Durable goods with quality differentiation

Roman Inderst *

University of Frankfurt (IMFS), Germany
London School of Economics, United Kingdom

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Abstract

We study the optimal strategy of a durable-goods monopolist who can offer goods in different qualities. The key finding is that the presence of the additional sorting variable further undermines the firm’s commitment problem, leading to results that contrast sharply with those of standard durable-goods models or those of models where the firm can commit.

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1. Introduction

The paper spells out the dynamic problem faced by a monopolist who faces a market of fixed size with heterogeneous consumers and can each period adjust both the price and the quality of his goods. Consequently, we combine two important strands of the literature on price discrimination: Second-degree price discrimination with commitment to a single offer (cf. Mussa and Rosen, 1978; Maskin and Riley, 1984) and intertemporal price discrimination in the “durable good” commitment problem (cf. Coase, 1972; Stokey, 1981; Bulow, 1982; Fudenberg et al., 1985; Gul et al., 1986). In our model with low- and high-valuation consumers, we thus allow a monopolistic firm to offer each period a range of qualities. Our key finding is that as the firm becomes sufficiently flexible in adjusting qualities and prices over time, it serves the whole market in the very first period, possibly even incurring a loss with the low-type consumers. Serving the low-type consumers by offering a low-quality version potentially at a loss in the first period represents a commitment that ensures higher profits from high-valuation consumers.

In the one-stage (second-degree price discrimination) commitment game, the firm would never sell below cost. (In fact, the virtual surplus with the low type must be strictly positive to ensure that the whole market is covered.) This feature clearly also holds in the standard durable-goods problem with only a single feasible quality level. (On the other hand, below-cost pricing also features in different contexts such as, for instance, in the presence of network externalities as in Farrell and Saloner, 1986 or with price wars as in the recent contribution by Marx and Shaffer, 2000.) In the standard durable-goods model, while the real time in which the market is served goes to zero as the time between periods shrinks, the number of periods it takes to clear the market also increases.

Other papers have also looked at the interaction of time-inconsistency problems and price discrimination. Some papers consider the successive introduction of goods of different qualities, e.g., as a sequence of upgrades (e.g., Villas-Boas, 1999). Typically, in these models the market operates for only two periods. A two-period version with quality differences is also considered in Takeyama (2002), which also generates below-cost pricing but cannot offer a comparison with the standard durable-goods case.1 An open time horizon is considered in a (menu) bargaining game by Wang (1998). As he only allows for a very

1 In this case, the seller can always circumvent the commitment problem by postponing trade until the second period. This is, however, the opposite of what we find, namely that if the time between periods is sufficiently small than all trade will take place in the very first period only.
specific payoff function, for which the virtual surplus is always strictly positive, the game always ends in the first period, with an outcome as under the optimal commitment mechanism.

2. The model

We consider a market for an infinitely durable and indivisible good. There is a continuum of mass one of infinitely-lived consumers. Consumers come in two types, \( t \in \{1, 2\} \) with respective masses \( q^0_t > 0 \). We call consumers of type \( t = 2 \) the ‘high-valuation’ segment and consumers of type \( t = 1 \) the ‘low-valuation’ segment. Consumers want to buy at most a single good. The market is served by a monopolistic firm, which chooses prices and qualities. Quality is measured by a real-valued variable \( x \). If a good’s quality equals \( x \) and is sold at the price \( p \) a buyer of type \( t \) realizes utility \( U_t(x, p) := V_t(x) - p \), while the firm realizes profits \( W(x, p) := p - C(x) \). Consumers have a reservation value of zero. As utility is transferable, we may call \( S(x) := V(x) - C(x) \) the surplus realized with type \( t \). We assume that \( V(x) \) and \( C(x) \) are three times differentiable satisfying: \( d^3S_j / dx^3 < 0 \) is bounded away from zero; \( V(x) > V_1(x) \); \( d^2V_j / dx^2 > dV_1 / dx; \) \( d^2V_j / dx^2 \geq d^2V_1 / dx^2 \). Total surplus \( S(x) \) is maximized by \( x_f \) such that \( S'_j := S(x_f) \) with \( S'_j > 0 \).

We consider discrete periods of length \( z > 0 \), which are numbered consecutively by \( n \in \{0, 1, \ldots\} \). We assume that all consumers and the monopolist apply the discount factor \( \delta = e^{-rz} \), where \( r > 0 \). The objective of the firm is to maximize discounted profits.

