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Contractual Signaling in a Market Environment[1]

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We consider a game of signaling where the informed sender proposes a contract, which can only be accepted or rejected by the receiver. While most of the literature considers a bilaterally monopolistic setting, we embed the game in a market environment where a sender may switch to another receiver in case of rejection. We analyze how this structural extension can be employed to restrict the set of (stationary) equilibrium allocations. *Journal of Economic Literature* Classification Number: C78, D82. © 2002 Elsevier Science (USA)

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1. INTRODUCTION

In the last two decades much effort has been invested in the analysis of games with incomplete information. Signaling games constitute a major contribution to this research. The canonical setting of a signaling game can be described as follows. A player with private information takes an action which is observed by a second (uninformed) player. The latter player seeks to infer something about the informed player's information from the action chosen and subsequently chooses an action himself. Typically, the first player is called the sender of a signal (or message), and the responding player is called the receiver. This set-up has been applied to many economic situations, including entry deterrence, insurance markets, and managerial compensation (see Mailath (1992) and Kreps and Sobel (1994) for recent summaries). A major drawback of the analysis of signaling games is the multiplicity of (perfect Bayesian) equilibria, which arises from the freedom to specify the receiver's beliefs following actions off the equilibrium path.

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Consequently, a large literature on refinements has developed criteria to reduce the set of equilibria. Probably the most popular refinements applied to games of signaling are the Intuitive Criterion by Cho and Kreps (1987) and the Divinity Criterion by Cho and Sobel (1990).[2] Below we explicitly relate our approach to the logic underlying the Intuitive Criterion. (For an overview on refinements in signaling games see Chapter 11 of Fudenberg and Tirole (1992).)

The present paper introduces a structural extension to signaling games which, among other things, provides a natural approach to restricting the set of equilibria. For the purpose of this paper we restrict our attention to games of signaling where the sender proposes a contract. The sender has private information about his type, which influences both his and the receiver's payoff if a contract is concluded. A receiver prefers to conclude a given contract with a "higher" type. The contract proposed by the sender specifies two variables, of which one may be a monetary transfer. The other variable has signaling properties, as the sender's utility function satisfies a standard sorting condition in this variable. The receiver is restricted to either accepting or rejecting the sender's proposal. (Kreps and Sobel (1994) call this the "take-it-or-leave-it setup.") The literature usually restricts its attention to a bilateral monopoly or, in the case of multiple receivers, to a one-shot game. In contrast, in our model the existence of multiple receivers allows the sender to switch to other receivers if the current match is not successful.[3] To be precise, we assume that homogeneous receivers are in fixed positions and are randomly visited by senders. A sender, who may be a worker or an insuree, can only conclude a single contract. We assume that the following three-stage game is played during a visit. The sender proposes a contract, which the receiver can only accept or reject. Rejection leads to a break-off, while after acceptance the sender can still decide whether to implement or to withdraw the contract.[4] Following withdrawal of the contract, the match is broken up. The sender may then leave the market or he can approach another receiver at some costs. To ensure that the outside option of switching to another receiver has constant value, we

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[2] The concept used in Cho and Sobel (1990) goes back to Banks and Sobel (1987). We should further mention the contributions by Sobel *et al.* (1990) and by Mailath *et al.* (1993), which develop refinements explicitly in the context of signaling games.

[3] This relates our paper to the search and matching market literature summarized in McMillan and Rothschild (1994). In the case of private information this literature, however, assumes either private values or that the contractual space does not contain a sorting or signaling variable.

[4] Adding the possibility of withdrawal is reminiscent of a similar approach in the theory of competition under adverse selection, where it is used to establish the existence of an equilibrium in pure strategies. See Hellwig (1987) for a strategic formulation of this argument dating back to Wilson (1977).

restrict our attention to stationary (perfect Bayesian) equilibria. Moreover, we introduce a mild additional equilibrium restriction. This ensures that the receiver accepts a proposal which is surely withdrawn by all types with whom he would not realize a payoff strictly exceeding his reservation value.[5]

Our first result is that the set of equilibrium contracts converges to the set of Rothschild–Stiglitz (or least-cost separating, LCS) contracts as the costs of switching vanish. Recall that for each type the LCS contract maximizes his utility subject to the receiver's participation constraint and the other types' incentive compatibility constraints. The logic underlying this convergence result is similar to the selection procedure in the Intuitive Criterion of Cho and Kreps (1987). In contrast to the refinement approach, which is supported merely by an introspective argument, the selection receives a strategic foundation in our extended signaling game. We also investigate how the equilibrium set depends on the size of switching costs and the prior distribution of types. In particular, we show for the case with two types that our selection exhibits a natural continuity property (in the prior distribution), which again sets it apart from an application of the Intuitive Criterion.

The rest of this paper is organized as follows. Section 2 introduces the model. In Section 3 we state some preliminary results. Section 4 investigates how switching costs shape the equilibrium set and derives a convergence result for low visiting costs. In Section 5 we explore how search costs and the distribution of types jointly determine the equilibrium set. Section 6 concludes by discussing alternative ways to structurally extend signaling games to reduce the multiplicity of equilibria. Some proofs are relegated to an Appendix.

2. THE MODEL

2.1. *Players and Payoffs*

There are two types of players called senders and receivers. A sender has private information about his type denoted by a natural number $i \in I = \{\mathbf{1}, \ldots, \bar{i}\}$ with finite $\bar{i} > 1$. A receiver has prior beliefs about a sender's type and assigns probability $\mu(i) > 0$ to a type $i \in I$, where $\sum_{i \in I} \mu(i) = \mathbf{1}$. A sender and a receiver can conclude a contract which specifies two variables t and y. We restrict our attention to applications where contracts $c = (t, y)$ are restricted to the set $c \in C = \Re \times \Re_{+}^{0}$. A contract c between a receiver

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[5] Hence, the existence of multiple receivers creates a type-dependent outside option. In the analyzed (stationary) equilibria, however, there will be no switching on the equilibrium path. Moreover, as a single sender contracts at most once, we analyze an essentially static game. For a dynamic model of equilibrium selection in (contractual) signaling games see the evolutionary analysis in Nöldeke and Samuelson (1997).

and a sender of type $i \in I$ realizes the utility $U^i(c)$ for the receiver and the utility $V^i(c)$ for the sender. Note that the receiver's utility is type-dependent. A sender of type i may also take up a mutually exclusive outside option realizing the utility $V_0^i \geq 0$. We normalize the receiver's reservation value to zero. We assume that U^i and V^i are continuously differentiable. Derivatives are denoted by subscripts. We invoke the following standard assumptions:

(A.1) (Monotonicity) $U_t^i < 0$ and $V_t^i > 0$, $U_y^i \geq 0$, and $V_y^i \leq 0$ for all $c \in C$.

(A.2) (Sorting) $(-V_y^i/V_t^i) > (-V_y^j/V_t^j)$ for $i < j$ and $c \in C$.

(A.3) (Common Values) $U^j(c) > U^i(c)$ for all $\mathbf{1} \leq i < j \leq \bar{i}$ and $c \in C$.

By (A.1) the two parties have conflicting preferences over the contractual variables. The conflict over t is intuitive if t represents a monetary transfer. (A.2) is a standard single-crossing condition. To higher types a marginal increase in y is less costly in terms of t. We therefore refer to y as the sorting or signaling variable. Finally, (A.3) specifies that the receiver strictly prefers to conclude a given contract with a higher type.

For an example where (A.1)–(A.3) are satisfied, consider a contract between an employer (receiver) and an employee (sender), where the employee's ability is his private information. Let t denote the wage and let y be a measure of effort. Contracted effort may either be exerted on the job or may relate to any other obligation such as training (see Spence (1973)). We assume that the activity is purely dissipative and specify $U^i(c) = i - t$ and $V^i(c) = t - y/i$. Reservation values are assumed to be type-independent and are normalized to $V_0^i = 0$. We will frequently refer to this example for illustrative purposes.

We make the following technical assumptions, which are similarly invoked in Maskin and Tirole (1992).

(A.4) There exists $\varepsilon > 0$ such that for all $c \in C$ and $i \in I$ it holds that $V_y^i(c) < -\varepsilon$, $V_t^i > \varepsilon$, and $U_t^i < -\varepsilon$. Moreover, along any indifference curve of the receiver, $-U_y^i/U_t^i$ goes to zero as $y \to \infty$.

(A.4) ensures that all considered programs have an interior solution. Moreover, it compacts the set of contracts which will become relevant for the further analysis. We define next a family of Rothschild–Stiglitz or "least-cost separating" contracts

DEFINITION. A family of contracts $\{c_L^i\}_{i \in I}$ is least-cost separating (LCS) if

(i) For $i = 1$: c_L^1 maximizes $V^1(c)$ subject to $c \in C$, $U^1(c) \geq 0$. Denote $V_L^1 = V^1(c_L^1)$.

