1^S) - v_1(x_1^S))],$$

which implies  $V_2^R - V_1^R \leq \check{\Delta}$  if and only if  $\delta \leq \check{\delta}$ . Collecting results and using Lemma 1 proves the assertion.  $\parallel$

To get more intuition for this result, observe that agents realize  $w_i^* - U^R$  with their own proposal, while we also know that low-type agents receive only their reservation value  $V_1^R$  if it is the principal's turn to make an offer. This implies that the difference in reservation values  $V_2^R - V_1^R$  is bounded from below by  $d^A(w_2^* - w_1^*)$ , where  $d^A$  is strictly increasing in  $\delta$  with  $d^A \rightarrow 1$  for  $\delta \rightarrow 1$ . In words, as agents become increasingly patient, the difference in their reservation values must reflect the difference in what they can get when being chosen as proposers. As the difference  $V_2^R - V_1^R$  increases, we know from Lemma 1 that the principal's trade-off between maximizing surplus and reducing the information rent left to the high type gradually vanishes. This reduces the distortion in the

offer made to the low type. Indeed, given some  $\theta$  and an equilibrium where all matches are successful, all contracts become efficient for  $\delta > \hat{\delta}$ .

Solving (7) for  $\hat{\delta}$  yields

$$\hat{\delta} = \frac{1}{1 + m^A(\theta)(1 - b)(\eta - 1)},$$

with

$$\eta = \frac{w_2^* - w_1^*}{v_2(x_1^*) - v_1(x_1^*)}.$$

For given  $\theta$  the boundary  $\hat{\delta}$  is thus decreasing in the agents' matching probability  $m^A(\theta)$  and their bargaining power  $1 - b$ . Intuitively, the higher these values are, the more closely is the difference in reservation values aligned to the difference  $V_2(c_2^A) - V_1(c_1^A) = w_2^* - w_1^*$ . Note next that  $v_2(x_1^*) - v_1(x_1^*)$  would be the high type's information rent for  $V_2^R = V_1^R$  and if the principal's offer specified  $x_1^P = x_1^*$ . The lower this term, the lower the boundary  $\hat{\delta}$ . This captures the principal's trade-off between maximizing the expected surplus and minimizing the high type's information rent.

Observe finally that the expressions for  $\hat{\delta}$  and  $\check{\delta}$  also illustrate the importance of granting agents a minimum bargaining power with  $b < 1$ . In fact, for  $b \rightarrow 1$  both boundaries converge to one. Below we have more to say on the limit case where  $b = 1$ .

#### 4.3. Analysis of the matching market: the main result

We proceed now in two steps to show that the set of equilibria under the two information regimes coincide for low frictions. We first show that an equilibrium under complete information is also an equilibrium if the agents' type is their private information. Subsequently, we prove the converse result.

If the agents' type is common knowledge, we know from the corollary to Proposition 1 that the difference in reservation values  $V_2^R - V_1^R$  must converge to  $w_2^* - w_1^*$  along any sequence of equilibria where  $\delta \rightarrow 1$ . From our previous discussion in Section 4.2 and, in particular, from Lemma 1 it is intuitive that the corresponding strategies of principals and agents can also be supported as an equilibrium if the agents' type is their private information.

**Proposition 2.** *For all sufficiently high values of  $\delta$  an equilibrium under complete information  $\psi \in \Psi_\delta$  is also an equilibrium under incomplete information  $\psi \in \bar{\Psi}_\delta$ .*

*Proof.* See Appendix D. ||

Proposition 2 establishes as a by-product that the set of equilibria under incomplete information is not empty for high  $\delta$ . In the remaining part of this section we prove that the converse of Proposition 2 holds. The proof proceeds in two steps. We first suppose as in Section 4.2 that all matches must be successful even if the agents' type is their private information. By appealing to Lemma 3, we can argue that the principals' proposal must be determined by (C3), implying that all offers are identical to those under complete information. In a second step, we rule out the possibility that principals do not make an acceptable proposal to both types. In the standard screening game with type-independent reservation values it is well known that principals may

indeed find it optimal to exclude the low type. Precisely, this is the case if the low type's virtual surplus is negative. (Recall that to determine the virtual surplus, one subtracts the information rent which has to be left to the high type). However, if the difference between reservation values  $V_2^R - V_1^R$  becomes sufficiently large, we already know that the trade-off between maximizing expected surplus and reducing the information rent vanishes. This allows us to conclude for low frictions that principals must indeed offer a pair of acceptable contracts.

