An "Image Theory" of RPM*

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Abstract

We show how a brand manufacturer’s control over retail prices can lead to efficiencies when consumers rely on prices as a signal of quality. For this we first show how higher prices can be associated with both higher quality perception as well as higher actual quality. We next identify a conflict of interest between retailers and manufactures. Retailers do not internalize the ensuing reputation spill-over that higher prices have on demand at all outlets. And they have less incentives to support brand image through higher prices as this erodes their own position in negotiations while increasing that of the manufacturer. Our efficiency defence for RPM thus applies even when retailers need not be incentivized to undertake non-contractible activities, as in our model the key opportunism problem, with respect to quality provision, lies between the manufacturer and consumers.

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1 Introduction

For many fast-moving consumer goods, notably in food retailing, quality depends on a manufacturer’s ongoing effort, such as to procure high-quality inputs or to secure high hygienic standards in production and handling. Consumers in turn may not always be able to assess a product’s quality before purchasing and may, amongst other things, rely on prices as a signal of quality. In this article we first set up a simple model that motivates a positive relationship between prices and both perceived as well as actual quality. We next show how this relationship gives rise to a conflict of interest between retailers and manufacturers, as manufacturers prefer a strictly higher price. Instead, when prices are set in retailers’ individual interests, they do not internalize the full reputation spill-over for the manufacturer’s good. The conflict of interest is further aggravated by the fact that prices are chosen also so as to thereby influence retailers’ and manufacturers’ bargaining position, which depends on consumers’ perception of the product’s quality. Overall, equilibrium quality and efficiency can be higher when, by means of retail price maintenance (RPM), manufactures exert greater control over retail prices.

Our contribution, notably to the literature on RPM, is thus to identify an efficiency gain that applies in particular to branded goods and that does not rely on non-contractible, sales-enhancing activities of retailers. In particular, it is thus applicable to many fast moving consumer goods, such as in food retailing, where even the staff of "bricks and mortar" retailers does not interact directly with consumers, e.g., to provide advice. Our theory thus provides a rationale for why branded goods manufacturers may be particularly eager to keep control over the retail price, even when retailers need not be incentivized to provide such services and even when, as we show, intrabrand competition is not the main concern. The reason, in our model, is the link between price and quality image together with the described conflicts of interest between retailers and the concerned manufacturer.

In this respect, our efficiency defence for RPM may apply also when various distribution channels, such as "offline" and "online", do not differ in the provision of other services to consumers, such as advice or shopping experience. Hence, in these cases there may not be an efficiency defence based the provision of incentives or, likewise, the protection of such incentives in one ("offline") channel against competition from another ("online") channel where such services are not provided. Attempts of brand manufacturers to control prices in online distribution have in fact been at the forefront in various recent antitrust cases.¹

¹For instance, Lindsay (2011) reports, in particular, two recent cases of, inter alia, minimum RPM provisions for online sales (Bioelements, a skin-care products manufacturer, and Tempur-Pedic, a seller of
The European Union’s recently redrafted block exemption regulation for vertical restraints and the respective guidelines consequently make specific reference to online sales. While the text recognizes the role of a product’s image as a justification for selective distribution systems, this does not extend to the control prices across channels to uphold such an image.\(^2\) Our theory suggests that this approach is too narrow to account for the relevance of a product’s image.

The relationship between price and quality perception, on which our theory builds, has been widely covered in the business literature, notably in marketing.\(^3\) An "image theory" of RPM has also been occasionally acknowledged in the legal literature,\(^4\) though to our knowledge it has been largely ignored by competition economics.\(^5\)

In our model, a higher price will be associated with higher quality and higher perception of quality by those consumers who do not directly observe quality choice. When the manufacturer controls the retail price, he will fully internalize the benefits from an overall higher perception of quality. Precisely, when consumers form beliefs based on their perception of prices across all retailers and shopping trips, the manufacturer internalizes premium mattresses). He observes a substantially different treatment of these clauses by the respective Californian and New York Attorney General, suggesting that also in the US the discussion about RPM is far from over even after the much discussed Leegin decision of the Supreme Court in 2007.

\(^2\)Also, a differential treatment of online distribution channels seems to be justified only when these lack the provision of additional services, thus mirroring theories that, in contrast to ours, build on retailer moral hazard. The Vertical Restraints Block Exemption Regulation and accompanying Guidelines came into force on 1 June 2010. For instance, with regards to online distributors manufactures may require the provision of after sales services or the presence of physical locations and showrooms. Cf. European Commission Regulation (EU) No 330/2010 of 20 April 2010 on the application of Article 101(3) of the Treaty on the Functioning of the European Union to categories of vertical agreements and concerted practices, OJ L 102, 23.4.2010 and European Commission Notice - Guidelines on vertical restraints, Official Journal C 130, 19.05.2010, p. 1.

\(^3\)In fact, the marketing literature often mentions, in this context, the dual role of prices, namely as a constraint to firms and as a product attribute that conveys information, rendering it the "most immediate and easiest to communicate marketing-mix variable" (Rao 1984; cf. Erickson and Johansson (1985) for another example and Leavitt (1954) for a very early contribution). Völckner and Hofmann (2007) provide a meta analysis of the empirical literature, which establishes a positive relationship between prices and perceived as well as actual quality, even beyond higher costs associated with the provision of higher quality.

\(^4\)Among legal scholars, for instance, Orbach (2008, 2010) builds a legal defence of RPM on such an “image theory” of prices. The argument there rests however on a more diffuse notion of "brand image", which is affected by price choice and which is assumed to generate consumer welfare. It relates to Leibenstein’s (1950) theory of “conspicuous consumption” and consumers’ willingness to pay a price “above the intrinsic value” to achieve exclusivity. (This goes directly back to Veblen (1899), as well as Taussig’s (1916) notion of "articles of prestige"). As we equate "brand image" with "quality image" in our model, we can however derive potential efficiency gains directly from more standard consumer preferences.

\(^5\)In Marvel and McCafferty (1984) low-cost retailers may free-ride on the quality certification provided by higher-cost retailers. While a higher margin promised to retailers thus ensures that also high-cost retailers stock (and thereby certify) a high-quality good, it is the availability of products at these retailers and not a particular price itself that generates the respective quality perception.

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this. In contrast, retailers tend to free-ride on the quality perception generated by other retailers’ choice of prices. We derive conditions for when an induced increase in quality dominates deadweight loss from a higher price, which is the case precisely when quality perceptions matter more as more consumers must rely on them for their purchases.

A second channel through which such a difference in interests between manufacturers and retailers affects prices and quality is the effect that the product’s (perceived) quality has on the outside options of manufacturers and retailers. Retailers may set a lower price also so as to thereby induce lower quality perceptions, which decreases the outside option of the manufacturer and enhances retailers’ own outside option in case they later stock a different product. Notably, under our chosen bargaining solution, such a conflict of interest would not arise when quality was exogenous, as then retailers and the manufacturer would choose the same price. Hence, the fact that quality is endogenous and affected by prices is essential also for this potential conflict of interest between retailers and manufacturers to arise in our model. Again, quality is higher when the manufacturer controls the price.

In our baseline model, a higher price induces higher quality when this choice is observed by at least some consumers (or when it is observed with positive probability by any given consumer). All other consumers form rational expectations. This model and its analysis serve two purposes. First, to some extent it is of interest in its own right as we clearly isolate, already in this simple model, three channels for why consumers may rightly anticipate a positive relationship between prices and quality. Second and more importantly, the model provides a simple starting point to introduce the conflict of interest between retailers and the considered manufacturer. That said, we are aware that a number of theoretical papers in the industrial organization literature have associated price and quality perception, albeit not in the context of vertical relationships and not applied to antitrust issues, notably Klein and Leffler (1981), Shapiro (1983), and Wolinsky (1983).

To model negotiations between a manufacturer and several, possibly competing retailers, we use a bargaining approach that extends an idea from Inderst and Wey (2003). Under this solution concept, each bilateral contract specifies a "fair" sharing rule to the

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6 Also in models where quality is exogenously given, high-quality firms may use high prices to signal the superiority of their products (e.g., Bagwell and Riordan (1991) or Milgrom and Roberts (1986), where high prices are used in combination with dissipative advertising). Noteworthy may also be a very early article in the Review of Economic Studies by Scitovszky (1944), who notes: “[…] in the United States ‘expensive’ is in the process of losing its original meaning and becoming a synonym for superior quality. Worse still, one of the largest American breweries uses the advertising slogan: ‘Michelob, America’s highest-priced beer!’” According to his reasoning and in line with the subsequent formalizations, including the one given in this paper, uninformed consumers can rely on a positive relationship between price and demand as "expert consumers” would not buy a high-price product if it was of low quality.

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net surplus generated under all possible agreements (coalitions). As we will discuss, the obtained solution is closely related to other recent solution concepts to multilateral negotiations, such as De Fontenay and Gans (2013). One of our contributions is to show the applicability of this solution concept, as in our setting it gives rise to relatively simple and intuitive expressions.

As noted above, our theory is different from extant theories that deal with the efficiency implications of manufacturers’ control over retail prices, notably through RPM, as these typically build on retailer non-contractible actions, such as the keeping of inventories (Deneckere et al. 1997) or the provision of services and other demand-enhancing activities (e.g., Telser 1960; Klein and Murphy 1988; Mathewson and Winter 1998; see, however, Romano 1994 for a setting where both parties make a non-price decision affecting demand). We already noted that our theory is also applicable to goods and markets where retailers do not provide this type of services, such as advice, or where this can be explicitly contracted for. In fact, for many branded products in areas such as grocery or cosmetics it may be manufacturers who must be incentivized to constantly ensure high quality. The key opportunism problem then lies between manufacturers and consumers.

