Prudence as a competitive advantage: On the effects of competition on banks’ risk-taking incentives

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ABSTRACT

This paper builds on the notion that corporate borrowers care about the overall riskiness of a bank’s operations as their continued access to credit may depend on the bank’s ability to roll over loans or to expand existing credit facilities. A key implication of this observation is that increasing competition among banks should have an asymmetric impact on banks’ incentives to take on risk: Banks that are already riskier will take on yet more risk, while their safer rivals will become even more prudent.

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1. Introduction

Deregulation and financial innovations have increased the options available to borrowers, leading to more intense competition among banks as well as between banks and alternative sources of finance (cf. Boot and Thakor, 2000). However, for corporate borrowers who shop for better deals the price (loan rate) should not be all that matters. Will they later be able to refinance a maturing loan and at what conditions? Will the existing lender roll over the loan, extend a credit facility, or provide additional finance at short notice in the future? In particular for businesses that are less mature, smaller, or more opaque, it could become quite costly (or even impossible) to replace an existing lending relationship at short notice. In fact, the existence of such “relationship capital” is a key notion in the large literature on relationship lending (cf. Boot, 2000).

Empirically, this is documented, for instance, in Slovin et al. (1993), who found that borrowers from Continental Illinois suffered an average 4.2% loss in their stock market value after the bank failed. Likewise, Djankov et al. (2005) show that bank closures in Indonesia, Korea, and Thailand decreased borrowers’ stock prices by 3.9%, while Yamori and Murakami (1999) document a 6.6% decrease for those borrowers who named the failed Japanese bank, Hokkaido Takusyoku, as their main lender. While outright bankruptcy is an extreme event, borrowers may also be adversely affected if liquidity problems force their main lender to call back loans or to refuse the expansion of existing credit facilities. To the extent that an existing relationship involves some degree of “lock-in”, e.g., due to an informational advantage of an existing lender, the borrower may not receive adequate funding elsewhere.

Based on these observations, this paper starts from the presumption that the riskiness of a bank’s existing operations, as well as its leverage, represent a key quality attribute in the eyes of potential borrowers. Banks that are perceived as being less aggressive in undertaking (on- and off balance sheet) risk would then be regarded by borrowers as a superior
choice. As I show, this can play an important role in making banks more “prudent”, i.e., conservative in their risk taking.

In fact, unless they are levered up sufficiently, e.g., through their deposit-taking activities, they may even choose to forgo positive-NPV “gambles”. The focus of this paper is, however, on the interaction of risk-taking and changes in competition.

I find that competition (e.g., through deregulation that limits the scope for horizontal differentiation) has an asymmetric impact on banks’ risk-taking incentives. As competition becomes more intense, it is likely that some banks become more prudent, while their rivals undertake riskier strategies. In particular, if banks already differ in the riskiness of their existing operations, more competition induces less (additional) risk taking by an already more prudent bank, while it has the opposite effect on rivals that already took on more risk.

What is crucial for results to hold is that borrowers can discern a bank at which they are less exposed to the risk of not receiving funding in the future. To some extent, as we discuss, banks that are less engaged in other, risky activities may be more willing or able to commit future funding. In addition, borrowers may be able to tell from banks’ overall activities whether they are exposed to different risks, e.g., arising from their underwriting or proprietary trading activities. In the recent financial crisis, some large universal banks were highly exposed to the subprime market through their own trading and investment activities (e.g., UBS) or through guarantees that they provided to special purpose vehicles that made such investments (e.g., German Landesbanken). Admittedly, the crisis also showed that some if not much of this exposure either was not adequately observed by market participants or it was not perceived as being highly risky. Then, the mechanism that is at work in my model cannot be active: To the extent that this cannot be credibly communicated to the market or to the extent that the market is not sufficiently wary of it, banks cannot gain competitive advantage in the lending market by being otherwise more prudent.

The recommendation to increase transparency if regulators are concerned about too much risk taking is not new and, in fact, a key pillar of the Basel 2 requirements. The findings of this paper have, however, additional implications for banking supervision in an increasingly competitive environment. They would support neither the view that supervision must be uniformly stepped up as competition increases nor the view that competition uniformly reduces the need for supervision. Instead, as competition for loans increases, I find that (additional) risk-taking decreases for banks that are already more prudent. To obtain this result, the strategic interaction of banks in the market for borrowers is important. Put differently, the paper’s results do not simply follow from the observation that banks may remain prudent so as to take advantage of future profit-making opportunities. As competition increases, I identify a tendency towards more “vertical differentiation”, given that in the model banks’ commercial borrowers care about how much risk banks take on through other operations. The asymmetry in risk-taking behavior that this paper identifies is also supported by recent evidence in Gropp et al. (2011). Though there the reason for why some banks become “less risky” is exogenous, as it depends on the existence or withdrawal of government guarantees, their finding that these guarantees “strongly increase the risk-taking of the competitor banks” would be predicted by our model.

Relation to the literature: In a recent study, Beck et al. (2003) find that different measures of competition, e.g., based on concentration or the degree of deregulation, predict different relationships between competition and the stability of the banking industry. Boyd et al. (2006) have shown that a more concentrated banking sector may be associated with less stability (cf. also De Nicolò and Loukoianova, 2007). This contrasts with earlier work as well as, more recently, with the findings of Jiménez et al. (2007) for a large sample of Spanish banks (cf. also the references therein). The theoretical results in this paper, though not being tied one-to-one to the hypotheses in the aforementioned empirical papers, suggest a rationale for why different studies may not find a robust relationship between competition and risk taking. We find that for a priori (and observationally) identical banks, competition may affect risk-taking incentives orthogonally, reflecting the increased benefits from vertical differentiation.

In terms of theory, much of the extant literature would assert a positive correlation between competition and banks’ incentives to take on (more) risk. Following Keeley (1990), one suggested channel works through a reduction in banks’ charter value. Perotti and Suarez (2002), by comparing a monopoly with a banking duopoly find that in the latter case banks are more prudent as either bank hopes to enjoy monopoly profits by being “the last bank standing”, once the other bank has failed.

It should also be noted that the paper focuses on banks’ incentives to take on risk. Clearly, holding all else constant, a reduction in banks’ profitability may also mechanically imply a higher risk of becoming insolvent. Another channel through which competition affects banks’ riskiness without affecting their own risk-taking incentives was identified in Boyd and De Nicolò (2005), where a lower interest rate induces less risk taking by borrowers, which in turn makes loans and thus banks’ balance sheets less risky (cf. also more recently Martinez-Miera and Repullo, 2010 for an extension and

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1 Cf. also Allen et al. (2009) on banks’ holding of capital buffers for competitive reasons.

