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Bargaining with a possibly committed seller ☆

Roman Inderst

London School of Economics, Department of Economics, Houghton Street, London WC2A 2AE, UK
INSEAD, CEPR

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Abstract

We consider negotiations with an open time horizon where a buyer has private information about his valuation and does not know whether the seller is committed to the advertised price. This setting combines two common specifications made in the non-cooperative bargaining literature: one side is privately informed about its valuation, which is drawn from a continuum, and the other side is possibly committed to a fixed offer. We analyze the game both in discrete and in continuous time and show convergence of the two settings, which extends results from Abreu and Gul [2000. Bargaining and reputation. Econometrica 68, 85–117]. One interesting result is that as time proceeds, the non-committed seller becomes less likely to concede in a given period, i.e., it appears as if he becomes more "stubborn." We further show that a seller may prefer to negotiate with a "worse" buyer as this enhances the value of his possible commitment.

Keywords: Bargaining; Commitment; Coase conjecture

1. Introduction

This paper considers bilateral negotiations between a buyer and a seller. The buyer has private information about his valuation. While the good is advertised at a fixed price, the buyer is not sure whether the seller is indeed committed to this price. For instance, the

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E-mail address: R.Inderst@lse.ac.uk.

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seller could be the actual owner of the good or, alternatively, he could be an agent of the owner without the right (or the incentives) to change the price. We analyze this setting by solving a bargaining game with open time horizon where the seller can repeatedly make a new offer. We consider the so-called "gap" case, i.e., there is a sure gain from trade between the buyer and the non-committed seller.[1]

We first analyze the game in continuous time and show that it has a unique equilibrium. The equilibrium exhibits delay in reaching an agreement and has the features of a war of attrition. We find that the non-committed seller concedes with a (weakly) declining hazard rate. Hence, it appears as if the non-committed seller becomes more "stubborn" over time.

The intuition for the seller's (weakly) declining hazard rate is as follows. Along the equilibrium path, a buyer type with a higher valuation concedes earlier than a type with a lower valuation. To keep the non-committed seller indifferent between continuing to pretend that he is committed and lowering the price, the buyer must concede with constant hazard rate (or "speed"). But this requires that the seller does not concede with constant speed. To see this, note that at a given time the marginal buyer type is just indifferent between accepting the commitment offer now and holding out marginally longer in the hope that the (non-committed) seller concedes in the meantime. As the loss from delay increases with the buyer's valuation, a high-valuation buyer is only indifferent between conceding now and holding out marginally longer if it is not too unlikely that the seller concedes in the next instant. In contrast, a buyer with a very low valuation is willing to hold out marginally longer even if the seller is quite unlikely to concede in the next instant.

We further use the equilibrium characterization to show that—somewhat surprisingly—the seller may strictly prefer to negotiate with a "worse" buyer, i.e., a buyer whose valuation exhibits a worse distribution in the sense of First-Order Stochastic Dominance.

In a second step, we analyze the model in discrete time. We prove existence of an equilibrium and show that it is generically unique. We finally show convergence for the discrete-time game as the time between offers goes to zero.

Most of our results are extensions of those in Abreu and Gul (2000). There, it is assumed that both the buyer and the seller are possibly committed to some fixed offer. With some probability, however, either party is not committed and has a known (reservation) value. After the first offer (and counter-offer), the number of commitment types shrinks to one. That is, the respective party is either committed to this offer or it is flexible. By the Coase Conjecture, revealing to be non-committed is equivalent to conceding immediately if the game is set in continuous time. In both Abreu and Gul (2000) and our model, it is this insight that generates a unique equilibrium, which furthermore has the structure of a war of attrition.

Our assumption that the seller is possibly committed to a fixed offer follows the approach taken recently in, e.g., Abreu and Gul (2000), Compte and Jehiel (2002), and Kambe (1999).[2] Abreu and Gul (2000) show that with this specification the set of all equilibria converges as time between offers vanishes—that is, even with two-sided private information and two-sided offers. This is different if players' have private information

[1] Formally, there is a strictly positive "gap" between the lowest possible valuation of the buyer and the known reservation value of the seller.

[2] For an earlier formulation see Myerson (1991).

about their reservation values but are free to make or accept any offer.[3] In this case, if private information is two-sided, there is a plethora of equilibria with often widely different outcomes, even if one restricts attention to the one-sided offer case (e.g., Ausubel and Deneckere, 1992).

Our setting is restrictive in that we only consider the case with a "gap." With one-sided private information, if such a gap between the seller's known reservation value and the buyer's lowest possible valuation exists, it is well known from the seminal work of Fudenberg et al. (1985) and Gul et al. (1986) that the Coase Conjecture holds. If there is no such gap, Ausubel and Deneckere (1989) have shown that even as the time between two consecutive offers goes to zero one can support equilibria with long expected delay where the seller's expected profit remains bounded away from zero.

The rest of this paper is organized as follows. Section 2 sets up and solves the game in continuous time. In Section 3 we analyze the game in discrete time and show convergence of the equilibrium outcomes as the time between two consecutive offers goes to zero. Section 4 concludes. All proofs are relegated to Appendix B.

2. The game in continuous time

2.1. The model

We analyze negotiations between a seller and a buyer. The seller has a single good and a reservation value of zero. There is an interval of possible types for the buyer, indexed by $q \in I = [0, 1]$. The seller holds the a priori beliefs that the buyer's type is uniformly distributed over I. The valuation of each type is specified by a nonincreasing and left-side continuous function $f : [0, 1] \to R_+$ with $f(0) = 1$ and $0 < f(1) < 1$. Hence, we restrict consideration to the "gap" case where from $\underline{f} := f(1) > 0$ there is a sure gain from trade.[4] The buyer's type is his private information. We assume that $f(q)$ is Lipschitz continuous at $q = 1$. That is, it holds for all q that $f(q) - f(1) \leqslant H(1 - q)$, where $H > 0$. This assumption allows us to apply standard results from the literature on the Coase Conjecture.[5] It also proves convenient to assume that the distribution of valuations has no mass point at p^c.[6] Define $q^c := \max\{q \mid f(q) \geqslant p^c\}$.

The seller may be committed to a fixed price p^c with a commonly known probability $\gamma_0 \in (0, 1)$. Hence, only with probability $1 - \gamma_0$ is the seller in a position to deviate from the commitment price p^c. We assume that p^c lies strictly between the highest and the lowest valuation for the buyer: $\underline{f} < p^c < 1$. Both players discount future payoffs by the factor $0 < \delta < 1$.

[3] See Kennan and Wilson (1993) for an early survey of the literature.

[4] If $F(v)$ denotes the distribution of the buyer's valuations, the function f is given by $f(q) = \inf\{v \mid F(v) \geqslant 1 - q\}$ for $q \in [0, 1]$.

[5] Precisely, this assumption follows Gul et al. (1986). For weaker conditions see Ausubel and Deneckere (1989).

[6] If there is a strictly positive mass on p^c, the equilibrium strategies of these types are not pinned down uniquely. This case is treated in the working paper version.