The rest of this paper is organized as follows. Section 2 introduces the baseline model and establishes a relationship between price and quality. Section 3 introduces retailers. Section 4 considers non-competing retailers and isolates a conflict of interest that arises from free-riding on quality (brand) image. In Section 5 we introduce retailer competition. Section 6 offers some concluding remarks.

2 The Relationship between Price and Quality

2.1 Model

In this section we consider first an auxiliary model to introduce the basic relationship between price and quality. In this model, a manufacturer sells directly to final consumers, so that we presently abstract from the presence of intermediaries. Demand for the firm’s product depends on the product’s price $p$ and a scalar indicator of quality $q \geq 0$: $D(p, q)$. The supplier’s costs depend on quantity $x = D(p, q)$ and quality: $c(x, q)$. As we will discuss in what follows, the positive relationship between price and quality that we derive in this section holds generally whenever $c_{qx} \geq 0$, i.e., when per-unit costs of production are weakly increasing in quality.\footnote{This is however not a necessary but only a sufficient condition.} Our motivational examples in the introduction suggest however a
specification where costs of ensuring higher quality change proportional with quantity: \( c(x, q) = k(q) x \). For instance, this should be the case when higher costs result from the procurement of higher-quality inputs or from ensuring higher hygienic and safety standards in production and shipment. Here, \( k(q) \) is assumed to be a twice differentiable function with \( k'(q) > 0 \) for \( q > 0 \) and \( k''(q) \geq 0 \). The supplier’s profits are \( \Pi = D(p, q) [p - k(q)] \).

We further specify that \( D_p < 0 \) and \( D_q > 0 \) where \( D > 0 \).

For the present auxiliary analysis, we now consider the following game. First, in \( t = 1 \), the firm chooses a price \( p \). Then, in \( t = 2 \), it chooses a quality \( q \). Finally, in \( t = 3 \), consumers decide whether to purchase or not. We discuss the sequence of timing in \( t = 1 \) and \( t = 2 \) below. Further, before purchasing a consumer only observes with probability \( \gamma \) the true quality. As we restrict consideration to equilibria in pure strategies, we denote an uninformed consumer’s beliefs by \( \widehat{q} \). These will depend on the observed price choice.

Before analyzing this game, we comment on the choice of the sequence of timing in \( t = 1 \) and \( t = 2 \). Our focus throughout this paper is on price as a (relatively) longer term choice (cf. also the additional discussion in Section 3). It is part of the overall positioning of the product, i.e., its branding. In practice, the decision on the overall price level must then be complementary to the other marketing choices such as the scope and content of the advertising campaign. While surely some key (quality) features of the product are also chosen for the long term, with a view particularly on fast moving consumer goods and notably branded food products, we consider decisions that must be made constantly so as to maintain high quality. As mentioned previously, this could concern the procurement of high-quality inputs or the overall conditions of production and handling of the product (e.g., with regards to food safety). The manufacturer could be tempted to save costs by reducing care or using cheaper inputs.

### 2.2 Equilibrium of the Auxiliary Game

We solve the game backwards. Consumers’ decision in \( t = 3 \) is already captured by the demand function. A given consumer is only informed with probability \( \gamma \) about the true quality choice. With probability \( 1 - \gamma \) a consumer is uninformed and has the beliefs \( \widehat{q} \). Demand is thus given by

\[
\widehat{x} = \gamma D(p, q) + (1 - \gamma) D(p, \widehat{q}),
\]

so that firm profits are

\[
\pi(p, q, \widehat{q}) = \widehat{x} [p - k(q)],
\]  

(1)
depending on price, actual quality, and quality beliefs. Actual quality affects demand only when the consumer is informed. For given beliefs $\hat{q}$, the optimal quality $q_{BR}$ is thus determined from (1) by the following first order condition:

$$\frac{d}{dq} \pi(p, q, \hat{q}) \bigg|_{q=q_{BR}} = \gamma[p - k(q_{BR})] D_q(p, q_{BR}) - \hat{x}_{BR} k'(q_{BR}) = 0,$$

(2)

where we used $\hat{x}_{BR} = \gamma D(p, q_{BR}) + (1 - \gamma) D(p, \hat{q})$. (Here, the notation $q_{BR}$ refers to the fact that this is the "best response" to a particular choice of the price and consumers’ beliefs.) We stipulate that $\pi$ is strictly concave in $q$ so that there is always a unique value $q_{BR}$. This, as well as uniqueness for the subsequently solved programs, holds, in particular, for the (linear-quadratic) specification that is used for illustration below.

In equilibrium, given the price $p$ that is set initially and that is observed by all consumers, beliefs must be rational. In a slight extension of notation, it is thus required that $\hat{q} = q_{BR}(\hat{q})$ (where it is convenient to suppress for now the dependency on the price). We denote this level, for given $p$, by $\hat{q}^*$. Using the condition for $q_{BR}$ from (2), $\hat{q}^*$ must solve

$$z(p, \hat{q}^*) := \gamma[p - k(\hat{q}^*)] D_q(p, \hat{q}^*) - D(p, \hat{q}^*) k'(\hat{q}^*) = 0.$$

(3)

Again, we assume that this gives rise to a unique interior solution and obtain from implicit differentiation

$$\frac{d\hat{q}^*}{dp} = \frac{1}{-z_{\hat{q}^*}} \left[ \begin{array}{c} \gamma D_q(p, \hat{q}^*) \\ -k'(\hat{q}^*) D_p(p, \hat{q}^*) \\ +\gamma(p - k(\hat{q}^*)) D_{pq}(p, \hat{q}^*) \end{array} \right],$$

(4)

where we have already split-up the three terms that we discuss below in turn. Note that $z_{\hat{q}^*} < 0$. Hence, the sign of $d\hat{q}^*/dp$ is determined by the expression in rectangular brackets in (4). This expression comprises three different effects that the price level has on quality, which we discuss next.

**Relationship between Price and Quality.** The term in the first line of (4) is strictly positive. When the price and thus also the margin is higher, the firm has more to gain by sustaining demand through choosing a higher quality. This is the most immediate effect that a higher price has on the manufacturer’s incentives to maintain quality at a high level. The strength of this effect hinges on the likelihood with which quality is observed (or, likewise, on the respective fraction of informed consumers). The second line in (4) is

\footnote{Precisely, this follows from inspection of the second-order derivative of firm profits, i.e., from differentiation of the first-order condition (2), and noting that $D_q > 0$ and $k' > 0$.}
also strictly positive from \( D_p < 0 \) and \( k' > 0 \). As the price increases and demand thereby decreases, an increase in the per-unit costs, when this is associated with higher quality, has a smaller negative impact on overall firm profits. This effect is independent of the observability of quality. We turn next to the last effect, as captured by the final line in (4). This term is zero when the marginal utility of an increase in quality is independent of the level of a consumer’s valuation. This holds, for instance, when demand is linear and separable in \( p \) and \( q \). But the effect is strictly positive when consumers with a higher absolute valuation also have a higher marginal valuation for quality.\(^\text{10}\) Then, as the price is higher, this increases the responsiveness of demand with respect to quality.

In what follows, we always stipulate that \( D_{pq} \geq 0 \), so that altogether, from the three discussed effects, a higher price will strictly increase the firm’s incentives to maintain high quality.

**Lemma 1** When \( D_{pq} \geq 0 \) holds, then at stage \( t = 2 \) in the auxiliary model quality, as obtained in (3), strictly increases with price: \( d\hat{q}^*/dp > 0 \).

As we are presently only interested in the relationship between price and quality, we are silent about efficiency. When \( \gamma = 1 \), we can, however, directly refer to the analysis in Spence (1975) and conclude that for any given price the resulting quality \( \hat{q}^* \) is inefficiently low. As we have immediately from (3) that \( \hat{q}^* \) is strictly increasing in \( \gamma \), this inefficiency becomes more severe as \( \gamma < 1 \).

**Equilibrium Price Level.** To determine the price that the manufacturer chooses in \( t = 1 \), it is instructive to first consider the case where \( \gamma = 1 \), in which case all consumers can observe quality. We already noted that then \( \hat{q}^* \), which is determined in (3), also solves the first-order condition (2). Turning to \( t = 1 \), we can thus conclude that for \( \gamma = 1 \) the optimal price \( p \) is then determined simply by setting the respective partial derivative with respect to \( p \) equal to zero. (Formally, this follows from the envelope theorem.) This is, however, no longer the case when \( \gamma < 1 \). Then, for a given price, the resulting equilibrium quality \( \hat{q}^* \) will be strictly below the value that would maximize ex-ante firm profits. The firm thus has a commitment problem vis-à-vis consumers, and the optimal choice of \( p \) takes

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\(^9\)This effect would be absent when, with general cost of production \( c(x,q) \), we had \( c_{qx} = 0 \), so that costs of quality provision were independent of actual output.

\(^{10}\)More formally, there could be a continuum of consumers, indexed by \( y \), with respective utility \( qy \). Then, the critical type is \( \bar{y} = p/q \), so that with distribution \( F(y) \) demand is \( D(p,q) = 1 - F(\bar{y}) \). When \( F(\cdot) \) is the uniform distribution, we have \( D_{pq} = 1/q^2 > 0 \).
this into account, so that now also the indirect effect through the incentives to choose (higher) quality matters.