**Proposition 3.** *For all sufficiently high values of  $\delta$  an equilibrium under incomplete information  $\psi \in \bar{\Psi}_\delta$  is also an equilibrium under complete information  $\psi \in \Psi_\delta$ .*

*Proof.* See Appendix E.  $\parallel$

We can now summarize Propositions 2–3 as follows.

**Theorem.** *For all sufficiently high values of  $\delta$  the set of equilibria is independent of the information regime, i.e.  $\psi \in \bar{\Psi}_\delta$  if and only if  $\psi \in \Psi_\delta$ .*

For sufficiently low frictions it is thus irrelevant whether principals have complete or incomplete information. Under both information regimes we obtain the same set of equilibria. All matches are successful and realize first-best surplus. We can thus conclude that a (matching) market with incomplete information is in this sense successfully decentralized if frictions become sufficiently low.

In Section 2.3 we have emphasized two properties of our bargaining game. Both sides can make a proposal and the respective proposer can commit to a take-it-or-leave-it offer. We argue now that the first ingredient is essential, whereas our results should carry over to a broader range of bargaining games.

As already indicated in the introduction, our results rely crucially on the assumption that agents have some (minimal) bargaining power expressed by  $b < 1$ . If only principals could make offers, we would encounter under both information regimes the well-known Diamond monopoly price paradox (1971). The presence of multiple principals does not lead to competition as offers are only made *ex post*, i.e. after the match has formed.<sup>16</sup>

On the other side, as long as agents have some bargaining power, we conjecture that our results remain qualitatively unchanged if we vary the contractual games. Recall that we assumed that either side may be chosen to make a take-it-or-leave-it offer. Consider some alternative specification of the contractual game which ensures under complete information that the difference  $V_2^R - V_1^R$  converge to  $w_2^* - w_1^*$  for  $\delta \rightarrow 1$ , as asserted in the corollary to Proposition 1. For instance, this is the case in a game as analysed by Rubinstein and Wolinsky (1985), where each period either player may be chosen as the proposer, while rejection leads to breakdown with some (strictly positive) probability. We have shown in a previous version that our results carry over to this specification.<sup>17</sup>

16. Given our specification  $V^0 > 0$ , the market would fail to open up if  $b = 1$ . By specifying  $V^0 = 0$  instead, we have shown in a previous version that, regardless of the level of frictions, the principals' proposal coincides with that in the bilaterally monopolistic case with type-independent reservation values.

17. Precisely, the equivalence of equilibria under both information regimes has been shown for the game where both players can offer menus. (Menu games with alternating offers have been explored first in Inderst (1998, 1999).)

5. CONCLUSION

This paper embeds contract design in a matching market environment. Following much of the screening literature, contracts specify a transfer and an additional (sorting) variable, utility is transferable, and an agent's type only affects his utility (private values). We show that, for sufficiently low frictions, the set of equilibria is independent of whether the agent's type is his private information or whether it can be observed by the respective trading partner. Particularly, all matches are successful and contracts maximize joint surplus. This result holds as long as there is a marginal probability that the agent is chosen as the proposer in a given match.

While we restrict attention to only two types, an extension to any finite set of types is obvious. Moreover, we have established in a technical note that a weaker result holds with a continuum of types. In particular, we can show that in any equilibrium with only successful matches the distortions in the principals' proposal must vanish as frictions disappear.

From a more general perspective, this paper is the first which considers contract design (with private information and a sorting variable) in an environment with search and matching. We feel that this approach should be further explored in different directions. The following routes seem to be promising. Observe first that we have restricted attention to payoffs with private values. With common values, where an agent's type directly affects a principal's utility, our setting would allow to analyse a market where signalling and screening coexist. Such a setting has been explored in Inderst (1997). Moreover, recall that in our matching market environment contracts are designed after the match has formed. An alternative approach would be a model of directed search. Principals compete by advertising menus, while coordination failure among agents gives rise to costly delay (frictions). Inderst and Müller (1999) study such a model. Finally, recall that this paper considers only contract design under *ex ante* private information (adverse selection). In Inderst and Müller (2000) we embed a contractual problem of (double-sided) moral hazard in a search market to analyse the impact of competition on efficiency.

