Why Sraffa was right not to publish his last article – a comment on Yoann Verger

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Yoann Verger (2019) needs to be congratulated on the discovery of Sraffa's unpublished paper on Manara. This draft is very interesting, since it concerns the beginnings of one of the major debates, namely on the viability of Sraffa's theory of joint production – a debate which has died down without all questions having been resolved. Verger's initiative might help to rekindle it. The matter is of special concern for me, since it provided the starting point for my academic career.

Manara's paper (Manara 1968) provided the first attempt to deal with Sraffa's theory of joint production in mathematical terms. He first gave conditions for joint production systems to result in semi-positive prices, but the conditions he proposed were little more than a restatement of the requirement that semi-positive prices had to result at some positive rate of profit. The formulation of economically meaningful conditions that would secure this outcome came later. Manara understood that such conditions could be derived in the single product case, using the well-known theories by Perron and Frobenius. They ensured essentially meaningful solutions, as soon as the system was capable of self-reproduction and contained a basic commodity, but the definition of basic commodities could not easily be extended to joint production, and the possibility of self-reproduction did not suffice to exclude negative prices. The last part of Manara's article showed how one of the formal definitions of basic commodities – the one which Sraffa had given in a footnote – could be used to isolate the basic commodities in this system by rearranging it and by applying multipliers. The non-basic processes could be separated from the basic system in such a way that a tax on basics would affect the prices of basics and non-basics, while a tax on non-basics would not affect the prices of basics. This was clarifying, but not really innovative.

Manara made the main point of his paper in its middle part by showing that a maximum rate of profit need not exist in a joint production system, in that the corresponding characteristic equation did not necessarily have a real, let alone positive solution. This was surprising from the economic point of view, since Manara's joint production system did have a definite physical surplus, and one had been accustomed to think, knowing the properties of single product systems, that the maximum rate of profit was the appropriate economic measure of the magnitude of the surplus. If there was no maximum rate of profit, there was no standard commodity either, and, with this, the specificity of Sraffa's approach in the attempt to revive classical economics seemed to have gone. This explains why Sraffa felt so much concerned about Manara's objection.

In his paper, Yoann Verger draws attention to the fact that there is a draft of a paper in the Sraffa archive (Sraffa Archive D3/14, listed by Verger), in which the author, now past his 70th year, tried to answer Manara's challenge. This was especially remarkable since Sraffa had published his book almost ten years earlier. A number of reviews and papers had appeared about it, but in most cases he would not answer his critics. He must have felt – and indeed he stated it in his draft – that this time a central point of his theory was at stake.

Verger's paper provides the chronology of the events and explains the details of what is to be found in the archive. It is clear from Verger's reproduction of Sraffa's draft that, while he was writing it, Sraffa was convinced that Manara's example was economically not legitimate. One of the processes seemed technically to be superior to the other so that they could not coexist for economic reasons. But Sraffa did not publish the paper in the end, although it seemed nearly complete. Verger speculates that Sraffa abandoned his draft when he read in my "Mr Sraffa on Joint Production" (Schefold 1971) that the question of technical superiority could be analysed by means of rates of reproduction of the commodities, as Sraffa himself had done in his draft, but the sufficient condition for guaranteeing the existence of a standard commodity on the basis of rates of reproduction was so restrictive that Sraffa, according to Verger's conjecture, decided not to pursue the approach further. Verger believes that Sraffa's approach can be rescued, however, if one extends the analysis of basics and non-basics in Sraffa systems according to the proposal by Dupertuis and Sinha (2003).
If non-basics are eliminated and, moreover, pathologies such as processes which are technically superior to all other processes, are excluded from a Sraffa system, it can be reduced to a so-called "atomic" core of processes and commodities that are all mutually interdependent, and then the standard commodity exists – at least according to the paper by Dupertuis and Sinha.

