Testing for Linear Trends in Dependent Heterogeneous Panels

Adina Ioana Tarcolea*
Goethe-University Frankfurt†

This version: October 12, 2006

Abstract

A panel test for the presence of a linear time trend is proposed. The test is applicable in cross-correlated, heterogenous panels, as long as the number of units is smaller than the number of time observations. The test is applied by means of subsampling when there is uncertainty about whether the innovations are stationary or integrated. Subsampling also allows for arbitrary number of units and time observations and works better in small samples than the asymptotic approximation. Finally, the method is applied to test whether the investment share in European GDPs has a trend or not.

JEL Classification: C23, C15, E22
Keywords: time trend, cross-dependent panels, subsampling, investment share

Word count: 3400 (approx.)

*I would like to thank Uwe Hassler, Matei Demetrescu and Vladimir Kuzin for very helpful comments. All remaining errors are mine.

†Statistics and Econometric Methods, Goethe-University Frankfurt, Gräfstr. 78 / PF 76, D-60054 Frankfurt, Germany, Tel: +49.69.798.28418, Fax: +49.69.798.23662, Email: tarcolea@wiwi.uni-frankfurt.de.

1 Introduction

In many branches of applied science, testing for the presence of a linear trend is a topic of great practical relevance. Global warming can be mentioned as a prominent, although non-economic example. In applied economics, many studies have been centered on the verification (or rejection) of the Prebisch-Singer hypothesis: among others, Kim et al. (2003) and Bunzel and Vogelsang (2003). Of course, other issues of importance for policy makers can be quantified in terms of deterministic trends as well.

One aspect that has been extensively discussed in the context of testing for a time trend, see for instance Canjels and Watson (1997) or Sun and Pantula (1999), is whether the stochastic component is stationary or integrated. Vogelsang (1998) suggested a method that works in both situations; Bunzel and Vogelsang (2003) provide an alternative.

All these papers deal with single time series. Although Kim et al. (2003) consider more time series, they do not provide an overall test, but study each of them separately. Considering this, the panel test proposed here is a natural and necessary development.

In panels, especially when N is not small compared to T, things become more complicated than for single time series, due to cross-dependence. A similar problem appears in panel unit root testing, see Pesaran and Breitung (2006). Here, we model this dependence by means of cross-correlation. The proposed test statistic can be used either for short-memory ($I(0)$) or for integrated ($I(1)$) panels. We show how to use subsampling in order to encompass both possibilities for panel testing, as an alternative to the proposals for single time series by Vogelsang (1998) or Bunzel and Vogelsang (2003).

To the structure of the paper: At first the model is described and a panel test statistic which tests the presence of a linear trend in either short-memory ($I(0)$) or in integrated ($I(1)$) panels is proposed. Then we show how to apply the test when a priori information about innovations is not available, by means of subsampling. In the end, the proposal is substantiated both by Monte Carlo simulations and an empirical application to the investment share of per capita GDP of 15 European countries.

2 Model and test

Let $y_{t,i}$, $t \in \{1, \ldots, T\}$, $i \in \{1, \ldots, N\}$ be our panel with N units, each with T time observations. Assume

$$y_{t,i} = \alpha_i + \beta_i t + x_{t,i},$$

where $\mathbf{x}_t = (x_{t,1}, \ldots, x_{t,N})'$ is a zero-mean $I(0)$ or $I(1)$ variable. Also, denote $\mathbf{y}_t = (y_{t,1}, \ldots, y_{t,N})'$.

In principle, testing for

$$\beta_i = 0 \text{ vs. } \beta_i \neq 0$$

in single units is not particularly difficult, even when $x_{t,i}$ is $I(1)$.[1] But in cross-dependent panels, this typically requires the non-parametric estimation

[1] As already mentioned, complications arise only when the order of integration is not known.

of the long-run covariance matrix Ω of $\mathbf{x}_t$ (or of $\Delta\mathbf{x}_t$, respectively). Although the problem was studied before in detail by Andrews in his paper from 1991 (see also Newey and West, 1987), the larger N is, the more unreliable the estimation of Ω becomes.

The basic idea is to avoid estimation by using a possibly random matrix, but one "proportional" to Ω, so that Ω cancels out. This idea was suggested by Kiefer, Vogelsang and Bunzel (2000). For an application to testing for uncorrelatedness of dependent time series, see Lobato (2001). Using the same approach, Breitung (2002) shows how to modify variance-ratio-type tests to consistently test for unit roots and cointegration.