To derive the optimal price $p$ for general $\gamma$, in a slight abuse of notation denote now $\pi(p, q^*) = \pi(p, q = q^*, \hat{q} = \hat{q}^*)$. With a slight transformation, the first-order condition in $t = 1$ becomes

$$
\frac{d\pi(p, q^*)}{dp} = \left[p - k (q^*)\right] D_p(p, q^*) + D(p, q^*) + \frac{dq^*}{dp} \left(1 - \gamma\right) \left[p - k (q^*)\right] D_q(p, q^*) = 0.
$$

As noted above, the second line in (5) is equal to zero when all consumers are informed so that then (and only then) the first-order condition is obtained from the partial derivative with respect to the price.

Lemma 2 In the equilibrium of the auxiliary model, the optimal price $p$ that is chosen by the manufacturer in $t = 1$ solves (5).

We again suppose that there is a unique price equilibrium. We denote this price by $p^*$ with corresponding quality $\hat{q}(p^*) = q^*$. We will make use of condition (5) in our extended model below. When we provide an illustration of the equilibrium characterization in the following section, we make further use of the presently solved auxiliary model.

3 Introducing Retailers and Negotiations

We now introduce retailers indexed by $n = 1, \ldots, N$. As will become clear from the following analysis, when there is only a single retailer, then the interests of the manufacturer and of the single retailer (with respect to how the retail price is set) will be perfectly aligned. This is no longer the case when there are multiple retailers, even when they are not in competition with each other. In fact, below we will isolate two different sources of such a conflict of interest, one arising from a free-riding problem and one arising from the way retail prices affect outside options and, thereby, the distribution of surplus through negotiations. For this analysis, we have to first set up and provide a solution to the bargaining problem between the manufacturer and retailers. This is done in the present section.
Timing. Our timing, which we discuss subsequently in detail, is as follows. Retail prices, now \( p_n \), are still set in the first period \( t = 1 \). Our key analysis will be the comparison of equilibrium outcomes when these are chosen individually by retailers or when they are perfectly controlled by the manufacturer (RPM). In \( t = 2 \) the manufacturer chooses quality. Before demand is realized, as consumers make their choice, in the presence of retailers there are now negotiations in \( t = 3 \). The respective solution concept is introduced below.

Our choice of timing, where retail prices are set first, essentially deprives wholesale contracts of their steering role, as these are used in \( t = 3 \) only to distribute profits. This implication is fully intended. We offer two main motivations.

Motivation 1: First, the role of (marginal) wholesale prices to control retailers’ pricing decisions even outside RPM arrangements should often be severely restricted in practice, which is not adequately reflected by games that consider a first-stage unconstrained commitment of a manufacturer to some arbitrary wholesale contract. For the purpose of this paper, notably the following is important. Antitrust authorities may consider manufacturers’ judicious choice of nonlinear wholesale contracts as "inducements" that are meant to ensure retailers’ adherence to high recommended retail prices and as such functionally equivalent to RPM - and thus as ultimately illegal. Direct evidence for such a strict practice comes from recent guidance of the German competition authority to manufacturers and retailers (Bundeskartellamt 2010, Becker 2013), which made explicit reference to restrictions on such inducements.\(^{11}\) Such a wide enforcement of a ban on RPM and on other means through which manufacturers may exert control over prices should further justify the comparison of the regimes of retailer "price ownership" and manufacturer "price ownership" in this paper.

Motivation 2: Second, the full commitment that the more standard timing presumes, where detailed wholesale contracts are set initially, may also lack realism in many settings. After all, outside fixed quantity commitments, even the signing of yearly framework agreements does not prevent retailers from continuously asking for (lump-sum) rebates and trying to renegotiate contracts. To back up such demands a retailer could then simply stop ordering the product ("running out of stock"). In fact, the respective demands

\(^{11}\)Other jurisdictions are arguably more permissive about the use of practices that are, to some extent, functionally equivalent to RPM. Notably in this respect is the use of recommended retail prices combined with a threat of suspension of delivery under the so called "Colgate doctrine" in the US, which is not permitted in Europe (cf. for a comparison Waelbroeck 2006).
made in particular by powerful retailers have been put repeatedly under antitrust scrutiny by agencies that are concerned with the exercise of buyer power. When at this stage retail prices have already been communicated (e.g., through prospects), they determine the stakes at this renegotiation stage, while initial wholesale contracts could be largely inconsequential. This again provides some justification for our choice of timing. Also, to the extent that wholesale contracts are open for renegotiations in the described sense, they are no substitute for a direct control over the retail price, so that RPM is, in this sense, also essential for the subsequently derived efficiencies.

Finally, our choice of timing has also the following additional benefit. It allows us to focus on the difference in retailers’ and the manufacturer’s preferences with respect to the role of the retail price as a means to communicate quality image. In our model, this is derived fully from primitives. As we show below, with exogenously fixed quality prices as well as the distribution of profits would not depend on the allocation of "price ownership", so that the difference is driven fully by the relationship between price and quality (image).

**Motivation of a Bargaining Solution Concept.** We now introduce multilateral negotiations at the beginning of $t = 3$. A large part of the literature typically assumes that one side, notably the manufacturer, can commit to make take-it-or-leave-it offers, thereby reducing all other parties to their outside options. An important shortcoming of this approach, in particular in light of our application, is that thereby the outside option of the proposing party and with this any (price-setting) strategies to affect it would not come into play. De Fontenay and Gans (2013), building on Stole and Zwiebel (1996), represents one recent contribution in the literature that allows for proposals by both sides in a noncooperative (open-ended) bargaining game. There is also a literature that applies cooperative game theory to, generally speaking, bargaining in networks with externalities (that is, in our context, when retailers are in competition).

It turns out that applied to our setting, the general approach that we propose next yields the same outcome as when we applied the non-cooperative approach of De Fontenay and Gans (2013) or also one of the existing cooperative approaches (discussed below). We

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12Retail prices could also be relatively persistent as, outside clearly specified promotional activities, retailers - and even more so manufacturers - may indeed want to provide such a consistent price image. There are also menu costs associated with a change of retail prices, as emphasized in the macroeconomic literature (cf. recently Nakamura and Steinsson 2008). The picture of a rather consistent price - in particular, outside promotions - is also confirmed by Hosken and Reiffen (2004). For a model that derives a relative persistence of prices from consumers’ preferences (given loss aversion) see Heidhues and Köszegi (2008).
feel, however, that our approach, which builds on Inderst and Wey (2003), is, at least in the present application, particularly straightforward and intuitive. To formalize it, denote the manufacturer by $M$ and each retailer by $R_n$ with $n = 1, ..., N$. As we will for $t = 1$ focus on a symmetric price equilibrium and as retailers will also be otherwise symmetric, we can restrict our subsequently introduced notation to the case where at most one retailer is different (through charging a different price). Without loss of generality we let this be retailer $R_1$, so that at most $p_1 \neq p$. A possible set of agreements can thus be described by the total number of agreeing retailers ($i \leq N$) and by whether this contains $R_1$. Denote by $\Pi_{in}^1(i)$ the joint profits of the ("Insider") coalition of $i$ retailers that agrees with $M$, where the superscript 1 denotes that this contains $R_1$. The respective notation when this does not contain $R_1$ is $\Pi_{in}^0(i)$, i.e., with superscript 0. For all non-agreeing outsiders denote the individual profit by $\pi_{out, 1}^1(i)$ when $R_1$ agrees and when $R_1$ does not agree by $\pi_{out, R_1}^0(i)$ for $R_1$ and by $\pi_{out, Rn}^0(i)$ for all other $N - i - 1$ non-agreeing retailers.

As at $t = 3$ contracts only serve the distribution of joint profits, we can suppress a separate notation for the respective transfers and characterize the outcome directly in terms of payoffs. A bilateral agreement now conditions, as in Inderst and Wey (2003), on what we call a "contingency", which is here the set of other retailers to which the manufacturer supplies. We can thus think of a bilateral agreement as, in our context, a vector of transfers that conditions on the manufacturer’s supply relations. While in equilibrium (cf. below) the manufacturer will supply all retailers, the transfers and resulting payoffs for all other contingencies will be payoff relevant as outside options. While we feel that such contingent contracting is also to some extent realistic, as we explore below, it can also be regarded more simply as part of a bargaining solution that thereby ensures that a party’s contribution to various subcoalitions also influences its equilibrium payoff.\footnote{After all, real-world bargaining will typically not follow a fixed protocol and may also vary considerably between different situations, so that, in our view, a solution concept should offer flexibility and incorporate much of the information of the specific economic situation.}

For a contingency where $M$ and $R_1$ agree, denote the respective payoff of the manufacturer by $V_{M}^1(i)$, that of $R_1$ by $V_{R_1}^1(i)$, and that of the other $i - 1$ agreeing retailers by $V_{Rn}^1(i)$. When $R_1$ and $M$ do not agree, the payoff of the manufacturer is denoted by $V_{M}^0(i)$ and that of the agreeing retailers by $V_{Rn}^0(i)$.

**Solution Concept.** For each contingency we require that the respective contracts and thus the resulting payoffs satisfy "balancedness": The incremental joint payoff, that is relative to the state without this additional agreement, must thereby be equal for both
sides. For an agreement with $R_1$ this is the case if and only if
\[ V_{M}^1(i) - V_{M}^0(i - 1) = V_{R_1}^1(i) - \pi_{Out,R_1}^0(i - 1), \tag{6} \]
which must hold for all $i$ (i.e., for all possible other agreements, as described by the number $i$ of all agreements). Recall also that $V_{M}^0(i - 1)$ represents the manufacturer’s disagreement payoff in this particular case, where there is no agreement with $R_1$, while the respective disagreement payoff for $R_1$ is the "outside option" $\pi_{Out,R_1}^0(i - 1)$. These values are subtracted to obtain the respective incremental payoff for $M$ (the left-hand side of (6)) and the respective incremental payoff for $R_1$ (the right-hand side of (6)).