2 This tendency may counteract the risk of “herding” that has been identified in other papers. For instance, such herding may follow from the expectation that the regulator will more likely bail out individual banks if more of them find themselves in a crisis (cf. Acharya and Yorulmazer, 2007 for such a “too-many-to-fail” argument).

3 Cf. also Flannery and Rangan (2008) on banks’ incentives to hold more capital when, as government guarantees are weakened, markets impose greater discipline.

4 This has been confirmed and extended by a number of other papers using dynamic models, e.g., Hellmann et al. (2000).
qualification of their results). On the other hand, for instance Cordella and Yeyati (2002) argue that under more intense competition banks would monitor less, thereby increasing the likelihood of bad loans.\footnote{In a similar vein, competition reduces the incentives to screen borrowers in Chan et al. (1986) or leads to more risk-taking by eroding informational rents originating from relationship banking in Besanko and Thakor (1993). See further Matutes and Vives (1996, 2000) for models of imperfect competition generating a positive relationship between competition and risk (as well as Carletti and Hartmann, 2003 for a recent survey).}

The model departs from the established literature in a number of ways. The mechanism that links competition to risk-taking is novel. In fact, the relevance of banks’ own riskiness for borrowers has been somewhat ignored in the (theoretical) literature.\footnote{For an exception see Detragiache et al. (2000), which seeks to explain the number of banking relationships that a firm entertains.} Also, the implications are different from those in the extant literature, which would typically predict a symmetric response of all banks to more competition.\footnote{The asymmetry is, however, shared with recent work by Boot and Marinč (2006). There, banks differ in their ability to monitor loans but can undertake investments to raise their overall level of monitoring. As competition becomes more intense, those banks that have a lower intrinsic ability to monitor expect to obtain a lower market share, which given the lump-sum nature of any investment into their monitoring capacity reduces their investment incentives. For banks with a higher intrinsic ability to monitor the opposite prediction holds.} While some of the arguments in the extant literature apply indiscriminately of whether there is more competition on the asset or liability side (say, for borrowers or depositors), e.g., given that they hinge simply on banks’ lower overall profitability, the new channel, which predicts an asymmetric change in risk-taking incentives, works exclusively through more competition in the markets for loans. That being said, however, the modeling framework may also prove more widely useful, for instance for the insurance literature. There, several papers have documented a negative relationship between the soundness of an insurer (default risk) and prices (insurance premia) (cf. Sommer, 1996; Phillips et al., 1998). The analysis shows, as an intermediary result, such a negative relationship between the overall riskiness of a bank’s operations and loan rates.

Finally, there is an interesting relationship between the present paper and the seminal contribution of Petersen and Rajan (1995) on relationship lending and competition. Their paper provides a model and offers evidence to support the hypothesis that competition can be inimical to the formation of long-term relationships. Their focus is on intertemporal risk and surplus sharing, which is hampered by opportunistic behavior in the presence of a competitive market for loans (in each period). As I discuss below, one interpretation of the present model is in terms of contractual commitments that banks are willing to make in the presence of alternative risk-taking opportunities. As competition in the loan-market increases, the key result states that for one bank it can become more attractive to abstain from such additional activities so as to thereby, offer a borrower more commitment to extend funds also in the future. Again, my results would thus speak in favor of a more nuanced effect of competition on relationship lending, which some relationships getting deeper and others looser as competition intensifies.

Organization of the paper. The rest of this paper is organized as follows. Section 2 introduces the model. In Section 3 I solve the baseline model and derive first results, albeit restricting attention to ex-ante symmetric banks. Section 5 analyzes the interaction of risk taking and competition, while Section 6 explores the role of (higher) leverage. Section 7 introduces ex-ante asymmetry to derive more implications. Section 8 compares the market outcome to a benchmark of efficiency. Section 9 concludes.

2. The model

I consider competition between two banks, indexed by \(i = 1, 2\). For the purpose of this paper I take each bank’s capital structure as given. As instance, for frequently done in the banking literature, we may suppose that it is determined by a bank’s access to (cheap) deposit finance. Originally each bank has capital \(C > 0\), of which some fraction was financed by debt and deposits. What matters for a bank’s risk-taking incentives is the total amount that the bank will have to repay to debtholders before equity holders are paid off. The bank is run in the interest of equity holders. Let \(D_i\) denote a bank’s total repayment obligations. I will later allow banks to differ in the riskiness of their existing business. For the time being, however, let us still abstract from this and take also banks’ capital structure as exogenously given. I next describe banks’ activities. Banks are able to engage in lending as well as in an additional risk-taking activity. Both choices are made in some initial period \(t = 1\), and I describe the respective timing further below.

Risk taking. Banks can choose a potentially risky activity. This could represent, for instance, proprietary trading, additional loan commitments (e.g., as back-up financing), guarantees, or other contingent obligations. For simplicity, I stipulate that initially this activity, to which I refer to as activity \(A\), does not require capital. Both banks have access to it. The outcome of this activity is known to the bank at the end of period \(t = 1\) and leads to payments in \(t = 2\). With probability \(p\) the bank will realize from activity \(A\) the payoff \(z’ > 0\), while with the residual probability \(1 - p\) the payoff \(z’’ < 0\) materializes. In other words, with probability \(1 - p\) the bank will have a net outflow of \(z’\).

It is convenient to denote \(F = |z’ - z’’|\) and \(f = z’\). This allows to analyze activity \(A\) in terms of some “fee” \(f\) that the bank receives for the risk of having to pay out \(F\) with probability \(1 - p\).\footnote{For the purpose of the present analysis it will be inconsequential whether and to what extent the outcomes of the two banks’ risky activities are correlated.} As I abstract from discounting and as all parties are risk neutral, the net present value of activity \(A\) is thus

\[
\eta_A := f(1 - p)F.
\]
It should be noted that all that is needed for the following analysis is that activity \( A \) may force the bank to preserve capital. Though I do not introduce regulation explicitly in the model, one could also imagine that this is due to regulatory constraints or that it is required by supervisors, provided that the bank's exposure from another activity, \( A \), proved to be more risky or less profitable than expected.