2.1 I(0) panels

Assume first:

$$T^{-0.5}\sum_{j=1}^{[sT]}\mathbf{x}_j \Rightarrow \Omega^{0.5}\mathbf{W}(s) \quad \text{for } s\in[0,1], \tag{1}$$

where "$\Rightarrow$" stands for weak convergence in a suitable metric space of cadlag random functions defined on $[0,1]$, $\mathbf{W}(s)$ is a vector of N independent standard Wiener processes and Ω is the long-run covariance matrix of the multivariate process $\mathbf{x}_t$. This requirement can be seen as the defining property of short-memory processes (cf. Lo, 1991).

We will use a demeaned statistic which reacts to the presence of a trend. Firstly, we will remove the mean from the data to obtain invariance with respect to α_i. With the demeaned data, we build partial sums,

$$\mathbf{S}_t = \sum_{j=1}^{t} (\mathbf{y}_j - \overline{\mathbf{y}}),$$

to obtain a statistic of the type used by Kwiatkowski et al. [KPSS] (1992):

$$\tau = \frac{1}{T^2} \sum_{t=1}^{T} \mathbf{S}_t' \mathbf{S}_t.$$

Under the null hypothesis, using the assumption on $\mathbf{x}_t$ and the Continuous Mapping Theorem, following distributional result arises imediately:

$$\tau \xrightarrow{d} \int_0^1 \mathbf{V}_\mu' (s) \, \Omega \mathbf{V}_\mu (s) \, ds,$$

where $\mathbf{V}_\mu$ is the so-called Brownian Bridge (see Kwiatkowski et al., 1992) and "$\xrightarrow{d}$" stands for convergence in distribution.

Remark 1 *If there is a deterministic trend in one of the units, it can be shown that*

$$\tau \xrightarrow{p} \infty \text{ with } \tau = O_p \left(T^3 \right),$$

and the test rejects for large values.

Its asymptotic null distribution is not free of nuisance parameters, but a pivotal variant could be obtained by using transformed $\mathbf{S}_t$,

$$\mathbf{S}_t^{\perp} = \Omega^{-0.5} \mathbf{S}_t$$

to compute the corresponding statistic

$$\tau^{\perp} = \frac{1}{T^2} \sum_{t=1}^{T} \mathbf{S}_t^{\perp\prime} \mathbf{S}_t^{\perp},$$

for which the following relationship holds:

$$\tau^{\perp} \xrightarrow{d} \int_0^1 \mathbf{V}_{\mu}'(s)\,\mathbf{V}_{\mu}(s)\,ds.$$

However, we do not use this pivotal variant, which, as said, can be rather unreliable.

Instead, we use some function of the data "proportional" to Ω, but invariant to the presence of a linear trend, such that the nuisance matrix Ω cancels out under both null and alternative hypothesis. We denote by $\mathbf{y}_t^{\tau}$ the $\mathbf{y}_t$ after demeaning and detrending by OLS. The corresponding cumulative sums are defined accordingly:

$$\mathbf{S}_u^{\tau} = \sum_{j=1}^{u} \mathbf{y}_j^{\tau}.$$

We can now suggest the use of a detrended KPSS-type statistic:

$$\omega = \frac{1}{T^2} \sum_{u=1}^{T} \mathbf{S}_u^{\tau} \mathbf{S}_u^{\tau\prime} \tag{2}$$

and it results that

$$\omega \xrightarrow{d} \Omega^{0.5} \left(\int_0^1 \mathbf{V}_{\tau}(v)\,\mathbf{V}_{\tau}'(v)\,dv \right) \Omega^{0.5},$$

with $\mathbf{V}_{\tau}$ the second-level Brownian Bridge (see Kwiatkowski et al., 1992). The condition $N < T$ is required for invertibility of ω in (2).

Then, we define the test statistic for the presence of linear trends,

$$\tau^{*} = \sum_{t=1}^{T} \mathbf{S}_t' \left(\sum_{u=1}^{T} \mathbf{S}_u^{\tau} \mathbf{S}_u^{\tau\prime} \right)^{-1} \mathbf{S}_t. \tag{3}$$

Proposition 1 *Assuming (1) holds for* $\mathbf{x}_t$*, it follows for* τ^* *from (3) under the null hypothesis of no linear time trends*

$$\tau^* \xrightarrow{d} \int_0^1 \mathbf{V}'_\mu(s) \left(\int_0^1 \mathbf{V}_\tau(v) \mathbf{V}'_\tau(v)\, dv \right)^{-1} \mathbf{V}_\mu(s)\, ds$$

as $T \to \infty$*.*

Proof: Follows directly with the Continuous Mapping Theorem.

Remark 2 *Remark 1 can be shown to hold for* τ^* *as well, so the test is consistent against the alternative of linear time trend.*

The test rejects for too large values; the distribution, being non-standard, requires simulation of critical values. The simulations were carried out in GAUSS with 100000 replications and with $T = 1000$. The disturbances $x_{t,i}$ are generated as independent standard normal and we set $\alpha_i = \beta_i = 0$. We tabulate the critical values for $N \in \{1, 2, 5, 10, 15\}$ and the 1%, 5% and 10% significance levels, see Table 1.