Verger thus has made a valiant attempt to rescue Srafa's theory with as small a modification as possible, but it fails. The proof of the existence of the "atomic" core with the required properties can be falsified by an example. Moreover, the "atomic" core is not necessarily unique. Verger does not even manage to explain clearly why Manara's example was, despite Sraffa's attempt to criticise it, economically significant. To get to the truth in this matter, one must start the story elsewhere.

(1) I convinced Sraffa and Pasinetti that Manara's example could be economically legitimate, provided it was interpreted in the right context, that is, taking appropriate costs of labour into account. This is what caused Sraffa to abandon his attempt; it will be shown below.

(2) It is true that the conditions, which ensure the existence of a standard commodity are restrictive, if there is joint production and if the only operation to be performed on the system consists in the elimination of non-basics. The conditions on rates of reproduction referred to above has this restrictive character. But there is a by now vast literature on the theory of joint production, mainly written in the1980s, showing that, by means of truncation and dual truncation, one can arrive at square Sraffa systems, from which negative prices, overproduced goods and inefficient processes have been eliminated so that, if only the system is capable of producing a surplus, one can, by gradually raising the rate of profit from zero, follow the wage curve of the system, which is then an envelope of individual wage curves of individual truncations succeeding each other as minimizing the cost of production, until eventually the wage is zero and a maximum rate of profit is reached. The only important restriction is that the truncations themselves have to be regular. Regularity, first defined in Schefold (1971, p. 20), is not a restrictive property, since it is generic.

(3) However, it is a question whether the standard commodity thus found still serves the purposes it serves in the case of single product systems.

(4) There is no room here to analyse fully whether the approach by Dupertuis and Sinha goes beyond the more familiar analysis of truncations of regular systems; they achieve rather less, it seems to me. At any rate, I here draw attention to the theory of intensive rent. An example was given already in 1971 for which the standard commodity does not

exist, and yet the basic processes *are* interdependent; we shall return to it at the end of this paper, after having discussed the other objections.

We first take up Manara's example. It is represented in table 1. As far as the numbers are concerned, it is as in the original publication, except that, like Verger, we preserve Manara's ordering of the processes, but exhibit the industries on the rows and the commodities on the columns.

	pikes	carps	labour		pikes	carps
breeding	1	1.1	x	$\rightarrow$	1.09	1.144
wild	1.1	1	$1- x$	$\rightarrow$	1.144	0.99

Table 1: Manara's example, with labour made explicit.

However, I here present an interpretation of the given numbers, which renders the economic meaning more transparent. There is fish production, of pikes and carps. It takes place in an artificial tank (first process) and in the "wild", in a lake (second process). The fish are periodically harvested. The input matrix shows with which quantities the reproduction begins, after the captured fish have been taken out, the output matrix shows the stocks immediately prior to the harvest. In the "wild", the pikes eat carps uncontrolled; they often catch the young so that the reproduction of the carps is slowed down, and taking the fishing activity into account, it is even necessary to replenish the stock of carps periodically by one per cent (compare the second input to the second output in the second process). In the tank, the pikes are kept apart and fed by means of labour. Hence they multiply more slowly and the carps more rapidly. Additional labour is required, apart from the fishing activity. The rates of reproduction thus become plausible. The system functions economically, because two processes are needed *to adapt output to demand*. The net outputs for the two kinds of fish differ somewhat in Manara's example, taken at unit activity levels, but net outputs could easily be changed within a certain range that is limited, as always with joint production (net output of pikes is 0.134 and of carps 0.034 at unit activity levels). The example is also economically meaningful *from the point of view of cost*. In terms of reproduction of *both* kinds of fish, breeding is more efficient than letting the fish grow in the "wild", but breeding requires more work, and it is easy to see, with appropriate values chosen for x, that the prices of both goods will be positive at some moderate level of the rate of profit. We choose $x = 0.9$.