**Lending competition.** Crucially, the set-up for the loan market must allow to deal with various degrees of competition in a tractable way. This is accomplished by using a standard model of horizontal differentiation, namely that of Hotelling competition. Here, differentiation could capture the extent to which a bank's more “local” customers are both able and willing to choose a more distant competitor. The associated costs (both for lenders and borrowers) could be influenced both by regulation and by changes in the lending technology.

To be precise, I stipulate that there is a single potential borrower whose preferences regarding the choice between the two banks are ex-ante unknown.\(^9\) His preferences are captured by a variable \( x \) that is uniformly distributed over \( x \in [0,1] \) such that when borrowing from bank \( i=1 \), the borrower will incur a disutility (measured in units of money) of \( tx \), while when borrowing from bank \( i=2 \) the respective disutility will be \((1-x)\tau \). One standard (Hotelling) interpretation is that \( x \) represents a measure of the experienced distance between the bank's and the borrower's premises. A decrease in \( \tau \) may be due to a change in technology, so that the physical distance between a borrower and his bank becomes less relevant. Also, a reduction in \( \tau \) may be due to a reduction in regulatory obstacles that make it difficult for banks and borrowers alike to enter into a relationship when they are located in different districts or states. (See, for instance, Degryse and Ongena, 2005 for a recent empirical application of the Hotelling approach or Degryse et al., 2009 for a wider interpretation.) Below I also discuss how a change in competition that is due to entry, so that the market share of the two considered banks erodes, would alter results.

From a loan the borrower obtains funds \( C \), which are fully invested into a long-term project. If the project is continued until its end, which is in \( t=3 \), then it pays off \( y \) with probability \( q \) and zero otherwise. The expected payoff \( qy \) is supposed to exceed \( C \). If liquidated prematurely in period \( t=2 \), however, the project's payoff is only \( by < C \).

**Loan contract.** The loan contract offered by bank \( i \) prescribes a total repayment of \( R_i \) comprising the principal \( C \) and interest \( C_r \) if the project is continued until \( t=3 \). If the project is terminated prematurely in \( t=2 \), all of its liquidation value \( by < C \) is seized by the bank. Importantly, note that the loan contract is only short term, allowing the bank to recall the loan in \( t=1 \).\(^10\) The specifications of the loan contract deserve some additional comments. In what follows, I will first discuss how to endogenize this restriction of contracts. Then, I will discuss the implications that this has and, finally, offer an alternative that would also generate the same set of results.

The fact that the contract is only short term is realistic. It can be endogenized by appealing to agency problems, either of moral hazard or adverse selection, between the borrower and the bank. For instance, I could imagine that the borrower can be of two types \( \xi = l,h \). The type is only privately known to the borrower originally, but revealed to the bank in \( t=2 \). Only a project of type \( \xi = h \) has a chance of realizing a payoff \( y \) in \( t=3 \), namely with probability \( q_h \), while the respective probability of a type-\( l \) project is \( q_l = 0 \). In addition, borrowers realize (arbitrarily small but strictly positive) private benefits in case the project is financed and continued until \( t=3 \). Facing such a problem of adverse selection, the bank would want to preserve the right to call back the loan in \( t=1 \) and, thereby, obtain at least the value of the seized assets \( by \). Otherwise, it would risk attracting also type-\( l \) borrowers. (Strictly speaking, we would also need that the pool of type-\( l \) borrowers, i.e. of so-called fraudulent applicants or “fly-by-night operators”, is sufficiently large.)

In the model, the bank will call back the loan early so as to prevent its own insolvency in case it needs the respective funds to honour its commitments under the activity \( A \). In fact, this will only be the case if the bank previously undertook the risky activity and if the underlying gamble resulted in an obligation to pay out \( F \) in \( t=2 \). (Note that I assume here also that \( by > F \).) What is key in the model is that borrowers can anticipate at which bank it is more or less likely that their funding will be recallled or not rolled over. For this purpose I stipulate that the choice of activity \( A \) is observable and that it is made before banks compete for the borrower.\(^11\)

Admittedly, in practice banks have a wide scope to take on risk in ways that is not easily observed by regulators, let alone market participants. However, as noted in the Introduction, we may regard the choice of activity \( A \) more broadly as a bank's overall choice of strategy, which should comprise, in particular, its decision whether to stay out of particular lines of business, such as trading and investing on its own account or underwriting activities, as well as being particularly active, e.g., through giving guarantees or providing commitments, in new, risky ventures.

I next offer, however, a slightly alternative set-up that generates the same results, but where observability is not an issue. I could allow banks to choose between two different types of contracts. One type of contract is exactly the previous one, where the bank can recall the loan or decide not to roll it over. The other type of loans would involve a sure, long-term commitment, e.g., as the borrower is allowed to draw down the funds immediately. While I assumed for convenience that the risky strategy does not consume capital at the outset, it is reasonable that a bank can only enter into it when it can ensure the respective counterparties that it has such funds at its disposal at the end of \( t=1 \). For instance, one may think of

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9 One could imagine that among a number of borrowers who are located at different distances to each bank only one may have financing needs and that his identity is ex-ante unknown.

10 For instance, this could be a short-term revolving loan facility or a revocable overdraft facility that is drawn down fully in \( t=1 \).

11 Though results extend to the case where this is only observed with some noise, the resulting signaling game would heavily complicate the analysis.
clearly represent feasible options, realistically both would have drawbacks on their own. First, if the bank raised more than $F > C > fy$, one may ask why the bank does not raise more funds up-front, namely at least $(F-f) + C$, or raise capital in $t=2$ so as to absorb the loss of $F$. Though these clearly represent feasible options, realistically both would have drawbacks on their own. First, if the bank raised more than $C$ up-front, then this could give rise to (“free cash-flow”) agency problems between the bank and its investors.\footnote{This or another reason for why bank capital is costly needs to be invoked also in other models where banks are capital constrained at some interim stage, i.e., where banks did not sufficiently large buffer capital initially.} Formally, I could imagine that the bank’s managers consume a fraction $\gamma > 0$ of all funds that have not been used up until $t=2$. As $\gamma$ becomes sufficiently large, it would not be optimal to raise more than $C$ up-front. Raising additional funds $F$ in $t=2$ may also prove too expensive as new investors may be at an informational disadvantage (“dilution problem”). Formally, I may suppose that at the end of $t=1$ the bank, possibly together with the borrower, learns about some features of the investment project, e.g., as captured by some type $\xi = l,h$. When the bank raises new capital on the back of its existing assets, a bank with a (relatively) bad loan may also want to raise new capital even without having immediate liquidity needs (cf. Stein, 1998; Winton, 2003).\footnote{A similar argument of adverse selection can be used to rule out the possibility that when having to repay his loan early, the borrower can successfully turn to another investor.} Note finally that the size of activity $A$ is exogenously given. In particular, it is not arbitrarily scalable, so that its “downside” could be continuously and deterministically adjusted. While adding such a strategic choice could be of interest, this feature captures in the most simple way the notion that by engaging in the additional, risky activity, the bank may always, with some probability, have to end its commitment to the borrower.