Level	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$
1%	17.89	30.45	76.05	176.10	308.19
5%	9.39	18.13	51.01	128.09	233.93
10%	6.48	13.54	40.80	107.49	200.70

Table 1: Critical values for τ^* in the I(0) case

We now study the behaviour of the test in small samples. For our Monte Carlo study, we employ following DGP:

$$\mathbf{x}_t = \mathbf{e}_t + \theta \mathbf{e}_{t-1}, \quad \mathbf{e}_t \sim iid\mathcal{N}(0, \Sigma),$$

β	T	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$
	50	6.40	6.66	10.78	22.76	53.22
0	100	5.74	6.90	8.10	11.84	27.38
	250	4.92	5.40	5.94	7.22	15.68
	50	11.28	11.62	15.26	26.56	57.00
0.01	100	41.84	40.68	36.40	38.68	56.04
	250	100.00	100.00	99.96	100.00	100.00
	50	84.88	85.22	84.48	87.16	95.88
0.05	100	100.00	100.00	100.00	100.00	100.00
	250	100.00	100.00	100.00	100.00	100.00
	50	99.96	99.98	99.84	99.92	100.00
0.1	100	100.00	100.00	100.00	100.00	100.00
	250	100.00	100.00	100.00	100.00	100.00

Table 2: Size (5%) and power of test based on τ^* in the I(0) case

with

$$\Sigma = \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{pmatrix}.$$

Since many economic series are positively autocorrelated, we specify $\theta = 0.5$. Also, there are usually positive correlations between similar series across countries, so we set $\rho = 0.5$. The observed $\mathbf{y}_t$ is generated by adding a linear trend, with coefficient $\beta_i = \beta \in \{0, 0.01, 0.05, 0.1\}$. The rejection frequencies at the 5% level based on 5000 replications are given in Table 2, for sample sizes $T \in \{50, 100, 250\}$. The critical values from Table 1 were used.

The size is close enough to the nominal level only for very small N. For large N, the test is extremely liberal; this is especially striking for $T = 50$, but it improves with growing T. Convergence to the asymptotic behaviour can be observed; the larger N is, the slower the convergence. Hence, it might be advisable to use exact finite-sample critical values when N is large. We however, favour a different solution, see subsection 2.3. In terms of power, the test behaves satisfactorily (however, given the size distortion, these rejection frequencies are not that meaningful).

2.2 I(1) panels

Should the stochastic components $x_{t,i}$ be integrated of order one (or, correspondingly, $\Delta\mathbf{x}_t$ have short memory), we assume:

$$T^{-0.5}\mathbf{x}_{[sT]} \Rightarrow \Omega^{0.5}\mathbf{W}(s), \quad \text{for } s \in [0,1]. \tag{4}$$

It turns out that the same test statistic τ^* has a well-defined limiting distribution under the null of no linear trends, see the Proposition below. The test statistic thus remains

$$\tau^* = \sum_{t=1}^{T} \mathbf{S}_t' \left(\sum_{u=1}^{T} \mathbf{S}_u^{\tau} \mathbf{S}_u^{\tau\prime} \right)^{-1} \mathbf{S}_t.$$

Proposition 2 *Assuming (4) holds for $\mathbf{x}_t$, following asymptotic distribution results for τ^* under the null hypothesis of no linear time trends as $T \to \infty$:*

$$\int_0^1 \left(\int_0^r \mathbf{W}_\mu(v)dv \right)' \left[\int_0^1 \left(\int_0^u \mathbf{W}_\tau(v)dv \right) \left(\int_0^u \mathbf{W}_\tau(v)dv \right)' du \right]^{-1} \left(\int_0^r \mathbf{W}_\mu(v)dv \right) dr,$$

where $\mathbf{W}_\mu$ is the demeaned Wiener process, $\mathbf{W}_\mu(v) = \mathbf{W}(v) - \int_0^1 \mathbf{W}(s)\,ds$, and $\mathbf{W}_\tau$ is the demeaned and detrended Wiener process (cf. Park and Phillips, 1988), $\mathbf{W}_\tau(v) = \mathbf{W}(v) + (6v-4)\int_0^1 \mathbf{W}(s)\,ds + (-12v+6)\int_0^1 s\mathbf{W}(s)\,ds$.

Proof: Follows directly with the Continuous Mapping Theorem.

Remark 3 *Under the alternative, the behaviour also changes compared to the $I(0)$ case: it is straightforward to show that*

$$\tau^* \xrightarrow{p} \infty, \text{ with } \tau^* = O_p(T)$$

as $T \to \infty$, so power reduction in comparison to the $I(0)$ case is to be expected.