In continuous time, we consider the following bargaining game. At each point of time a (non-committed) seller and a buyer can choose whether to hold out or whether to concede. If only the seller concedes at some time t, the price is $\underline{f}$. If only the buyer concedes, the price is p^c. If both sides concede at the same time, we set the price equal to $(p^c + \underline{f})/2$, though this will be without consequences as in equilibrium a mutual concession at the same time will be a zero-probability event.[7]

Our motivation for these specifications is the following. Below we set out a bargaining game in discrete time where in each period the seller can make an offer, to which the buyer can respond by accepting or rejecting. A non-committed seller can offer any price. We show convergence between the outcome of the game in discrete and that in continuous time. The main intuition is that by the Coase Conjecture a non-committed seller who reveals his type by offering a price different to p^c will almost instantaneously offer a price that is arbitrarily close to $\underline{f}$ as the time between offers goes to zero.

In the game in continuous time the strategy of the non-committed seller is a distribution function $H_S(t)$ over $t \in [0, \infty)$, where each realization t denotes a time of concession. A committed seller never concedes. A buyer with type $q > q^c$, who has a valuation strictly below the commitment price p^c, also never concedes. For a buyer with valuation $f(q) \geqslant p^c$, i.e., for all $q \in [0, q^c]$, a pure strategy is a function $t_B : [0, q^c] \to [0, \infty]$, which denotes the time at which type q concedes by accepting the commitment price p^c. By optimality, given $\delta < 1$ a buyer with a higher valuation must not concede later in any given equilibrium ("skimming property"). Without loss of generality, we may thus assume that t_B is nondecreasing and left-continuous.[8]

2.2. Analysis

Define $G_S(t)$ by $G_S(t) = (1 - \gamma_0)H_S(t)$ for $t < \infty$ and $G_S(\infty) = 1$. Note that G_S includes the behavior of the committed type. Define next $G_B(t)$ by $G_B(t) = \max\{q \mid t_B(q) \leqslant t\}$ for $t < \infty$ and $G_B(\infty) = 1$. Again, G_B includes also the strategy of all types $q > q^c$. If the seller chooses to concede at t, his expected payoff given the buyer's strategy G_B equals

$$U^S(t, G_B) := p^c \int\limits_{t'<t} \mathrm{e}^{-rt'}\,\mathrm{d}G_B(t') + \mathrm{e}^{-rt}\big[1 - G_B(t)\big]\underline{f} + \mathrm{e}^{-rt}\bar{G}^B(t)\frac{1}{2}\big(p^c + \underline{f}\big),$$

where we use $\bar{G}_B(t) := G_B(t) - \lim_{t' \to t} G_B(t')$. For a buyer of type $q \leqslant q^c$ the expected utility from conceding at t equals

$$U^B(q, t, G_S) := \int\limits_{t'<t} \mathrm{e}^{-rt'}\big(f(q) - \underline{f}\big)\,\mathrm{d}G_S(t') + \mathrm{e}^{-rt}\big(1 - G_S(t)\big)\big(f(q) - p^c\big) + \mathrm{e}^{-rt}\bar{G}_S(t)\left[f(q) - \frac{1}{2}\big(p^c + \underline{f}\big)\right],$$

[7] The game has the features of a war of attrition (see, e.g., Hendricks et al., 1986).

[8] If $f(\cdot)$ is not strictly decreasing, the "skimming property" holds if types in the flat segment are adequately perturbed.

where $\bar{G}_S(t) := G_S(t) - \lim_{t' \to t} G_S(t')$. We look for a sequential equilibrium. We find a unique equilibrium, in which all possible gains from trade (with the non-committed seller) are realized for sure in finite time.

Proposition 1. *The game in continuous time has a unique equilibrium, which has the following characteristics*:

(i) *The probability of immediate concession can only be strictly positive for one player, i.e.,* $G_B(0) > 0$ *implies* $G_S(0) = 0$ *and vice versa. These probabilities satisfy*

$$\frac{\gamma_0}{1 - G_S(0)} = \mathrm{e}^{\int_{G_B(0)}^{q^c} -\frac{f(q)-p^c}{\underline{f}} \frac{1}{1-q} \, \mathrm{d}q}. \tag{1}$$

(ii) *There is a finite time* $\bar{t}$ *by which there is sure trade with all buyers whose valuations exceed* p^C, *where* $\bar{t}$ *is given by*

$$\bar{t} = \left(\frac{p^c - \underline{f}}{r\underline{f}}\right) \ln\left(\frac{1 - G_B(0)}{1 - q^c}\right). \tag{2}$$

(iii) *The buyer's path of concessions over* $0 \leqslant t \leqslant \bar{t}$ *is given by*

$$\frac{1 - G_B(t)}{1 - G_B(0)} = \mathrm{e}^{-r \frac{\underline{f}}{p^c - \underline{f}} t}, \tag{3}$$

while the seller's path of concessions is given by

$$\frac{1 - G_S(t)}{1 - G_S(0)} = \mathrm{e}^{\int_{G_B(0)}^{G_B(t)} -\frac{f(q)-p^c}{\underline{f}} \frac{1}{1-q} \, \mathrm{d}q}. \tag{4}$$

Proof. See Appendix A.

Equation (3) characterizes the buyer's strategy. If no sale has taken place by some time $t \leqslant \bar{t}$, then the non-committed seller must be kept indifferent between conceding or holding out further. To keep the seller indifferent, types with valuations $f(q) > p^c$ must concede with constant "speed" (hazard rate). This requirement gives rise to condition (3). For buyer types who do not concede instantaneously at $t = 0$ and who concede for sure by $t = \bar{t}$, i.e., all q satisfying $G_B(0) < q \leqslant q^c$, we can derive from (3) the strategy[9]

$$t_B(q) = \left(\frac{p^c - \underline{f}}{r\underline{f}}\right) \ln\left(\frac{1 - G_B(0)}{1 - q}\right). \tag{5}$$

We come next to the seller's strategy. To ensure that buyer types $q \leqslant q^c$ concede with constant speed, the seller must not concede with constant speed—that is, unless $f(q)$ is constant for all $q \leqslant q^c$. In Appendix A, we derive the seller's path of concessions in (4) from the requirement

$$\frac{g_S(t_B(q))}{1 - G_S(t_B(q))} = r \frac{f(q) - p^c}{p^c - \underline{f}}, \tag{6}$$

[9] Note that substituting $q = q^c$ into $t_B(q)$ yields (2).

where g_S is the density of G_S. Hence, by (6) and as $f(q)$ is (weakly) decreasing, the hazard rate for the seller's concession (weakly) decreases over time. Moreover, if $f(q^c) = p^c$ holds the hazard rate converges to zero as $t \to \bar{t}$.

In our model, the non-committed seller thus appears to become more stubborn over time. This feature distinguishes our results from those in Abreu and Gul (2000). The difference follows naturally as we allow the buyer to have a range of different valuations, while in Abreu and Gul (2000) a non-committed buyer can only have a single reservation value. In contrast to our model, they allow, however, for multiple commitment types for both the seller and the buyer.

How does the equilibrium change in the initial probability that the seller is committed? To answer this question, note first that by Proposition 1 we have to distinguish between two cases. By inspection of (1), there exists a threshold $0 < \overline{\gamma}_0 < 1$ such that $G_B(0) = 0$ and $G_S(0) > 0$ for $\gamma_0 < \overline{\gamma}_0$, $G_B(0) = 0$ and $G_S(0) = 0$ for $\gamma_0 = \overline{\gamma}_0$, and $G_B(0) > 0$ and $G_S(0) = 0$ for $\gamma_0 > \overline{\gamma}_0$.