(ii) For $i \in \{2, \ldots, \bar{\imath}\}$, c_L^i maximizes $V^i(c)$ subject to $c \in C$, $U^i(c) \geq 0$, $V^{i-1}(c) \leq V_L^{i-1}$. Denote $V_L^i = V^i(c_L^i)$.[6]

A LCS contract for a type $i > 1$ maximizes the utility of this type subject to the receiver's participation constraint and the incentive compatibility constraint of the adjacent lower type. In the labor example with $U^i(c) = i - t$ and $V^i(c) = t - y/i$, we obtain $y_L^1 = 0$ and $y_L^i = i - \mathbf{1} + y_L^{i-1}$ for $i > 1$, while $t_L^i = i$ holds for all types. The lowest type chooses the first-best value $y_i^1 = 0$, while all higher types choose y sufficiently large to separate it from the next lower type.

We restrict our attention to the case where all types will find it profitable to enter the market as $V_L^i > V_0^i$.

(A.5) $V_L^i > V_0^i$ holds for all $i \in I$.

Moreover, it is convenient to ensure that the family $\{c_L^i\}_{i \in I}$ is uniquely determined. This holds if we invoke the following concavity assumption:

(A.6) For all i, V^i and U^i are quasi-concave (one strictly).[7]

Given (A.1)–(A.6), we can derive the following standard result (see, for instance, Maskin and Tirole (1992)).

LEMMA 1. *The LCS family of contracts* $\{c_L^i\}_{i \in I}$ *satisfies overall incentive compatibility* (*i.e.*, $V_L^i \geq V^i(c_L^j)$ *for all* $i, j \in I$) *and is separating with* $y_L^j > y_L^i$ *for* $j > i$. *Moreover,* $U^i(c_L^i) = 0$ *holds for all* $i \in I$.

2.2. *Market Environment and Signaling Game*

As noted in the Introduction, we intend to use the presence of more than one receiver to create an outside option for a sender who fails to conclude a contract with a given receiver. Suppose therefore that there is a continuum of receivers in the market. If we assume that each receiver can conclude arbitrarily many contracts, the number of senders is irrelevant. For convenience, however, we consider a single (representative) sender. This sender can enter the market to visit a receiver or he can realize his exogenous reservation value V_0^i. If he decides to enter the market, he picks

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[6] Observe that (A.4) ensures that the programs in (i) and (ii) indeed have a solution. For $i = 1$ this follows as the optimal y is surely finite by $V_y^i(c) < -\varepsilon$ and $\lim_{y\to\infty}(-U_y^i/U_t^i) = 0$. For $i > 1$ the set of incentive-compatible contracts is non-empty as $V_t^i > \varepsilon$.

[7] To see how uniqueness is established, consider the respective program for some type i. It can be shown (see also Lemma 1 below) that optimality implies $U^i(c_L^i) = 0$. If the incentive compatibility constraint for type $i - 1$ is not binding, maximizing $V^i(c)$ subject to $U^i(c_L^i) = 0$ indeed yields a unique solution. If the constraint for $i - 1$ binds, we can invoke (A.1)–(A.2) and concavity to show that y_L^i is the unique minimal value satisfying $V^{i-1}(t, y) = V_L^{i-1}$, where t is determined by $U^i(t, y) = 0$.

one receiver randomly. If his current visit is successful, a contract is implemented and the sender leaves the market. Otherwise, he faces the choice of leaving the market or visiting another receiver. In the latter case the sender again randomly chooses a receiver and incurs a fixed "search" cost $s > 0$ (in utility terms).[8] Observe that the first visit is assumed to be costless, which avoids case distinctions if s becomes large. We assume that all visited receivers do not observe the sender's previous history. During a visit the following game Γ is played:

Stage 1. The sender offers a single deterministic contract $c \in C$.

Stage 2. The receiver either accepts or rejects the offer.

Stage 3. In case of rejection, the match dissolves. In case of acceptance, the sender may implement the contract or withdraw it, in which case the match dissolves.

3. EQUILIBRIUM CONCEPT AND PRELIMINARY RESULTS

As indicated in the introduction, we restrict our attention to the case where the market environment creates an outside option with constant value. We therefore consider only stationary strategies. We further require that players' strategies are sequentially optimal and that receivers' beliefs are consistently updated. We have to invoke next an additional equilibrium restriction which we regard to be rather mild. Suppose a receiver observes a deviating offer c. When deciding whether to accept or to reject the offer, he anticipates the sender's optimal strategy in Stage 3. Given some offer c, denote the set of types i satisfying $U^i(c) \leq 0$ by I_1. Suppose next that *all* types $i \in I_1$ would strictly prefer to withdraw c as visiting another receiver is more profitable than realizing $V^i(c)$. Moreover, assume that there exists a non-empty set of types I_2 with $U^i(c) > 0$ such that all $i \in I_2$ will surely implement c as $V^i(c)$ strictly exceeds their (continuation) utility after withdrawal. If the receiver accepts c, his expected utility will therefore be nonnegative, regardless of his out-of-equilibrium beliefs. Moreover, it will be strictly positive if beliefs put some probability on I_2. In what follows, we want to ensure that the receiver accepts c in these circumstances. We do this by restricting consideration to equilibria where a receiver will accept a proposal in the case of indifference.

Alternatively, we could employ one of the following assumptions. First, we could enlarge the contractual space to include the payment of an "indemnity" to the receiver if a contract is withdrawn. Given some positive

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[8] All results of this paper extend to the case where frictions are modeled by discounting, in which case we have to assume additionally $V_0^i > 0$ for all types.

indemnity, if all types i with $U^i(c) < 0$ still withdraw the contract, the receiver must now accept the proposal, as his expected utility is strictly positive regardless of his beliefs. Second, requiring that out-of-equilibrium beliefs have full support would ensure that the receiver puts positive probability on I_2 and has therefore positive expected utility from acceptance. Third, as switching to another receiver is costly, a weak forward induction refinement could also be employed to ensure that the receiver does not put probability one on I_1.

Given stationarity and the restriction that the receiver accepts in the case of indifference, we can conclude that the game ends in the first period, i.e., with the first visit or with the decision not to enter the market at all. Note that for sufficiently high values of s, switching to another receiver will not be optimal. In this case we are essentially back to the standard one-shot analysis. This will be explored more formally in Section 4.2 below.

We now introduce some additional notation. We say that the null contract $\emptyset$ is implemented if the sender chooses the latter option. Denote $C_0 = C \cup \{\emptyset\}$ and define an allocation as a mapping $\psi(i, c)$ on $I \times C_0$ which specifies for each type a probability distribution over C_0. Denote for given costs s the set of equilibrium allocations by Ψ_s. All further results will be stated with respect to (equilibrium) allocations. Let supp denote the support of a probability function. We make the following definitions:

$$\begin{aligned} C^i_\psi &= \operatorname{supp} \psi(i, \cdot), \\ C_\psi &= \bigcup_{i \in I} C^i_\psi, \\ I^{\mathrm{E}}_\psi &= \{i \mid \exists c \in C \cap C^i_\psi\}, \\ I^c_\psi &= \{i \mid c \in C^i_\psi\} \qquad \text{for all } c \in C_\psi. \end{aligned}$$

I^{E}_ψ denotes the set of types who possibly enter the market, and C_ψ denotes the set of possibly implemented contracts. Any $c \in C_\psi$ may be either pooling ($|I^c_\psi| > 1$) or separating ($|I^c_\psi| = 1$). Observe that $\max I^c_\psi$ is the highest type in a pool. The singlecrossing property (A.2) now allows us to apply standard arguments for the following result.

LEMMA 2. *Consider an equilibrium allocation ψ and a contract $c \in C_\psi \cap C$. For all types $\min I^c_\psi \le i \le \max I^c_\psi$, $i \in I^{\mathrm{E}}_\psi$ implies $i \in I^c_\psi$. Moreover, for any type i there do not exist more than two contracts $c \in C^i_\psi \cap C$ with $|I^c_\psi| > 1$.*

Proof. In contradiction to the first assertion, consider the following case: $c \in C$; $i < j < k$; $i, k \in I^c_\psi$; $j \notin I^c_\psi$; $\hat{c} \in C^j_\psi \cap C$; $\hat{c} \neq c$. Optimality now implies $V^j(\hat{c}) \ge V^j(c)$, $V^i(\hat{c}) \le V^i(c)$, and $V^k(\hat{c}) \le V^k(c)$. It must also hold that $\hat{y} \neq y$. Otherwise, by (A.1) $\hat{c} \neq c$ would imply strict preference for the same contract for all three types, contradicting at least one of the three inequalities. If $\hat{y} > y$ ($\hat{y} < y$), $V^j(\hat{c}) \geq V^j(c)$ and $V^i(\hat{c}) \leq V^i(c)$ ($V^j(\hat{c}) \geq V^j(c)$ and $V^k(\hat{c}) \leq V^k(c)$) contradict the single-crossing condition (A.2). (By (A.2), given $l < m$ and $c_1, c_2 \in C$ with $y_1 < y_2$, $V^l(c_1) \leq V^l(c_2)$ implies $V^m(c_1) < V^m(c_2)$.) This proves the first assertion. The second assertion is now implied by the first. Otherwise, two different types $j, k \in I^{\mathrm{E}}_{\psi}$ would have to be indifferent between two contracts $c, \hat{c} \in C$ with $c \neq \hat{c}$ and (by A.1) $y \neq \hat{y}$, which contradicts (A.2). ■

We provide next an existence result. Recall that by (A.5) all types realize strictly more under their LCS contract than if they decide not to enter the market, which yields V_0^i. Define the LCS allocation ψ_L such that for each type $i \in I$, $\psi_L(i, \cdot)$ puts probability mass one on $c = c_L^i$.