As an alternative method, one might difference the series and test for a non-zero mean by a similar combination of KPSS-type statistics. However, this alternative test statistic would have a degenerate distribution if the series were $I(0)$, so we do not elaborate this topic here.

We have to give new critical values for the case of integrated disturbances, see Table 3.

Level	$N = 1$	$N = 2$	$N = 5$	$N = 10$	$N = 15$
1%	351.53	1036.08	7568.4	48612.3	163848.8
5%	129.77	452.02	3934.5	29322.1	107222.5
10%	74.99	278.88	2771.4	22245.2	84373.4

Table 3: Critical values for τ^* in the I(1) case

Unfortunately, these differ strongly from those in the $I(0)$ case. Therefore, special care is to be taken if the integration order of the stochastic component $\mathbf{x}_t$ is not known, see the next subsection.

β	T	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$
	50	5.54	5.16	5.58	6.66	8.14
0	100	5.08	5.04	5.72	5.36	5.60
	250	5.56	4.28	5.10	5.30	4.84
	50	5.76	5.22	5.22	6.38	7.80
0.01	100	5.30	4.46	4.96	5.20	6.00
	250	5.44	5.18	5.60	5.16	5.34
	50	5.96	5.02	5.32	6.62	8.78
0.05	100	5.22	5.70	5.68	5.14	5.42
	250	6.64	6.36	6.40	6.12	5.20
	50	6.42	6.10	6.40	7.28	8.72
0.1	100	8.50	6.88	6.38	6.52	5.96
	250	11.58	10.86	9.46	6.90	7.38

Table 4: Size (5%) and power of test based on τ^* in the I(1) case

The corresponding Monte Carlo results (for which $\mathbf{x}_t$ is based on the same MA process as before, but using its partial sums) are provided in Table 4.

Compared to Table 2, the simulations provide good results regarding the size properties. The low power is determined by the fact that the small trend is "hidden“ by the stochastic trend (note also the reduced convergence rate under the alternative hypothesis).

2.3 Unknown integration order

Since the test statistic τ^* has well-defined asymptotic null distributions under both $I(0)$ and $I(1)$ possibilities, while being consistent under the alternative of a linear trend, the subsampling method, proposed by Politis and Romano (1994), works in both $I(0)$ and $I(1)$ cases.

The idea of the subsampling method is to approximate the sampling distribution of a test statistic by recomputing it on subsamples of smaller size of the observed data. One uses blocks of size l of consecutive observations in order to preserve within each block the dependence structure of the underlying model. Note that there are $M = T - l + 1$ such blocks. Only a very weak assumption on l will be required; typically, $l/T \to 0$ and $l \to \infty$ as $T \to \infty$.

Compared to the bootstrap methods, for which much work has to be done in order to demonstrate their validity in a given situation, subsampling only requires the existence of a nondegenerate limiting distribution of the respective statistic and some bound on the serial dependence of the underlying sequence. In the context of time series this can be for instance a mixing condition, see Politis, Romano and Wolf (1999, p. 70).

In the panel situation, the series modelling the disturbances must satisfy a mixing condition corresponding to the multivariate framework. Together with a moment condition, this implies weak convergence to Brownian Motion (see, for instance, Herrndorf, 1984), which is required for the test to have a valid asymptotic distribution. Of course, in the case of integrated disturbances, the mixing condition does not hold. One may still use subsampling, since, when *the differences* are mixing, subsampling these would be justified. Because the test statistic is invariant w.r.t. non-zero mean, subsampling differences and building their partial sums is equivalent to subsampling the disturbances themselves. Hence, one may subsample the data in both $I(0)$ and $I(1)$ cases.

Similarly, to simplify the computation, we suggest as panel test statistic the sum of single test statistics:

$$\tau^S = \sum_{i=1}^{N} \tau_i^*,$$

where τ_i^* are the univariate analogues to (3). The distribution of this modified τ^S is then subsampled.

Summing single test statistics removes the requirement that N be smaller than T. The distribution of τ^S depends indeed on Ω, but, by using subsampled critical values (as described below), the test becomes invariant to Ω without requiring any kind of orthogonalization. More importantly, cointegration across units is now allowed for $I(1)$ series by the same argument.

We detail here the used subsampling procedure: First build $M = T-l+1$ overlapping blocks of length l, the first one being $\{\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_l\}$ and the last one being $\{\mathbf{y}_{T-l+1}, \mathbf{y}_{T-l+2}, ..., \mathbf{y}_T\}$. Let B_m be a generic block (of size l) of the consecutive data $\{\mathbf{y}_m, ..., \mathbf{y}_{m+l-1}\}$, $m = 1, 2, ..., M$; we compute for each of the blocks the corresponding τ_m^S statistic and then use the M ordered realizations τ_m^S in order to estimate the distribution function. The critical value at level α is then $\tau^S_{[(1-\alpha)M]}$.