In the case $\gamma_0 > \overline{\gamma}_0$, we have

$$-\ln \gamma_0 = \int_{G_B(0)}^{q^c} \frac{f(q) - p^c}{\underline{f}} \frac{1}{1-q} \, \mathrm{d}q \tag{7}$$

such that $G_B(0)$ is continuous and strictly increasing in γ_0 with $G_B(0) \to q^c$ as $\gamma_0 \to 1$. That is, the more likely it is that the seller is committed, the more types concede immediately at $t = 0$. As $\gamma_0 \to 1$ almost all types whose valuation exceeds p^c concede immediately. Moreover, by (2) we have that $\bar{t}$ is strictly decreasing in γ_0 with $\bar{t} \to 0$ as $\gamma_0 \to 1$. As an immediate consequence, the expected delay with a non-committed seller converges to zero. Finally, as $G_B(0)$ changes continuously in γ_0 by (7), we have from Proposition 1 that also equilibrium strategies change continuously.

In the case $\gamma_0 < \overline{\gamma}_0$ such that $G_B(0) = 0$, it is immediate from (1) that $G_S(0)$ and thus also all equilibrium strategies are continuous and decreasing in γ_0 with $G_S(0) \to 1$ as $\gamma_0 \to 0$. Hence, as it becomes arbitrarily unlikely that the seller is initially committed, in equilibrium the non-committed seller concedes with almost probability one at the first instance. (More formally, from $G_S(0) = (1 - \gamma_0) H_S(0)$ we have that $G_S(0) \to 1$ implies $H_S(0) \to 1$.) It is now interesting to note from (2) that $\bar{t}$ does not change in γ_0 when $G_B(0) = 0$. But as $H_S(0) \to 1$ from $G_S(0) \to 1$, the expected delay with a non-committed seller converges to zero as $\gamma_0 \to 0$.

One upshot of this analysis is that both for $\gamma_0 \to 1$ and for $\gamma_0 \to 0$ the expected delay with the non-committed seller converges to zero. Unfortunately, we were not able to establish more generally how the expected delay varies in γ_0 over $\gamma_0 \in (0, 1)$. Finally, we turn to the expected profits of a non-committed seller. As the non-committed seller is indifferent over $0 < t < \bar{t}$ between conceding and holding out (marginally) longer, his expected profits are simply given by

$$G_B(0) p^c + \left[1 - G_B(0)\right] \underline{f}. \tag{8}$$

Substituting for $G_B(0)$ from our just derived results, we have that (8) is continuous and (weakly) increasing in γ_0. The respective limits are $\underline{f}$ as $\gamma_0 \to 0$ and $q^c p^c + (1 - q^c) \underline{f}$ as $\gamma_0 \to 1$. Hence, while for $\gamma_0 \to 0$ the non-committed seller does not gain from the presence of buyer types with valuation above $\underline{f}$, he can for $\gamma_0 \to 1$ extract the full difference $p^c - \underline{f}$ from all types with valuation exceeding p^c.

We summarize the key insights from this comparative analysis as follows.

Corollary 1. *The equilibrium of the game in continuous time is continuous in γ_0. The non-committed seller's expected profit is* (*weakly*) *increasing in γ_0, with the limits $\underline{f}$ for $\gamma_0 \to 0$ and $q^c p^c + (1 - q^c)\underline{f}$ for $\gamma_0 \to 1$, while the expected delay with the non-committed seller approaches zero both as $\gamma_0 \to 0$ and as $\gamma_0 \to 1$.*

2.3. Example

By way of an example we now want to analyze how the equilibrium changes as the "gap" becomes smaller. For this we consider a piecewise linear valuation function $f(q)$. Suppose $f(q) = 1 - q$ for $0 \leqslant q \leqslant \underline{q} < 1$ and $f(q) = 1 - \underline{q}$ for $q \geqslant \underline{q}$ with $1 - \underline{q} < p^c$. Note that $\underline{f} = 1 - \underline{q}$, while $q^c = 1 - p^c$.

We consider a change in the "gap" $\underline{f}$, while leaving the valuation function unchanged for $q \leqslant \underline{q}$, i.e., for all types who would be willing to buy from a committed seller. By inspection of (1) it is immediate that $G_S(0) = 0$ and $G_B(0) > 0$ must hold if $\underline{f}$ is sufficiently close to zero. Moreover, $G_B(0)$ strictly increases as $\underline{f}$ decreases with the limit $G_B(0) \to q^c$ as $\underline{f} \to 0$. Hence, as the gap becomes smaller it becomes more likely that the buyer concedes immediately. As the gap becomes arbitrarily small, almost all types with valuation exceeding p^c must concede immediately. To ensure that this is optimal for the buyer it must become very unlikely that the non-committed seller concedes soon. In fact, we obtain from (2) that $\bar{t} \to \infty$ as $\underline{f} \to 0$, while from (6) we have for any finite t that $G_S(t) \to 0$ as $\underline{f} \to 0$.

These results imply that as $\underline{f} \to 0$ the outcome converges to one where the seller is surely committed, i.e., where a buyer with valuation exceeding p^c immediately accepts the fixed offer while there is no trade with a buyer whose valuation is below p^c.

We finally analyze how the non-committed seller's profit changes in $\underline{f}$. Differentiating (8), we obtain the derivative $(p^c - \underline{f})(\mathrm{d}G_B(0)/\mathrm{d}\underline{f}) - G_B(0)$, where we can show from (1) that $\mathrm{d}G_B(0)/\mathrm{d}\underline{f} \to -\infty$ as $\underline{f} \to 0$ and thus $G_B(0) \to q^c$.[10] Hence, if the gap is already small, the non-committed seller strictly gains if it is further decreased. Intuitively, as $\underline{f}$ decreases, the seller has more to lose when revealing that he is of the non-committed type. This makes him tougher and, consequently, the buyer weaker. Note finally that (in the considered class of piecewise linear functions) a lower value of $\underline{f}$ represents a buyer whose distribution of values is worse in the sense of First-Order Stochastic Dominance. In this sense, our result implies that the seller may prefer to face a "worse" buyer.

[10] Formally, we obtain from (8) the requirement $-\underline{f} \ln \gamma_0 = (1 - p^c - G) + p^c[\ln(p^c) - \ln(1 - G)]$, such that by implicit differentiation $\mathrm{d}G_B(0)/\mathrm{d}\underline{f} = \ln \gamma_0 / [1 - p^c/(1 - G_B(0))]$. Note that as $G_B(0) \to q^c$ we have from $p^c = 1 - q^c$ that $1 - p^c/(1 - G_B(0))$ goes to zero.

3. The game in discrete time

In discrete time, the seller can make an offer at times $\{0, z, 2z, \ldots\}$ spaced equally apart in real time, where $z > 0$. The buyer can either accept or reject the current offer. Both players discount future payoffs by $\delta = \mathrm{e}^{-rz}$ with $r > 0$. We solve again for a sequential equilibrium.[11]

It is now useful to review some of the results for the (standard) game where the seller is surely non-committed.[12] The game ends in finite time and has a (generically unique) equilibrium. Along the equilibrium path, the seller strictly decreases his offer and each period the buyer accepts with strictly positive probability. Moreover, the Coase Conjecture holds. That is, as z converges to zero the first offer of the seller becomes arbitrarily close to $\underline{f}$ and the game ends almost immediately in real time.