PROPOSITION 1. *For all s it holds that $\psi_L \in \Psi_s$.*

Proof. By Lemma 1 the family $\{c_L^i\}_{i \in I}$ is incentive compatible, while $U^i(c_L^i) = 0$. By (A.5) each type indeed prefers to visit a receiver instead of taking up his outside option. It therefore remains to consider the possibility that a sender visits a receiver and proposes an out-of-equilibrium contract $c \in C \backslash \{c_L^i\}_{i \in I}$. Define

$$\begin{aligned} C_B^1 &= \{c \in C \mid V^1(c) \geq V_L^1\} \backslash \{c_L^2\}, \\ C_B^i &= \{c \in C \mid V^i(c) \geq V_L^i\} \Big\backslash \Big\{ \{c_L^{i+1}\} \cup \bigcup_{j<i} C_B^j \Big\} \qquad \text{for } i < \bar{\imath}, \\ C_B^{\bar{\imath}} &= \{c \in C \mid V^{\bar{\imath}}(c) \geq V_L^{\bar{\imath}}\} \Big\backslash \Big\{ \bigcup_{j<\bar{\imath}} C_B^j \Big\}. \end{aligned}$$

Recall next that we have so far restricted the characterization of an equilibrium to the respective allocation ψ. To check whether a particular allocation ψ indeed represents an equilibrium allocation, we have to introduce some notation for receivers' (out-of-equilibrium) beliefs. We specify that all receivers share the same belief function μ_ψ mapping $C \times I$ into Δ_I, the $I - 1$-dimensional simplex over the set of types I. Hence, $\mu_\psi(i \mid c)$ is the probability assigned to type $i \in I$ if c is observed. Consider now again the case $\psi = \psi_L$. Define $\mu_\psi(i \mid c) = 1$ for all $c \in C_B^i$. Contracts $c \in C \backslash \{C_B^i\}_{i \in I}$ are basically irrelevant, and we may choose for completeness $\mu_\psi(\bar{\imath} \mid c) = \mathbf{1}$. Let C^{A} denote the set of contracts which is accepted by a receiver given his beliefs μ_ψ. For completeness we explicitly derive this set, where we specify that a sender implements an accepted contract if he is indifferent between implementation and withdrawal. Define

$$\overline{U}^i(c) = \begin{cases} 0 & \text{if } V^i(c) < V_L^i - s \\ U^i(c) & \text{otherwise,} \end{cases}$$

$$\overline{U}^{\mathrm{E}}(c) = \sum_{i \in I} \mu_\psi(i \mid c) \overline{U}^i(c).$$

It is therefore optimal for a receiver to accept an offer c iff $\overline{U}^{\mathrm{E}}(c) \geq 0$. Hence, we obtain $C^{\mathrm{A}} = \{c \in C \mid \overline{U}^{\mathrm{E}}(c)\} \geq 0$. We prove the following claim by induction: For any $i \in I$ there exists no $c \in C^{\mathrm{A}}$ yielding $V^i(c) > V^i_L$.

For $i = 1$ the claim is true given the construction of c^1_L and C^1_B, and as $\mu_\psi(1 \mid c) = \mathbf{1}$ holds for $c \in C^1_B$. Suppose now the assertion holds for all types up to $i - \mathbf{1} < \bar{\imath}$. Define $\widetilde{C} = C^{\mathrm{A}} \cap \bigcup_{j \leq i} C^j_B$. By inductive assumption, it holds that $V^{i-1}(c) \leq V^{i-1}_L$ for all $c \in \widetilde{C}$. Moreover, by the construction of C^j_B for $j \leq i$, there exists for all $c \in \widetilde{C}$ a type $j \leq i$ such that $U^j(c) \geq 0$. (Otherwise, $c \notin C^{\mathrm{A}}$.) Given common values from (A.3), we thus obtain $U^i(c) \geq 0$ for all $c \in \widetilde{C}$. Hence, all $c \in \widetilde{C}$ satisfy the constraints of the program used to derive the LCS contract c^i_L, which implies $V^i_L \geq V^i(c)$ for all $c \in \widetilde{C}$. Finally, by construction of C^i_B, there exists no $c \in C^{\mathrm{A}} \cap \bigcup_{j > i} C^j_B$ yielding $V^i(c) \geq V^i_L$. We have thus proved by induction that there exists no profitable deviation given the specified beliefs. ■

Recall that a contract will be withdrawn by all types who are better off realizing $V^i_\psi - s$ if they visit another receiver. This is anticipated by the receiver. When ψ_L is supported as an equilibrium allocation in Proposition 1, this is taken into account when appropriate out-of-equilibrium beliefs are constructed.

The analysis of equilibria proceeds now in two steps. Section 4 provides a convergence result for $s \to 0$. We show that the set of supported equilibrium allocations converges to the LCS outcome. The underlying argument will be similar in spirit to the argument used to support the well-known Intuitive Criterion in standard signaling models (see Cho and Kreps (1987)). However, while the Intuitive Criterion rests on an introspective argument, our convergence result has an explicit strategic foundation. In Section 5 we explore additional attractive features of our approach. In particular, we point out that, for given $s > 0$, the set of equilibria satisfies a reasonable condition of continuity.

4. EQUILIBRIUM ANALYSIS

4.1. *Convergence for Low Frictions*

Our main result in this section is that the set of equilibrium allocations converges to the LCS allocation ψ_L as the cost of visiting another receiver s vanishes. (Recall that $\psi_L(i, c^i_L) = 1$ for all $i \in I$.)

PROPOSITION 2. *For any $\varepsilon > 0$ we can choose $\bar{s} > 0$ such that for all $s \leq \bar{s}$, $\psi \in \Psi_s$, and $i \in I$, it holds that*

$$\varnothing \notin C^i_\psi.$$

$\psi(i, \cdot)$ assigns at least probability $1 - \varepsilon$ to contracts in an ε-neighborhood of c^i_L.

Proof. See Appendix 1.

To prove Proposition 2, we use that a receiver accepts a (deviating) proposal if this is surely withdrawn by all types with whom he would not realize positive utility. The decision of some type i to withdraw a proposal c depends on this type's (continuation) payoff after withdrawal. Suppose, for instance, that $\psi \in \Psi_s$ is supported by an equilibrium in pure strategies. After withdrawal, type i realizes $V^i(c) - s$, where c is the unique element in C^i_ψ. As s approaches zero, type i withdraws all contracts $\bar{c}$ satisfying $V^i(\bar{c}) < V^i(c)$. Suppose now additionally that there are only two types and take a pooling allocation $\bar{\psi}$ implementing $c \in C$. In equilibrium it must hold by (A.3) that $U^2(c) > 0$, which by (A.2) implies the existence of a contract $\bar{c}$ with $\bar{y} > y$ such that $U^2(\bar{c}) > 0$, $V^2(\bar{c}) > V^2(c)$, and $V^1(\bar{c}) < V^1(c)$. For sufficiently low values of s it will hold additionally that $V^1(\bar{c}) < V^1(c) - s$. By construction, $\bar{c}$ is therefore accepted for low s and thus constitutes a profitable deviation for the high type. As a consequence, the pooling allocation $\bar{\psi}$ is not supported by an equilibrium as s becomes sufficiently small.

In Section 4.2 we will explore more fully the implications of visiting costs on the set of equilibria. For the remainder of this section we continue to focus on the limit case where s approaches zero. In fact, for $s = 0$ the selection procedure implied by our strategic extension is analogous to that underlying the Intuitive Criterion of Cho and Kreps (1987). Recall that for an equilibrium (of the standard one-shot signaling game) surviving the Intuitive Criterion there must not exist a deviating proposal such that at least one type strictly gains under any best response of the receiver, where the receiver's beliefs are restricted as follows. Beliefs must assign zero probability to types who are strictly better off under their equilibrium strategy compared with the deviation followed by any best response of the receiver. For games with transferable utility, Kreps and Sobel (1994) show that this criterion uniquely selects the family of LCS contracts. The arguments used to prove Proposition 2 immediately extend this result to signaling games with nontransferable utilities satisfying (A.1)–(A.5).