In the main analysis I also set banks’ leverage sufficiently low such that this by itself does not generate additional risk-taking incentives, allowing us to focus on the novel contribution of our model. I show subsequently that the equilibrium characterization survives if leverage adds additional risk-taking incentives (albeit, as is well known, expressions then quickly become unwieldy). Still, the model is focused much on risk-taking in banking rather than, say, risk-taking in any non-financial corporation. This is the case as the “knock-on” effect of the bank’s additional risky activity works through its commercial lending operation, where potential borrowers care about the overall soundness of the lender’s financial position.\footnote{Having said this, the literature has also drawn attention to cases where other “stakeholders” such as customers or employees should be concerned about a firm’s balance sheet, at least if there is some (relationship-specific) lock-in (e.g., Titman, 1984). Arguably, in a relationship between a lender and a commercial borrower we can rightly assume that the latter is sufficiently sophisticated to learn from analysts whether the former is relatively more or less in good shape compared to its peers. In addition, the risk imposed on the borrower in case of a subsequent solvency problem of its main bank could be large (as, for instance, evidenced by the findings that were discussed in the Introduction).}

3. Equilibrium

As noted previously, I first derive results for the benchmark case where the level of outstanding debt, including deposits, does not itself affect banks’ risk-taking incentives. Hence, for the moment I can thus presume that the bank operates to maximize total firm value. Abstracting thereby from (more standard) risk-taking incentives that follow from leverage, this allows us to focus on the novel mechanism that is at work in our model.

As also noted above, in the baseline case banks have no other ongoing activities in place. Consequently, the first decision is made at the beginning of $t=1$, where banks can choose whether to undertake the risky activity $A$. Subsequently, they compete for the borrower by making offers $R_i$. In what follows, I proceed backwards by first solving for the equilibrium in the loan market. Subsequently, I determine the equilibrium at the initial stage, where the risky activity $A$ can be undertaken.

3.1. Competition for loans

Banks compete for the borrower at the end of period $t=1$. For the moment, it is convenient to denote more generally by $p_i$ the commonly known probability with which bank $i$ will be able to roll over the loan in $t=1$. (In equilibrium, it will thus hold that $p_i \in \{p,1\}$.)

A borrower who is located at $x \in [0,1]$ and takes out a loan with bank $i=1$ realizes the expected payoff

$$p_1[q(y-R_1) - xt].$$

(1)

Note that (1) takes into account that the loan will be recalled with probability $1-p_1$. If the loan is, instead, continued, then the borrower realizes with probability $q$ the project’s payoff $y$ minus the contractually stipulated repayment $R_1$. Finally, the last term $xt > 0$ in (1) captures the degree of banks’ horizontal differentiation.
In analogy to (1), the borrower’s expected payoff when turning to bank \( i = 2 \) equals
\[
p_i^2 q (y - R_2) - (1 - x) y. \tag{2}
\]
Comparing (1) with (2), the borrower is just indifferent between the two offers if \( x \) is equal to some critical value \( \tilde{x} \) given by
\[
\tilde{x} = \frac{1}{2} \left( 1 + \frac{p_1 q (y - R_1) - p_2 q (y - R_2)}{2 \tau} \right). 
\tag{3}
\]
Note that \( \tilde{x} \) is also the ex-ante probability with which, for given \((p_i, R_i)\), bank \( i = 1 \) will attract the borrower, while the respective ex-ante probability for bank \( i = 2 \) equals \( 1 - \tilde{x} \).

Using \( \tilde{x} \), the expected profit of bank \( i = 1 \) from the loan business is thus given by
\[
\tilde{x} [p_i q R_i + (1 - p_i) \beta y - C]. 
\tag{4}
\]
As I so far assume that the bank’s own leverage does not affect decision making, the optimal choice of \( R_i \) for a given risk profile, as captured by the respective values of \( p_i \), maximizes (4). Likewise, the optimal choice of \( R_2 \) for the rival bank \( i = 2 \) maximizes
\[
(1 - \tilde{x}) [p_2 q R_2 + (1 - p_2) \beta y - C]. \tag{5}
\]
By assuming that
\[
1 - p \gamma (q - \beta) < 3 \tau \tag{6}
\]
holds, I can ensure for \( p_i \in (p, 1) \) that in equilibrium both banks attract the borrower with positive probability as \( 0 < \tilde{x} < 1 \).

**Proposition 1.** For a given risk profile, as captured by the respective values of \( p_i \), banks offer loans that stipulate a required repayment of
\[
R_i = \frac{1}{2 \tau^{p_i}} \left[ C + \frac{\gamma (1 - p_i) y - p_i q R_i}{3} \right]. 
\tag{7}
\]
Expression (7) is intuitive. Note first that if neither bank engages in the risky activity \( A \) such that \( p_1 = p_2 = 1 \), then this simplifies to \( R_1 = R_2 = R \) with \( q R = C + \tau \). In other words, the expected total repayment, \( q R \), is equal to the principal \( C \) plus a margin that is equal to the measure of horizontal differentiation, \( \tau \). Suppose that both banks choose the risky activity, such that \( p_1 = p_2 = p < 1 \), in which case expression (7) becomes \( q p R + \beta y (1 - p) = C + \tau \). In this case, the expected total repayment, which now includes the liquidation value \( \beta y \) in case the loan is recalled, is again equal to the principal \( C \) plus \( \tau \). Finally, note that if one bank, say bank \( i = 1 \), chooses to take on additional risk while bank \( i = 2 \) stays prudent, such that \( p_1 = p < p_2 = 1 \), I have from (7) that \( R_1 < R_2 \). The more prudent bank will thus charge a higher interest rate. Still, it is straightforward to show that the more prudent bank, namely \( i = 2 \) in this case, will attract the borrower with a higher probability \( (\tilde{x} < 0.5) \). More formally, substituting the equilibrium loan rates \( R_i \) from (7) into (3), I obtain that
\[
\tilde{x} = \frac{1}{2} + \frac{1}{2 \tau} (p_1 - p_2) \frac{1}{3} (q - \beta) y. \tag{8}
\]

From **Proposition 1** we next have the following result.