It really was Manara's fault to forget labour in his analysis of the example. One might try to defend the omission by arguing that labour could be implicit in the means of production as in Sraffa's second model (production with a surplus). But that would be a mistake: the purpose of the standard commodity is to provide a measure for variations of distribution between wages and profits. Sraffa, when discussing Manara's example, could be content with leaving Manara in his error if he just wanted to show that there was a snag in Manara's argument. But, as reported by Verger, I discussed the example, which had been presented to me by Pasinetti, with Pasinetti and Sraffa in the spring of 1970. Although the first process is superior in terms of rates of reproduction, it had become clear to me that Manara's example was economically viable, if labour was introduced in appropriate relative amounts and received a positive wage. I had not yet begun to think about existence theorems for the standard commodity, based on assumptions about rates of reproduction; this I began to do only some time after I had been confronted with Manara's challenge. I only argued in the discussion that the standard commodity was there to explain changes of relative prices as caused by changes in distribution between wages and profits. Manara's example, complemented by a labour vector, with more labour allocated to the first process, was economically viable; relative prices then would move with the rate of profit. That would have to be discussed. Was it a problem for Sraffa's theory if the standard commodity for this kind of analysis was not there? If an appropriate labour vector complemented it, one could not object to Manara's example. Sraffa and Pasinetti took the point, but they never really told me what that meant for them. In particular, as already stated, I was not told that Sraffa had made a prior attempt to reply to Manara in a paper to be published.

Of course, neither they nor I knew that I would go to work on joint production for many years. I was just one of the students on whom they could try their arguments. Amartya Sen once recounted that he had been used as a test person on a much grander scale.

He was, as a student at Trinity College, one of the few persons, to whom Sraffa showed the manuscript of *Production of Commodities by Means of Commodities* before publication. Actually, Amartya Sen was asked to sit at a table in Sraffa's rooms, Sraffa would bring the manuscript and watch how Amartya Sen would read it, during several days. Amartya Sen told me that he thought he understood what he was reading under such circumstances. I am still impressed by that.

It is not easy to grasp the movement of relative prices intuitively in a model such as that of table 1, where there are four rates of reproduction and two labour inputs. A surplus is being produced, but a falling wage need not result in a rise of a rate of profit up to a maximum, where wages are zero; it may just as well happen in such models that one of the prices turns negative. Abraham-Frois and Edmond Berrebi (1976, p.119) have called *Théorème de Schefold* the proposition (Schefold 1971, p. 33) that, for any joint production system with positive prices at a given rate of profit, either "a maximum rate of profit and a standard commodity exist and/or prices in any standard do not remain positive for all positive r". The theorem meant that even if no maximum rate of profits existed, the rate of profits could not rise indefinitely, because negative prices would come in.

What could negative prices mean? It was Joan Robinson who would provide the relevant example in her lectures with reference to her experience in India. The lower the real wage, she said, the longer lorries would be kept on the street, because repair work was cheap. This sounded like neoclassical theory, but substitution works in some contexts, and the effect could be reproduced in Sraffa systems with fixed capital, with lorries as machines, and with, as they grow older each year, the older machine being a joint product of transportation. Sraffa had used subsystems to explain negative values. He showed that more could be produced of some commodity, if a negative labour value was involved, by reducing the amount of labour employed (assuming constant returns to scale). The paradoxical character of the consideration disappears with fixed capital. If a machine needs an increasing amount of repairs, as it grows older, it pays to scrap it and to replace it by a new machine. The old machine will have had a negative labour value, for the new machine will allow to save labour (less repair work) and the old machine, since it is not used any more in production, will appear as a newly produced output. Since there is no demand for it, it will receive a zero price.

The generalisation of the idea that a negative labour value indicates the possibility to increase output and to reduce labour input by scrapping a commodity is known under the heading “truncation”. Assuming constant returns and making the golden rule assumption, it turns out that, at the rate of profit where the price of some commodity turns negative, a process can be eliminated, saving labour and producing as much as before, but overproducing the commodity, which thus receives a zero price and ceases to be a commodity, as the old lorry in the example.