While being similar in spirit to the application of the Intuitive Criterion, our approach exhibits an important conceptual difference. To evaluate whether a deviating proposal represents a "credible signal" of a high type, the Intuitive Criterion compares the (candidate) equilibrium utility with the utility derived under the new proposal. This procedure has been criticized for a lack of "global consistency." As Mailath *et al.* (1993) argue, a type may fare worse under a deviation, but he may still be better off than with his supposed equilibrium proposal if the receiver adjusts his beliefs to account for the (credible) deviation of other types. This criticism does not apply to the result of Proposition 2. The main difference is that a low type who withdraws his proposal can indeed realize his (candidate) equilibrium utility (minus s) by visiting *another* receiver. In short, one could say that our approach to embedding the signaling game in a market environment (with low frictions) represents a *strategic* foundation of the selection procedure underlying the Intuitive Criterion.

In Section 5 we will argue that our approach has additional attractive features which depend on the role of $s > 0$. To conclude the discussion of the convergence result, we finally point out a further implication regarding the support of equilibrium allocations. We show in the proof of Proposition 2 that any $c \in C_\psi$ belongs to an ε-neighborhood of c_L^i with $i = \max I_\psi^c$, where $\varepsilon > 0$ becomes arbitrarily small as s vanishes. This auxiliary result can be restated in a more precise form to obtain a convergence result on the support of equilibrium allocations. By extending any standard metric on C to C_0 (recall that $C_0 = C \cup \{\emptyset\}$), we can define the Hausdorff metric on the family of closed sets $\widehat{C} \subseteq C_0$ (see, for instance, Berge (1963)). Denote by $\overline{C}_s$ the closure of $\bigcup_{\psi \in \Psi_s} C_\psi$, i.e., of the set of possibly implemented contracts for given costs s. We obtain the following corollary, where convergence is with respect to the Hausdorff metric.

COROLLARY. $\lim_{s \to 0} \overline{C}_s = \bigcup_{i \in I} c_L^i$.

4.2. *Monotonicity in Switching Costs*

While we have so far only considered the case of (arbitrarily) low switching costs, this section investigates more generally how the choice of s affects the set of equilibria. While the derived results are interesting in their own right, they also prepare the discussion of Section 5, where we study the interaction of s with the (prior) distribution of types μ.

We prove that the set of equilibria is nonincreasing in s. Moreover, under an additional requirement it is also shown to be strictly decreasing. To formulate this requirement, we define the family of first-best contracts. By (A.1)–(A.6) there exists for each type i a unique contract c_F^i which maximizes $V^i(c)$ subject to $U^i(c) \geq 0$. We make the following assumption:

(A.7) $c_L^i \neq c_F^i$ for all $i > \mathbf{1}$.

Hence, under (A.7) the incentive compatibility constraint in the definition of the LCS contracts becomes binding for all types $i > 1$.

PROPOSITION 3. *If $\psi \in \Psi_s$, then $\psi \in \Psi_{s'}$ holds for all $s' > s$. Moreover, given* (A.7), *there exists $\bar{s} > 0$ such that for all $s < s' < \bar{s}$ we find some ψ satisfying $\psi \in \Psi_{s'}$ and $\psi \notin \Psi_s$.*

Proof. See Appendix 2.

The monotonicity in s is intuitive from our previous remarks in Section 4.1. It reinforces our selection idea. Observe also that for strict monotonicity the compared levels of visiting costs s and s' must not become too high. Clearly, for sufficiently high values we are simply back to the case of a bilateral monopoly, where a marginal adjustment of visiting costs has no effect on the set of equilibrium outcomes.

5. DEPENDENCY ON THE PRIOR DISTRIBUTION

By Proposition 3 the set of equilibria gradually decreases as s becomes smaller. We investigate next how s and the prior beliefs μ interact. In doing so, we will again compare our approach with the selection obtained from applying the Intuitive Criterion to the standard one-shot game. Recall from Section 4.1 that in the bilateral monopoly case the Intuitive Criterion selects the LCS allocation regardless of the choice of the prior distribution. Hence, the refined set of equilibria is not sensitive to an important ingredient of the economic setting. A major implication of this observation is that the selection of the Intuitive Criterion lacks a natural property of continuity in the prior beliefs μ on the boundary of the simplex Δ_I. To make this precise, we focus in what follows on the two-type case where $I = \{1, 2\}$. We abbreviate the high type's probability by $\mu = \mu(2)$.

Consider for a moment the example where $U^i(c) = i - t$ and $V^i(c) = t - y/i$, for which we obtained the LCS choice $y_L^2 = 1$. If μ becomes equal to one, implying that there is no longer private information, the sender's proposal specifies the first-best choice $y = 0$ (and the transfer $t = 2$). We argue now that this discontinuity is absent in our model, which is due to the interaction of positive visiting costs $s > 0$ with the prior distribution. We explore this interaction in two steps. Denote for given costs s and given prior beliefs μ the set of equilibrium allocations by $\Psi_s(\mu)$. We first analyze how the survival of (pure-strategy) pooling equilibria in $\Psi_s(\mu)$ depends on the choice of s and μ. This result is then used to prove, for a given choice of s, a natural form of continuity of $\Psi_s(\mu)$ as $\mu \to 1$.

Consider some (pure-strategy) pooling allocation where $\psi(i, c^P) = 1$ for $i \in I$ and some $c^P \in C$. If (A.7) holds, one can show for the standard one-shot game of signaling that a pooling equilibrium exists at least if μ is sufficiently high. Precisely, as argued in more detail in the proof of Proposition 4 below, existence is ensured for all $\mu \in [\underline{\mu}, 1]$ where $\underline{\mu} < 1$. In the market environment, we show an analogous result, where this time the threshold on the prior distribution depends on s and is thus denoted by $\underline{\mu}(s)$. Proposition 4 derives now some intuitive properties of the function $\underline{\mu}(s)$. These properties are subsequently illustrated with an example.

PROPOSITION 4. *Consider the two-type case and suppose that* (A.7) *holds. Then a pooling equilibrium exists if and only if* $\mu \in [\underline{\mu}(s), 1]$, *where* $\underline{\mu}(s)$ *is continuous and nondecreasing, satisfying* $\underline{\mu}(s) < 1$ *for* $s > 0$ *and* $\underline{\mu}(0) = 1$.

Proof. See Appendix 2.

Proposition 4 illustrates an intuitive trade-off. By (A.3) the high type's benefits from "separating away" from a pooling allocation clearly decrease as μ increases. By previous arguments the high type can indeed credibly signal his type by deviating to a contract which is surely withdrawn by the low type. However, as s increases, the low type is increasingly less prepared to withdraw a contract to visit another receiver. In other words, the high type's costs for (credible) separation are increasing in s. Proposition 4 illustrates this trade-off between the benefits and the costs of separation.

EXAMPLE. Suppose again that $U^i(c) = i - t$ and $V^i(c) = t - y/i$. Observe that in the one-shot game there always exists a pure-strategy pooling equilibrium where $y^P = 0$ and $t^P = 1 + \mu$. Indeed, it is straightforward that we can support any pooling equilibrium where $V^1(c^P) \geq V_L^1 = 1$. In the market environment, we obtain additionally the following test. Consider a possible deviation c (for the high type). We can restrict consideration to the case where the respective sorting variable satisfies $y > y^P$. To ensure that this proposal would be withdrawn by the low type, it must hold that $t - y < t^P - y^P - s$. In this case the receiver accepts the proposal if $t \leq 2$. Moreover, the new contract is profitable for the high type if $t - y/2 > t^P - y^P/2$. Combining these inequalities, we can rule out the existence of a pure-strategy pooling equilibrium if

$$s < (1 - \mu)/2. \tag{1}$$

This also represents a necessary condition. Hence, for given costs s, we can only exclude pooling equilibria if and only if μ is sufficiently low such that (1) holds. This threshold converges to $\mu = 1$ as $s \to 0$. Observe finally that, using the terminology of Proposition 4, we obtain $\underline{\mu}(s) = \min\{0, 1 - 2s\}$.

We conclude this section with a direct implication of Proposition 4. If the pair of first-best contracts is not separating, we know from Proposition 4 that we can support for some $s > 0$ a pooling allocation if μ is sufficiently high. Using the continuity of payoffs, it can be shown next that we can in particular support a pooling equilibrium where the respective contract c^{P} lies arbitrarily close to the high types first-best contract c_F^2 if μ becomes close to one. For fixed s we can then prove the following form of lower semincontinuity of $\Psi_s(\mu)$ at $\mu = 1$.

PROPOSITION 5. *For any* $\varepsilon > 0$ *we can find a value* $\bar{\mu} < 1$ *such that for all* $\bar{\mu} < \mu < 1$ *there exists* $\psi \in \Psi_s(\mu)$ *specifying* $\psi(2, c) = \mathbf{1}$ *for a contract* $c \in C$ *in an* ε*-neighborhood of* c_F^2.