**Proposition 2.** For a given risk profile, as captured by the respective values of \( p_i \), banks’ expected equilibrium profits from making loans are given by
\[
\pi_i = \frac{1}{2 \tau} \left[ \tau + \frac{\gamma (q - \beta) (p_1 - p_2)}{3} \right]^2. \tag{9}
\]
This result gives rise to the following immediate observations. Intuitively, both banks’ expected profits from the loan market are higher the more they are horizontally differentiated (as captured by a larger value of \( \tau \)). Moreover, a given bank’s expected profits increase in its own value \( p_i \) and decrease in its competitor’s value \( p_j \). In other words: If bank \( i \) is perceived by a potential borrower to be of “higher quality” as it will more likely roll over the initial loan, then this allows the respective bank to realize higher profits; but if the bank’s competitor is perceived to be of “higher quality”, then this reduces the profits of the former bank. It is useful to collect these results more formally.

**Corollary 1.** It holds that \( \frac{d \pi_i}{dp_i} > 0 \) and \( \frac{d \pi_i}{dp_j} < 0 \). Moreover, the likelihood with which bank \( i \) will make a loan in equilibrium is strictly increasing in \( p_i \) and strictly decreasing in \( p_j \).

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15 It should be noted that (3) only applies if it satisfies \( 0 \leq \tilde{x} \leq 1 \), which in turn is the case if \( [p_1 q (y - R_1) - p_2 q (y - R_2)] / \tau \leq 1 \). Otherwise, we clearly have that either \( \tilde{x} = 0 \) or \( \tilde{x} = 1 \). Below I will invoke a condition that ensures that the solution is indeed interior.

16 Note that this also confirms that (6) indeed ensures that in equilibrium both banks are active with positive probability.
3.2. Risk-taking incentives

The expression for profits in Proposition 2 is now instrumental to solve for the equilibrium strategies at the beginning of period \( t = 1 \), where banks can choose whether to undertake the risky activity. I first derive a set of auxiliary comparative results, which all are immediate from undertaking the respective differentiation of expression (9).

**Lemma 1.** Banks’ expected profits from making loans, \( \pi_i \), satisfy

\[
\frac{d^2 \pi_i}{dp_i} > 0 \quad \text{and} \quad \frac{d^2 \pi_i}{dp_i dp_j} < 0 \quad \text{for} \quad i \neq j.
\]

It is worthwhile to discuss in more detail the comparative results of Lemma 1. We already know that the higher \( p_i \), the higher are the profits of bank \( i \). From \( \frac{d^2 \pi_i}{dp_i^2} > 0 \) this effect is stronger if \( p_i \) is already high. In words, when a bank becomes relatively more attractive to the borrower, this has a larger (positive) effect on profits if the bank is already regarded as a relatively safe choice. From \( \frac{d^2 \pi_i}{dp_i dp_j} < 0 \), on the other hand, the effect on own profits is smaller if also the rival bank is regarded as being a safe choice (high \( p_j \)).

The bank’s risk attributes, \( p_i \), thus represent strategic substitutes. This finding is not specific to the chosen Hotelling model but hold in most other models of oligopolistic competition with vertical differentiation.\(^{17}\) I argue next that Lemma 1 is also intuitive. Take first the case of \( \frac{d^2 \pi_i}{dp_i^2} > 0 \). Recall from Corollary 1 that in equilibrium bank \( i \) is more likely to make a loan the higher is \( p_i \). For instance, for \( i = 1 \) a higher value \( p_1 \) will translate into a higher value of \( \bar{x} \). If now \( \bar{x} \) is already high, then the bank will benefit more if it can further raise \( R_i \), following a further increase in \( p_1 \). In other words, an increase in the bank’s perceived “quality”, as expressed by \( p_1 \), is thus more profitable if the bank’s “market share” \( \bar{x} \) is already high, given that this allows the bank to earn a higher margin (from a higher \( R_1 \)) on a larger volume of transactions (or, more precisely, with a higher probability in our setting).

An analogous intuition applies to \( \frac{d^2 \pi_i}{dp_i dp_j} < 0 \). An increase in, say, \( p_2 \) pushes down the threshold \( \bar{x} \) and, thereby, the “market share” of bank \( i = 1 \). By the previous arguments, this makes an increase in \( p_1 \) less profitable. Moreover, there is now an additional effect at work. As is intuitive, banks aggregate profits in the loan market are lower as they compete more on equal grounds, i.e., as their respective values \( p_i \) are more similar. Further below I will study in detail how these forces are affected by the prevailing degree of competition.

The fact that banks’ choice of the risky activity gives rise to strategic substitutes is the driving force behind the asymmetries in the equilibrium outcome that I derive next. As noted above, it is a rather general feature in Industrial Organization models of vertical differentiation. (Note that this observation relates to firms’ choice of the respective vertical attribute, such as quality, and not to prices or quantities, in which they subsequently compete.) Still, in the specific context of banking there may be countervailing forces, so that, more generally, risk-taking represents a strategic complement. This is the case when a bank considers it more likely to be bailed out in case also other banks find themselves in a precarious condition (cf. Acharya and Yorulmazer, 2007). To the extent that this is the case, this would work against the asymmetries that I discuss next.

4. Risk taking in equilibrium

At \( t = 1 \), a bank’s expected profits when not undertaking the risky activity is given by \( \Pi_i = \pi_i \), where we have to substitute \( p_1 = 1 \) into expression (9) for \( \pi_i \). When undertaking activity \( A \), instead, expected profits equal

\[
\Pi_i = \pi_i + \eta_A,
\]

where now \( p_1 = p \) and where \( \eta_A \) represents the expected payoff from \( A \).

Intuitively, if \( \eta_A \) is sufficiently high, then both banks strictly prefer to undertake activity \( A \). On the other hand, it is equally intuitive that for low values of \( \eta_A \), which can also be negative, neither bank will choose activity \( A \). In both cases, banks stay symmetric.