Each period the monopolist may offer a range of products. A product is fully characterized by a price \( p \) and a quality \( x \). Any (remaining) buyer can decide to buy one of the products or to wait for a better deal. Though we do not a priori restrict the number of different quality levels, we specify that each period at most two different levels are offered. Our equilibrium concept is that of subgame perfect equilibrium, with pure strategies for consumers. An equilibrium path can therefore be described by a sequence of offers \( \{(x_1^t, p_1^t), (x_2^t, p_2^t)\} \) and a sequence of residual masses \( \{q_1^t, q_2^t\} \) for the two types of consumers. It is convenient to denote the fraction of high-valuation buyers in the residual market by \( \nu := q_2^t / q^0 \). Note that \( 0 < \nu < q_2^0 \). The firm cannot directly discriminate between the two market segments, i.e., we rule out third-degree price discrimination. Moreover, to rule out first-degree price discrimination we make the standard assumption that the firm cannot condition its strategy on the actions taken by individual (or mass zero) consumers.

One limitation of our model is the restriction to two types of consumers only. Our method of proofs does not allow for an immediate generalization. Also, in our model a given consumer can purchase at most once.

For a first benchmark in the analysis consider the case with a durable good of a single quality. Here, the monopolist serves the whole market in finite time by posting a sequence of decreasing prices. If it takes more than one period to clear the whole market, which is the case if the fraction of high-valuation consumers is sufficiently large, low-valuation consumers purchase in the last period, while at least some high-valuation consumers accept a higher price in earlier periods. Moreover, as the real time \( z \) between two periods shrinks, the number of periods it takes to clear the market typically increases, while the real time to do so decreases.

A second benchmark is that of price discrimination with commitment (or, likewise, with a non-durable good). Here, if only the upper segment of the market is served, the firm offers the quality \( x = x_2^t \) at the price \( p = V_2(x_2^t) \), extracting all surplus from high-type consumers. If the firm serves the whole market, it offers the product in two quality levels, \( x_1^t \), with respective prices, \( p_1^t \). If \( \nu \) denotes the fraction of high-valuation consumers in the (residual) market, qualities and prices are determined as follows:

\[
\begin{align*}
\frac{dS_1(x)}{dx} \bigg|_{x=x_1^t} &= \nu \frac{dS_2(x)}{dx} \bigg|_{x=x_2^t}, \\
p_1^t &= V_2(x_2^t) - V_1(x_2^t) - V_1(x_1^t)(1-\nu), \\
p_1^t &= V_1(x_1^t).
\end{align*}
\]

The bottom-range quality is determined by the trade-off between maximizing surplus and minimizing the information rent obtained by high-valuation consumers. We write the bottom-range quality as a function \( x_1^t(\nu) \) of the fraction of high-valuation consumers. By Eq. (1) \( x_1^t(\nu) \) is continuous and strictly decreasing in \( \nu \).

Comparing payoffs, the whole market is served if the low-valuation consumers’ virtual surplus, which is given by

\[
(1 - \nu)S_1(x_1^t(\nu)) - \nu [V_1(x_1^t(\nu)) - V_1(x_1^t(\nu))],
\]

is positive. This expression is strictly decreasing in \( \nu \), it becomes strictly negative as \( \nu \rightarrow 1 \), and it is strictly positive at \( \nu = 0 \).

Lemma 0. If the firm sells a non-durable good, there exists a threshold value \( 0 < \bar{\nu} < 1 \) for the fraction of high-valuation consumers such that the following holds.

If the fraction of high-valuation consumers falls below \( \bar{\nu} \), the firm serves the whole market with the range of qualities and prices \( \{(x_1^t, p_1^t), (x_2^t, p_2^t)\} \) as characterized in Eq. (1). If the fraction of high-valuation consumers exceeds \( \bar{\nu} \), the firm only serves the high-valuation segment.

As a final benchmark, let us suppose that in our setting the durable-goods monopolist could commit to a price and product policy for all future periods? It is straightforward to show that this problem is analogous to the non-durable goods case. In particular, the firm does not use delay for price discrimination. The whole market is either served immediately or low-valuation consumers will never buy. From now on we will, therefore, refer to the products and prices characterized in Eq. (1) as the ‘commitment offer’.

3. Analysis

We state next our main result.