Turning to an agreement with any other retailer $R_n$, we must make the following distinction: For a contingency where there is also agreement with $R_1$, equal division of the incremental joint payoff, now between $M$ and $R_n$, requires that
\[ V_{M}^1(i) - V_{M}^1(i - 1) = V_{R_n}^1(i) - \pi_{Out,R_n}^1(i - 1), \tag{7} \]
while for a contingency without an agreement with $R_1$ we have likewise that
\[ V_{M}^0(i) - V_{M}^0(i - 1) = V_{R_n}^0(i) - \pi_{Out,R_n}^0(i - 1). \tag{8} \]
Conditions (6) to (8) complete the characterization of our solution concept.

The following characterization will be restricted to prices set in $t = 1$ that ensure that joint profits are maximized with the "grand coalition", $i = N$.\(^{14}\) In light of future calculations we also derive a simplified characterization for the case where all retailers are symmetric as also $p_1 = p$. Then, profits of "stand-alone" retailers, which have not agreed to be supplied by $M$, only depend on the number of agreeing retailers and can be more simply denoted by $\pi_{Out}(i)$, while joint profits for the agreeing coalition can, again by symmetry, be written as $\Pi_{In}^1(i) = i \pi_{In}(i)$.

**Proposition 1** When there are $N$ retailers, suppose that at $N - 1$ retailers the same price $p$ prevails, while at most one retailer chooses potentially a different price. Assume without loss of generality that the latter retailer is $R_1$. The bargaining outcome in $t = 3$ must satisfy the imposed property of "balancedness" across all contingencies (i.e., all possible sets of agreements). Then, the manufacturer’s payoff equals
\[ V_{M}^* = \frac{1}{N(N + 1)} \left[ \sum_{i=0}^{N-1} (i + 1) \left[ \Pi_{In}^1(i + 1) - \pi_{Out,R_1}^0(i) - i \pi_{Out}(i) \right] + \sum_{i=1}^{N-1} \sum_{j=1}^{i} \left[ \Pi_{In}^0(j) - j \pi_{Out,R_n}^0(j - 1) \right] \right]. \tag{9} \]
\(^{14}\)This implies, in particular, that the possibly deviating price $p_1$ does not fall too short of $p$. This restriction is without loss of generality, provided the alternative supply option is sufficiently unattractive.
and with symmetric retailers and thus also $p_1 = p$ abbreviates to

$$V_M^* = \frac{1}{N + 1} \sum_{i=1}^{N} i [\pi_{In}(i) - \pi_{Out}(i - 1)] .$$

(10)

The payoff of retailer $R_1$ is

$$V_{R1}^* = V_M^* + \pi_{Out, R1}^0 (N - 1) - \frac{1}{N} \sum_{i=1}^{N-1} [\Pi_{In}^0(i) - i \pi_{Out,Rn}^0(i - 1)] ,$$

(11)

and with symmetry it equals the payoff of all other retailers $R_n$ and can be written as

$$V_R^* = V_M^* - \frac{1}{N} \left[ \sum_{i=1}^{N-1} i \pi_{In}(i) - \sum_{i=1}^{N} i \pi_{Out}(i - 1) \right] .$$

(12)

**Proof.** See Appendix.

The composition of payoffs is particularly transparent in case of symmetry, i.e., with expressions (10) and (12). The manufacturer’s payoff from negotiations thus represents a weighted average, that is over all possible coalitions, of the incremental value that is generated at any given retailer. With respect to $R_1$, whose price choice we will later consider, in (12) we have expressed the equilibrium payoff $V_{R1}^*$ so that the difference to that of the manufacturer, $V_M^*$, is immediately seen. We comment on this difference in detail below, as it will drive the difference in price setting incentives. That is, the following term will essentially describe the difference in incentives: $V_M^* - V_R^*$, which is

$$\frac{1}{N} \left[ \sum_{i=1}^{N-1} i \pi_{In}(i) - \sum_{i=1}^{N} i \pi_{Out}(i - 1) \right] .$$

(13)

Intuitively, the manufacturer’s payoff relative to that of any retailer is higher when his product generates a higher incremental surplus, i.e., when it generates a higher gross surplus in each bilateral relationship (i.e., the first term in (13)) and when a retailer’s alternative is less profitable (i.e., the second term in (13)), again weighted over all possible coalitions.\(^{15}\) Our subsequent analysis reveals that our qualitative insights are more general, relying on the insight that quality and quality perceptions affect differently retailers’ and the manufacturer’s payoffs both with and without an agreement.

\(^{15}\)Navarro (2007) derives a bargaining value for general networks with externalities by extending the “fair allocation” or “balancedness” rule of Myerson (1977a/b). This requirement mirrors the equal sharing of net surplus, which we impose for all contingencies (on which the agreement conditions in our approach). Applying Navarro’s bargaining value to our specific setting yields Proposition 1. As noted already above, this value is also obtained in De Fontenay and Gans (2013) from a non-cooperative approach, where an individual disagreement restarts negotiations. All these specifications thus ensure that equilibrium payoffs depend more broadly on the potential profits achieved under various possible agreements (or coalitions).
**Auxiliary Case: Exogenous Quality.** Suppose for a moment that quality was exogenously given. We now argue that then at \( t = 1 \) the same price equilibrium is obtained regardless of whether prices \( p_n \) are chosen by individual retailers or by the manufacturer. To see this, we consider a variation in \( p_1 \). Observe now that in the difference in profits

\[
V^*_M - V^*_R1 = \frac{1}{N} \sum_{i=1}^{N-1} \left[ \Pi^0_{rn}(i) - i\pi^0_{Out,Rn}(i-1) \right] - \pi^0_{Out,R1}(N-1)
\]

(14)

none of the terms depends directly on \( p_1 \) simply as they all relate to contingencies where there is no agreement with retailer \( R1 \). Consequently, we have indeed

\[
\frac{\partial}{\partial p_1} V^*_M - V^*_R1|_{q \text{ fixed}} = 0.
\]

(15)

This observation is important as it ensures that the subsequently discussed conflict of interest between retailers and the manufacture with regards to retail prices is due only to the implications that prices have for quality and quality perception.

4 Image Spillover

In this section, we abstract from downstream retail competition. Without competition, the profits that are realized at any given retailer do not depend directly on whether the manufacturer’s product is also sold at other retailers (and at what price). This allows to isolate the aforementioned free-riding problem, on which this section focuses. We still suppose that consumers are aware of the prices that have been set at all other retailers. For instance, this could be through advertising, but also through other shopping trips that are, however, not considered to be substitutes (e.g., as the product is not storable and consumers only decide on the basis of convenience at any given instance).

As in Section 2 the first step in the analysis is to derive the equilibrium quality \( \hat{q}^* \) at \( t = 2 \) for given prices \( p_n \). This is accomplished by requiring that the manufacturer’s optimal choice \( q_{BR} \) equals the expectations of uninformed consumers \( \hat{q} \). The only difference is that now, in the presence of retailers, \( q_{BR} \) maximizes \( V^*_M \), as given in Proposition 1. We relegate the formal details to the proof of Proposition 2. For given \( \hat{q}^* \), as a function of prices \( p_n \), we can next use the present specification that retailers serve separate markets so as to heavily simplify the expressions for payoffs and to subsequently solve for equilibrium prices.

Without competition, profits realized at an individual retailer do not directly depend on choices at other retailers and we can thus write, even when \( p_1 \neq p \), each bilateral profit
as

$$\pi(p_n, \hat{q}^*) = [p - k(\hat{q}^*)][D(p, \hat{q}^*) - k(\hat{q}^*)].$$

Note also that with competition "outside option" payoffs (without a bilateral agreement with the manufacturer $M$) for each retailer are independent of prices at all other retailers and also of the manufacturer’s quality (and can thus be simply denoted by $\pi_{Out}$). With this at hands, without competition we have from Proposition 2 for the manufacturer

$$V_M^* = \frac{1}{2} [\pi(p_1, \hat{q}^*) + (N - 1)\pi(p, \hat{q}^*) - N\pi_{Out}].$$

For $R1$ we obtain

$$V_{R1}^* = V_{R1}(N) = V_M^* + \pi_{Out} - \frac{1}{N} \sum_{i=1}^{N-1} i [\pi(p, \hat{q}^*) - \pi_{Out}],$$

so that we have for the difference in the respective price-setting incentives

$$\frac{d}{dp_1} [V_M^* - V_{R1}^*] = \frac{N - 1}{2} \frac{d\hat{q}^*}{dp_1} \left[ \frac{\partial}{\partial \hat{q}^*} \pi(p, \hat{q}^*) \right]. \quad (16)$$

When expression (16) is strictly positive, then this implies that the manufacturer prefers a strictly higher price. Appealing to symmetry, the respective choice $p_n = p_M^*$ at $t = 1$ is then strictly higher than the equilibrium outcome when retailers individually choose prices, $p_n = p_R^*$. In the proof of Proposition 2 we show that still $\frac{d\hat{q}^*}{dp_1} > 0$, as expected from the analysis in Section 2. And we also show that $\frac{\partial}{\partial \hat{q}^*} \pi(p, \hat{q}^*) > 0$ holds if and only if from $\gamma < 1$ there is a commitment problem, again in complete analogy to the analysis without retailers in Section 2. Hence, in the present case without competition between retailers, a wedge between $p_M^*$ and $p_R^*$ exists if and only if quality perceptions indeed matter as quality is not perfectly observed by all consumers. Then, expression (16) captures the fact that only the manufacturer, but not individual retailers, internalizes the positive effect that a change in quality and quality perceptions has on the sales at all (other) retailers.