The rise of the rate of profit in joint production systems thus is not necessarily limited by a maximum rate of profit, with a standard commodity associated with it, but it can also be limited by a price turning negative, and it can rise further, if there is profit maximisation, only after truncation so that the remaining commodities have positive prices. Sraffa’s attempt to refute Manara’s example by pointing to the inferiority of one of the processes without taking wage costs into account was formally not mistaken, because Manara had made such assumptions. But Sraffa’s attempted answer would not be effective, for as soon as somebody asked what the standard commodity was for, the answer had to be: to analyse effects of changes of distribution on proces, but where was such change in a model without labour? And there is another point on which Sraffa was only formally right. In the draft, as here published by Verger, Sraffa points to the fact that he had made the assumption of a positive real root of the characteristic equation explicitly, in order to have the standard commodity, but that he had done it at the advice of Besicovic; the problem to justify the assumption economically remained. In later conversations, Sraffa was annoyed that the standard commodity did not necessarily exist in the general case, but he had accepted the fact. It did exist in the fixed capital case; he was happy and visibly content when I could confirm that he had described the composition of the standard commodity correctly, regarding the proportions in which machines of different ages appear in it, when it is a matter of finding the standard commodity for pure fixed capital systems.

Of course, one can find a “true” maximum rate of profit by applying truncations to a given joint production system, which will then be “square”, with a number of commodities produced equal to the number of positive prices and to the number of processes activated. This came out in my paper “On Counting Equations” of 1978, after an important suggestion made by Steedman, under the assumption of an equality of the rate of profit and the rate of balance growth g (golden rule), and this was extended to cost minimizing systems (where r and g could be different) in papers by Lippi, Salvadori and myself. The classical assumption is $r > g = 0$. Of course, one needs to make regularity assumptions. For me, this research was essentially closed with the second edition of my thesis (Schefold 1989). The procedure may be illustrated by means of the model with pikes and carps. As long as the rate of profit is low enough and the wage cost of raising fish in the basin is high, both processes will be operated in a stationary state at low rates of profit. But if the rate of profit rises and wages fall, harvesting fish in the wild will be become unprofitable. Carps will tend to be overproduced, their price drops to zero. If the workers in the breeding industry receive a sufficiently low wage, harvesting in the wild thus will be abandoned, carps are overproduced and a free good, and the rate of profit can rise up to a level determined by the rate of reproduction of pikes. The standard commodity then consists of pikes only.

The modern reader is invited to program the Manara example on his or her laptop to confirm this insight. One finds, with a labour input to tank production of 9/10 and one of 1/10 to lake production, that prices in terms of labour commanded are at first positive. The price of pikes rises monotonically up to a distinct sharp maximum near *r*=11%. The price of carps in terms of labour commanded *falls* and becomes zero near *r*=3.3%. Hence the movement depends on the composition of the wage. We measure it in terms of the surplus produced in the stationary state at unit activity levels. The wage rate then is equal to unity at r=0 and falls monotonically. Truncation obviously must take place at *r*=3.3%, for the system consisting of both processes and both kind of fish ceases to be viable, as soon as the price of carps has turned negative. Only the tank process will be used at high rates of profit, when wages are so low that their cost does not compensate for the relative inefficiency of the production of fish in the lake. Truncation means that the surplus of pikes will then be produced by activating only the first process, carps will be overproduced and the wage rate becomes linear. It falls off and becomes zero at what may now be called the true maximum rate of profit at *r*= 9%. The second process, lake production, now is not used, because labour has become cheap. This means that lake production is not profitable at the wage resulting from the truncation, and this remains true up to the rate of profit, where the wage rate is zero.