Proposition. There exists a unique equilibrium in which the durable-goods monopolist serves the whole market in the first...
period by offering the range of qualities and prices \( \{(x'_t, p'_t)\in T \) characterized in Eq. (1) if either of the following conditions holds:

i) the initial fraction of high-valuation consumers falls below the threshold \( \bar{\nu} \),

ii) or, regardless of the composition of the market, the real time between two consecutive periods, \( z \), becomes sufficiently small.

If the fraction of high-valuation consumers falls below the threshold \( \bar{\nu} \), which is used in Lemma 0, the prediction for the durable-goods case is identical to that for the non-durable goods case. This is intuitive as clearing the market immediately with the commitment offer is also what the firm would optimally do in case it could commit. More interesting is, however, the case where the firm would not want to serve the market if it could commit to do so, that is the case where \( \nu^0 > \bar{\nu} \). Only in this case does the firm face a true commitment (or time-inconsistency) problem, which is at the heart of our analysis. If the firm can change its policy sufficiently fast, then the unique equilibrium is as in the case where \( \nu^0 < \bar{\nu} \), i.e., the whole market is served immediately with the offers determined in Eq. (1). For \( S_t(x'_t, \nu^0) < 0 \) the low-type segment is served at a loss.

**Corollary.** For sufficiently low values of \( z \), the durable-goods monopolist sells to low-valuation consumers below costs if, holding everything else constant, either the fraction of high-valuation consumers is sufficiently large, the first-best surplus level \( S^*_1 \) realized with low-valuation consumers is sufficiently low, or the ‘difference’ between high- and low-valuation consumers is sufficiently large.\(^3\)

We next provide some intuition for the Proposition. Suppose the durable-goods monopolist serves some high-valuation consumers without immediately clearing the whole market. This necessarily decreases the fraction of high-valuation consumers in tomorrow’s (residual) market. Applying the logic from the non-durable goods case, a decrease in the fraction of high-valuation consumers will induce the firm to offer lower-quality buyers a more attractive product. As this is now rationally anticipated by all consumers, it reduces the price high-valuation consumers are willing to pay today. In other words, if the monopolist does not serve the whole market immediately, quality as a means for price discrimination cannot be used to its full effect.

What drives our result of immediate market clearing is, therefore, the interaction of the commitment (or time-inconsistency) problem in the two dimensions of the firm’s strategy: price and quality. Neither can the monopolist commit that he will stick to his current prices nor can he commit that he will stick to his current product range. If the firm can offer a range of qualities, its commitment problem becomes more severe: neither can the firm commit not to offer a (much) lower price in the following period(s), nor can it commit to offer a (much) higher quality also at the lower end of its product range.

**Appendix A. Proof of the Proposition.**

We first state a series of auxiliary results. Proofs are heavily abbreviated or omitted if they relate to standard results from the durable-goods case with a single quality level.

**Lemma 1.** Consider some residual market with distribution \( \nu \). For \( \nu < 1 \) all remaining low-valuation consumers realize zero utility. If \( \nu = 1 \) holds, then all remaining consumers, who all have a high valuation, realize zero utility.

Using Lemma 1, we obtain the following results.

**Lemma 2.** If the market is fully served in some final period \( N \), then this is done by the commitment offer \( \{(x'_t, p'_t)\in T \). If the fraction of high-valuation consumers drops below \( \bar{\nu} \) in some period \( n \), then the whole residual market is served in \( n \). Finally, at any point of time, the profit realized under the commitment offer represents a lower boundary for the payoff that can be extracted from the residual market.

For Lemma 2 note that the definition of the commitment offer in Eq. (1) naturally extends to the boundary cases \( \nu = 0 \) and \( \nu = 1 \) if we specify that only a single (first-best) offer is made in this case. Lemma 2 already proves the assertion in the Proposition relating to the case \( \nu^0 < \bar{\nu} \). We show next that the game ends in finite time.

**Lemma 3.** The game ends in finite time.

**Proof.** If \( q^2_1 < q^0_1 \) holds for some finite \( n \), this implies by optimality for high-valuation consumers that \( q^2_1 = 0 \) must hold for some sufficiently large \( n' \). In this case the game ends no later than in period \( n' + 1 \). It therefore remains to rule out the case where low-valuation consumers are never served. In this case the market distribution \( \nu^0 \) forms a nonincreasing sequence. If it drops below \( \bar{\nu} \), we already know from Lemma 2 that the game must end in that period. Using the arguments of Fudenberg, Levine, and Tirole (1985) it is straightforward to show that this follows from the firm’s optimality. \( \square \)

Before continuing with the proof of the Proposition, it should be noted that it is straightforward to show existence of an equilibrium where all low-type consumers are served in the final period \( N \). (This can be established iteratively as in Ausubel and Deneckre, 1989.)