These results prove useful also for our model: they apply to the continuation game after the seller makes an offer other than p^c, revealing that he is non-committed. After the seller revealed that he is non-committed, i.e., after making an offer different from p^c, these results apply to the continuation equilibrium. In particular, this implies that once the non-committed seller reveals his type the final price will converge to $\underline{f}$ as $z \to 0$. In the limit, the price that is offered by a non-committed seller who reveals his type is thus $\underline{f}$. This was precisely what we assumed in the game set in continuous time.

These observations suggest that the outcome of the game in continuous time can be obtained as a limit of the game in discrete time. Building on arguments in Abreu and Gul (2000) we can indeed confirm this result. To formalize this, denote an equilibrium of the game in discrete time by $\sigma \in \Sigma(z)$, where we have made explicit that the equilibrium set depends on z. We denote the random outcome corresponding to σ by $\tilde{\theta} = (\tilde{p}, \tilde{t})$, where a realization (p, t) denotes a price paid at some real time t at which agreement is reached.

Proposition 2. *Let $\{\sigma^k\}$ be a sequence of equilibria where $\sigma^k \in \Sigma(z^k)$ and $z^k \to 0$. Let $\tilde{\theta}^k$ denote the random outcome associated with σ^k, and let $\tilde{\eta}$ denote the random outcome associated with the unique equilibrium of the game in continuous time. Then $\tilde{\theta}$ converges in distribution to $\tilde{\eta}$.*

Proof. See Appendix B.

The proof differs from that in Abreu and Gul (2000) in only one aspect. In their model, after the first round of offers the seller and the buyer can both be only non-committed or committed to a single known price, while their reservation value is common knowledge. In the proof of Proposition 2 we must, in contrast, derive convergence for all types $q \leqslant q^c$, who have potentially different valuations $f(q)$.

What is still missing is to show existence of an equilibrium of the game in discrete time for any z. Our final result closes this gap.

Proposition 3. *$\Sigma(z)$ is non-empty and generically a singleton.*

11 The formal description of strategies and equilibrium requirements is standard and thus omitted.

12 For proofs see Gul et al. (1986).

Proof. See Appendix B.

4. Conclusion

This paper analyzes a strategic situation which seems to be common in markets where anonymous agents trade. If the product represents a standardized good, there should be much more information asymmetry regarding the buyer's reservation value than regarding a seller's costs of production. However, the seller may have private information about his possibility to renegotiate his advertised price. For instance, he may refuse to do so claiming that he is an employee of a distant principal.

This paper analyzes this game both in discrete and continuous time and shows additionally convergence between these cases. One interesting feature of the game is that the non-committed seller concedes with a strictly decreasing hazard rate, making it less likely that agreement will be reached as time proceeds. With a piecewise linear example we have also derived two interesting comparative results. As the gap in the buyer's valuation closes, the outcome approaches that of a game where the seller is surely committed, while the seller is strictly better off if the gap becomes smaller.

One restriction of our analysis is that the buyer cannot make counteroffers. A non-committed seller hesitates to reduce his price as this immediately reveals his type and thus immediately drives down the final agreement price. This feature, which basically excludes the possibility to make gradual concessions, makes it credible that the seller plays tough. If the buyer could make counterproposals instead of only replying to the seller, this reasoning may no longer work as the buyer could counter with an *intermediate* price offer. However, as the buyer's counter-proposal may be interpreted as a signal of high valuation, this strategy may not be used in some of the equilibria of a game with alternating offers. These issues remain to be explored.

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Appendix A. The game in continuous time

We first derive a number of properties that must be satisfied in any equilibrium. We omit or abbreviate some arguments as they are well known from the literature on the war of attrition (see also Abreu and Gul, 2000).

(i) *G_S and G_B have no common discontinuity at $t < \infty$.*

(ii) *The support of H_S has a finite upper bound $\bar{t} > 0$, where also $G_B(\bar{t}) = q^c$.* As $\underline{f} > 0$ and $q^c < 1$, the boundedness of the support of H_S follows immediately from optimality for the seller. For a buyer with valuation exceeding p^c it is also not optimal to hold out if he puts probability one on the committed seller, while a buyer with valuation below p^c will never concede. This establishes $G_B(\bar{t}) = q^c$.

(iii) *G_S and G_B are continuous and strictly increasing over* $[0,\bar{t}]$.[13]

By properties (i)–(iii), $t_B(q)$ is continuous and invertible on $t\in(0,\bar{t}]$ (with $G_B(t)$ its inverse). Optimality implies that $U^S(t,G_B)$ is constant over $t\in(0,\bar{t}]$ and thus differentiable. This yields for the density g_B of G_B the requirement

$$\frac{g_B(t)}{1-G_B(t)}=r\frac{\underline{f}}{p^c-\underline{f}}.$$

Using properties (i)–(iii), we next rewrite for all $t>0$ the buyer's payoff as $U^B(q,t,G_S)=f(q)\tau_1(t)-\tau_2(t)$, where we define $\tau_1(t):=\int_0^t \mathrm{e}^{-rt'}\,\mathrm{d}G_S(t')+\mathrm{e}^{-rt}(1-G_S(t))$ and $\tau_2(t):=\underline{f}\int_0^t \mathrm{e}^{-rt'}\,\mathrm{d}G_S(t')+\mathrm{e}^{-rt}(1-G_S(t))p^c$. For $t>0$, τ_1 and τ_2 are continuous and monotonic and thus a.e. differentiable. Optimality implies now for any pair $0\leqslant q\leqslant q'\leqslant q^c$ that

$$f(q)\big[\tau_1\big(t_B(q)\big)-\tau_1\big(t_B(q')\big)\big]\geqslant\tau_2\big(t_B(q)\big)-\tau_2\big(t_B(q')\big).\tag{9}$$

As $t_B(q)$ is for $q>G_B(0)$ a.e. differentiable and as f is a.e. continuous, (9) yields a.e. for $G_B(0)<q<q^c$ the requirement

$$f(q)=\frac{g_S(t_B(q))(p^c-\underline{f})+r[1-G_S(t_B(q))]p^c}{r[1-G_S(t_B(q))]},\tag{10}$$

where g_S denotes the density of G_S. By (10) and as G_S has no mass on $t>0$, we can thus derive for $q>G_B(0)$ a.e. the requirement

$$\frac{g_S(t_B(q))}{1-G_S(t_B(q))}=r\frac{f(q)-p^c}{p^c-\underline{f}}.$$

By the invertibility of $t_B(q)$ on $0<t\leqslant\bar{t}$ we obtain next (4), where we substituted from (5) that a.e. $\mathrm{d}t_B(q)/\mathrm{d}q=\frac{p^c-\underline{f}}{r\underline{f}}\frac{1}{1-q}$. Finally, (1) and (2) follow from (3), (4), and property (ii).[14]

We argue finally that the characterization is unique. Observe first that, given $G_S(0)$ and $G_B(0)$, the final period $\bar{t}$ and the distribution functions over $t\in(0,\bar{t}]$ are uniquely determined. Moreover, uniqueness of $G_S(0)$ and $G_B(0)$ follows from the requirement in (1) together with property (i). This completes the proof of Proposition 1.

Appendix B. The game in discrete time

It is useful to consider first the game where the seller has already revealed by making an offer $p\neq p^c$ that he is not committed. If the current state, i.e., the truncation of buyer types, is q, define $M(q)$ as the set of optimal states following q. Define also $t(q):=\inf M(q)$ and $P(t(q))$ as the reservation price for the respective type $t(q)$. The set

[13] Observe that this admits atoms at $t=0$.