The analysis can be rendered exact, using the chapter on the analysis of prices in steady states in Schefold (1989, pp. 114 – 123); it discusses a similar numerical example in detail. Here it should suffice to say, using the concepts introduced in that chapter, that the wage curve in the stationary state is given by two cost-minimizing systems successively. The first consists of both processes and both commodities, the second only of the first commodity and the first process. This truncation is q-feasible in the stationary state (the second commodity is overproduced) and p-feasible for all rates of profit from the point onwards, where the price of the second commodity turns zero (the second process is unprofitable). To complete the analysis, one must look also at the other three truncations that are formally possible and show that they are economically not relevant, because they are not q-feasible or not p-feasible.

So the standard commodity consists of pikes only. *But what is the use of such a standard commodity*? The standard commodity shall simplify the wage curve for a given technique and render the movement of relative prices transparent. It also is a physical analogue (Eatwell 1975) of a corn model and helps to analyse distribution as if we were in a one commodity world. The analogy has its limits: one needs prices to define a given real wage. The standard commodity finally is most useful in relation to what I call the Ricardian exercise (Ricardo 1966 [1951], p.35) for single product systems: If the rate of profit rises, the wage rate must fall, whatever the composition of the numéraire, so that the cost of production, including normal profits, rises more in a capital intensive industry than in a labour intensive industry. For, to the extent that profits rise, the price must rise, but, to the extent that wages fall, labour costs fall. Hence one expects the prices of capital-intensive goods to rise relative to the prices of labour-intensive goods, the cost of which, in the given numéraire, will *fall*. Sraffa, in presenting this Ricardian exercise, has a reservation to make: If the prices of the capital goods used in the production of the capital intensive good fall strongly during the process, the conclusion can not be drawn, hence one must look for the capital composition of the means of production of the goods currently used in production, and so further on backwards, and unaffected are only baskets of a composition of outputs which is the same as that of the inputs. The ideal numéraire, the standard commodity, therefore is that commodity, for which the causes for changes of relative prices are *absent*. This is the invariability of the standard of value, for every numéraire is otherwise invariant by definition.

This entire derivation, which can be explained well only in a full paper (Schefold 1986), has always been presented for single product systems only. The intensity of capital $\mathbf{a}_i\mathbf{p}/l_i$ (where $\mathbf{a}_i$ is the vector of inputs, $\mathbf{p}$ the vector of Sraffa prices, expressed in some provisional numéraire, and l_i the amount of labour used in the industry under consideration, i) explains the influence of distribution on the price of the output, if we are dealing with single products, but what, if the changing costs have to be ascribed to different outputs? It is true that one can reduce joint production systems to vertically integrated systems by the procedure explained in Schefold (1971, p. 30) – the term “vertical integration” was introduced later – but the standard commodity looses much intuitive appeal.

Now back to our example: What is the use of a standard commodity consisting only of pikes, if the task is to analyse the movement of relative prices (here the relative price of pikes *and* carps) at low rates of profit, i.e. in the system consisting of *both* processes? This is the problem of truncation again. A joint production system can not easily be regarded as one technique, as in the case of a single product system, since the possibility of truncation is inherent. If we have, to use another example, a pure fixed capital system with one machine which grows ten years old at the maximum rate of profit, lowering this rate of profit and having a positive wage then may entail that the machine grows only five years old. This is like having a different technique. What does the standard commodity, containing ten old machines, mean at that level of distribution, where there are positive prices only for five machines? We know that the standard commodity is suitable as a standard of value, if there is one given technique. If there are many techniques to be compared, because there is a choice to be made, a numéraire must be common to all the techniques. To take the standard commodity of one of the techniques as the numéraire is a possibility, but it is not very useful. A large system of joint production is, because of the possibility of many truncations, like a spectrum of techniques, and that is why the standard commodity here is not of much help. It is probably agreed that the standard commodity must be unique, given a system. But what if a system, which is economically viable at one rate of profit, yields another truncation as a solution, as soon as the rate of profit is somewhat changed in a joint production example? If each is associated with a standard commodity, which is the relevant one?