APPENDIX

A. *Proof of Proposition 1.* Recall first that we only consider equilibrium candidates where the market opens up. If type  $i$  enters, optimality implies  $c_i^P = \emptyset$  or  $x_i^P = x_i^*$ ,  $t_i^P = V_i^R - v_i(x_i^*)$ . To satisfy  $V_i^R \geq V^0$ , it must therefore hold that  $c_i^A \neq \emptyset$ , implying by optimality  $x_i^A = x_i^*$ ,  $t_i^A = u(x_i^*) - U^R$ . Observe next that  $c^P = \emptyset$  and  $c_i^A \neq \emptyset$  would imply by optimality  $w_i^* = V_i^R + U^R$ . Substitution into (3) reveals that this cannot be the case. In summary, if type  $i$  enters, all matches with  $i$  are successful and the implemented contracts are uniquely determined. We prove next an auxiliary claim.

**Claim.** *For all sufficiently high values of  $\delta$  the following results hold.*

- (i) *If  $n^P > n_2^A + n_1^A$ , then  $e^P = n^P - n_1^A - n_2^A$ ,  $e_2^A = n_2^A$ , and  $e_1^A = n_1^A$ , while  $U^R = U^0$ .*
- (ii) *If  $n^P < n_2^A + n_1^A$ , then  $e^P = n^P$ ,  $e_2^A = n_2^A$ , and  $e_1^A = n^P - n_2^A$ , while  $V_1^R = V^0$ .*
- (iii) *If  $n^P = n_2^A + n_1^A$ , then  $\mathbf{e} = \mathbf{n}$ .*

*Proof.* Turn first to assertion (i). Stationarity and optimality of the principals' entry decision imply  $U^R = U^0$ . If an agent of type  $i$  enters the market and waits to offer a contract specifying  $x = x_i^*$  and  $t = u(x_i^*) - U^R$ , his expected utility is bounded from below by  $V_i^E = d^A(w_i^* - U^0)$ . It thus remains to show that  $V_i^E > V^0$  holds for sufficiently high values of  $\delta$ , which by (A.3) is indeed the case if  $d^A$  becomes close to one. We argue by contradiction. By  $d^A \in [0, 1]$  we can then find a sequence of equilibria  $\psi_\delta \in \Psi_\delta$ , where  $\delta \rightarrow 1$ , for which the respective values  $d_\delta^A$  converge to a value  $d_1^A < 1$ . As a consequence, the respective values  $d_\delta^P$  must converge

to one. Given the optimal contracts derived above, the reservation value of some type  $i$  satisfying  $e_i^A > 0$  is equal to  $d_\delta^A(w_i^* - U^0)$ . By  $d_\delta^P \rightarrow 1$ ,  $d_\delta^A \rightarrow d_1^A < 1$ , (A.3), and  $U_\delta^R = U^0$ , this implies that (3) cannot be satisfied for sufficiently high values of  $\delta$ .<sup>18</sup>

Having proved that (i) holds for sufficiently high values of  $\delta$ , we turn to assertion (ii). In case  $e^P < n^P$  holds, optimality of entry implies  $U^R = U^0$ . By the argument in the proof of assertion (i) this would lead to the entry of all agents for sufficiently high  $\delta$ , which by  $n^P > n_1^A + n_2^A$  contradicts stationarity. Hence, all principals must enter for high  $\delta$ . By (A.1), stationarity, and optimality of entry for agents, this implies  $e_2^A = n_2^A$ ,  $e_1^A = n^P - n_2^A$ , and  $V_1^R = V^0$ .

For assertion (iii) we can now combine the previous arguments. If not all principals enter, we already know from  $U^R = U^0$  that all agents must enter for high  $\delta$ , which by  $n^P = n_1^A + n_2^A$  contradicts stationarity. On the other side, if not all agents enter, it follows from stationarity and  $n^P = n_1^A + n_2^A$  that  $e^P < n^P$ . This implies  $U^R = U^0$ , which again must induce all agents to enter for high  $\delta$ . This completes the proof of the claim.  $\parallel$

In what follows, we restrict consideration to sufficiently high values of  $\delta$  such that the claim holds. By (A.3) and inspection of (3), there exists for sufficiently high  $\delta$  a unique value  $\bar{\theta}(\delta)$  solving  $U^R = U^0$ . Similarly, there exists for sufficiently high  $\delta$  a unique value  $\underline{\theta}$  solving  $V_1^R = V^0$ . We restrict consideration to sufficiently high values of  $\delta$  such that existence of  $\bar{\theta}$  and  $\underline{\theta}$  is ensured, while it also holds that  $\bar{\theta} > \underline{\theta}$ .