To prove the Proposition, we consider the two last periods \( N - 1 \) and \( N \) in case the market is not served in \( n = 0 \). By Lemma 3 such a final period \( N \) always exists. We analyze first the case where the residual market in \( N - 1 \) is served by a skimming policy. Subsequently, we rule out all other non-skimming cases. We need the following intuitive auxiliary result.

**Lemma 4.** There exists a value \( \tilde{\nu} < 1 \) such that for all \( z \) and in any equilibrium the fraction of high-valuation consumers in the final period satisfies \( \nu^N < \tilde{\nu} \).

**Proof.** Suppose this was not the case, implying existence of a sequence of equilibria where it holds in the respective final

\(^3\) Admittedly, we have not introduced a (continuous) parameter capturing buyers’ types. Recall that \( x'_t \) is determined, for given \( \nu \), by the requirement \( \frac{dV(x'_t)}{dx} \mid_{x=x'_t} = \frac{dV(x)}{dx} \mid_{x=x'_t} \). The difference in types is then captured by the difference in the marginal valuations \( dV(x)/dx \).
periods $N_a$ that $v^{N_a} \to 1$. By Lemma 2 and the construction of the commitment offer this implies for the respective utilities of high-valuation consumers, which we denote by $U_a$, that $U_a \to 0$. By optimality for high-valuation consumers this gives us a sequence of thresholds $\bar{x}_a$ with $\bar{x}_a \to \infty$ such that all goods sold to low-valuation consumers in an equilibrium indexed by $a$ must have a quality not above $\bar{x}_a$. To complete the argument we can now make use of the fact that surplus is strictly concave, implying that selling to low-valuation consumers becomes increasingly loss making as $\bar{x}_a$ decreases. It is straightforward to show that this cannot constitute an optimal strategy for high values of $a$. □

Recall for the next claim that a skimming sequence of offers and purchases over the last periods $N-1$ and $N$ implies that no low-valuation consumer purchases strictly before some high-valuation consumers, i.e., that $q^N_1 < q^{N-1}_1$ implies $q^N_2 = 0$.

**Lemma 5.** There exists $z > 0$ such that for all $z < z$ there is no equilibrium where the market is served over more than one period and where the firm follows a skimming policy over the last two periods $N-1$ and $N$.

**Proof.** We argue to a contradiction. By Lemma 2 the payoff from serving the whole market already in $N-1$ is bounded from below by the commitment offers. Denote this payoff by

$$W_1 = q^{N-1}P \left( \alpha_1^N \left( v^{N-1}, v^N \right) \right). \tag{3}$$

Suppose now first that no low-valuation consumer buys in $N-1$, implying $q^{N-1}_1 = q^N_1$. By Lemma 2 the firm makes the respective commitment offer in $N$. By incentive compatibility for high-valuation consumers, who must therefore realize at least the utility $\delta A(x^*_1 \left( v^N \right))$, the firm’s payoff from serving the market over two more periods is then bounded from above by

$$W_2 = S_2 \left[ q^{N-1}_2 - (1 - \delta) q^N_2 \right] + \delta q^{N-1}_1 S_1 \left( x_1^N \left( v^N \right) \right) - \delta q^N_1 A(x^*_1 \left( v^N \right)) \tag{4}.$$  

From Eqs. (3)–(4) we obtain

$$\frac{W_2 - W_1}{q^{N-1}} = \delta \left[ \frac{1}{1 - v} \left( v^{N-1} S_1^N + \Omega \left( v^{N-1}, x^*_1 \left( v^N \right) \right) \right) \right] - \left[ \frac{1}{1 - v} \left( v^{N-1} S_2^N + \Omega \left( v^{N-1}, x^*_1 \left( v^{N-1} \right) \right) \right) \right], \tag{5}$$

where we used $q^N_2 / q^{N-1} = (1 - v^{N-1}) v^N / (1 - v^N)$. We show that Eq. (5) becomes strictly negative for high values of $\delta$. As $\Omega(\left( v^{N-1}, x_1^N \left( v^{N-1} \right) \right)$ strictly exceeds $\Omega(\left( v^{N-1}, x_1^N \left( v^N \right) \right)$ for all $v^{N-1} \neq v^N$ due the construction of $x_1$ and as $\delta < 1$, we only have to discuss the case where

$$\left( \frac{1}{1 - v^{N-1}} - \frac{1}{1 - v^N} \right) S_2^N + \Omega \left( v^{N-1}, x^*_1 \left( v^{N-1} \right) \right) < 0. \tag{6}$$