All this seems to be ignored in the paper by Dupertuis and Sinha. They make much, on the other hand, of the idea that commodities could come as packets of inputs which stand in the same proportion in all lines of production. But this means that two columns of inputs of the input matrix are proportional. Hence the system would not be regular (it is one of the assumptions of regularity that the determinant of matrix are does not vanish).

Pasinetti wrote to Sraffa on 11 December 1969 (Sraffa Archive D3/14/24) and advised him for his "nota sul M" rather to speak of "assoluta inferiorità (absolute inferiority) than of "incompatibilità". The notion chosen by Dupertuis and Sinha in order to exclude such cases is "interdependence". By means of linear combinations of rows and columns according to certain rules, they want to reduce the system to its "atomic" core where all processes are interdependent. Then, they assert, a standard commodity must exist.

They overlooked that I gave an example in my thesis where such interdependence results from the construction of the basic system, and yet no standard commodity exists. It seems worthwhile to reproduce this example fully, taken from Schefold (1971, p.88). Corn is produced on *one* kind of land by means of *two* different techniques (intensive rent) by means of corn and ploughs, and ploughs are produced by means of ploughs and corn in a single product process (table 2).

ploughs	corn	land		ploughs	corn
$2/5$	1	0	$\rightarrow$	1	0
$1/10$	$2/5$	1	$\rightarrow$	0	1

$2/5$	$1/5$	2	$\rightarrow$	0	1

Table 2: Intensive rent, no standard commodity.

Land is eliminated and the basic system is obtained by doubling the coefficients of the second process and deducting the result from the third. One obtains the basic system shown in table 3.

ploughs	corn		ploughs	corn
$2/5$	1	$\rightarrow$	1	0
$-1/5$	$3/5$	$\rightarrow$	0	1

Table 3: The basic system, pertaining to the system of table 2.

It is not obvious that a surplus is being produced according to table 3, but we can reassure ourselves by looking at table 2. We can see that, at unit activity levels, the surplus of ploughs is equal to $1/10$ and the surplus of corn equal to $2/5$. Moreover, it must be assumed that the land is used up, hence that three units of land are available in total. The economy is therefore viable from the point of view of quantities. It is also viable from the point of view of prices, if the labour inputs stand in the following proportion: $2:25:1$. The proportions can not be changed much; otherwise, the rent turns negative. Again, the reader is invited to program the system on a laptop. The solution laboriously calculated by hand in Schefold (1971) will be confirmed.

The characteristic equation, following from table 3, is $(11/25)(1+r)^2 - (1+r)+1=0$.
The discriminant is negative: $1-44/25=-19/25$, hence there is no maximum rate of profit nor a real standard commodity. But it is clear from table 3 that there is a maximum of interdependence, in that the basic system is a single product system: each process produces one of the commodities that are necessary for reproduction. There are two

anomalies in the basic system: one input is negative (ploughs in corn production) and the sum of the corn inputs is greater than the corn output at unit activity levels. This is the consequence of the introduction of negative multipliers, which Sraffa defended in his book. In fact, what matters for reproduction is the existence of a surplus in the original system, according to table 2.

Hence Verger seems to err in asserting that the problem of the missing standard commodity could be overcome by reducing the system to a state in which there is full interdependence of the processes. The mistake must be traced back to the original paper by Dupertuis and Sinha.

Sraffa himself said in one of the drafts of his attempted reply to Manara (Sraffa Archive, D/14: pp. 72-74): "Please note that the example given by M. not only was excluded from the real solutions condition that I had explicitly stated: it would have also been excluded from the elementary economic condition of convenience. It would be interesting if M. or some one more patient or luckier, found an example of a basic product system economically possible, that doesn't have any real solution for the standard commodity". Such an example has here been given, after reducing the processes using land to the corresponding basic system.

Yoann Verger has done us a favour by publishing Sraffa's draft and by clarifying some of the circumstances surrounding its origin. But his paper would be better if he had concentrated on the history of economic thought and avoided presenting an analytical solution, which makes Sraffa's mistake worse.

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