Consider now the case  $n^P > n_2^A + n_1^A$ . Given the claim, the specification of contracts, and the definition of reservation values,  $\theta = \bar{\theta}$  must hold in equilibrium. For  $n^P < n_2^A + n_1^A$  it must likewise hold that  $\theta = \underline{\theta}$ . With  $n^P = n_2^A + n_1^A$  the requirements  $U^R \geq U^0$  and  $V_1^R \geq V^0$ , which ensure that entries specified in the claim are optimal, are satisfied if and only if  $\theta \in [\underline{\theta}, \bar{\theta}]$ . Finally, observe that given our specification of entries and contracts, the proposed equilibrium candidates can indeed be supported by equilibrium strategies for all players. This completes the proof of Proposition 1.  $\parallel$

For further reference we introduce a threshold  $\bar{\delta}_1 < 1$  such that the assertions hold for all  $\delta > \bar{\delta}_1$ .

**B. Proof of Lemma 1.** Note first that we can restrict attention to differences  $0 \leq V_2^R - V_1^R \leq w_2^* - w_1^*$ . We now argue first that the solution to the screening programmes satisfies the conditions of one of the three cases discussed in the main text. Recall that we distinguished between the following cases:

- Case (C1) Only  $IR_1$  and  $IC_2$  are binding.
- Case (C2) Only  $IR_1$ ,  $IR_2$  and  $IC_2$  are binding.
- Case (C3) Only  $IR_1$  and  $IR_2$  are binding.

We proceed in two steps.

**Claim 1.**

- (i)  $IR_i$  binds for at least one  $i \in I$ .
- (ii)  $IC_i$  and  $IC_j$  are not simultaneously binding for  $i \neq j$ .
- (iii) If  $IR_i$  is not binding,  $IC_i$  must be binding and vice versa for  $i \in I$ .

*Proof.* If (i) does not hold in a supposed solution, define  $\varepsilon_i = V_i^R - V_i(c_i^P)$  and  $\varepsilon = \min(\varepsilon_1, \varepsilon_2) > 0$  to obtain a new menu  $\{(x_i^P, t_i^P - \varepsilon)\}_{i \in I}$ . This satisfies  $IR_i$  by construction and leaves  $IC_i$  unchanged, while the principal strictly gains. To prove (ii), note first that both  $IC_i$  can only bind if  $x_1^P = x_2^P = \bar{x}$ , which must imply  $t_1^P = t_2^P = \bar{t}$ . Consider such a pooling offer and suppose first that  $\bar{x} < x_2^*$ . There exists a contract  $\hat{c} = (\hat{x}, \hat{t})$  in the neighbourhood of  $\bar{c}$  satisfying  $\hat{x} > \bar{x}$  and  $V_2(\hat{c}) = V_2(\bar{c})$ . By (A.1) offering  $\hat{c}$  to  $i = 1$  and  $\bar{c}$  to  $i = 2$  is incentive compatible, while the principal is strictly better off as  $\mu > 0$  and  $dw_2(x)/dx|_{x=\bar{x}} > 0$ . This contradicts the optimality of the pooling offer  $\bar{c}$  in case  $\bar{x} < x_2^*$ . The argument for  $\bar{x} \geq x_2^*$  is analogous as we can this time construct a new contract  $\hat{c}$  with  $\hat{x} < \bar{x}$  for the low-type agent. To prove (iii), consider first  $i = 1$ . If  $IR_1$  does not bind, we can decrease  $t_1^P$  marginally by  $\varepsilon_1 > 0$ . Similarly, a slack in  $IC_1$  admits a decrease of  $t_1^P$  by  $\varepsilon_2 > 0$ . Define  $\varepsilon = \min(\varepsilon_1, \varepsilon_2)$ . Offering  $(x_1^P, t_1^P - \varepsilon)$  instead of  $c_1^P$ , where all constraints are still satisfied, is thus strictly profitable for the principal. The argument for  $i = 2$  is analogous.  $\parallel$