We argue first that Eq. (6) implies $v^{N-1} - v^N > \Delta v$ for some threshold $\Delta v > 0$. To see this, note that by Lemma 2 and Lemma 4 we have $v^{N-1} \in \left[ \bar{v}, \bar{v} \right]$, where $\bar{v} > 0$ and $\bar{v} < 1$, while the skimming policy implies $v^N \leq v^{N-1}$. At $v^{N-1} = v^N$ the left-hand side of Eq. (6) is equal to $\Pi(v^{N-1}, x^*_1 \left( v^{N-1} \right))$, which exceeds $S_1^N$ for all choices of $v^{N-1}$. Recall also that $S_1^N > 0$. Using continuity in $v^{N-1}$ and $v^N$ as well as the restriction on $v^{N-1}$, this implies existence of the threshold $\Delta v$. Using the definition of $x^*$ this immediately implies that the difference $x^*_1 \left( v^{N-1} \right) - x^*_1 \left( v^N \right) > 0$ is bounded away from zero over all feasible choices for $v^{N-1}$ and $v^N$, and that finally the same holds for the difference $\Omega(\left( v^{N-1}, x^*_1 \left( v^{N-1} \right) \right) - \Omega(\left( v^N, x^*_1 \left( v^N \right) \right)) > 0$. From this it finally follows that Eq. (5) must become negative for all sufficiently high $\delta$ and thus for all values $z$ falling below some threshold $z_1 > 0$.

It remains to be discussed that the second kind of skimming policy according to which low-valuation consumers are also served in $N-1$. As the previous expressions do not fully apply to this case, we treat it separately, though the same logic applies. By definition this implies $q^N_2 = 0$. Using $v^N = 0$, we know from Lemma 2 that the firm offers low-valuation consumers the first-best quality $x^*_1$ in the last period. High-valuation consumers must therefore receive over the last two periods at least the utility $\delta A(x^*_1)$ and this obtains as an upper boundary for the firm’s payoff from serving the residual market over periods $N-1$ and $N$

$$W_2 = q^{N-1}_2 \left[ S_2 - \delta A(x^*_1 \left( v_1 \right)) \right] + (q^{N-1}_1 - q^N_1) S_1 \left( x^*_1 \left( v_1 \right) \right) + \delta q^N_1 S_1^N. \tag{7}$$

Using Eqs. (3) and (7) we can again derive $(W_2 - W_1) / q^{N-1}$ in analogy to Eq. (5). At $\delta = 1$ this becomes $\Omega(\left( v^{N-1}, x^*_1 \left( v_1 \right) \right) - \Omega(\left( v^N, x^*_1 \left( v^N \right) \right))$, which is strictly negative by $v^{N-1} \geq v_1 > 0$ and by definition of $x^*$. Using the continuity of $(W_2 - W_1) / q^{N-1}$ in $\delta$ and the restriction $v^{N-1} \geq v_1$ thus obtains a second threshold $z_2 > 0$ such that for $z < z_2$ it is strictly less profitable to serve the market in two more periods in case all high-valuation consumers by in $N-1$. Choosing $z = \min \left\{ z_1, z_2 \right\}$ proves the claim. □

**Lemma 6.** The firm cannot obtain higher profits from serving the residual market over the two more periods $N-1$ and $N$ if it applies a non-skimming policy.

**Proof.** Suppose first that only low-valuation consumers buy in $N-1$. Denote the purchased quality by $x^N_1$. From Lemma 2 we know that the firm makes the commitment offer in the final period $N$. By the high-valuation consumers’ incentive compatibility constraint this implies the requirement $\Lambda(x^N_1) \leq \Delta A(x^*_1 \left( v^N \right))$. By concavity of $S_1$, an upper boundary for the firm’s payoff is obtained by choosing $x^N_1$ such that $\Lambda(x^N_1) = \delta A(x^*_1 \left( v^N \right))$, holds, implying also $x^N_1 < x^*_1 \left( v^N \right))$. □

**References**