The optimal choice of the block size l can be data-driven if the integration order of the series is known. Since our assumption is that we do not know the true integration order, we have to run a Monte Carlo experiment to determine optimal values for l. We must note here the fact that, although l converges to infinity together with T, the rate of convergence of τ^S is slower than in the parametric case and hence the power of the subsampled test is expected to be lower.

l	β	$I(\cdot)$	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$
	0	0	3.52	1.82	0.86	0.26	0.18
6	0	1	10.02	10.80	10.90	11.10	10.50
	0.1	0	99.88	99.96	100.00	100.00	100.00
	0.1	1	11.92	15.74	17.08	19.36	22.82
	0	0	**5.94**	5.20	3.08	1.68	1.70
8	0	1	**10.92**	10.42	12.00	13.88	14.52
	0.1	0	**99.88**	99.98	100.00	100.00	100.00
	0.1	1	**14.54**	15.10	19.78	24.24	27.58
	0	0	7.52	**6.50**	5.20	4.32	3.32
10	0	1	10.58	**10.80**	12.44	13.06	15.06
	0.1	0	99.88	**100.00**	100.00	100.00	100.00
	0.1	1	14.50	**16.50**	18.58	23.66	27.06
	0	0	6.90	6.00	**4.74**	3.64	3.40
12	0	1	8.92	9.54	**10.36**	10.68	10.88
	0.1	0	99.40	99.98	**100.00**	100.00	100.00
	0.1	1	10.98	13.06	**14.88**	18.56	21.02
	0	0	7.52	6.24	6.02	**5.40**	**4.52**
14	0	1	9.26	9.90	9.38	**10.46**	**11.96**
	0.1	0	99.00	99.94	100.00	**100.00**	**100.00**
	0.1	1	11.10	12.56	13.70	**18.12**	**19.36**
	0	0	8.52	7.14	6.48	6.56	6.42
16	0	1	9.60	9.98	10.86	11.46	11.52
	0.1	0	98.44	99.90	100.00	100.00	100.00
	0.1	1	12.20	12.52	14.12	17.78	20.34

Table 5: Size (5%) and power of test based on τ^S as function of block length, T=50

The results for the Monte Carlo simulations for $T = 50, 100$ and 250 are given in Tables 5, 6 and 7, respectively. A few words to the Monte Carlo setup: since our goal is only to determine an optimal l, and not to test, we simplify the framework as much as possible. So we set $\theta = \rho = 0$, which also ensures "neutrality" of the Monte Carlo experiment, and let β take only two possible values, $\beta \in \{0, 0.1\}$. The simulations are carried out with 5000 replications and each table contains the rejection frequencies at the 5% level for both $I(0)$ and $I(1)$ cases.

l	β	$I(\cdot)$	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$
	0	0	**5.62**	4.90	4.10	2.94	2.94
12	0	1	**8.08**	8.64	9.42	9.98	10.86
	0.1	0	**100.00**	100.00	100.00	100.00	100.00
	0.1	1	**13.18**	15.94	21.72	29.42	34.88
	0	0	6.56	**5.60**	5.46	4.26	4.38
14	0	1	8.66	**8.56**	10.44	10.50	10.22
	0.1	0	100.00	**100.00**	100.00	100.00	100.00
	0.1	1	13.48	**16.68**	21.14	26.64	33.32
	0	0	6.78	6.88	**5.46**	5.46	5.42
16	0	1	9.46	9.10	**9.14**	9.52	9.80
	0.1	0	100.00	100.00	**100.00**	100.00	100.00
	0.1	1	14.34	16.70	**20.50**	26.00	32.26
	0	0	7.66	7.24	6.18	**5.62**	**4.98**
18	0	1	9.12	9.40	9.04	**9.82**	**9.62**
	0.1	0	100.00	100.00	100.00	**100.00**	**100.00**
	0.1	1	14.60	15.44	19.94	**26.52**	**30.84**
	0	0	7.98	7.88	7.46	6.02	6.18
20	0	1	9.92	9.02	8.80	9.74	10.38
	0.1	0	100.00	100.00	100.00	100.00	100.00
	0.1	1	13.80	16.42	20.88	24.70	29.32