**Claim 2.** If  $IC_2$  does not bind,  $IC_1$  is also not binding.

*Proof.* If  $IC_2$  does not bind,  $IR_2$  must bind by Claim 1. By optimality it must hold that  $c_2^P = (x_2^*, V_2^R - v_2(x_2^*))$ , in case this specification satisfies  $IC_1$ . The constraint  $IC_1$  transforms then to  $V_1(c_1^P) \geq V_2^R + v_1(x_2^*) - v_2(x_2^*)$ . This is satisfied with a slack as  $V_1(c_1^P) \geq V_1^R$  by  $IR_1$ ,  $v_2(x_2^*) - v_1(x_2^*) > w_2^* - w_1^*$ , and  $V_2^R - V_1^R \leq w_2^* - w_1^*$ , which holds by assumption.  $\parallel$

18. Note that this holds irrespective of the value of the distribution  $\mu_\delta$  determined in  $\psi_\delta$ .

The cases (C1)–(C3) are now obtained by combining Claims 1 and 2. To prove Lemma 1, it remains to analyse when these cases apply. We do so in two steps.

**Claim 3.**

- (i) For  $\check{\Delta} < V_2^R - V_1^R$  (C1) cannot apply.
- (ii) For  $0 \leq V_2^R - V_1^R \leq \check{\Delta}$  the offer determined by (C1) satisfies all constraints.
- (iii) For  $0 \leq V_2^R - V_1^R < \hat{\Delta}$  (C3) cannot apply.
- (iv) For  $\hat{\Delta} \leq V_2^R - V_1^R \leq w_2^* - w_1^*$  the offer determined by (C3) satisfies all constraints.
- (v) The offer determined by (C2) satisfies all constraints, while a solution to (5) exists for  $\check{\Delta} \leq V_2^R - V_1^R \leq \hat{\Delta}$ .

*Proof.* Regarding assertions (i)–(ii), recall that in (C1) the constraints  $IC_1$  and  $IR_2$  are neglected. Recall now that  $x_1^P = x_1^S < x_1^* < x_2^* = x_2^P$ . As  $IC_2$  binds, this implies by (A.1) that  $IC_1$  is satisfied. Turn next to  $IR_2$ . Substitution of the binding constraints  $IR_1$  and  $IC_2$  yields  $V_2(c_2^P) = V_1^R + v_2(x_1^S) - v_1(x_1^S)$ , which by (A.1) satisfies  $V_2(c_2^P) \geq V_2^R$  for  $V_2^R - V_1^R \leq \check{\Delta}$ , while  $IR_2$  does not hold for  $V_2^R - V_1^R > \hat{\Delta}$ . This proves assertions (i)–(ii).

Turn next to assertions (iii)–(iv). Recall that in (C3) both incentive compatibility constraints are neglected. Substitution of the binding participation constraints into  $IC_1$  yields the requirement  $V_1^R \geq V_2^R + v_1(x_2^*) - v_2(x_2^*)$ , which is satisfied for all considered differences  $V_2^R - V_1^R \leq w_2^* - w_1^*$ . Similarly,  $IC_2$  transforms to  $V_2^R \geq V_1^R + v_2(x_1^*) - v_1(x_1^*)$ , which by (A.1) holds if and only if  $V_2^R - V_1^R \geq \hat{\Delta}$ . This proves assertions (iii)–(iv).

Regarding assertion (v), recall that only  $IC_1$  is neglected in (C2). As both participation constraints bind and as  $x_2^P = x_2^*$ , we know already from the previous arguments for (C3) that  $IC_1$  is satisfied. At this point we must also discuss existence of a solution  $x_1^P$  to (5) if  $\check{\Delta} < V_2^R - V_1^R < \hat{\Delta}$ . By (A.1) we indeed get a unique solution satisfying  $x_1^S < x_1^P < x_1^*$ .

For the following arguments it is also useful to note that substituting  $V_2^R - V_1^R = \check{\Delta}$  into (5) yields  $x_1^P = x_1^S$ , such that the solution candidates of (C1) and (C2) coincide, while substituting  $V_2^R - V_1^R = \hat{\Delta}$  yields  $x_1^P = x_1^*$ , such that the solution candidates of (C3) and (C2) coincide.  $\parallel$

**Claim 4.** For  $V_2^R - V_1^R < \check{\Delta}$  and  $V_2^R - V_1^R > \hat{\Delta}$  (C2) cannot apply.

*Proof.* Take first the range  $V_2^R - V_1^R > \hat{\Delta}$ , where by (5) (C2) specifies  $x_1^P > x_1^*$ . The principal can now profitably deviate by offering the menu determined in (C3), which specifies  $x_1^P = x_1^*$ . To see this, recall from Claim 1 that all constraints of the programme are still satisfied, while the offer solves a less constrained programme. For  $V_2^R - V_1^R < \hat{\Delta}$  we obtain under (C2)  $x_1^P < x_1^S$ . As (C1) solves a less constrained programme, while the menu satisfies all constraints by Claim 1, (C2) cannot be optimal.  $\parallel$