In contrast, a key implication of Lemma 1 is that for intermediate values of \( \eta_A \) the equilibrium involves an asymmetric choice: one bank becoming riskier and the other one stays safe. This follows as from Lemma 1 risk taking becomes relatively more profitable if the rival bank stays prudent, while if the rival bank is expected to undertake the risky strategy, then the safe strategy of foregoing activity \( A \) becomes relatively more attractive.\(^{18}\)

\(^{17}\) For a technical discussion see Athey and Schmutzler (2001).

\(^{18}\) As is standard, I restrict consideration to pure-strategy equilibria. As the outcome in a mixed-strategy equilibrium would also be asymmetric with positive probability, the insights extend, however, also to this case. Note also that for brevity’s sake I do not comment separately on the (non-generic) case where \( \eta_A \) takes on the value of either threshold in the proposition, in which case both symmetric and asymmetric equilibria exist.
Proposition 3. Expressed in terms of the net present value $\eta_A$ from the risky activity (A), the following equilibrium outcome in $t = 1$ obtains: There exist two thresholds $0 < \eta_A < \eta_A^*$ satisfying

$$\eta_A = \frac{\tau}{2} - \frac{1}{2\tau} \left[ \tau - \frac{y(q - \beta)(1-p)}{3} \right],$$

and

$$\eta_A^* = \frac{1}{2\tau} \left[ \tau + \frac{y(q - \beta)(1-p)}{3} \right] - \frac{\tau}{2},$$

such that for $\eta_A \leq \eta_A^*$ neither bank chooses the risky activity, for $\eta_A \geq \eta_A^*$ both banks choose the risky activity, and for $\eta_A^* \leq \eta_A \leq \eta_A^*$ only one bank chooses the risky activity.

Proof. See Appendix.

Proposition 3 is interesting in itself as it shows in a relatively simple model that otherwise symmetric banks may end up choosing different risk profiles for strategic reasons. Incidentally, note also that this makes it more difficult to draw inferences on the state of the whole banking system from observations (e.g., critical incidences or site visits) at a single bank or at only few banks.

5. Risk taking and competition

Using Proposition 3, I can now proceed to the key comparative analysis on the impact of competition. For this recall first that the degree of horizontal differentiation is lower and competition is thus more intense as $\tau$ decreases. From differentiating the expressions for $\eta_A$ and $\eta_A^*$ in (11) and (12) with respect to $\tau$, I immediately obtain the following result.

Proposition 4. As competition increases, the threshold $\eta_A$ from Proposition 3 decreases, while the threshold $\eta_A^*$ increases. With more intense competition it thus becomes less likely that both banks either take on additional risk through activity A or stay prudent, while it becomes more likely that only one bank chooses A.

Proposition 4 formalizes the assertion from the Introduction that more intense competition has an asymmetric effect on risk-taking incentives. Under more intense competition banks are more likely to become asymmetric in terms of the additional risk they take on, as well as in terms of their offers and market shares when competing for borrowers (cf. Corollary 1).

Clearly, for very low and very high values of $\eta_A$ a change of $\tau$ will not affect the respective symmetric equilibria. An observable shift occurs instead around the respective thresholds $\eta_A$ and $\eta_A^*$, where a reduction of $\tau$ leads to a switch from a symmetric to an asymmetric equilibrium. Depending on the attractiveness of the “gambling” strategy A, more competition could thus indeed be associated both with more and with less risk taking in the banking industry. This may help to explain some of the conflicting and ambiguous empirical findings (cf. the Introduction). In the subsequent Sections I will obtain additional results in case banks are already asymmetric, thereby obtaining sharper predictions on which bank will become more or less prudent.

Discussion. An intuitive expression of Proposition 4 is in terms of the interaction of vertical and horizontal differentiation. As horizontal differentiation is reduced, without vertical differentiation banks’ profits on the loan market erode even if they are both (equally) attractive for a borrower. This reduces the incentives for a bank to still “play prudent” if this is also the strategy of its rival. These observations provide further intuition for the reduction of the lower threshold $\eta_A^*$. On the other hand, if one bank is expected to undertake the risky strategy, then following an increase in competition it becomes more profitable for its rival to differentiate itself by being instead more prudent. This provides an intuition for the increase in the upper threshold $\eta_A^*$.

Note that as competition increases, the joint market share of the considered two banks remains unchanged. (Likewise, the respective loan volume stays unchanged, namely at C.) As discussed above, more intense competition may be due to changes in regulation or technology, for instance. Alternatively, an increase in competition may result from entry into the loan market. In this case, the market shares of the incumbents banks that we consider so far would naturally decrease. This would now have an additional, more standard effect on the banks’ incentives to engage in the additional, risky activity, as I explore next. To capture this in the most simple way, suppose that a borrower only shows up with probability $\psi$. Then, banks’ profit function becomes $Pi = \psi \tau_1 + \eta_A$. With this exogenous variation in $\psi$, which thus captures only a rescaling of the market, the equilibrium on the loan market is not affect. Further, the respective thresholds $\eta_A$ and $\eta_A^*$ are generalized simply by multiplying the respective expressions with $\psi$. From this it follows immediately that the impact of a reduction in $\psi$ is that it unambiguously increases for both banks the incentive to undertake activity A. This is intuitive as

\textbf{19} In particular, equilibrium loan rates are unchanged. Note that this would clearly not be the case when we added, for instance, another firm on the Salop circle, as this would, in addition, reduce the distance between firms. The latter effect is, however, the same as a reduction in “transportation cost” on the Hotelling line, which was our previous analysis.
the only change that a reduction in $\psi$ engenders is to make the loan-making business less relevant, in terms of bank profits. Therefore, it pays less for each bank to forego profits from $A$ so as to increase its appeal to a potential borrower.

An alternative, standard way to formalize an increase in competition is by exogenously increasing the number of competitors. For instance, this is done in a Salop circle, where banks are equally distant, by increasing the number of competing firms from $N$ to $N+1$. This has two implications. First, in each “local market”, over which the adjacent two firms compete, firms become essentially less horizontally differentiated, as the average distance of the respective consumers decreases. This is akin to a reduction in $\tau$ in the model’s two-firm Hotelling set-up. Proposition 4 states the respective implications for firms’ choices of their vertical differentiation, namely risk-taking through activity $A$ in the present model. The second implication of the increase of the number of firms in the presently discussed Salop circle model is to reduce the market share of each firm. In the preceding paragraph, I have just discussed in a much simplified set-up the implication that this has on banks’ strategy, namely to unambiguously and symmetrically increase incentives to undertake strategy $A$ in the model. This thus exerts a countervailing force to the asymmetry obtained in Proposition 4. In this paper, I refrain from solving the discussed Salop-circle model. While in more standard applications the two-firm Hotelling analysis is readily extended in this way, as the outcome is typically symmetric, with endogenous asymmetry this is no longer so, given that the number of possible case distinctions increases.