Table 6: Size (5%) and power of test based on τ^S as function of block length, T=100

l	β	$I(\cdot)$	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$
	0	0	5.40	4.98	4.88	4.18	3.74
18	0	1	6.78	6.64	7.20	8.14	8.04
	0.1	0	100.00	100.00	100.00	100.00	100.00
	0.1	1	19.00	24.02	37.44	52.24	64.62
	0	0	**5.36**	**5.12**	**5.04**	4.56	4.30
20	0	1	**6.58**	**6.58**	**7.36**	7.86	7.78
	0.1	0	**100.00**	**100.00**	**100.00**	100.00	100.00
	0.1	1	**20.32**	**23.46**	**35.76**	49.88	63.42
	0	0	5.74	5.92	5.56	**4.48**	**4.58**
22	0	1	7.14	6.70	7.54	**7.44**	**7.52**
	0.1	0	100.00	100.00	100.00	**100.00**	**100.00**
	0.1	1	19.20	23.60	35.30	**50.38**	**60.86**
	0	0	6.16	5.94	5.50	5.32	4.94
24	0	1	7.26	7.46	7.78	7.32	7.34
	0.1	0	100.00	100.00	100.00	100.00	100.00
	0.1	1	16.38	19.46	28.22	39.16	47.18

Table 7: Size (5%) and power of test based on τ^S as function of block length, T=250

We choose the values of l which realize a reasonable trade-off between correct size and power. In Tables 5, 6 and 7, the bolded cases represent our choice of optimal l. From the simulations results we see that the optimal block length also depends on N and that it would be appropriate to choose larger l for larger N. This happens because the overall variability in data increases with growing N, except for the case when T is large and the number of observations compensates the increase in data variability.

Finally, we provide the Monte Carlo simulation results for the behaviour of test itself in Tables 8 and 9, which are the "subsampling equivalents" of Tables 2 and 4.

β	T	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$
	50	4.20	3.60	4.00	4.60	3.90
0	100	2.60	3.00	3.40	3.60	4.60
	250	3.30	3.00	2.80	4.30	2.80
	50	6.40	6.80	6.80	8.50	8.20
0.01	100	27.60	33.60	49.00	52.40	57.00
	250	99.80	100.00	100.00	100.00	100.00
	50	56.20	71.70	79.70	87.20	88.10
0.05	100	99.90	100.00	100.00	100.00	100.00
	250	100.00	100.00	100.00	100.00	100.00
	50	94.80	98.20	99.60	99.40	99.70
0.1	100	100.00	100.00	100.00	100.00	100.00
	250	100.00	100.00	100.00	100.00	100.00

Table 8: Size (5%) and power of test based on τ^S in the I(0) case, l as bolded in Tables 5, 6 and 7.

Compared to Table 2, subsampling in the $I(0)$ case stabilizes the size (the test is even somewhat conservative). This comes at a cost of power losses, which is a common problem in the subsampling framework. However, one should note that, for the subsampled version, power increases with N, which does not seem to be the case with the parametric version of the test.

In the $I(1)$ case, subsampling leads to an oversized test for small number of units. For a large number of units, the size remains close to the nominal level, which was not the case with the parametric version of the test. Furthermore, the power gains over Table 4 are significant for $N \geq 2$. Again, power increases with growing N.

To summarize these results, our advice to practitioners is to use the subsampled test, since it works better in most cases and does not impose any

β	T	$N=1$	$N=2$	$N=5$	$N=10$	$N=15$
	50	6.00	7.90	7.60	7.20	6.50
0	100	6.90	7.00	7.90	8.70	6.80
	250	5.00	6.40	7.60	6.20	5.60
	50	9.40	9.10	5.60	7.20	5.60
0.01	100	6.00	6.40	7.00	8.60	6.50
	250	6.80	7.20	6.60	6.60	5.00
	50	6.80	8.60	7.00	7.50	7.00
0.05	100	6.30	6.80	8.00	10.00	9.00
	250	6.50	8.20	9.40	9.20	9.60
	50	7.40	10.60	8.80	8.00	10.30
0.1	100	9.40	9.10	10.70	11.20	11.40
	250	9.80	14.00	14.70	15.50	19.20

Table 9: Size (5%) and power of test based on τ^S in the I(1) case, l as bolded in Tables 5, 6 and 7.

conditions on N and T; additionally, it provides trustworthy results in cases when the assumption about the integration order is wrong or not certain.

3 Investment shares of per capita GDP

As an empirical application, we examine the investment shares of per capita GDP, since investment shares are a relevant measure for an economy. Even the simplest endogenous growth model incorporates this quantity as a key variable. Along this line, Levine and Renelt (1992) find investment shares to correlate with growth and with the ratio of international trade to GDP. A more recent contribution in this field of research is due to Madsen (2002), who analyzes the interdependence between economic growth and investment.

We employ annual data from the Penn World Table (Heston, Summers and Aten, 2002). There is strong reason to assume the investment shares of the GDP not to be integrated and we use therefore critical values from Table 1. More precisely, we examine 15 European economies, members of the European Union (the former EU 15)[2]: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and the United Kingdom. The used data, covering the period from 1970 to 2000, is plotted in Figure 1.