The asserted case distinction follows now by combining Claims 1 and 2 with Lemma 1.  $\parallel$

*C. Proof of Lemma 2.* We start with the second assertion and assume that in equilibrium agents of both types enter the market. Observe first that  $V_2^R - V_1^R > 0$  holds by (A.1) for  $\delta > 0$ . We argue next to a contradiction and suppose that  $V_2^R - V_1^R \geq w_2^* - w_1^*$ . In this case it is straightforward to show that in any optimal menu offered in the screening game the high type does not realize more than his reservation value, *i.e.*  $c_2^P = \emptyset$  or  $t_2^P = V_2^R - v_2(x_2^P)$  in case  $c_2^P \in C$ . To ensure  $V_2^R \geq V^0$ , this requires  $c_2^A \in C$ , where optimality implies  $x_2^A = x_2^*$  and  $t_2^A = u(x_2^*) - U^R$ . This yields  $V_2^R = d^A(w_2^* - U^R)$ . As a low-type agent can always follow the strategy to wait until he is the proposer in order to implement  $x_1^A = x_1^*$  and  $t_1^A = u(x_1^*) - U^R$ , we obtain  $V_2^R - V_1^R \leq d^A(w_2^* - w_1^*)$ . As  $d^A < 1$  holds from  $\delta < 1$ , we obtain a contradiction to the assumption that  $V_2^R - V_1^R \geq w_2^* - w_1^*$ . This proves assertion (ii).

Turn now to assertion (i) and suppose first that  $n^P > n_2^A + n_1^A$ . Stationarity and optimality imply  $U^R = U^0$ , while an agent of type  $i$  can realize at least  $V_i^P = d^A(w_i^* - U^0)$  by waiting until he is in the role of the proposer. We argue to a contradiction. We distinguish between two cases. If  $e_1^A = 0$ , there is complete information about the type of agents in the market. We can therefore apply the argument in Proposition 1 to show that a deviating low-type agent who enters the market can realize strictly more than  $V^0$ , *i.e.*  $V_1^P > V^0$  follows if  $\delta$  becomes sufficiently high. This yields a contradiction for the case where  $e_1^A = 0$ . Suppose next that  $0 < e_1^A < n_1^A$ , implying  $V_1^R = V^0$ . If this is the case for all high values of  $\delta$ , we can find a sequence of equilibria  $\psi_\delta \in \Psi_\delta$  where  $\delta \rightarrow 1$  and where the respective values  $d_\delta^A$  converge to a value  $d_1^A < 1$ . As a consequence, we obtain  $d_\delta^P \rightarrow 1$ . Using assertion (ii) and the assumption that low types realize  $V^0$ , a principal can realize at least  $w_1^* - V^0$  in his screening offer, regardless of the type he faces. Hence, by waiting until he can make an offer, a principal can realize at least  $d_\delta^P(w_1^* - V^0)$ . By  $d_\delta^P \rightarrow 1$  and (A.3), this yields for high  $\delta$  a contradiction to the requirement that principals realize exactly  $U^0$ , which is implied by  $n^P > n_2^A + n_1^A$ , optimality of entry, and stationarity.

Having proved assertion (i) for the case  $n^P > n_2^A + n_1^A$ , we turn to the remaining two cases. The further argument is now completely analogous to that provided in Proposition 1 under complete information. It is therefore omitted.  $\parallel$

For further reference we introduce a threshold  $\bar{\delta}_2 < 1$  such that the claims hold for all  $\delta > \bar{\delta}_2$ .

D. *Proof of Proposition 2.* Define  $\hat{d}^A$  by  $\hat{d}^A(w_2^* - w_1^*) = \hat{\Delta}$ . Consider next the case of complete information. By the argument in the proof of the corollary to Proposition 1, there exists a threshold  $\bar{\delta}_3 < 1$  such that for all  $\delta > \bar{\delta}_3$  and all equilibria under complete information  $\psi \in \Psi_\delta$  it holds for the respective pairs  $(\theta, \delta)$  that  $d^A \geq \hat{d}^A$ .<sup>19</sup> We prove that  $\psi \in \Psi_\delta$  implies  $\psi \in \bar{\Psi}_\delta$  for  $\delta \geq \bar{\delta}_3$ . For this we have to check whether the strategies specified under complete information remain optimal under incomplete information. This is surely the case regarding entry, the agents' proposals, and all responses. Regarding the principals' proposals, observe that by construction  $V_2^R - V_1^R \geq \hat{\Delta}$ . By Lemma 1 this implies that principals cannot gain by offering a deviating menu which is acceptable to both types, while the menu specified in (C3) satisfies all four constraints. As  $w_i^* > U^R + V_i^R$ , it is also not profitable to deviate to an offer which is not acceptable to both types.  $\parallel$