**Proposition 2** Consider the case where the manufacturer sells through $N$ non-competing retailers. Then, the manufacturer’s preferred retail price, $p_M^*$, is the same as that chosen individually by retailers, $p_R^*$, only when there is no role for quality perceptions as $\gamma = 1$. Otherwise, both price and quality are strictly higher when the manufacturer controls retail prices: $p_M^* > p_R^*$ and $q_M^* > q_R^*$.

**Proof.** See Appendix.
Proposition 2 is silent about implications for welfare and consumer surplus, to which we turn below. It embodies our first channel through which a manufacturer’s control over retail prices, notably through RPM, has implications for prices and quality even when retailers do not perform non-contractible actions, as the opportunism problem lies between the manufacturer and consumers, and even when intrabrand competition is not an issue, as seen by the fact that with fixed quality prices do not differ \( p_M^* = p_R^* \). The reason is a free-riding problem of retailers on the effect that individual retail prices have on quality perceptions. This is the essence of the "image theory" of RPM, as discussed in the Introduction, where in our model "image" relates to quality perceptions.

Note next that the manufacturer’s preferred choice, \( p_M^* \), is not affected by the number of non-competing retailers, while the retailers’ free-riding problem is aggravated as their number \( N \) increases.

\textbf{Corollary 1} When quality perceptions matter as \( \gamma \in (0, 1) \), the difference \( p_M^* - p_R^* > 0 \) is strictly increasing in \( N \). Consequently, also the difference in equilibrium quality, \( q_M^* - q_R^* > 0 \), is strictly increasing in \( N \).

\textbf{Proof.} See Appendix.

It is worthwhile to note that while \( p_M^* \) maximizes the manufacturer’s profits, the price that prevails under retailer "price ownership", \( p_R^* \), does not maximize the joint profits of retailers. In fact, retailers would jointly be best off when setting \( p_n = p_M^* \), i.e., exactly the price that would prevail under manufacturer "price ownership"! As is the essence of a free-riding problem, they have however a private incentive to deviate by setting a strictly lower price at their own market. The coincidence of \( p_M^* \) and the symmetric price that would maximize total retailer profits does no longer hold when we introduce retailer competition in Section 5.

\textbf{Welfare Comparison.} We now consider the case of linear demand \( D(p_n, q) = q - p_n \) and constant marginal cost of production \( \frac{1}{2} q^2 \). We relegate a full derivation of expressions to the subsequent proof. For \( t = 2 \) we obtain now explicitly

\[
\hat{q}^* = \frac{\bar{p} + \sqrt{\bar{p}^2 + 4\bar{p}2\gamma (2 + \gamma)k}}{2 + \gamma}.
\]

where \( \bar{p} = \frac{1}{N} \sum_{n=1}^{N} p_n \). The first-order condition for \( p_n = p = p_R^* \) becomes

\[
\left( \frac{1}{N} \right) \left( \gamma k + \hat{q}^* \right) \left( p - \frac{\hat{q}^* (3\hat{q}^* - 2p)}{2k} \right) + \frac{(\hat{q}^*)^2}{2k} - 2p + \hat{q}^* = 0.
\]
The respective condition for $p_n = p = p^*_M$ is also obtained from (18) when setting $N = 1$. This follows simply as, first, with $N = 1$ there is no difference between the two cases and as, second, $p^*_M$ does not depend on $N$, as noted above.

For a particular numerical specification, Figure 1 plots the two prices $p^*_M$ and $p^*_R$ as a function of the degree of transparency about quality, $\gamma$. For this example only we have made the difference between the two scenarios largest, namely by letting $N \rightarrow \infty$. Then, the retailers’ equilibrium choice of $p^*_R$ does not internalize at all the implications that prices have on quality perception.

Prices are always the same when $\gamma = 1$, as then quality perceptions do not matter. For all lower values of $\gamma > 0$ we have $p^*_M > p^*_R$. In Figure 1 we have also characterized the prices that would maximize total welfare and consumer surplus, $p^*_W$ and $p^*_CS$. Intuitively, it holds that $p^*_W > p^*_CS$, as total welfare is not affected by mere transfers from consumers to firms. Further, at $\gamma = 1$ note that both efficiency benchmark prices are strictly lower than the price that would be obtained in the market, $p^*_M = p^*_R$. We now compare the retailers’ individually preferred price $p^*_R$ with the two efficiency benchmarks. As $\gamma$ becomes sufficiently low, $p^*_R$ falls below both the price $p^*_W$ that would maximize total welfare (when $\gamma < \gamma_W$) and the lower price $p^*_CS$ that would maximize consumer surplus (when $\gamma < \gamma_{CS}$). The deadweight loss that a higher price would entail and even the direct loss to consumers, at least for price increases that are not too large, is more than compensated by the resulting increase in equilibrium quality. Proposition 3 shows that these results hold generally for our specification.

The implications for consumer surplus are also brought out more clearly in Figure 2. The non-monotonic line plots the difference between consumer surplus at $p^*_R$ and $p^*_CS$, $CS(p^*_R) - CS(p^*_CS)$. It is zero at at $\gamma = 0$, where demand becomes zero, and at $\gamma = \gamma_{CS}$, while it is otherwise strictly negative. More important is the intersection with the difference $CS(p^*_M) - CS(p^*_CS)$, indicating that $CS(p^*_M) > CS(p^*_R)$ holds for low levels of transparency $0 < \gamma < \gamma'_{CS}$ and $CS(p^*_M) < CS(p^*_R)$ holds for high levels of transparency $\gamma'_{CS} < \gamma < 1$. While to save space we do not plot this separately, the same picture is obtained when we consider total welfare: For all $\gamma$ below some threshold $\gamma'_W$ welfare is higher when the manufacturer controls the price. This threshold is intuitively (considerably) higher than $\gamma'_{CS}$.

**Proposition 3** In the chosen linear-quadratic specification, the following comparison of prices and efficiency (in any given market) holds:

i) For low levels of transparency $\gamma$, the price that prevails when retailers set prices is strictly
Figure 1: This figure shows $p^*_M$ and $p^*_R$, next to the prices that maximize welfare, $p^*_W$, and consumer surplus, $p^*_CS$, as a function of $\gamma$. Parameter values are $k = 2$ and $N = \infty$.

Figure 2: Consumer surplus for $p^*_M$ and $p^*_R$, net of maximum consumer surplus at $p^*_CS$, as a function of $\gamma$. Parameter values are $k = 2$ and $N = \infty$. 
below both the price that would maximize consumer surplus and the price that would maximize total welfare \( (p^*_R < p^*_CS \text{ and } p^*_R < p^*_W) \), while for high values it is strictly higher than both \( (p^*_R > p^*_CS \text{ and } p^*_R > p^*_W) \). The price prevailing when the manufacturer sets prices is always strictly higher than both efficiency benchmarks \( (p^*_M > p^*_CS \text{ and } p^*_M > p^*_W) \).

ii) For low levels of transparency \( \gamma \), welfare and also consumer surplus are strictly higher under the price that the manufacturer sets \( (W(p^*_M) > W(p^*_R) \text{ and } CS(p^*_M) > CS(p^*_R)) \), while for high values the opposite holds such that welfare and consumer surplus are strictly higher under the price that prevails when retailers set prices \( (W(p^*_M) < W(p^*_R) \text{ and } CS(p^*_M) < CS(p^*_R)) \).

**Proof.** See Appendix.

Together, the comparison of consumer surplus and welfare brings out two key insights. The first is that from the perspective of both consumer surplus and welfare, efficiency can be higher under \( p^*_M \) even though this exceeds \( p^*_R \). The reason is the thereby induced increase in quality. The second insight is the role of quality perceptions and transparency. In the linear-quadratic specification, the free-riding problem under retailer "price ownership" becomes sufficiently important when the fraction of consumers who do not observe quality is sufficiently large. In this case consumer surplus and total welfare are both strictly lower when retailers individually choose prices in their own interest. The opposite holds when transparency is sufficiently high, so that perceptions matter less.

## 5 Retailer Competition

Given our preceding analysis, we turn right to the choice of prices in \( t = 1 \). Recall that we are interested in the difference in price setting incentives between the manufacturer and individual retailers. As we already observed that for some exogenously fixed quality level incentives are the same, again only the indirect effect matters, i.e., the effect that prices have on quality. That is:

\[
\frac{d}{dp_1} [V^*_M - V^*_R_1] = \frac{d \bar{q}^*}{dp_1} \frac{d}{dq^*} [V^*_M - V^*_R_1] \\
= \frac{d \bar{q}^*}{dp_1} \left[ \frac{1}{N} \sum_{i=1}^{N-1} \frac{d}{dq^*} \pi_{In}(i) - \frac{1}{N} \sum_{i=1}^{N} \frac{d}{dq^*} \pi_{Out}(i-1) \right].
\]

(19)

We assume again concavity of profits, now of \( \pi_{In}(i) \) and \( \pi_{Out}(i) \) in \( \bar{q}^* \). The second sum in (19) captures the effects that a higher induced quality level has on the profits of non-agreeing retailers under various contingencies (i.e., depending on the number of agreeing
Without competition, these terms were equal to zero. With competition, these terms generate a wedge between the preference of the manufacturer and that of any retailer when

\[ \frac{d}{dq_i} \pi_{Out}(i) < 0 \text{ for } 1 \leq i \leq N - 1, \]

as then a higher quality of the manufacturer’s product undermines the "outside option" of a non-agreeing retailer. This makes the manufacturer prefer a higher retail price as the thereby induced higher quality reduces retailers’ profits when they do not come to an agreement with the manufacturer. This ultimately benefits the manufacturer. The presently discussed effect thus isolates another mechanism through which a conflict of interest between retailers and the manufacturer can lead to different preferences regarding the retail price.