A final observation concerns the fact that for the preceding comparative analysis the profitability of activity $A$ has been left unchanged. In particular, this implies that more intense competition in the lending market does not reduce the respective NPV $\eta_A$. Intuitively, this may be the case as the particular change in technology or regulation indeed only affects lending, but not the additional activity, such as underwriting. If this was no longer the case, note first that the previous results regarding the thresholds for $\eta_A$ would clearly not be affected. However, while the reduction in horizontal differentiation in the lending market induces banks to become more differentiated vertically through a different choice regarding activity $A$, a reduction in the NPV of this activity would counteract this asymmetry and reduce both banks’ incentives to choose $A$.

6. Leverage

So far I abstracted from the impact of leverage on risk taking. This allowed to clearly work out the new effects that are at the focus of this paper. In addition, the present analysis would also be justified if banks had sufficiently low leverage. In view of the fact that banks tend to have high leverage, however, it seems warranted to consider also the case where leverage is sufficiently high to possibly create additional risk-taking incentives.\(^{20}\) I show that this is indeed the case also in our model and that in this case the previous characterization of the equilibrium survives.

In more standard models, where there is no additional activity to be undertaken, there would only be three possible cash-flow realizations for the bank: zero (if a loan is made and is not repaid), $C$ (if no loan is made), and $R$ (if a loan is made and successfully repaid). Clearly, if $C$ exceeds the bank’s repayment obligation, $D_0$ then a loan would be made indiscriminately. I abstract from this case by assuming that $D_0 < C$. In the present setting, additional cash-flow states are possible as banks can also undertake activity $A$. If this is undertaken but realizes a loss, the bank’s cash flow equals $\beta y + f - F$ in case also a loan was made and had to be subsequently recalled. If no loan was made, the bank realizes either $C + f$ or $C + f - F$, depending on the outcome of the “gamble” from strategy $A$, provided this was undertaken. In equilibrium, if the commercial loan market is sufficiently important (in terms of profits) compared to strategy $A$, the ordering of the bank’s possible cash flow realizations is then as follows:

$$f < \beta y + f - F < C + f - F < C + f < R + f.$$  \hfill (13)

Recall now from the Introduction that there may be two instances when the bank does not roll over the loan or extend new credit facilities. This may simply be the outcome of insolvency, or it may be necessary to prevent – in our case otherwise unavoidable – insolvency. In what follows, I want to focus (also in the light of reducing case distinctions) on the first case. I thus assume that $D_0 > \beta y + f - F$: The bank cannot meet its repayment obligations even if it recalls the loan to fulfill its contractual obligations under strategy $A$.\(^{21}\) I also assume symmetry with $D_0 = D$.

Proposition 5. The preceding characterization of equilibria continues to hold if banks’ repayment obligations, $D_0 > D > 0$, affect their risk-taking incentives. That is, both banks undertake the risky activity if $\eta_A \geq \eta_A^*$, both banks stay prudent if $\eta_A \leq \eta_A^*$, and only one bank undertakes the risky activity if $\eta_A^* \leq \eta_A \leq \eta_A^*$. Furthermore, competition decreases $\eta_A$ and increases $\eta_A^*$. In addition, as leverage increases, both thresholds decrease, making it overall more likely that either bank undertakes the risky activity.

Proof. See Appendix.

The proof of Proposition 5 also shows that as leverage increases, banks become more aggressive in the market for borrowers. They thus demand lower interest rates, leading to lower values of $R$ and thus lower profits.

\(^{20}\) One should note one caveat regarding the use of the term leverage in this context. In an asymmetric equilibrium, banks’ expected profits will differ, implying that also the market value of their equity is different. Consequently, banks would then have different leverage in terms of debt value to total firm value (equity plus debt value). For simplicity I focus, however, on differences in banks’ outstanding repayment obligations.

\(^{21}\) Recall also that in this section we want to focus, in contrast to the previous analysis, on the case of relatively high leverage.
There are two remaining cases that must be distinguished in the proof. The most simple case is that where \( D < C + f - F \). Here, the bank fails either if its loan was not repaid or if it undertook both activities, i.e., that of making a loan and strategy \( A \), while the “gamble” from strategy \( A \) was not successful. If both banks stay prudent (\( \eta_A \leq \eta_A^* \)) equityholders realize in this case an expected payoff (net of \( C \)) that is just equal to \( \tau / 2 - D \), while if both “gamble” (\( \eta_A \geq \eta_A^* \)) their expected payoff is \( \tau / 2 - D + \eta_A^* \). Interestingly, as the repayment obligation (“face value of debt”) \( D \) increases, say by \( A_D \), the value of equityholders’ claims decreases by exactly the same amount, \( A_D \). While the increase in the value of debt is strictly smaller than \( A_D \), given that it is risky, the resulting difference is exactly made up by the reduction in each bank’s overall profits, following a reduction of the interest rate that is charged to borrowers, given that banks compete more aggressively.

The second remaining case is that where \( D > C + f - F \). Here, the bank fails also if it did not make a loan, but the gamble failed. If both banks stay prudent, then a marginal increase in \( D \) does not affect the outcome, such that equityholders still realize (net of \( C \)) \( \tau / 2 - D \). If both banks gamble, instead, then given the high probability with which debt will not be repaid in full, the value of equity now decreases by less than \( A_D \) (namely, by only \( pA_D \)).

### 7. Heterogeneous banks

I next extend the analysis to the case where banks have already initially different risk profiles. For the purpose of this analysis, I model it in a parsimonious way by stipulating that even without undertaking the risky activity \( A \), bank \( i \) will with probability \( 1-s_i \) have an outflow of funds denoted by \( S \) in \( t=2 \). (Again, \( s_i \) is thus a measure for the quality of a bank in terms of its initial appeal to the borrower.) All that matters for the analysis is that this outflow is not too large so that \( \beta y \) is still sufficient to cover \( S+f-F \). For simplicity, I further stipulate that both potential shocks are uncorrelated. If bank \( i \) undertakes activity \( A \), it will thus be able to roll over the loan only with probability \( p_{Si} \), while without strategy \( A \) this is the case with probability \( s_i \).