The investment shares have a visible downward time trend in all countries except for Ireland, Luxembourg, Portugal and the UK, but the degree of co-movement seems considerable. The panel test statistic has a value of 4338.27, which is highly significant even at the 1% level. Recall, however, that the test based on asymptotic critical values is extremely oversized for $N = 15$. If the data were integrated, this would not be significant anymore, not even at the 10% level. In this case, however, the co-movement would most likely appear due to cointegration, case in which the critical values in Table 3 would not be valid anymore.

To make sure the finding of a trend is substantiated, we apply the subsampling procedure, first at the country level and then for the whole panel. The simulations to obtain an optimal l are repeated for $T = 30$ and $N = 1$, as well as $N = 15$. For the single tests, we find significant trends only in Austria, Finland, France, Italy and Sweden, which is somewhat surprising, given the graphical evidence. In this case, $l = 6$ is the optimal block length

[2]Eight of the ten omitted new EU members, being during the 70's and the 80's members of the communist block, exhibit data that is not comparable to the EU 15.

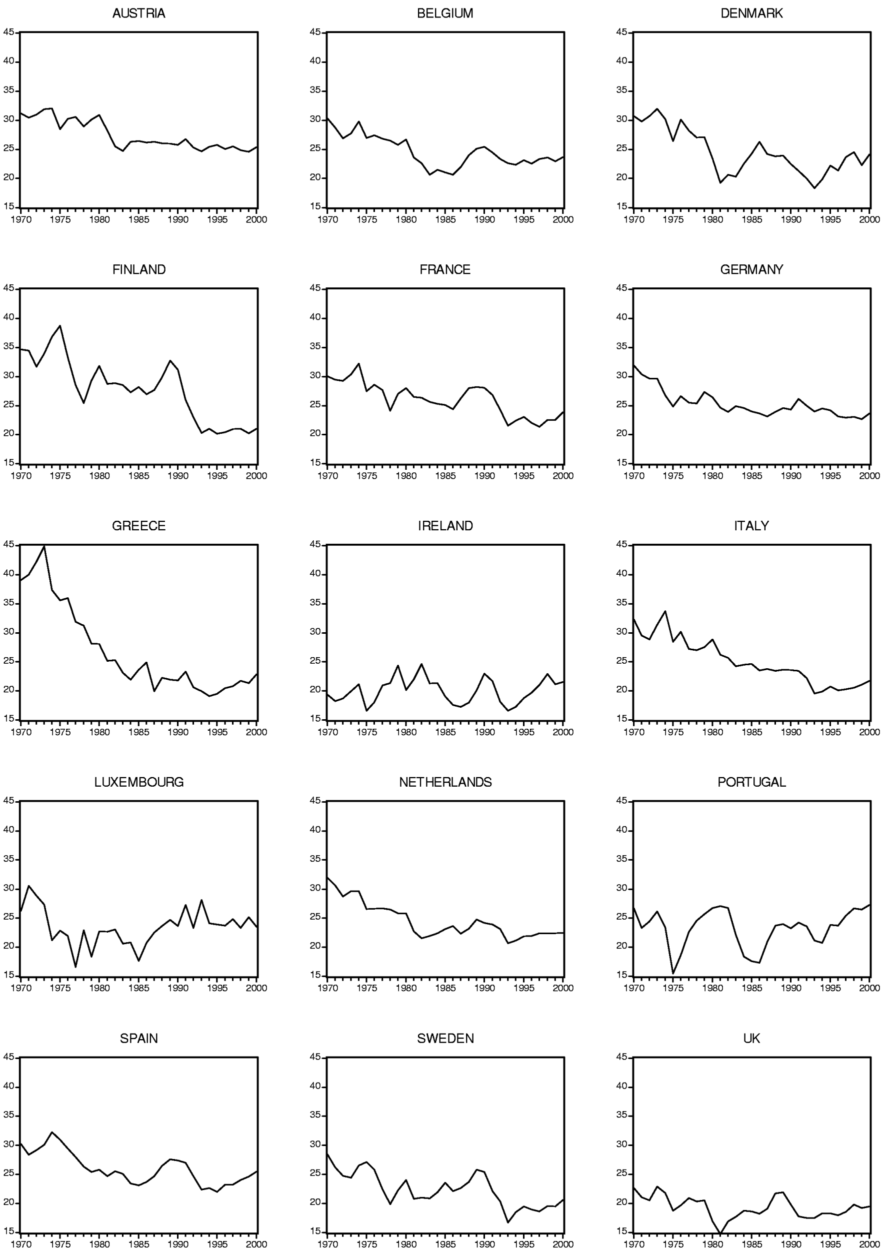

Figure 1: Investment shares of per capita GDP in EU15

for which the test keeps its size, while the power properties are satisfactory.