Without competition, we know already that the first term in (19) is zero when \( \gamma = 1 \) and otherwise strictly positive ("free riding" problem). More generally, it captures the effect that quality has on the joint profits of agreeing retailers, again summed up over all possible contingencies, that is, whether one or up to \( N - 1 \) retailers agree.\(^\text{17}\) At first it may seem intuitive that all expressions \( \frac{d}{dq_i} \pi_{In}(i) \) should be positive, i.e., that a higher quality increases "insiders'" profits, in particular when there is competition. However, this ignores the fact that the provision of quality is costly. To sign also this term and thus ultimately the whole expression, we undertake two separate analysis. We first derive general clear-cut implications when the level of transparency is low, so that perceptions matter a lot. Then, we derive implications for the specific case with linear demand.

Before turning to these analyses, the overall nature of the game considered in this paper should be recalled, as the present "reduced form" representation may mask the underlying complexity. We consider and compare the equilibrium outcomes when either the manufacturer or all retailers simultaneously set retail prices in \( t = 1 \), which affect demand both directly and indirectly through the induced quality equilibrium. The latter in turn is shaped by both the manufacturer’s incentives, which depend on the outcome of the subsequent negotiations with all \( N \) retailers, as well as the quality perceptions that the choice of all prices together induce at consumers who do not directly observe quality. Perceptions and the actual chosen quality must, of course, coincide in equilibrium.

\(^\text{16}\)Note for completeness that clearly \( \frac{d}{dq_i} \pi_{Out}(0) = 0 \) as then there is no retailer that offers the incumbent manufacturer’s product.

\(^\text{17}\)The respective term for \( N \) agreeing retailers dropped out as here we consider the difference in profits between the manufacturer and a retailer.
Transparency. Recall that for $\gamma < 1$ there is an opportunism problem vis-à-vis consumers who do not directly observe quality. Precisely, while for given prices the choice of $q_{BR}$ maximizes the manufacturer’s profits, $V^*_M$, this is not so for the resulting equilibrium quality $\bar{q}^*$. While in equilibrium it holds that $\bar{q}^* = q_{BR}$, $\bar{q}^*$ only satisfies the respective first-order condition when $\gamma = 1$. Consider now instead the auxiliary problem to choose quality $\bar{q}^*$ so as to maximize the profits of an agreeing coalition with $i$ retailers, $\pi_{In}(i)$. As $\gamma$ becomes sufficiently small, the equilibrium quality is always too low, given the opportunism problem, regardless of the choice of $i = 1, ..., N$. Appealing to concavity, also the first term in (19) is then strictly positive.

**Proposition 4** Consider the general case where retailers can be in competition. When $\gamma > 0$ is sufficiently small so that a large fraction of consumers must rely on quality perceptions, the manufacturer’s preferred retail price and the thereby induced quality are both strictly higher than when retailers individually choose their preferred retail price.

Hence, when transparency is low so that quality perceptions matter a lot, we can sign both terms in expression (19). The manufacturer’s optimal choice of the retail price will then be strictly higher than that of retailers, implying also a strictly higher quality, both as the manufacturer cares about the positive impact of quality on his bargaining position (the second sum in (19)) and as the manufacturer cares more about the problem of underprovision of quality due to the opportunism problem vis-à-vis consumers (the first term in (19)).

Linear Demand. We now return to our specification with linear demand, for which we extend the quadratic utility function to the case with $N$ differentiated retailers. Demand at retailer $n$ is given by

$$(q_n - p_n) - \delta \sum_{m \neq n} (q_m - p_m). (20)$$

We are now specific about the choices made by non-agreeing retailers. Also applying the same timing of moves, we suppose that then consumers can buy a good of fixed quality $q_0$ and price $p_0$ at these retailers. Considering again symmetric prices for the manufacturer’s products, joint profits generated with one of $i$ agreeing retailers are then given by

$\pi_{In}(i) = [(1 - \delta (i - 1)) (\bar{q}^* - p) - \delta (N - i) (q_0 - p_0)] (p - k (\bar{q}^*)).$

---

18That is, a representative consumer derives utility of $u(x, q) = \sum_{n=1}^{N} [q_n x_n - \frac{1}{2} \beta x_n^2] - \phi \sum_{m \neq n} x_n x_m$, where $x_n$ denotes the quantity bought at retailer $n$. Setting $\partial u/\partial x_n = p_n$ and solving for $x_n$ yields (20), where $\delta = \phi / (\beta - \phi) (\beta + (N - 1) \phi)$ and $\beta$ and $\phi$ are such that $\frac{\beta + (N-2) \phi}{(\beta - \phi)(\beta + (N-1) \phi)} = 1$. 

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Profits of each one of the \((N - i)\) non-agreeing retailers are given by
\[
\pi_{Out}(i) = [(1 - \delta (N - i - 1)) (q_0 - p_0) - \delta i (\bar{q} - p)] (p_0 - k_0).
\]

**Proposition 5** Consider the case with linear demand and competition. Then for all \(\gamma > 0\) the manufacturer’s preferred retail price and the thereby induced quality are both strictly higher than when retailers individually choose their preferred retail price.

**Proof.** See Appendix.

Incidentally, competition can now also lead to "gold-plating" in terms of quality by the manufacturer, as he thereby improves his bargaining position. That is, the quality choice that is induced by the higher price set by the manufacturer strictly exceeds an efficiency benchmark. To illustrate this, take \(\gamma = 1\), such that equilibrium quality maximizes \(V_M^*\) and, by the envelope theorem, that the manufacturer’s preferred price is determined by setting the partial derivative with respect to price equal to zero. With the linear demand specification, note that a marginal price reduction has the same demand expanding effect as a marginal increase in quality. Therefore, by optimality, also the marginal cost of an increase in quality \((k'(q))\) has to be equal to the "marginal cost" of a price reduction, which is equal to one. Using again \(k'(q) = \frac{1}{2k}q^2\), this yields \(q_M^* = k\) (provided that \(p = p_M^*\)). Note next that total welfare is now
\[
W = N (1 - \delta) \left[\frac{1}{2} (q - p)^2 + (q - p) (p - k(q))\right].
\]
For given symmetric price \(p\) the welfare maximizing quality would thus be
\[
q_W = \frac{2}{3} (k + p),
\]
which is strictly decreasing in \(p\). Evaluated at \(p = p_M^*\), as this is strictly decreasing in \(\delta\), this suggests that quality may be indeed too high when there is sufficient competition. This can be confirmed with respective threshold
\[
\delta'' = \left(\frac{3}{2(N - 1)}\right) \frac{k}{k + q_0 - p_0 + 2(p_0 - k_0)}.
\]
Still, despite such "gold plating", here for \(\delta > \delta''\), we can show that the outcome may still be more efficient than when price and quality are lower under the respective prices

\[^{19}\text{Precisely: } \frac{d p_M^*}{d \delta} = \frac{-3}{2(3 - 25)^2} [(q_0 - p_0) + 2(p_0 - k_0)].\]
choices of retailers.\(^{20}\) However, in the linear specification there is no monotonic relationship between the welfare comparison and the degree of competition. Still, our observations in this section further confirm the paper’s main message of identifying an efficiency rationale for RPM through the described "image theory", though we can not provide guidance as to whether this should be more or less applicable depending on the degree of downstream competition.\(^{21}\)

6 Concluding Remarks

We consider a manufacturer’s incentives to choose quality in an environment where this is not observed directly by all consumers, so that their quality perceptions matter. We further ask how these incentives are influenced by the product’s price. Here, we derive various channels through which a higher price can induce higher quality and quality perception by those consumers who do not directly observe quality but form rational expectations. This set-up is then embedded into a game where either retailers or the manufacturer control the retail price ("price ownership").

We isolate different sources of a conflict of interest between retailers and the manufacturer, which lead to different prices and qualities depending on which side exerts control over prices. One channel that supports such a conflict of interest works through a reputation spill-over across retailers. Each individual retailer does not take into account how his price choice affects the overall perception of the product’s quality and, thereby, also equilibrium quality choice. Then, "price ownership" of the manufacturer leads to higher quality. Though this is associated with a higher price, we identify when efficiency and consumer surplus are both still higher. A second channel supporting a conflict of interest between the manufacturer and retailers operates when there is competition between retailers. Then, a higher (perceived) quality, which would be induced by a higher retail price, reduces a retailer’s but increases the manufacturer’s outside option in bilateral negotiations. Again, the manufacturer’s preferred price and the thereby induced equilibrium quality are higher than when retailers control prices.

\(^{20}\)Such an example is obtained for \(k = 2, N = 2\), as well as \(p_0 = 0.5, q_0 = 1.5, k_0 = 0.1\) for the outside option, in which case we have \(\delta'' \approx 0.79\). Welfare is then higher under \(p^o_M\) than under \(p^o_R\) if \(\delta > 0.24\).