E. *Proof of Proposition 3.* We show first that we can restrict consideration to the case where a match may only be broken up unsuccessfully if a principal makes a proposal to a low-type agent. Observe that throughout the proof we restrict consideration to values  $\delta > \bar{\delta}_2$ , which was defined in the proof of Lemma 2. This ensures that both types enter.

**Claim 1.** For all  $\psi \in \bar{\Psi}_\delta$  it holds that  $c_i^A \in C$  for both  $i \in I$ , while  $c_2^P \in C$ .

*Proof.* Consider first the game where agents make a proposal. We argue to a contradiction and assume that  $c_i^A = \emptyset$ , which by optimality implies  $w_i^* \leq U^R + V_i^R$ . To ensure  $V_i^R \geq V^0$ , it must hold that  $c_i^P \in C$ , which again is only optimal if  $w_i^* \geq U^R + V_i^R$ .<sup>20</sup> This implies  $x_i^P = x_i^*$  and  $w_i^* = U^R + V_i^*$ . As this yields  $V_i^R = d^P(w_i^* - U^R) < w_i^* - U^R$ , we obtain a contradiction.

Turn next to the game where principals propose. We argue again to a contradiction and assume  $c_2^P = \emptyset$ . Optimality implies then  $V_i^R = V^0$  and thus  $V_i^R = d^A(w_i^* - U^R)$ , from which it follows that  $w_i^* > U^R + V_i^R$  holds for both  $i \in I$ . Clearly, this implies that setting  $c_i^P = \emptyset$  for both  $i \in I$  is not optimal. As  $c_2^P = \emptyset$ , it follows from optimality that  $x_1^P = x_1^*$  and  $t_1^P = V_1^R - v_1(x_1^*)$ . Principals can now profitably deviate by offering additionally  $i = 2$  the contract  $c$ , where  $x = x_2^*$  and  $t_2^P = V_2^R - v_2(x_2^*)$ , in case the new menu is incentive compatible. As by assumption type  $i = 2$  previously (weakly) preferred to reject  $c_1^P$ , it remains to check incentive compatibility for type  $i = 1$ . By  $V_2(c) = V_2^R$  and  $V_1(c_1^P) = V_1^R$ , this follows from  $V_i^R = d^A(w_i^* - U^R)$  and (A.1).  $\parallel$

By Claim 1 we are left with two equilibrium candidates. If all matches are successful, the distribution in the market is given by  $\mu = \mu^0$ . If  $c_1^P = \emptyset$ , low-type agents circulate longer and we obtain

$$\mu = \frac{\mu^0(1-b)}{1-b(1-\mu^0)} < \mu^0. \quad (8)$$

We discuss now the two remaining candidates in turn. Consider first the case where all matches are successful.

**Claim 2.** There exists  $\hat{\delta}_1 < 1$  such that for all  $\delta > \hat{\delta}_1$ , any  $\psi \in \bar{\Psi}_\delta$  where all matches are successful satisfies also  $\psi \in \Psi_\delta$ .

*Proof.* If the principals' offer is determined by (C3), it is obvious that the respective equilibrium where all matches are successful is also an equilibrium under complete information. By Lemma 1 this is the case if  $V_2^R - V_1^R \geq \hat{\Delta}$ . Denote now  $\Delta V^R = V_2^R - V_1^R$  and  $E^i = w_i^* - V_i^R$ . We argue to a contradiction, which implies existence of a sequence of equilibria  $\psi_\delta \in \bar{\Psi}_\delta$ , where  $\delta \rightarrow 1$ , all matches are successful, and  $\Delta V_\delta^R < \hat{\Delta}$ . With the tightness in  $\psi_\delta$  given by  $\theta_\delta$ , we denote  $m_\delta^P = m^P(\theta_\delta)$  and  $m_\delta^A = m^A(\theta_\delta)$ , and we also use  $U_\delta^R, E_\delta^i, d_\delta^A$  and  $d_\delta^P$  for the respective choices in  $\psi_\delta$ . Observe next that  $\Delta V_\delta^R < \hat{\Delta}$  transforms to  $E_\delta^2 - E_\delta^1 > v_2(x_2^*) - v_2(x_1^*)$ , while it surely holds that

$$U_\delta^R \leq d_\delta^P[\mu^0 E_\delta^2 + (1-\mu^0)E_\delta^1]. \quad (9)$$