[14] Note that the local incentive compatibility used for the construction of t_B implies by monotonicity of f and continuity of t_B that any type $q\leqslant q^c$ with $t_B(q)\in(0,\bar{t})$ prefers $t_B(q)$ to some (alternative) $t\in(0,\bar{t})$. The extensions to $t=\bar{t}$ and $t=0$ are immediate.

of the seller's optimal next offers is denoted by $\Pi(q)$. From Gul et al. (1986) we know that $t(q) \in M(q)$ and thus $P(t(q)) \in \Pi(q)$, while $P(t(q))$ is unique and $M(q)$ is generically unique. Given some state q and that the non-committed seller has revealed himself, we denote his expected payoff in a continuation equilibrium by $R(q)$. It is also useful to define $\bar{R}(q) := (1-q)R(q)$. We know from Gul et al. (1986) that $R(q)$ is continuous, which clearly also extends to $\bar{R}(q)$.

The following results used in the proofs of Propositions 2 and 3.

Claim 0. (i) *The game with a non-committed seller ends in finite time.*

(ii) *If p^c has been offered in all previous periods and the state is q, then the next offer must be $p \in \Pi(q) \cup p^c$. Following any offer $p \neq p^c$, there exists a unique continuation equilibrium.*

(iii) *In equilibrium, if the non-committed seller has previously offered only p^c, he must not offer p^c with probability one in the next period. Each period the buyer must accept with positive probability until $q = q^c$ has accepted.*

Proof. We omit the proofs of assertions (i) and (ii) as the arguments are standard. Consider next assertion (iii). We argue by contradiction. Assume that the non-committed seller offers p^c with probability one over some periods $\{n, \ldots, n+m\}$, where m must be finite from assertion (i). After rejection in period $n+m$, the seller by assumption reveals his type with positive probability and offers a price $p \neq p^c$. From assertion ii) his equilibrium payoff is then $R(q_{n+m+1}) > 0$, where q_{n+m+1} denotes the respective state. Optimality for the buyer now implies zero acceptance probability between n and $n+m$ such that $q_{n+m+1} = q_n$. Hence, by $R(q_{n+m+1}) = R(q_n) > 0$, the seller would be strictly better by offering p in n. This proves the first part of assertion (iii). The second part follows directly from results in Gul et al. (1986) if the non-committed seller has been revealed, while along a sequence of offers p^c it follows immediately from the preceding arguments. □

By Claim 0, the non-committed seller's equilibrium strategy is thus described as follows. In periods $n = 0$ to $n = \bar{n} \geqslant 0$ the seller offers a price from the set $\{p^c\} \cup \Pi(q)$, i.e., he either continues to offer p^c, thereby pretending that he is committed, or he reveals his type. We denote by ρ_n^S the probability that the non-committed seller offers p^c in some period n and by γ_n the updated probability that the seller is of the commitment type at the start of period n, both conditional on p^c having been offered in all previous periods. Hence, after p^c has been offered in all periods up to n, the probability that p^c is offered in period n is given by $\pi_n := \gamma_n + (1-\gamma_n)\rho_n^S$. In period $n = \bar{n}+1$ the seller finally reveals his type for sure.

B.1. Convergence: proof of Proposition 2

It is useful to restate the Coase Conjecture formally.

Claim 1. *For any $\varepsilon > 0$ there exists a value $z(\varepsilon) > 0$ such that for all $z < z(\varepsilon)$ it holds that $R(q) < \underline{f} + \varepsilon$ and $\sup \Pi(q) < \underline{f} + \varepsilon$.*

We now consider a sequence of values $\{z^k\}$ with $z^k > 0$ and $z^k \to 0$ and a corresponding sequence of equilibria $\{\sigma^k\}$ such that $\sigma^k \in \Sigma(z^k)$. For each equilibrium we make the dependency on the respective value z^k explicit, e.g., by writing for the seller's payoff $R(q, z^k)$. We furthermore choose all values z^k sufficiently small such that $\sup \Pi(0, z^k) < p^c - \Delta$ holds for some $\Delta > 0$.

In σ^k the non-committed seller is surely revealed for the first time at the beginning of period $\bar{n}^k + 1$. Denote the respective finite real time by $T^k = (\bar{n}^k + 1)z^k$. We next choose for given t the smallest integer n such that nz^k does not exceed t and define $Q^k(t) := q_n^k$. Define also $\tau^k(q) := \min\{t \mid Q^k(t) \geqslant q\}$ and $Q^{R,k}(t)$ with $Q^{R,k}(t) := \lim_{t' \to t} Q^k(t)$ for $t > 0$ and $Q^{R,k}(0) = 0$ for $t = 0$. Note that $Q^k(t)$ and $Q^{R,k}(t)$ differ only at points where $t = nz^k$ holds for integers $0 \leqslant n \leqslant \bar{n}^k + 1$. Using next $\bar{Q}^k(t) := \min\{Q^k(t), q^c\}$, we then have for all high-valuation types the distribution of concessions $\Gamma_B^k(t) := \bar{Q}^k(t)/q^c$. For the non-committed seller define by $\Gamma_S^k(t)$ the aggregate probability with which he reveals himself by real time t. Note that $\Gamma_S^k(T^k) = 1$, while the upper bound of the support of Γ_B^k is either T^k or $T^k - z^k$.

We next extract subsequences in the following order. First, as by optimality there exists some upper bound in real time until which the non-committed seller has surely conceded in any equilibrium and for any z, we can extract a subsequence such that $T^k \to T$ with finite T. (For convenience, we suppose that the original sequence converges.) Second, by Helly's theorem we can further extract subsequences such that $\Gamma_B^k(t)$ and $\Gamma_S^k(t)$ converge in distribution to some distribution functions $\Gamma_B(t)$ and $\Gamma_S(t)$.[15] Given $\Gamma_B(t)$, define next $\bar{Q}(t) := q^c \Gamma_B(t)$ and, for $0 \leqslant q \leqslant q^c$, $\tau(q) = \min\{t \mid \bar{Q}(t) \geqslant q\}$. To establish properties of the limits, we first derive the following assertions for the equilibria along the sequence.

Claim 2. (i) $T > 0$.

(ii) *For any* $\varepsilon > 0$ *we can find* $\bar{k}$ *and* $\bar{\varepsilon} > 0$ *such that for any pair* $\check{t}, \hat{t}$ *with* $\hat{t} \geqslant \check{t} + \varepsilon$ *and* $\hat{t} \leqslant T$ *and for all* $k > \bar{k}$ *it holds that* $Q^{R,k}(\hat{t}) - Q^{R,k}(\check{t}) > \bar{\varepsilon}$.

Proof. Consider first assertion (i). We argue to a contradiction. Suppose thus that for any $\varepsilon > 0$ it holds for all high values of k that $T^k + z^k < \varepsilon$. Observe next that $Q^k(T^k + z^k) \geqslant q^c$ and $\Gamma_S^k(T^k) = 1$. Consider now the type $q = 0$, which is supposed to accept p^c before real time ε in case only p^c has been offered so far. By $\Gamma_S^k(\varepsilon) = 1$ and Claim 1, rejecting any offer of p^c until $T^k + z^k$ yields type $q = 0$ a payoff not below $e^{-r\varepsilon}(1 - \underline{f} - \varepsilon)(1 - \gamma_0) + e^{-r\varepsilon}(1 - p^c)\gamma_0$ in case $z^k < z(\varepsilon)$. This yields a contradiction if we choose ε sufficiently low and thus k sufficiently high.