\(^{21}\)It may be conjectured that in the presence of upstream competition, there may be less scope for welfare losses resulting from higher prices. However, the interaction of asymmetric information about quality, up- and downstream competition, and multilateral negotiations has so far proved to be insufficiently tractable. Modifying our approach or complementing the analysis with a different modelling set-up should be an interesting avenue for future research.
As we discussed in detail, we chose a particular timing of strategies for our model. Notably, this deprived wholesale prices of their role to better align incentives. We argued why this could follow from a strict implementation of the prohibition of RPM. We also noted that the pressure from continuous negotiations, for instance, may make retail prices indeed more persistent than wholesale arrangements. The key implication that derives from our modeling specification is that retail prices affect the manufacturer’s incentives and thereby both quality and quality perceptions. Future work could model the dynamic interplay between wholesale and retail prices and the continuous choice of quality, taking into account that all these choices are to some extent (and only to some extent) persistent. In particular, quality and quality perceptions would then be the result of current as well as of past price choices at all retailers.

7 Appendix A: Omitted Derivations and Proofs

Proof of Proposition 1. For contingencies without an agreement with $R_1$, we have the joint profit condition

$$V_M^0(i) + iV_{Rn}^0(i) = \Pi_{In}^0(i),$$

which together with (8) yields

$$(i + 1)V_M^0(i) = iV_M^0(i - 1) + [\Pi_{In}^0(i) - i\pi_{Out,Rn}^0(i - 1)].$$

From this we obtain for the manufacturer

$$V_M^0(i) = \frac{1}{i + 1} \sum_{j=1}^i [\Pi_{In}^0(j) - j\pi_{Out,Rn}^0(j - 1)].$$

(21)

When there is an agreement with $R_1$, we have the joint profit condition

$$V_M^1(i) + (i - 1)V_{Rn}^1(i) + V_{R1}^1(i) = \Pi_{In}^1(i).$$

Here, we can substitute (6) and (7), which yields

$$V_M^1(i) = \frac{1}{i(i + 1)} \sum_{j=0}^{i-1} (j + 1) [\Pi_{In}^1(j + 1) - j\pi_{Out}^1(j) - \pi_{Out,R1}^0(j) + V_M^0(j)].$$

Making use of (21) we obtain $V_M^1(i)$ and, together with (6) also $V_{R1}^1(i)$. Choosing $i = N$ yields the final characterization.\footnote{Again, we suppose here that the incremental profit for each agreement is, for all contingencies, positive.} Q.E.D.
Proof of Proposition 2. Denote by \( \pi(p_n, q, \hat{q}) \) the bilateral profits for an agreement with retailer \( n \), which depend only on the respective price \( p_n \) as well as actual and perceived quality. We will first make use of this notation to derive equilibrium quality at stage \( t = 2 \), \( \hat{q} = \hat{q} = q \). Subsequently, as done in the main text, we can further simplify notation by substituting for this choice. At \( t = 2 \) the manufacturer chooses \( q \) to maximize his own payoff. With the preceding observations and making use of Proposition 1, without competition we have

\[
V_M^* = \frac{1}{2} \left[ \sum_{n=1}^{N} \pi(p_n, q, \hat{q}) - N \pi_{Out} \right],
\]

so that

\[
q_{BR} = \arg \max_q \sum_{n=1}^{N} \pi(p_n, q, \hat{q}).
\]

Or, with the first-order condition as in (2), \( q_{BR} \) solves

\[
\gamma \left[ \sum_{n=1}^{N} (p_n - k(q_{BR})) D_q(p_n, q_{BR}) \right] - k'(q_{BR}) \sum_{n=1}^{N} \left[ \gamma D(p_n, q_{BR}) + (1 - \gamma) D(p_n, \hat{q}) \right] = 0.
\]

(23)

Recall that at this stage we solve for the equilibrium where a consumer who does not observe quality holds rational beliefs, so that \( q_{BR} = \tilde{q} = \tilde{q}^* \). Note again that profits in one retail market currently depend on the price in another retail market only through the effect that the other price has on the manufacturer’s incentives to adjust quality - and, for \( \gamma < 1 \), on the respective beliefs of uninformed consumers. Hence, the equilibrium requirement for \( \tilde{q}^* \) at \( t = 2 \) is that, in analogy to condition (3),

\[
z(p_n, \tilde{q}^*) := \gamma \left[ \sum_{n=1}^{N} (p_n - k(\tilde{q}^*)) D_q(p_n, \tilde{q}^*) \right] - k'(\tilde{q}^*) \sum_{n=1}^{N} D(p_n, \tilde{q}^*) = 0,
\]

(24)

so that from implicit differentiation

\[
\frac{d\tilde{q}^*}{dp_n} = \frac{1}{-z_{\tilde{q}^*}} \left( \gamma D_q(p_n, \tilde{q}^*) - k'(\tilde{q}^*) D_p(p_n, \tilde{q}^*) + \gamma (p_n - k(\tilde{q}^*)) D_{pq}(p_n, \tilde{q}^*) \right).
\]

As in Lemma 1 this is strictly positive when, where we have rearranged terms,

\[
z_{\tilde{q}^*} = \gamma \left[ \sum_{n=1}^{N} [(p_n - k(\tilde{q}^*)) D_{aq}(p_n, \tilde{q}^*) - 2k'(\tilde{q}^*) D_q(p_n, \tilde{q}^*)] \right] - k''(\tilde{q}^*) \sum_{n=1}^{N} D(p_n, \tilde{q}^*)
\]

\[-(1 + \gamma)k'(\tilde{q}^*) \sum_{n=1}^{N} D_q(p_n, \tilde{q}^*)
\]

\(< 0.
\]
This holds both as the first term is strictly negative from concavity of the program for \(q_{BR}\) and the second term is strictly negative from \(k' > 0\) and \(D_q > 0\).

For the manufacturer’s optimal price choice we obtain

\[
\frac{dV^*_M}{dp_M} = D_p(p_n, \hat{q}^*) (p_n - k(\hat{q}^*)) + D(p_n, \hat{q}^*) + \sum_{n' = 1}^{N} [D_q(p_{n'}, \hat{q}^*) (p_{n'} - k(\hat{q}^*)) - D(p_{n'}, \hat{q}^*) k'(\hat{q}^*)] \frac{d\hat{q}^*}{dp_n} = 0. \tag{25}
\]

When some retailer \(n\) (notably \(R1\)) could optimally choose \(p_n\), after substituting into Proposition 1, this would satisfy the requirement

\[
\frac{dV^*_R}{dp_n} = D_p(p_n, \hat{q}^*) (p_n - k(\hat{q}^*)) + D(p_n, \hat{q}^*) + [D_q(p_n, \hat{q}^*) (p_n - k(\hat{q}^*)) - D(p_n, \hat{q}^*) k'(\hat{q}^*)] \frac{d\hat{q}^*}{dp_n} = 0. \tag{26}
\]

For \(\gamma = 1\) we have from (23) that

\[D_q(p_n, \hat{q}^*) (p_n - k(\hat{q}^*)) - D(p_n, \hat{q}^*) k'(\hat{q}^*) = 0,\]

so that from the respective first-order conditions and focusing on a symmetric outcome we have \(p^*_M = p^*_R\). For \(\gamma < 1\) note first that, again at the symmetric choice \(p_n = p\), it holds that

\[D_q(p_n, \hat{q}^*) (p_n - k(\hat{q}^*)) - D(p_n, \hat{q}^*) k'(\hat{q}^*) > 0,\]

from which, together with (26) and (25) as well as \(\frac{d\hat{q}^*}{dp_n} > 0\), we have in this case that \(p^*_M > p^*_R\). Q.E.D.

**Proof of Corollary 1.** An immediate and intuitive way to see that \(p^*_M\) is independent of \(N\) is to maximize \(V^*_M\) directly with respect to a symmetric price \(p\). Note first that at symmetric prices \(p_n = p\), we have that \(\frac{d\hat{q}^*}{dp_n} = N \frac{d\hat{q}^*}{dp}\) and further that \(\frac{d\hat{q}^*}{dp}\) is simply obtained from, with a slight abuse of notation,

\[z(p, \hat{q}^*) = \gamma [(p - k(\hat{q}^*)) D_q(p, \hat{q}^*)] - k'(\hat{q}^*) D(p, \hat{q}^*) = 0.\]

With this at hands, the first-order condition for \(p = p^*_M\) can be written as

\[D_p(p, \hat{q}^*) (p - k(\hat{q}^*)) + D(p, \hat{q}^*) + [D_q(p, \hat{q}^*) (p - k(\hat{q}^*)) - D(p, \hat{q}^*) k'(\hat{q}^*)] \frac{d\hat{q}^*}{dp} = 0 \tag{27}\]

and is thus indeed independent of \(N\). We next argue that, instead, \(p^*_R\) is strictly decreasing in \(N\). Using concavity of the program, from (26) this is the case when, at symmetric prices
\( p_n = p = p^*_R \), this holds for \( \frac{dq^*}{dp_n} \). As \( z_{pn} \) is independent of \( N \), we only need to verify that \( |z_{\tilde{q}}| \) is increasing, which holds as, using \( p_n = p \), it is proportional to \( N \). \textbf{Q.E.D.}

**Proof of Proposition 3.** Joint profits of the manufacturer together with retailer \( n \) are given by

\[
\pi(p_n, q, \tilde{q}) = (q_n - p_n) \left( p_n - \frac{1}{2k} \tilde{q}^2 \right),
\]

where we use \( q_n = \gamma q + (1 - \gamma) \tilde{q} \). The linear demand structure allows us to write the equilibrium condition for \( \tilde{q}^* \) in (24) in terms of the average retail price \( \bar{p} = \frac{1}{N} \sum_{n=1}^{N} p_n \):

\[
\gamma \left( \bar{p} - \frac{1}{2k} (\tilde{q}^*)^2 \right) - \frac{1}{k} \tilde{q}^* (\tilde{q}^* - \bar{p}) = 0, \tag{28}
\]

which can be solved for \( \tilde{q}^* \) as given by (17) and used in the first-order condition (18).