Proof. See Appendix 2.

6. CONCLUSION

We embed a standard contractual game of signaling into a market environment. If an informed sender visits an uninformed receiver, a three-stage game is played. The sender proposes a contract. Rejection by the receiver leads to a break-up, while the sender can still choose whether to withdraw or implement an accepted proposal. The market environment provides the sender with the possibility to switch to another receiver. By restricting our attention to stationary equilibria where the "outside option" of switching has a constant value, we can show under a mild additional assumption that the set of contracts which may be implemented in an equilibrium converges to the set of least-cost separating contracts as the costs of visiting another receiver vanish. While the underlying logic is similar to that supporting the Intuitive Criterion of Cho and Kreps (1987), we emphasize three attractive features of our approach to embed the game of signaling in a market environment. First, the selection process receives a strategic foundation, while it is only supported by an introspective argument in the Intuitive Criterion. Second, for positive search costs the set of equilibria depends on the primitives, i.e., in particular on the prior distribution of types. Finally, this also implies a natural form of continuity as the private information content vanishes, e.g., as the prior probability put on the low type vanishes.

As the paper discusses only the case of a finite set of types, it remains to show that a similar convergence result holds for a continuum. (See Mailath (1987) for the treatment of the continuum case in a standard model of monotonic signaling.) Moreover, the idea of using structural extensions to signaling games to reduce the set of equilibria may be used in ways different from that proposed in this paper. To illustrate just one route for further research, suppose that before proposing a contract to the receiver, the sender may interact with a third party. For instance, a firm proposing the details of an equity or bond issue to the market may be contacted by an investment bank offering the firm a publicly known (side) contract which promises a transfer conditional on the firm's offer and the market's reaction. (Indeed, deals along this line are common in the underwriting business.) In essence, such an extension would add a stage of screening to prune the set of equilibria of the subsequent signaling game.

7. APPENDIX 1: PROOF OF PROPOSITION 2

Define by (A.4) and (A.5) the compact and non-empty set

$$C^R = \{c \in C \mid \exists i \in I \text{ such that } U^i(c) \geq 0, V^i(c) \geq V_0^i\}.$$

As utility functions are continuously differentiable, (A.1)–(A.4) imply the existence of two finite values $\bar{k}, \underline{k} > 0$ such that for all contracts $c \in C^R$ and all pairs $i < j$ it holds that

$$\begin{aligned} &\left|U_t^i\right| \le \bar{k}; \qquad \left|V_y^i\right| \le \bar{k}; \qquad V_t^i \ge \bar{k}; \\ &(-V_y^i/V_t^i) - (-V_y^j/V_t^j) \ge \underline{k}; \qquad U^j(c) - U^i(c) \ge \underline{k}. \end{aligned} \tag{2}$$

In what follows, we will be able to restrict our consideration to contracts in C^R. Applying the boundaries in (2) will ensure that our convergence results apply uniformly. We start by deriving a lower boundary on equilibrium utilities.

Claim 1. For any $\varepsilon > 0$ there exists a value $\bar{s} > 0$ such that for all $s \le \bar{s}$, $\psi \in \Psi_s$, and $i \in I$, it holds that $V_\psi^i \ge V_L^i - \varepsilon$

Proof. The proof proceeds by induction. By construction of c_L^1 it follows immediately for $i = 1$. Assume thus that the assertion holds up to $i - 1 < \bar{i}$, i.e., for any $\varepsilon_1 > 0$ we find some $\bar{s}_{i-1}^{\varepsilon_1} > 0$ such that $V_\psi^j \ge V_L^j - \varepsilon_1$ holds for all $s \le \bar{s}_{i-1}^{\varepsilon_1}$, $\psi \in \Psi_s$, and $j < i$. We argue to a contradiction and assume that the claim does not extend to i, i.e., for all $\tilde{s} > 0$ we can find a value $s < \tilde{s}$ such that $V_\psi^i < V_L^i - \varepsilon$ holds for some $\psi \in \Psi_s$. This implies the existence of two sequences $\{\psi_n\}$ and $\{s_n\}$ with $s_n \to 0$, $\psi_n \in \Psi_{s_n}$, and $V_{\psi_n}^i < V_L^i - \varepsilon$. Observe next that continuity and (A.1) imply the existence of a contract $\tilde{c} = (\tilde{t}, y_L^i)$ with $\tilde{t} < t_L^i$ such that $U^i(\tilde{c}) > 0$ and $V_i(\tilde{c}) > V_L^i - \varepsilon$. By (A.3), this also implies $U^j(\tilde{c}) > 0$ for all $j \ge i$. Choose next $\varepsilon_1 = \underline{k}(t_L^i - \tilde{t})/2$ and N such that $s_n \le \bar{s}_{i-1}^{\varepsilon_1}$ holds for all $n > N$. Using global incentive compatibility of $\{c_L^i\}_{i \in I}$, $s_n \le \bar{s}_{i-1}^{\varepsilon_1}$, and the inductive assumption, this yields for all $n > N$ and $j < i$

$$V^j(\tilde{c}) \le V_L^j - \underline{k}(t_L^i - \tilde{t}) \le V_{\psi_n}^j + \varepsilon_1 - \underline{k}(t_L^i - \tilde{t}) = V_{\psi_n}^j - \varepsilon_1.$$

Observe that $\tilde{c}$ is withdrawn by all types $j < i$. Choose now M such that $s_n < \varepsilon_1$ holds for all $n > M$. For all $n \ge \max\{N, M\}$ the deviation $\tilde{c}$ is thus accepted by the receiver and constitutes for i a profitable deviation.

By the finiteness of I, we can finally specify $\bar{s} = \bar{s}_{\bar{i}}^{\varepsilon} > 0$ to conclude the proof. ■

By (A.5), Claim 1 also implies that $\varnothing \notin C_\psi^i$ holds for all types $i \in I$ as s becomes sufficiently small. To put also an upper boundary on equilibrium utilities, we must proceed in several steps. We first (virtually) rule out pooling as s becomes small. For this purpose we need the following auxiliary result.

Claim 2. For any $\varepsilon > 0$ there exists a value $\bar{s} > 0$ such that for all $s \le \bar{s}$, $\psi \in \Psi_s$, and $c \in C_\psi$, it holds that $U^i(c) \le \varepsilon$ for $i = \max I_\psi^c$.

Proof. We restrict our attention to sufficiently low values of s such that, by Claim 1, $\emptyset \notin C_\psi$. The assertion is immediate if $\max I_\psi^c = 1$. For $i > 1$ we argue to a contradiction and assume the existence of sequences $\{s_n\}$, $\{c_n\}$, and $\{\psi_n\}$ with $s_n \to 0$ and, for all n, $\psi_n \in \Psi_{s_n}$, $c_n \in C_{\psi_n}$, $i = \max I_{\psi_n}^{c_n} > 1$, and $U^i(c_n) > \varepsilon$. Note that $c_n \in C^R$. Fix n for a moment and denote for simplicity $c = c_n$, $\psi = \psi_n$. As utility functions satisfy (A.1) and are continuously differentiable, we can apply the implicit function theorem to construct a function $\hat{t}(\tilde{y})$ in a neighborhood of y (with $\tilde{y} \geq y$) such that $V^i(\hat{t}(\tilde{y}), \tilde{y}) = V^i(c)$ and $\hat{t}(\tilde{y})$ is differentiable with $d\hat{t}(\tilde{y})/d\tilde{y} = -dV_y^i(\tilde{c})/dV_t^i(\tilde{c}) \geq 0$. As long as we stay on the indifference curve of type i (implying $V^i(\tilde{c}) = V^i(c) = V_\psi^i > V_0^i$) and as long as $\tilde{c}$ satisfies $U^i(\tilde{c}) \geq 0$, we ensure $\tilde{c} \in C^R$, which allows us to apply the boundaries (2). Choose now $\hat{y} - y = \varepsilon \underline{k}/(2\bar{k}^2)$ such that, by (2),

$$U^i(\hat{c}) > \varepsilon - \frac{\bar{k}^2}{\underline{k}}(\hat{y} - y) + \varepsilon > \frac{\varepsilon}{2},$$

which uses $|U_t^i| \leq \bar{k}$, $|V_y^i| \leq \bar{k}$, $V_t^i \geq \underline{k}$, and $U_y^i/V_t^i \leq 0$. In analogy, we obtain for all $j < i$

$$V^j(\hat{c}) \leq V_\psi^j - \underline{k}^2(\hat{y} - y) \leq V_\psi^j - \frac{\varepsilon \underline{k}^3}{2\bar{k}^2}, \tag{3}$$

which uses $V_t^i \geq \underline{k}$ and $|(-V_y^i/V_t^i) - (-V_y^j/V_t^j)| \geq \underline{k}$. Recall that we abbreviated $c = c_n$ and $\psi = \psi_n$. Choose now N and $\bar{s} > 0$ such that for all $n > N$ it holds that $s_n < \bar{s} < \varepsilon \underline{k}^3/(4\bar{k}^2)$. From the preceding argument this implies for all $n > N$ the existence of a contract $\hat{c}_n \in C$ such that $V^i(\hat{c}_n) = V_{\psi_n}^i = V^i(c_n)$ for $i = \max I_{\psi_n}^{c_n}$, $U^i(\hat{c}_n) > \frac{\varepsilon}{2}$, and $V^j(\hat{c}_n) < V_{\psi_n}^j - \varepsilon \underline{k}^3/4\bar{k}^2$ for all $j < i$. This allows us to marginally adjust the transfer to obtain a contract $\check{c}_n$ where $V^i(\check{c}_n) > V_{\psi_n}^i$ holds strictly, while the other inequalities are still satisfied.[9] In analogy to the procedure in Claim 1, we have thus constructed a profitable deviation for type i for all $n > N$, which concludes the proof. ■