Returning to panels, in the $I(0)$ case the size properties are good for $l = 9$ and $l = 10$; the test is undersized for $l = 11$, while loosing power at the same time. Similarly to Table 5, the test is oversized (around 10% for the 5% significance level) in the $I(1)$ case; $l = 10$ or $l = 11$ seem here to be the optimal choices. The power properties are improved compared to the single tests, as expected.[3]

To the test results: in this application, the importance of l becomes obvious at the panel level. If choosing $l = 9$, the subsampled test rejects at the 5% level. If choosing $l = 10$ or $l = 11$, the subsampled test only rejects at the 10% level.

To sum up, there is statistical evidence that investment activity has been reducing over the past 30 years in the EU area. One explanation that seems plausible is that the accelerated development of new technologies in the past 40 years reduced the level of investment necessary to keep growth at a given level.

4 Conclusions

We study a panel test for the presence of linear trends. The test works for cross-correlated, heterogeneous panels which are either stationary or integrated, as long as $N < T$.

Further, it is shown how to use subsampling to apply the test when the

[3]The complete results are available upon request from the author.

order of integration is not known. Subsampling also allows for arbitrary N and improves the small sample behaviour over the asymptotic approximation in terms of both size and power. Hence, the subsampled version of our test should be preferred in practical applications.

In an empirical application, it is found that the investment share of per capita GDP has been declining since 1970 in the EU area as a whole; for individual countries, the evidence is mixed. This result also emphasizes the importance of panel tests like the one proposed here.

References

[1] Andrews, D.W.K. (1991). 'Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation', *Econometrica* 59, 817-858.

[2] Breitung, J. (2002). 'Nonparametric Tests for Unit Roots and Cointegration', *Journal of Econometrics* 108, 343-363.

[3] Breitung, J. and M. H. Pesaran (2006). 'Unit Roots and Cointegration in Panels', forthcoming in: Matyas, L. and P. Sevestre (ed.), *The Econometrics of Panel Data*, 3^{rd} edition, Kluwer Academic Publishers.

[4] Bunzel, H. and T.J. Vogelsang (2003). 'Powerful Trend Function Tests That are Robust to Strong Serial Correlation with an Application to the Prebisch-Singer Hypothesis', Working Papers Series, Department of Economics of Iowa State University.

[5] Canjels, E. and M.W. Watson (1997). 'Estimating Deterministic Trends in the Presence of Serially Correlated Errors', *Review of Economics and Statistics* 19, 271-288.

[6] Herrndorf, N. (1984). 'A Functional Central Limit Theorem for Weakly Dependent Sequences of Random Variables', *Annals of Probability* 12, 141-153.

[7] Heston, A., R. Summers and B. Aten (2002). 'Penn World Table Version 6.1', Center for International Comparisons at the University of Pennsylvania (CICUP), October 2002.

[8] Kiefer, N.M., T.J. Vogelsang and H. Bunzel (2000). 'Simple Robust Testing of Regression Hypothesis', *Econometrica* 68, 695-714.

[9] Kim, T.-H., S. Pfaffenzeller, T. Rayner and P. Newbold (2003). 'Testing for Linear Trend with Application to Relative Primary Commodities Prices', *Journal of Time Series Analysis* 24, 539-551.

[10] Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin (1992). 'Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root', *Journal of Econometrics* 54, 159-178.

[11] Levine, R. and D. Renelt (1992). 'A Sensitivity Analysis of Cross-Country Growth Regressions', *The American Economic Review* 82, 942-963.

[12] Lo, A.W. (1991). 'Long-Term Memory in Stock Market Prices', *Econometrica* 59, 1279-1313.

[13] Lobato, I.N. (2001). 'Testing that a Dependent Process is Uncorrelated', *Journal of the American Statistical Association* 96, 1066-1076.

[14] Madsen, J.B. (2002). 'The Causality Between Investment and Economic Growth', *Economics Letters* 74, 157-163.

[15] Newey, W.K. and K.D.West (1987). 'A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix', *Econometrica* 55, 703-708.

[16] Park, J.Y. and P.C.B. Phillips (1988). 'Statistical Inference in Regressions with Integrated Processes: Part1', *Econometric Theory* 4, 468-498.

[17] Politis, D.N. and J.P. Romano (1994). 'Large Sample Confidence Regions Based on Subsamples under Minimal Assumptions', *Annals of Statistics* 22, 2031-2050.

[18] Politis, D.N., J.P. Romano and M. Wolf (1999). *Subsampling*, Springer-Verlag, New York.

[19] Sun, H. and S.G. Pantula (1999). 'Testing for Trends in Correlated Data', *Statistics and Probability Letters* 41, 87-95.

[20] Vogelsang, T.J. (1998). 'Trend Function Hypothesis Testing in the Presence of Serial Correlation', *Econometrica* 66, 123-148.