Further, to calculate the example, for \( N \to \infty \) expression (18) simplifies to

\[
\frac{(\tilde{q}^*)^2}{2k} - 2p^*_R + \tilde{q}^* = 0.
\]

To compare efficiency, from the utility function of the respective representative consumer we obtain in each market the consumer surplus

\[
CS = \frac{1}{2} (q - p)^2
\]

and total welfare

\[
W = \frac{1}{2} (\tilde{q}^* - p)^2 + (\tilde{q}^* - p) (p - k (\tilde{q}^*)).
\]

Consider assertion i). We will first show that \( p^*_M > p^*_{CS} \). To this end rewrite condition (27) for \( D(p, \tilde{q}^*) = \tilde{q}^* - p \) to obtain

\[
\left. \frac{dq^*}{dp} \right|_{p^*_M} = \frac{(p - k (\tilde{q}^*)) - (\tilde{q}^* - p)}{(p - k (\tilde{q}^*)) - (\tilde{q}^* - p) k' (\tilde{q}^*)} < 1. \tag{29}
\]

This follows from (17) which implies that \( D(p, \tilde{q}^*) > 0 \) only if \( p < 2k \), such that \( \tilde{q}^* < 2k \) and thus \( k' (\tilde{q}^*) < 1 \). The assertion that \( p^*_M > p^*_{CS} \) then follows from

\[
\left. \frac{dCS}{dp} \right|_{p^*_M} = (\tilde{q}^* - p) \left( \left. \frac{dq^*}{dp} \right|_{p^*_M} - 1 \right) < 0. \tag{30}
\]

Recall next from the proof of Proposition 2 that \( p^*_R = p^*_M \) for \( \gamma = 1 \), which implies that also \( p^*_R > p^*_{CS} \) for large values of \( \gamma \). Consider next the opposite end: \( \gamma = 0 \). There, we
have $p^*_{CS} = 0$ and that $\frac{dp^*_{CS}}{d\gamma} \to \infty$ as $\gamma \to 0$.\(^{23}\) Likewise, for $\gamma = 0$ we obtain $p^*_R = 0$ and
\[
\left. \frac{dp^*_R}{d\gamma} \right|_{\gamma = 0} = k \left( \frac{3\sqrt{N(9N - 8)} - \sqrt{\frac{9N - 8}{N}} + 9 - 7N}{4(N - 1)} \right),
\]
which is equal to $5/4 \left( \sqrt{5} - 1 \right) k$ for $N = 2$, it strictly decreases in $N$, and it converges to $k/2$ as $N \to \infty$. Given the boundedness of $\frac{dp^*_R}{d\gamma}$, we have thus shown that $p^*_{CS} > p^*_R$ must hold for small values of $\gamma$, regardless of the choice of $N$. (In fact, for $N \to \infty$ one can even solve in closed form for the threshold $\gamma_{CS} = \frac{2}{3}$.)

Furthermore, from (17) we obtain for $\gamma = 0$ that $\hat{q}^* = p$ and thus $\frac{d\hat{q}^*}{dp} = 1$, which in turn implies from (30) that $\left. \frac{dCS}{dp} \right|_{p^*_M} = 0$ so that since $W = CS + \pi$, we get $p^*_W = p^*_M$. Furthermore, solving the system of (18) (with $N = 1$) and (28) for the equilibrium values $p^*_M$ and $q^*_M$ yields that $p^*_M = \frac{3}{2} k$ for $\gamma = 0$. Hence, we have that $p^*_W > p^*_R$ for small values of $\gamma$. Since $W = \pi + CS$ it follows from (30) that $p^*_W < p^*_M$ for all $\gamma > 0$ and therefore, since $p^*_R = p^*_M$ for $\gamma = 1$, also $p^*_W < p^*_R$ for large values of $\gamma$.

Now turn to assertion ii) and consider again $\gamma = 1$ where it holds that $p^*_R = p^*_M$ so that $CS (p^*_R) = CS (p^*_M)$ (and also $W (p^*_R) = W (p^*_M)$). However, since at $\gamma = 1$, it holds that
\[
\frac{dW (p^*_R)}{d\gamma} = \frac{dCS (p^*_R)}{d\gamma} = \left( \frac{1}{N} \right) \frac{k^2}{36} < \frac{k^2}{36} = \frac{dCS (p^*_M)}{d\gamma} = \frac{dW (p^*_M)}{d\gamma},
\]
it follows that $CS (p^*_M) < CS (p^*_R)$ and $W (p^*_M) < W (p^*_R)$ for high values of $\gamma$.

Now consider $\gamma = 0$, where $CS (p^*_R) = CS (p^*_M) = 0$ and, further, $\frac{dCS (p^*_R)}{d\gamma} = \frac{dCS (p^*_M)}{d\gamma} = 0$. However, $\frac{d^2CS (p^*_R)}{d\gamma^2} = \frac{d^2CS (p^*_M)}{d\gamma^2} = \frac{4}{9} k^2$ and
\[
\frac{d^2CS (p^*_R)}{d\gamma^2} = k^2 \left( \frac{\sqrt{9N - 8} - 5\sqrt{N}}{16N} \right)^2,
\]
which, for $N = 2$ equals $0.477k^2 > \frac{4}{9} k^2$, but for $N = 3$ this equals $0.385k^2 < \frac{4}{9} k^2$. Furthermore, $\frac{d^2CS (p^*_R)}{d\gamma^2}$ is strictly decreasing in $N$ and converges to $1/4k^2$ as $N \to \infty$. Therefore, if $N > 2$, $CS (p^*_R) < CS (p^*_M)$ for small values of $\gamma$. Finally, we have $W (p^*_R) = W (p^*_M) = 0$ but $\frac{dW (p^*_R)}{d\gamma} = 0$ and $\frac{dW (p^*_M)}{d\gamma} = \frac{8}{27} k^2$ at $\gamma = 0$, so that $W (p^*_R) < W (p^*_M)$ for small values of $\gamma$. Q.E.D.

**Proof of Proposition 5.** With linear demand, we have
\[
V^*_M = \frac{N}{6} \left\{ \begin{array}{ll}
\frac{1}{2} \left[ (3 - 2\delta (N - 1)) (\hat{q}^* - p) - \delta (N - 1) (q_0 - p_0) \right] (p_k - k^* (\hat{q}^*) )
\end{array} \right\}
\]
\[
\left. \begin{array}{ll}
- \left[ (3 - \delta (N - 1)) (q_0 - p_0) - 2\delta (N - 1) (\hat{q}^* - p) \right] [p_0 - k_0]
\end{array} \right\}
\]

\(^{23}\)Precisely: $p^*_CS = (1 + \gamma) \sqrt{\gamma (2 + \gamma)} k - \gamma (2 + \gamma) k$. 

29
and
\[ V_M^* - V_R^* = \frac{(N + 1)}{6} \left\{ \frac{N+1}{N+1} (3 - 2\delta (N - 2)) (\tilde{q}^* - p) - \delta (N - 1) (q_0 - p_0) [p - k (\tilde{q}^*)] \right\} \]
\( - [(3 - \delta (N - 1)) (q_0 - p_0) - 2\delta (N - 1) (\tilde{q}^* - p)] [p_0 - k_0] \). \hspace{1cm} (31)

Now observe that from the first-order condition for \( q_{BR} \) we obtain with \( q_{BR} = \tilde{q} = \tilde{q}^* \) that
\[
(3 - 2\delta(N - 1)) [\gamma (p - k (\tilde{q}^*)) - (\tilde{q}^* - p) k' (\tilde{q}^*)] \\
+ \delta (N - 1) [(q_0 - p_0) k' (\tilde{q}^*) + 2\gamma (p_0 - k_0)] \\
= 0.
\]
This implies that
\[
(3 - 2\delta(N - 1)) [(p - k (\tilde{q}^*)) - (\tilde{q}^* - p) k' (\tilde{q}^*)] \\
\geq -\delta (N - 1) [(q_0 - p_0) k' (\tilde{q}^*) + 2(p_0 - k_0)], \hspace{1cm} (32)
\]
with a strict inequality for \( \gamma < 1 \). Differentiating (31) with respect to \( q \) yields
\[
\left. \frac{d (V_M^* - V_R^*)}{dq} \right|_{\tilde{q}^*} = \frac{(N + 1)}{6} \left\{ \frac{N+1}{N+1} (3 - 2\delta (N - 2)) [p - k (\tilde{q}^*) - (\tilde{q}^* - p) k' (\tilde{q}^*)] \\
+ \delta (N - 1) (q_0 - p_0) k' (\tilde{q}^*) + 2\delta (N - 1) [p_0 - k_0] \right\},
\]
from which, by using (32) and after some transformations, we obtain
\[
\left. \frac{d (V_M^* - V_R^*)}{dq} \right|_{\tilde{q}^*} \geq \delta (N - 1) \left( \frac{6 - 5\delta(N - 1)}{3 - 2\delta(N - 1)} \right) [(q_0 - p_0) k' (\tilde{q}^*) + 2(p_0 - k_0)] \\
> 0,
\]
where by (32) the first inequality holds even strictly for \( \gamma < 1 \) and the second inequality follows from \( \delta (N - 1) < 1 \). Hence, we can conclude that \( p_M^* > p_R^* \) and consequently \( q_M^* > q_R^* \) Q.E.D.
References