Suppose next that assertion (ii) does not hold for some ε, $\check{t}$, and $\hat{t}$. As $T^k \to T$ and $z^k \to 0$, we can for sufficiently large k choose periods $\check{n}^k$ and $\hat{n}^k$ such that $q_{\check{n}^k} \leqslant Q^k(\check{t})$, $q_{\hat{n}^k} \geqslant Q^k(\hat{t})$, $\check{n}^k > 0$, $\hat{n}^k < \bar{n}^k$, and $z^k(\hat{n}^k - \check{n}^k) > \varepsilon/2$. We know from Claim 0 that the expected payoff of the non-committed seller equals $R(q_{\check{n}^k}, z^k)$ if he has offered p^c un-

[15] See Abreu and Gul (2000) for details.

til $\check{n}^k - 1$. Similarly, the payoff in $\hat{n}^k$ equals $R(q_{\hat{n}^k}, z^k)$. As $R(\cdot) \geqslant \underline{f} > 0$, we have by optimality for the seller that

$$R\big(q_{\check{n}^k}, z^k\big) - R\big(q_{\hat{n}^k}, z^k\big) \geqslant \underline{f}\mathrm{e}^{-rz^k(\hat{n}^k - \check{n}^k)}. \tag{11}$$

Moreover, optimality for the buyer implies for all values $q' > q$ that

$$R\big(q_{\check{n}^k}, z^k\big)(1 - q_{\check{n}^k}) - R(q_{\hat{n}^k}, z)(1 - q_{\hat{n}^k}) \leqslant q_{\check{n}^k} - q_{\hat{n}^k}, \tag{12}$$

where we use that $f(0) = 1$ represents the maximum valuation. To complete the proof we show a contradiction between (11) and (12). To see this, note that by assumption we can find a subsequence k_i such that $q_{\check{n}^{k_i}} - q_{\hat{n}^{k_i}} \to 0$, while $q_{\check{n}^{k_i}} \leqslant q^c$ remains bounded away from one. This implies by (12) that $R^B(q_{\check{n}^k}, z^k) - R^B(q_{\hat{n}^k}, z) \to 0$, which by $z^k(\hat{n}^k - \check{n}^k) > \varepsilon/2$ contradicts (11). □

It is helpful to rephrase assertion (ii) in Claim 2. It simply states that the buyer must concede sufficiently fast as long as the non-committed seller has not revealed himself. Intuitively, it would otherwise not be optimal for the non-committed seller to hold out—which is also what we have used to prove the claim by contradiction.

Claim 3. (i) *$\Gamma_B(t)$ is strictly increasing up to $t = T$.*
(ii) *The support of $\Gamma_B(t)$ and $\Gamma_S(t)$ is $[0, T]$.*
(iii) *$\Gamma_B(t)$ and $\Gamma_S(t)$ are continuous at $t > 0$.*
(iv) *$\Gamma(t)$ and $\Gamma_S(t)$ do not have both a discontinuity at $t = 0$.*
(v) *The supports of $\Gamma_B(t)$ and $\Gamma_S(t)$ are convex.*

Proof. Regarding assertion (i), we argue to a contradiction. Suppose that $\Gamma_B(t') = \Gamma_B(t'')$ holds for some $t' < t'' \leqslant T$. As $\Gamma_B(t)$ is a distribution function, we can find two points $t' \leqslant \tilde{t}'$ and $\tilde{t}'' \leqslant t''$ in the immediate neighborhood of t' and t'' respectively (implying $\tilde{t}' < \tilde{t}''$) at which Γ_B is continuous, implying $\Gamma_B^k(\tilde{t}') \to \Gamma_B(\tilde{t}')$ and $\Gamma_B^k(\tilde{t}'') \to \Gamma_B(\tilde{t}'')$. By Claim 1 it must hold that $Q^k(\tilde{t}'') - Q^k(\tilde{t}') > \varepsilon$ for some $\varepsilon > 0$ and all sufficiently large k. By construction of Γ_B^k and Γ_B this contradicts convergence.

Regarding assertion (ii), note first that it holds for Γ_S by construction of T. If $\Gamma_B(T) < 1$ this would contradict convergence at some t in the right-side neighborhood of T where Γ_B is continuous. For assertion (iii) suppose Γ_S has a discontinuity at some $0 < t \leqslant T$. Denote the size of the respective atom at t by ε. Choose next $0 < t' < t < t''$ in the respective neighborhoods such that Γ_S^k converges at t' and t''. As a consequence, $\Gamma_S^k(t'') - \Gamma_S^k(t') > \varepsilon/2$ holds for sufficiently high k. By standard arguments, optimality for the buyer implies existence of $\varepsilon' > 0$ such that for all sufficiently high k it holds that $Q^k(t') - Q^k(t' - \varepsilon') = 0$, contradicting Claim 1. (Observe that we can use that any revealing offer p satisfies $p \leqslant p^c - \Delta$.)

Consider next assertion (iv) and suppose that both distributions have an atom at $t = 0$. Suppose, in particular, that $\Gamma_S(0) > \varepsilon$ and $\Gamma_B(0) > \varepsilon$ for some $\varepsilon > 0$. For high k this implies from the Coase Conjecture that by optimality the non-committed seller must offer p^c with probability one in $n = 0$. By $\Gamma_S(0) > \varepsilon$ and convergence, we can thus for arbitrarily small $t > 0$ choose k sufficiently high such that the non-committed seller reveals himself with probability not below ε between $n = 1$ and real time t. As revelation implies offering a price $p \leqslant p^c - \Delta$, acceptance of p^c in $n = 0$ is not optimal for type $q = 0$ if t is chosen sufficiently low and k is chosen sufficiently high (implying that e^{-rtz^k} becomes close to one). Summing up, $\Gamma_S(0) > \varepsilon$ and $\Gamma_B(0) > \varepsilon$ imply for high k that with probability one there is no trade in $n = 0$, which contradicts Claim 0.

Assertion (v) follows from assertion (i) for the buyer. Suppose it does not hold for the seller such that $\Gamma_S(t') = \Gamma_S(t'')$ for some $0 < t' < t'' < T$ and $t'' - t' > \varepsilon$, $\varepsilon > 0$. Consider now the time interval $[t'' - \varepsilon/2, t'']$. As k becomes arbitrarily high, we can use by now standard arguments to show that the buyer will concede with zero probability in $[t'' - \varepsilon/2, t'']$ as k becomes sufficiently high. Intuitively, as delay is costly, he will prefer to do so before $t'' - \varepsilon/2$. This contradicts Claim 0. □

Observe that Claim 2 implies that Γ_S^k and Γ_B^k converge at all $t > 0$, while at least one sequence converges additionally at $t = 0$, i.e., everywhere.