Denote now for a given equilibrium allocation ψ a receiver's *consistent* beliefs when observing an offer $c \in C_\psi$ by the function $\mu_\psi(i \mid c)$ mapping $C \times I$ into Δ_I. We can now use Claim 2 to put an upper boundary on pooling.

Claim 3. For any $\varepsilon > 0$ there exists a value $\bar{s} > 0$ such that for all $s \leq \bar{s}$, $\psi \in \Psi_s$, and $c \in C_\psi \cap C$, it holds that $\mu_\psi(\max I_\psi^c \mid c) > 1 - \varepsilon$.

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[9] Formally, we have first to extend the boundaries in (2) to $V_t^i \leq \bar{k}$. We can then increase the transfer by $\Delta t = \frac{1}{2\bar{k}} \min[\varepsilon/2, \varepsilon \underline{k}^3/4\bar{k}^2] > 0$, which ensures $U^i(\hat{c}_n) > \frac{\varepsilon}{4}$, $V^j(\hat{c}_n) < V_{\psi_n}^j - \varepsilon \underline{k}^3/8\bar{k}^2$, and $V^i(\check{c}_n) \geq V_{\psi_n}^i + \Delta t \underline{k}$.

Proof. The claim is trivially satisfied for all c, where $\max I_\psi^c = 1$. Observe next that all $c \in C_\psi \cap C$ satisfy $c \in C^R$ and therefore $U^j(c) - U^i(c) > \underline{k} > 0$ for all $j > i$. Claim 2 allows us to choose for any $\bar{\varepsilon} > 0$ a value $s^{\bar{\varepsilon}} > 0$ such that for all $s \le s^{\bar{\varepsilon}}$, $\psi \in \Psi_s$, and $c \in C_\psi$, it holds that $U^i(c) \le \bar{\varepsilon}$ with $i = \max I_\psi^c$. Choosing $\psi \in \Psi_s$ for $s \le s^{\bar{\varepsilon}}$ and using (A.3), we thus obtain for all $c \in C_\psi^i$ with $i = \max I_\psi^c > \mathbf{1}$

$$\sum_{j \in I} \mu_\psi(j \mid c) U^j(c) < \mu_\psi(i \mid c)\bar{\varepsilon} + (\mathbf{1} - \mu_\psi(i \mid c))(\bar{\varepsilon} - \underline{k}).$$

Hence, the receiver's participation constraint $\sum_{j \in I} \mu_\psi(j \mid c) U^j(c) \ge 0$ is only satisfied if $\mathbf{1} - \mu_\psi(i \mid c) < \bar{\varepsilon}/\underline{k}$. Choosing $\bar{\varepsilon} = \varepsilon\underline{k}$ and $\bar{s} = \bar{s}^{\bar{\varepsilon}}$ proves the claim. ■

Define $\mu^{\mathrm{m}} = \min_{i \in I} \mu(i) > 0$ and $\mu^{\mathrm{M}} = \max_{i \in I} \mu(i) < 1$. As any type i can be at most in two pools by Lemma 1, Claim 3 implies by straightforward calculations the upper boundary $2\varepsilon\mu^{\mathrm{M}}/((1-\varepsilon)\mu^{\mathrm{m}})$ for the aggregate probability with which i may choose contracts $c \in C_\psi^i$, where $i < \max I_\psi^c$. This yields the following corollary to Claim 3.

Claim 4. For any $\varepsilon > 0$ there exists a value $\bar{s} > 0$ such that for all $s \le \bar{s}$, $i \in I$, and $\psi \in \Psi_s$, $\psi(i, \cdot)$ must attribute at least aggregate probability $1 - \varepsilon$ to contracts $c \in C$, where $\max I_\psi^c = i$.[10]

We can now put Claim 4 to work and derive an upper boundary for equilibrium utilities. For this purpose, the following optimization programs are useful. For any type $i > 1$ and a real value v, define the program $P^i(v)$ which maximizes $V^i(c)$ subject to $c \in C$, $U^i(c) \ge 0$, and $V^{i-1}(c) \le v$. By (A.1) and (A.4), a solution exists. As it is also unique by (A.6) (which is, however, not essential), we can denote it by $c_*^i(v)$. Denote the realized value of the objective function by $V_*^i(v)$, which is continuous.

Claim 5. For any $\varepsilon > 0$ there exists a value $\bar{s} > 0$ such that for all $s \le \bar{s}$, $\psi \in \Psi_s$, and $i \in I$, it holds that $V_\psi^i \le V_L^i + \varepsilon$.

Proof. By Claims 1 and 4, we can restrict our consideration to the case where for any $i \in I$ there exists some $c \in C$ with $V_\psi^i = V^i(c)$ and $i = \max I_\psi^c$. By (A.3), c must satisfy $U^i(c) \ge 0$. The proof proceeds by induction. It is immediate for $i = 1$. Assume the assertion holds up to $i - 1$, i.e., for any $\bar{\varepsilon} > 0$ there is a value $\bar{s}_{i-1}^{\bar{\varepsilon}} > 0$ such that $V_\psi^j \le V_L^j + \bar{\varepsilon}$ holds for all $j < i$, $s \le \bar{s}_{i-1}^{\bar{\varepsilon}}$, and $\psi \in \Psi_s$. Any contract $c \in C_\psi^i$ must satisfy $V^{i-1}(c) \le V_\psi^{i-1}$ and $U^i(c) \ge 0$. (Recall $c \in C$, $i = \max I_\psi^c$.) By construction of $P^i(\cdot)$, we obtain $V_\psi^i \le V_*^i(V_\psi^{i-1}) \le V_*^i(V_L^{i-1} + \bar{\varepsilon})$, which uses the inductive assumption. Using

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[10] More formally, we claim that $\int_{\{c \mid \max I_\psi^c = i\}} d\psi(i, c) \ge 1 - \varepsilon$.

$V_L^i = V_*^i(V_L^{i-1})$, the continuity of $V_*^i(\cdot)$, and the inductive assumption, we can indeed find a value $\bar{s}_i^\varepsilon$ such that the assertion extends to i for given ε. From the finiteness of I we finally obtain $\bar{s} = \bar{s}_i^\varepsilon > 0$. ■

We are now in a position to prove Proposition 2. By previous results, we can choose for any $\tilde{\varepsilon} > 0$ a value $\tilde{s}^{\tilde{\varepsilon}} > 0$ such that for all $s \leq \tilde{s}^{\tilde{\varepsilon}}$ and $\psi \in \Psi_s$ the following claims apply: $\emptyset \notin C_\psi$; $V_\psi^j \leq V_L^j + \tilde{\varepsilon}$ for all $j \in I$; for any $c \in C_\psi$ and $i = \max I_\psi^c$ it holds that $U^i(c) \geq 0$, $V^i(c) \geq V_L^i - \tilde{\varepsilon}$, and $V^{i-1}(c) \leq V_\psi^{i-1}$. The first assertion follows from Claim 1, the second from Claim 5, and the third from the receiver's participation constraint, Claim 1, and incentive compatibility.