19. Of course, we choose  $\bar{\delta}_3 \geq \bar{\delta}_1$ , which was defined in the proof of Proposition 1, to ensure that  $\Psi_\delta$  is non-empty by Proposition 1.

20. Note that this would be different with common values.

If a principal deviates and offers only an acceptable contract to high types, he can realize at least

$$U_\delta^D = \frac{\delta m_\delta^P \mu^0 b}{1 - \delta(1 - m_\delta^P \mu^0 b)} E_\delta^2. \quad (10)$$

To ensure that  $U_\delta^D \leq U_\delta^R$ , which is required in an equilibrium, it must thus hold by (9)–(10) that  $E_\delta^2 - E_\delta^1 \leq E_\delta^1(1 - \delta)/(\delta b m_\delta^P \mu^0)$ . The right side of this inequality is only higher than  $v_2(x_2^*) - v_2(x_1^*)$  if

$$\delta < \frac{E_\delta^1}{E_\delta^1 + b m_\delta^P \mu^0 [v_2(x_2^*) - v_2(x_1^*)]}.$$

By  $\delta \rightarrow 1$  this implies  $m_\delta^P \rightarrow 0$  and thus  $d_\delta^A \rightarrow 1$ . By the definition of  $V_i^R$  it follows that  $\Delta V_\delta^R \geq \hat{\Delta}$  holds for sufficiently high values of  $\delta$ . This yields a contradiction, which completes the proof.  $\parallel$

**Claim 3.** *There exists  $\hat{\delta}_2 < 1$  such that for all  $\delta > \hat{\delta}_2$  there exists no  $\psi \in \bar{\Psi}_\delta$  where not all matches are successful.*

*Proof.* By Claim 1 we must only consider a single case, where  $c_1^P = \emptyset$ ,  $x_2^P = x_2^*$ , and  $t_2^P = V_2^R - v_2(x_2^*)$ . This implies  $V_i^R = d^A(w_i^* - U^R)$  for both  $i \in I$ . A principal can profitably deviate by offering additionally for  $i = 1$  the contract  $c$  with  $x = x_1^*$  and  $t = V_1^R - v_1(x_1^*)$ , in case this menu is incentive compatible. Incentive compatibility is satisfied if  $V_2^R - V_1^R \geq \hat{\Delta}$ . The remaining argument is now similar to that in Claim 2. We argue to a contradiction and assume existence of a corresponding sequence of equilibria  $\psi_\delta \in \Psi_\delta$ , where  $\delta \rightarrow 1$  and where the difference of reservation values satisfies  $\Delta V_\delta^R < \hat{\Delta}$ . Observe also that the distribution of types is given by (8). Denote next

$$\tilde{d}_\delta^P = \frac{\delta m_\delta^P \mu^0 b}{1 - \delta[1 - b m_\delta^P \mu^0]}.$$

We obtain  $U_\delta^R = \tilde{d}_\delta^P [w_2^* - d_\delta^A (w_2^* - U_\delta^R)]$ , which transforms to

$$U_\delta^R = \frac{\tilde{d}_\delta^P (1 - d_\delta^A)}{1 - \tilde{d}_\delta^P d_\delta^A} w_2^*. \quad (11)$$

If  $d_\delta^A$  does not converge to one,  $\tilde{d}_\delta^P$  converges to one, which by (11) implies that  $U_\delta^R$  converges to  $w_2^*$ . For high  $\delta$  this already yields a contradiction as both types must realize at least  $V^0 > 0$  in the market. On the other hand, if  $d_\delta^A$  converges to one, we obtain for sufficiently high  $\delta$  a contradiction to the assumption that  $\Delta V_\delta^R < \hat{\Delta}$ . Hence, we obtain a contradiction for both cases, which proves the claim.  $\parallel$

Choosing  $\delta > \max\{\hat{\delta}_1, \hat{\delta}_2\}$  completes the proof.  $\parallel$

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