Claim 4. *$\tau^k(q) \to \tau(q)$ uniformly for all $0 \leqslant q \leqslant q^c$.*

Proof. The proof is by contradiction. If the claim does not hold, we can extract the following subsequences. First, there exists $\varepsilon > 0$ such that we can extract $\{q^{k_i}\}$ and a sequence of equilibria such that $\left|\tau^{k_i}(q^{k_i}) - \tau(q^{k_i})\right| \geqslant \varepsilon$. We can further extract subsequences where q^{k_i} converges towards some value $\bar{q}$. Observe also that $\tau(q^{k_i}) \to \tau(\bar{q})$ holds by Claim 3, which asserts continuity of τ. Finally, by the finiteness of all $\tau(\cdot)$ for given f we can further extract subsequences such that $\tau^{k_i}(\bar{q})$ converges towards some finite value $\bar{\tau} \geqslant 0$. We suppose for simplicity that the original sequences satisfy all these properties.

Summing up, we have that $\tau^k(\bar{q}) \to \bar{\tau}$ holds for some $\bar{q}$, where $|\bar{\tau} - \tau(\bar{q})| \geqslant \varepsilon$. We distinguish between two cases. In Case (i) we can extract a sequence where $\tau^{k_i}(\bar{q}) \geqslant \tau(\bar{q}) + \varepsilon$. In Case (ii) we can extract a sequence where $\tau^{k_i}(\bar{q}) \leqslant \tau(\bar{q}) - \varepsilon$. Consider first Case (i). If $\tau(\bar{q}) = 0$, choose some arbitrarily close time $t' > 0$ to ensure that Γ_B^k converges at t'. Otherwise, set $t' = \tau(\bar{q})$. By construction it holds for high k_i that $Q^{k_i}(\bar{\tau}) < \bar{q}$, while $Q(t') \geqslant \bar{q}$. As $Q(\cdot)$ is strictly increasing by Claim 2 (due to the strict monotonicity of Γ_B), this contradicts the convergence at t' and $\bar{\tau}$. (Recall from assertion (iii) in Claim 2 that Γ_B is continuous for all $t > 0$.) Consider next Case (ii). If $\bar{\tau} = 0$, take some $t' > 0$ to ensure convergence at t'. Otherwise, set $t' = \bar{\tau}$. By construction it holds for high k_i that $Q^{k_i}(t') \geqslant \bar{q}$ and $Q(t') < \bar{q}$, where we use continuity of $Q(\cdot)$. By the strict monotonicity of the functions Q^{k_i} for high k_i due to Claim 1, this yields a contradiction. □

Denote now $G_S^k(t) = \Gamma_S^k(t)(1 - \gamma_0)$ and $G_B^k(t) = \Gamma_B^k(t)(1 - q^c)$.

Claim 5. *For any $\varepsilon > 0$ and any $0 < q \leqslant \bar{q}^c$ there exists a finite $\bar{k}_B(\varepsilon)$ such that for all $k > \bar{k}_B(\varepsilon)$ and all $t > 0$ it holds that*

$$U^B\big(q, \tau_B(q), G_S\big) > U^B(q, t, G_S) - \varepsilon. \tag{13}$$

Proof. We heavily abbreviate the proof at points where we use arguments that are fully identical to those in Abreu and Gul (2000). We distinguish between two cases.

Case (i) $G_S(0) = 0$. In this case G_S^k converges everywhere by Claim 3. Moreover, observe that $U^B(q, t, G_S)$ is bounded and continuous in t. By the arguments from Abreu and Gul (2000) this implies $U^B(q, t, G_S^k) \to U^B(q, t, G_S)$ (uniformly) over all t and q. Recall next that $\tau^k(q) \to \tau(q)$ holds uniformly by Claim 4, while for $k \to \infty$ all prices $p \neq p^c$ offered in σ^k converge to $\underline{f}$ by Claim 0 and the Coase Conjecture. This allows us to apply the arguments of Abreu and Gul (2000) to prove (13). Observe also that the inequality (13) extends to $t = 0$ and $q = 0$ in Case (i).

Case (ii) $G_S(0) > 0$. By Claim 3 this implies $G_B(0) = 0$ and thus $\tau_B(q) > 0$ for all $q > 0$. The rest of the argument is analogous to Case (i). □

We consider next the seller. The following result can be proved by complete analogy to Claim 5.

Claim 6. *For any $\varepsilon > 0$ there exists $\bar{k}_S(\varepsilon)$ such that for all $k > \bar{k}_S(\varepsilon)$ and all $t, t' > 0$ it holds that*

$$U^S(t, G_B) > U^S(t', G_B) - \varepsilon \quad \text{if } t \leqslant T.$$

As Claims 5–6 hold for all ε and as $G_B(T) = q^c$ and $G_S(T) = 1 - \gamma_0$ hold for the same value T, the distribution of concessions for the non-committed seller and types $q \leqslant q^c$, and the final real time T must be identical to those derived for the unique equilibrium of the game set in continuous time. Finally, as all offered and accepted prices $p \neq p^c$ converge to $\underline{f}$, we have finally proved Proposition 2.

B.2. Existence: proof of Proposition 3

We use that as soon as the seller offers a price $p \neq p^c$, there exists by Claim 0 a unique continuation equilibrium. Moreover, the first revealing offer is generically unique. Precisely, we know from Claim 0 that $p \in \Pi(q) \cup p^c$, where generically $\Pi(q)$ is a singleton. To establish existence, we take in case $\Pi(q)$ is not a singleton the (maximum) offer $P(t(q))$. Hence, it remains to specify for all periods $n \leqslant \bar{n}$ a probability ρ_n^S that the seller offers p^c, conditional on having offered p^c in all previous periods. This sequence of probabilities must make the corresponding strategy of the buyer, i.e., the respective sequence of states q_n, optimal.

Claim 7. *In any period $n \geqslant 0$ such that p^c has been offered in all previous periods and the current state q_n satisfies*

$$\bar{R}(q_n) > p^c\big(q^c - q_n\big) + \delta \bar{R}\big(q^c\big), \tag{14}$$

the non-committed seller offers p^c with probability zero, i.e., $\rho_0^S = 0$.

Proof. The payoff from offering p^c is at most

$$\frac{q^c - q_n}{1 - q_n} p^c + \delta \frac{1 - q^c}{1 - q_n} R\big(q^c\big).$$

Since offering $P(t(q_n)) \neq p^c$ yields $R(q_n)$ by Claim 0, (14) implies that offering p^c cannot be optimal. □

Define next for any $q \geqslant 0$ such that $\bar{R}(q) < p^c(q^c - q) + \delta \bar{R}(q^c)$ the value $Q^*(q)$ by the equation

$$\bar{R}(q) = p^c\big(Q^*(q) - q\big) + \delta \bar{R}\big(Q^*(q)\big). \tag{15}$$

Note that such a value exists since the right-hand side of (15) is too high at $Q^*(q) = q^c$, too low at $Q^*(q) = q$ and continuous in q by continuity of $\bar{R}(q)$. Moreover, we next show that $p^c q + \bar{R}(q)$ is increasing, from which it follows that $Q^*(q)$ is unique and also increasing in q. Given the buyer's (reservation-price) strategy in the continuation game after the seller has been revealed to be non-committed, it is immediate that $0 < \bar{R}(q) - \bar{R}(q') \leqslant P(q)(q' - q)$ holds for any $q' > q$, while we have $P(q) \leqslant p^c$ as otherwise type q would have already accepted the commitment offer. These two inequalities jointly imply that $\bar{R}(q) - \bar{R}(q') < p^c(q' - q)$ and thus that $p^c q + \bar{R}(q) < p^c q' + \bar{R}(q')$.