Take $\varepsilon > 0$ and a type $i > 1$. We prove by contradiction that there exists $\bar{s}_i < 0$ such that in all $\psi \in \Psi_s$ with $s \leq \bar{s}_i$ any $c \in C_\psi^i$ with $i = \max I_\psi^c$ must lie in the ε-neighborhood of c_L^i. Denote the ε-neighborhood of c_L^i by $\Omega_\varepsilon(c_L^i)$ and introduce the compact set $C^D = C^R \backslash \Omega_\varepsilon(c_L^i)$. Recall now the construction of $\tilde{s}^{\tilde{\varepsilon}} > 0$ for any $\tilde{\varepsilon}$. We choose a sequence $\{\tilde{\varepsilon}_k\}$ converging to zero. For each $\tilde{\varepsilon}_k$ there exists a value $\tilde{s}_k > 0$ ($\tilde{s}_k = \tilde{s}^{\tilde{\varepsilon}_k}$). Suppose now the asserted value $\bar{s}_i > 0$ does not exist for given ε. This implies that we can find for any $\tilde{s}_k > 0$ a value $s \leq \tilde{s}_k$ with $\psi \in \Psi_s$, $c \in C_\psi^i$, $i = \max I_\psi^c$, and $c \notin \Omega_\varepsilon(c_L^i)$. Denote this contract by c_k, which lies in C^D. Hence, the sequence $\{c_k\}$ has a subsequence converging to a contract $\check{c} \in C^D$. We can take the original sequence as the subsequence. By construction, along this subsequence it holds that $V^i(c_k) \geq V_L^i - \tilde{\varepsilon}_k$, $U^i(c_k) \geq 0$, and $V^{i-1}(c_k) \leq V_\psi^{i-1} \leq V_L^{i-1} + \tilde{\varepsilon}_k$. As $\lim_{k\to\infty} \tilde{\varepsilon}_k = 0$, $\check{c}$ satisfies $V^i(\check{c}) \geq V_L^i$, $U^i(\check{c}) \geq 0$, and $V^{i-1}(\check{c}) \leq V_L^{i-1}$. As $\check{c}$ also satisfies the constraints of program $P^i(V_L^{i-1})$, $\check{c} \notin \Omega_\varepsilon(c_L^i)$ contradicts the fact that $V_*^i(V_L^{i-1}) = V_L^i$ is (uniquely) realized by c_L^i. This completes the proof for some type $i > 1$. The argument for $i = 1$ is completely analogous. By the finiteness of I, we thus obtain $\bar{s}^1 = \min_{i\in I} \bar{s}_i > 0$ for given ε.

To complete the proof of the proposition, now use Claim 4 to choose another threshold $\bar{s}^2 > 0$ such that $s \leq \bar{s}^2$ and $\psi \in \Psi_s$ ensure that any type i chooses contracts c where $i \neq \max I_\psi^c$ with not more than ε probability. The proposition follows by setting $\bar{s} = \min[\bar{s}^1, \bar{s}^2]$.

8. APPENDIX 2: PROOF OF PROPOSITIONS 3–5

Proof of Proposition 3. We prove first (weak) monotonicity. Consider a pair $s' > s$ and some allocation $\psi \in \Psi_s$. In analogy to the proof of Proposition 1, denote the set of deviating contracts $c \notin C_\psi$ which are accepted by the receiver by C_s^{A}. To support ψ it is necessary that $V_\psi^i \geq V^i(c)$ holds for all $i \in I$ and $c \in C_s^{\mathrm{A}}$. Define next the set of types which are willing to withdraw c by I_s^c, i.e., $i \in I_s^c$ if and only if $V^i(c) - s \geq V_\psi^i$. In analogy to

Proposition 1 define $\overline{U}^i(c) = 0$ for $c \in I_s^c$ and $\overline{U}^i(c) = U^i(c)$ otherwise. Hence, by definition we find for all $c \notin C_s^{\mathrm{A}}$ beliefs $\mu_\psi^s(\cdot \mid c)$ ensuring that $\sum_{i\in I} \mu_\psi^s(i \mid c)\overline{U}^i(c) < 0$. In particular, there must exist some type $i^c \in I$ such that $i \notin I_s^c$ and $\overline{U}^{i^c}(c) < 0$, and we can specify as well $\mu_\psi^s(i^c \mid c) = 1$. Consider now $s' > s$. We argue that we can also support $\psi \in \Psi_{s'}$ with the same beliefs. It is immediate that we can restrict ourselves to the consideration of deviating strategies $c \notin C_\psi$ for the sender. Moreover, it is sufficient to construct beliefs $\mu_\psi^{s'}(\cdot \mid c)$ for $c \notin C_\psi$ which ensure that the respective set of accepted contracts $C_{s'}^{\mathrm{A}}$ satisfies $C_{s'}^{\mathrm{A}} \subseteq C_s^{\mathrm{A}}$. Define again for all $c \notin C_\psi$ the set of types $I_{s'}^c$, i.e., $i \in I_{s'}^c$ if and only if $V^i(c) - s' \geq V_\psi^i$. Clearly, it holds that $I_s^c \subseteq I_{s'}^c$. The claim is thus proven by choosing under s' the same beliefs as for s, i.e., $\mu_\psi^{s'}(i^c \mid c) = 1$.

Turn now to strict monotonicity. Define the allocation ψ where all types $i \leq \bar{\imath} - 1$ implement their respective LCS contract c_L^i with probability one, while the highest type $\bar{\imath}$ implements with probability one some contract $\bar{c}$ which we construct next. Consider the program to maximize $V^{\bar{\imath}}(c)$ subject to $U^{\bar{\imath}}(c) = 0$ and $V^{\bar{\imath}-1}(c) = V_L^{\bar{\imath}-1} - s'$ for some $s' > 0$. Recall that this program was also used for the proof of Proposition 2, where we claimed that it has a unique solution, which is now denoted by $\bar{c}$. If (A.7) holds, the respective choice of the sorting variable $\bar{y}$ satisfies $\bar{y} > y_L^{\bar{\imath}}$ by (A.2). Recall now that we denoted the realized utility under this program by $V_*^{\bar{\imath}}(V_L^{\bar{\imath}-1} - s')$, which is continuous in s' and by (A.7) is strictly decreasing. Define $\bar{s} > 0$ by the requirement $V_*^{\bar{\imath}}(V_L^{\bar{\imath}-1} - \bar{s}) = \max\{V_0^{\bar{\imath}}, V^{\bar{\imath}}(c_L^{\bar{\imath}-1})\}$ and assume that $s' < \bar{s}$. We argue next that we can support ψ as an equilibrium allocation for s', i.e., that $\psi \in \Psi_{s'}$. For contracts $c \in C_\psi$ the receiver's acceptance strategy is clearly optimal, while the sender cannot profitably deviate to contracts in this set by the global incentive compatibility of the LCS offers and the construction of $\bar{c}$ (satisfying in particular $V^{\bar{\imath}}(\bar{c}) \geq V^{\bar{\imath}}(c_L^{\bar{\imath}-1})$). Hence, it remains to consider deviations of the sender to contracts $c \notin C_\psi$. To define appropriate out-of-equilibrium beliefs for the receiver, we construct the following sets:

$$
\begin{aligned}
C_B^1 &= \{c \in C \mid V^1(c) \geq V_L^1\} \setminus \{c_L^2\}, \\
C_B^i &= \{c \in C \mid V^i(c) \geq V_L^i\} \Big\backslash \Big\{\{c_L^{i+1}\} \cup \bigcup_{j<i} C_B^j\Big\} \qquad \text{for } i < \bar{\imath} - 1, \\
C_B^i &= \{c \in C \mid V^i(c) \geq V_L^i - s'\} \Big\backslash \Big\{\{\bar{c}\} \cup \bigcup_{j<i} C_B^j\Big\} \qquad \text{for } i = \bar{\imath} - 1, \\
C_B^{\bar{\imath}} &= \{c \in C \mid V^{\bar{\imath}}(c) \geq V^{\bar{\imath}}(\bar{c})\} \Big\backslash \Big\{\bigcup_{j<\bar{\imath}} C_B^j\Big\}.
\end{aligned}
$$

(Observe that the definitions are not identical to those used in Proposition 1.) Define next the beliefs $\mu_\psi(i \mid c) = 1$ for all $c \in C_B^i$, which are shared by all receivers. Moreover, for completeness define $\mu_\psi(\bar{\imath} \mid c) = 1$ for all $c \in C \backslash \{C_B^i\}_{i \in I}$. As in Proposition 1 we can now define the set C^{A} of contracts accepted by the receiver given his beliefs μ_ψ. (We specify again that the sender withdraws a contract only if he strictly prefers to do so.) To support ψ, we only have to consider deviations $c \in C^{\mathrm{A}}$. The argument for types $i \leq \bar{\imath} - 1$ is now completely analogous to that in Proposition 1. Consider thus $\bar{\imath}$, where we claim that $c \in C^{\mathrm{A}}$ implies $V^{\bar{\imath}}(c) \leq V^{\bar{\imath}}(\bar{c})$. This follows immediately from the construction of $\bar{c}$ and the set $C_B^{\bar{\imath}}$.[11]