For all $n \geqslant 1$ we also define $Q_n^*(q)$ recursively by $Q_n^*(q) := Q^*(Q_{n-1}^*(q))$.

Claim 8. *If*

$$\bar{R}(0) < p^c q^c + \delta \bar{R}\big(q^c\big), \tag{16}$$

then in period zero the seller offers p^c with positive probability, i.e., $0 < \rho_0^S \leqslant 1$. This is accepted by types $q \in [0, q_0]$, where $q_0 \geqslant Q^*(0)$, and either $q_0 = Q^*(0)$ or $\rho_0^S = 1$.

Proof. By (16) the seller must offer p^c with positive probability. Otherwise, his payoff would be at must $\bar{R}(0)$, while deviating and offering p^c would yield $p^c q^c + \delta \bar{R}(q^c)$. By definition of $Q^*(\cdot)$, offering p^c is optimal only if it is accepted with probability $q_0 \geqslant Q^*(0)$; and if this inequality is strict, offering p^c yields a strictly higher payoff than any price $p \neq p^c$. □

Claim 9. *For any $q_0 \geqslant Q(0)$, define n^* as the largest integer satisfying*

$$\bar{R}\big(Q_{n^*-1}^*(q_0)\big) \leqslant p^c\big[q^c - Q_{n^*-1}^*(q_0)\big] + \delta \bar{R}\big(q^c\big). \tag{17}$$

This integer n^* exists and is unique and finite for $\delta < 1$.

Proof. If $\bar{R}(0) \geqslant p^c q^c + \delta \bar{R}(q^c)$, then set $n^* = 0$. Otherwise, existence and uniqueness follow as by Claim 0 the game ends after a finite number of periods and as $Q^*(\cdot)$ is strictly increasing. □

Claim 10. *Consider any period $1 < n \leqslant n^*$ such that p^c was offered in all previous periods, and the state in period 1 was $q_0 \geqslant Q^*(0)$. Then*

(i) *the current state is $Q_{n-1}^*(q_0)$;*

(ii) *the monopolist charges p^c with probability $\rho_n^S \in (0, 1)$, and $P(t(Q_{n-1}^*(q_0))$ with probability $1 - \rho_n^S$;*

(iii) *the price p^c is accepted by all types $q \in (Q_{n-1}^*(q_0), Q_n^*(q_0)]$.*

Proof. If p^c has been offered in all previous periods, then it follows from Claim 1 that the non-committed seller cannot offer p^c with probability one. (Otherwise, any type willing to accept p^c would have done so earlier so the seller could deviate by "jumping ahead" and offer $p \neq p^c$ with positive probability.) The seller also cannot offer p^c with probability zero as the resulting payoff would be $\bar{R}(1)/(1-q)$, while in this case deviating to p^c would "convince" the buyer that he was surely committed, yielding him from $\bar{R}(q) < p^c(q^c - q_n) + \delta \bar{R}(q^c)$ a strictly higher payoff. Together with Claim 0 this implies that the seller must strictly randomize between p^c and $P(t(q))$. By definition of $Q^*(q)$, this randomization is indeed optimal if the price p^c is accepted by all buyer types $q' \in (q, Q^*(q)]$. By iterating this argument (and starting from $q_0 \geqslant Q^*(0)$ in $n=1$) we thus have that the state in period n must be $Q^*_{n-1}(q_0)$. This proves parts (i), (ii), and (iii), noting that by definition of n^*, the state $Q^*_{n-1}(q_0)$ in any period $1 < n \leqslant n^*$ indeed satisfies the inequality $\bar{R}(Q^*_{n-1}(q_0)) < p^c[q^c - Q^*_{n-1}(q_0)] + \delta \bar{R}(q^c)$. □

To conclude the proof of existence, it remains to show that there exists a state q_0 (following the first offer of p^c) together with a sequence of probabilities $\{\rho^S_n\}^{n^*}_{n=0}$ which make the buyer's behavior (as described in Claim 10) optimal. Moreover, to complete the proof of Proposition 3 we also have to show that these values are generically unique.

Consider any period $1 < n \leqslant n^*$ such that p^c has been offered in all previous periods, and suppose that the seller again offers p^c. Then the current state is $Q^*_{n-1}(q_0)$, and the buyer expects the next offer to satisfy $p \in P(t(q)) \cup p^c$. Optimality requires that all buyer types $q \in (Q^*_{n-1}(q_0), Q^*_n(q_0)]$ accept p^c, i.e., that

$$\begin{aligned}
(1-\delta) f\big(Q^*_n(q_0)\big) &\geqslant p^c - \delta\big[\pi_{n+1} p^c + (1-\pi_{n+1}) P\big(t\big(Q^*_n(q_0)\big)\big)\big], \\
(1-\delta) \lim_{Q^*_n(q_0) \longleftarrow q} f(q) &\geqslant p^c - \delta\big[\pi_{n+1} p^c + (1-\pi_{n+1}) P\big(t\big(Q^*_n(q_0)\big)\big)\big].
\end{aligned} \tag{18}$$

To satisfy (18) for all $n = 0, 1, \ldots, n^*$, we choose $\{\rho^S_n\}^{n^*}_{n=0}$ such that for all $n = 1, 2, \ldots, n^* + 1$,

$$\begin{aligned}
\pi_n &\equiv \frac{\gamma_0 + (1-\gamma_0)\rho^S_n \prod_{m=0}^{n-1} \rho^S_m}{\gamma_0 + (1-\gamma_0) \prod_{m=0}^{n-1} \rho^S_m} \\
&= \frac{p^c - \delta P(t(Q^*_{n-1}(q_0))) - (1-\delta) f(Q^*_{n-1}(q_0))}{\delta(p^c - P(t(Q^*_{n-1}(q_0)))}
\end{aligned} \tag{19}$$

where we use that $\rho^S_{n^*+1} = 1$. Note that the first equality in (19) just uses Bayes' rule, while the second equality chooses the sequence of values π_n so as to satisfy the first equation in (18) with equality. Generically, i.e., with functions $f(q)$ that are also right-continuous at the respective points, (19) is also the only sequence that satisfies (18). Solving the system of equations in (19) yields for $n = 1, 2, \ldots, n^*$,

$$\rho^S_n = \frac{\prod_{m=n}^{n^*} \frac{\delta[p^c - P(t(Q^*_m(q_0)))]}{p^c - P(Q^*_m(q_0))} - 1}{\prod_{m=n-1}^{n^*} \frac{\delta[p^c - P(t(Q^*_m(q_0)))]}{p^c - P(Q^*_m(q_0))} - 1} \tag{20}$$

and

$$\rho_0^S = \frac{\gamma_0}{1-\gamma_0}\left(\prod_{m=0}^{n^*}\frac{\delta[p^c - P(t(Q_m^*(q_0)))]}{p^c - P(Q_m^*(q_0))} - 1\right). \tag{21}$$

Recall now that we need either $\rho_0^S = 1$, or $\rho_0^S \in (0,1)$ and $q_0 = Q^*(0)$. If the right-hand side of (21) is below one if evaluated at $q_0 = Q^*(0)$, then set $q_0 = Q^*(0)$ and evaluate (20) and (21) at this value. Otherwise, set $\rho_0^S = 1$ and evaluate (20) and (21) at the value of q_0 which solves (21). This completes the proof of Proposition 3.

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