Having supported $\psi \in \Psi_{s'}$, it remains to show that $\psi \notin \Psi_s$ for all $s < s'$. Following the procedure repeatedly applied in the proof of Proposition 2, we show that regardless of the choice of out-of-equilibrium beliefs we find for $\bar{\imath}$ a profitable deviation c. Observe first that there exists a contract c satisfying $U^{\bar{\imath}}(c) > 0$, $V^{\bar{\imath}-1}(c) < V_L^{\bar{\imath}-1} - s$, and $V^{\bar{\imath}}(c) > V^{\bar{\imath}}(\bar{c})$. By choosing $y < \bar{y}$ this follows from (A.2), (A.6), and the construction of $\bar{c}$, which implied $\bar{y} > y_L^{\bar{\imath}}$ due to (A.7). Using the global incentive compatibility of the LCS offers, $V^i(c) < V_L^i - s$ holds also for all types $i < \bar{\imath}$. Hence, the deviation c is indeed accepted by the receiver, which completes the proof that $\psi \notin \Psi_s$. ■

Proof of Proposition 4. Consider the program to maximize $V^2(c)$ subject to $U^1(c) \geq 0$ and denote the realized payoff by $\widehat{V}$. Denote next the set of contracts c^{P} satisfying

$$\begin{aligned} V^1(c^{\mathrm{P}}) &\geq V_L^1, \\ V^2(c^{\mathrm{P}}) &\geq \max\{\widehat{V}, V_0^2\}, \\ \mu U^2(c^{\mathrm{P}}) + (1-\mu) U^1(c^{\mathrm{P}}) &\geq 0, \end{aligned}$$

by $C^{\mathrm{P}}(\mu)$. Clearly, by (A.3) $c^{\mathrm{P}} \in C^{\mathrm{P}}(\mu)$ implies $c^{\mathrm{P}} \in C^{\mathrm{P}}(\mu')$ for all $\mu' > \mu$. Moreover, by continuity of the payoff functions and by (A.4), C^{P} is compact and changes continuously in μ (as long as the set is non-empty). Together with (A.7) this implies that $C^{\mathrm{P}}(\mu)$ remains non-empty if and only if $\mu \in [\underline{\mu}, 1]$ where $\underline{\mu} < 1$. Observe now that there exists a pooling allocation implementing c^{P} in the one-shot game if and only if $c^{\mathrm{P}} \in C^{\mathrm{P}}(\mu)$. This is also a necessary requirement for a pooling equilibrium in the market environment. Additionally, it follows from by now standard arguments that in the market environment there must not exist some contract c satisfying

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[11] Precisely, we have to add the following observation. Recall that $\bar{c}$ maximizes $V^{\bar{\imath}}(c)$ subject to $U^{\bar{\imath}}(c) = 0$ and $V^{\bar{\imath}-1}(c) = V_L^{\bar{\imath}-1} - s'$. By standard arguments (used, for instance, to prove Lemma 1), (A.2) and (A.7) ensure that $\bar{c}$ solves also uniquely the program with (inequality) constraints $U^{\bar{\imath}}(c) \geq 0$ and $V^{\bar{\imath}-1}(c) \geq V_L^{\bar{\imath}-1} - s'$.

$V^1(c) < s + V^1(c^{\mathrm{P}})$, $V^2(c) > V^2(c^{\mathrm{P}})$, and $U^2(c) \geq 0$. (It is easily checked that the stated conditions are both necessary and sufficient for a pooling equilibrium to exist.) Denote the set of surviving pooling contracts c^{P} by $C^{\mathrm{P}}(\mu, s)$. As $c^{\mathrm{P}} \in C^{\mathrm{P}}(\mu, s)$ implies $c^{\mathrm{P}} \in C^{\mathrm{P}}(\mu', s')$ for all $\mu' \geq \mu$ and $s' \geq s$, existence is thus ensured if and only if $\mu \in [\underline{\mu}(s), 1]$. By the arguments in Proposition 2 it holds that $\underline{\mu}(0) = 1$, while (A.7) implies $\underline{\mu}(s) < 1$ for all $s > 0$. Finally, the continuity of $\underline{\mu}(s)$ follows from the continuity of $C^{\mathrm{P}}(\mu, s)$. ■

Proof of Proposition 5. Suppose first that $V^1(c_F^2) \leq V_L^1$ and specify the allocation $\psi(2, c_F^2) = 1$ and $\psi(1, c_L^1) = 1$. It is straightforward to support this as an equilibrium allocation regardless of the choice of s and $\mu < 1$. Hence, it remains to consider the case where $V^1(c_F^2) > V_L^1$, i.e., where (A.7) holds. Consider the program to maximize $V^2(c)$ subject to $c \in C$ and $\mu U^2(c) + (1 - \mu)U^1(c) \geq 0$. Existence of a solution follows from (A.4). (However, (A.6) no longer ensures uniqueness.) Denote the maximum by $\widetilde{V}(\mu)$ and the (non-empty) compact set of solutions by $\widetilde{C}(\mu)$. $\widetilde{V}(\mu)$ is continuous and $\widetilde{C}(\mu)$ is upper semicontinuous in μ. Observe also that $\widetilde{V}(1) = V_F^2$. Choose now a contract $\tilde{c}(\mu) \in \widetilde{C}(\mu)$ for given μ. Using continuity of $\widetilde{V}(\mu)$, upper semicontinuity of $\widetilde{C}(\mu)$, and continuity of $V^1(\cdot)$, we can choose for any $\bar{\varepsilon} > 0$ a sufficiently large value $\bar{\mu}^{\bar{\varepsilon}} < 1$ such that for all $\bar{\mu}^{\bar{\varepsilon}} < \mu \leq 1$ the following claims hold:

$$\begin{aligned} \tilde{c}(\mu) &\in \Omega_{\bar{\varepsilon}}(c_F^2), \\ \widetilde{V}(\mu) &> V_F^2 - \bar{\varepsilon}, \\ V^1(\tilde{c}(\mu)) &< V^1(c_F^2) + \bar{\varepsilon}. \end{aligned} \tag{4}$$

(Recall that $\Omega_{\bar{\varepsilon}}(c_F^2)$ denotes the $\bar{\varepsilon}$-neighborhood of c_F^2.) Construct now an allocation with $\psi(2, \tilde{c}(\mu)) = 1$ and $\psi(1, \bar{c}) = 1$, where $\bar{c} = c_L^1$ if $V_L^1 > V^1(\tilde{c}(\mu))$ and $\bar{c} = \tilde{c}(\mu)$ otherwise. To prove $\psi \in \Psi_s(\mu)$ for sufficiently high values of μ, we show the existence of an equilibrium where $i = \mathbf{2}$ visits a receiver and proposes the accepted contract $\tilde{c}(\mu)$, while $i = \mathbf{1}$ implements $\bar{c}$ with probability one. To construct beliefs, which are shared by all receivers, define the sets

$$\begin{aligned} C_B^1 &= \{c \in C \mid V^1(c) \geq V^1(\bar{c}) - s\} \backslash \{\tilde{c}(\mu)\}, \\ C_B^2 &= C \backslash \big\{C_B^1 \cup \{\tilde{c}(\mu)\}\big\}. \end{aligned}$$

Following the procedure in Propositions 1 and 3, we specify $\mu_\psi(i \mid c) = \mathbf{1}$ for $c \in C_B^i$, $\mu_\psi(2 \mid \tilde{c}(\mu)) = \mu$ if $\bar{c} = \tilde{c}(\mu)$, and $\mu_\psi(2 \mid \tilde{c}(\mu)) = 1$ otherwise. If ψ is an equilibrium allocation, the low type will only withdraw an accepted contract c for sure if $c \notin C_B^1$. The acceptance set of the receiver C^{A} may now be constructed as in the proofs of Propositions 1–3. Clearly, strategies are optimal for receivers and the low-type sender. Consider next the high type. By (A.5) and (5) we can find $\varepsilon_1 > 0$ such that $V^2(\tilde{c}(\mu)) \geq V_0^2$ holds for all $\mu > \bar{\mu}^{\bar{\varepsilon}}$ and $\bar{\varepsilon} \leq \varepsilon_1$. Observe next that we can restrict our consideration to deviations $c \in C_B^2 \cap C^{\mathrm{A}}$. As this implies $V^1(c) < V^1(c_F^2) - s + \bar{\varepsilon}$ and $U^2(c) \geq 0$, it follows from the continuity of the payoffs that we can find some ε_2 such that all contracts $c \in C_B^2 \cap C^{\mathrm{A}}$ satisfy $c \notin \Omega_{\varepsilon_2}(c_2^F)$ in the case $\bar{\varepsilon} < \varepsilon_2$. By the construction of c_2^F any $c \notin \Omega_{\varepsilon_2}(c_2^F)$ satisfies $V^2(c) < V^2(c_2^F)$ for some $\varepsilon_3 > 0$. Summing up results and using (5), we can thus indeed support ψ with $\tilde{c}(\mu) \in \Omega_{\bar{\varepsilon}}(c_F^2)$ as an equilibrium allocation if prior beliefs satisfy $\mu > \bar{\mu}^{\bar{\varepsilon}}$, where $\bar{\varepsilon} = \min[\varepsilon, \varepsilon_1, \varepsilon_3] > 0$. ■

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