**Equilibrium analysis.** Turning first to competition in the loan market, I work, as previously, with a slightly more general notation: \( p_i \in (p,1) \). A borrower located at \( x \in [0,1] \) now realizes the expected profits

\[
p_1s_1q(y-R_i) - x\tau
\]

or

\[
p_2s_2q(y-R_2) - (1-x)\tau.
\]

when obtaining a loan from bank \( i=1 \) or bank \( i=2 \), respectively. The critical threshold at which the borrower is just indifferent becomes thus

\[
\hat{x} = \frac{1}{2} \cdot \frac{q_1p_1s_1(y-R_1) - q_2p_2s_2(y-R_2)}{2\tau}.
\]

Profits from competition on the loan market are then given by

\[
\hat{x}(p_1s_1qR_1 + (1-p_1s_1)\beta y - S(1-s_1)) - C_i,
\]

for bank \( i=1 \), while for bank \( i=2 \) equal

\[
(1-\hat{x})(p_2s_2qR_2 + (1-p_2s_2)\beta y - S(1-s_2)) - C_i.
\]

I confine a derivation of the resulting equilibrium profits to the proof of Proposition 6. To characterize banks’ incentives to undertake the risky activity \( A \), I have to introduce some additional notation in analogy to Proposition 3. For a given bank \( i \), there exist two thresholds for the net present value \( \eta_i \) of activity \( A \): For \( \eta_i < \eta_i^* \) bank \( i \) would never want to undertake the risky activity; for \( \eta_i \geq \eta_i^* \) bank \( i \) would always want to undertake the risky activity; and for \( \eta_i^* \leq \eta_i \leq \eta_i^* \) bank \( i \) would remain prudent if its rival undertakes the risky activity and would itself undertake the risky activity if its rival stays prudent instead. Clearly, if banks are initially symmetric with \( s_1 = s_2 \), then the thresholds for both banks are also symmetric: \( \eta_i^* = \eta_i^* \) and \( \eta_i^* = \eta_i^* \). We are thus back to the case analyzed in Proposition 3. If banks are, however, initially asymmetric, then the respective thresholds are different. In this case, I find that the initially more risky bank, say \( i=2 \) when \( s_1 > s_2 \), has higher incentives to take on additional risk. Formally, both thresholds are then lower for bank \( i=2 \): \( \eta_i^* > \eta_i^* \) and \( \eta_i^* > \eta_i^* \). (For a formal statement see the subsequent Proposition 6.)

The intuition for this result follows immediately from the previous observations on the properties of banks’ profit functions \( \pi_i \) (cf. Lemma 1). Recall, in particular, that \( \pi_i \) is convex in the probability with which bank \( i \) will be able to roll over the loan in \( t=2 \) (i.e., previously \( p_i \) and now \( s_i p_i \)). The higher is \( s_i \), the higher are thus also the benefits from staying prudent by not undertaking activity \( A \), i.e., from choosing \( p_1 = 1 \) instead of \( p_1 = p < 1 \).

The full characterization of all equilibria depends now on the size of the difference \( s_1 - s_2 > 0 \). Intuitively, if this difference is sufficiently large, then in an asymmetric equilibrium, which in analogy to Proposition 3 applies for intermediary values of \( \eta_i \), only the initially more risky bank \( i=2 \) will take on additional risk. Otherwise, i.e., if \( s_1 - s_2 > 0 \) remains small, then for intermediary values of \( \eta_i \) there will still be a multiplicity of equilibria: Either bank \( i=1 \) or bank \( i=2 \) undertakes activity \( A \), while the other bank stays prudent.

\[22\] The expressions for payoffs in the asymmetric equilibrium are somewhat more complicated and contained in the Appendix.
Proposition 6. Suppose banks have already initially different risk profiles with bank \( i = 2 \) being more risky as \( s_1 > s_2 \). Then banks’ incentives to take on additional risk in \( t = 1 \) are given as follows: (i) If the difference in \( s_1 - s_2 > 0 \) is sufficiently large, then \( \eta^*_{A1} > \eta^*_{A2} \) holds. In this case, for low net present value \( \eta_A \) of activity \( A \) \((\eta_A < \eta^*_{A2})\) neither bank undertakes \( A \), for high values of \( \eta_A \) \((\eta_A > \eta^*_{A1})\) both banks undertake \( A \), while for intermediate values of \( \eta_A \) \((\eta^*_{A2} < \eta_A < \eta^*_{A1})\) only the initially more risky bank, \( i = 1 \), undertakes \( A \). (ii) If instead \( \eta^*_{A1} < \eta^*_{A2} \) holds as the difference \( s_1 - s_2 > 0 \) is relatively small, then the only difference to case (i) is that there now exists an interval \( \eta^*_{A1} < \eta_A < \eta^*_{A2} \) with multiple asymmetric equilibria: Either one of the banks may undertake activity \( A \), while the other bank does not.

Proof. See Appendix.

Competition and risk taking. Again, we are mainly interested in the implications of an increase in competition, as captured by a lower value of \( \tau \). I take first Case (i) of Proposition 6. For the interval \( \eta^*_{A2} < \eta_A < \eta^*_{A1} \), where only bank \( i = 2 \) undertakes the additional risky activity \( A \), more competition reduces the lower boundary, \( \eta^*_{A2} \), and increases the upper boundary, \( \eta^*_{A1} \). (Formally, this follows immediately from differentiating the respective thresholds in the proof with respect to \( \tau \).) In this case, I can thus unambiguously conclude that more intense competition makes it more likely both that the initially more risky bank, \( i = 2 \), additionally undertakes the risky activity \( A \) and that the initially less risky bank, \( i = 1 \), stays prudent.

Recall next that in Case (ii) of Proposition 6 we still obtain multiple equilibria over an intermediary interval of values \( \eta_A \), though the size of this interval decreases as banks become ex ante more heterogeneous. If I then still choose the equilibrium where the initially more risky bank undertakes activity \( A \), then the comparative analysis in \( \tau \) from the previous Case (i) still fully applies. For the comparative analysis in the following Proposition I select this case for the following reason. Suppose instead that in Case (ii) and for \( \eta^*_{A1} < \eta_A < \eta^*_{A2} \) we would select the equilibrium where only the initially less risky bank, \( i = 1 \), undertakes activity \( A \). As I change \( \eta_A \), our predictions for the risk-taking incentives of both banks would then be non-monotonic in the following way: We would predict that bank \( i = 2 \) stays prudent for very low as well as some intermediary values of the net present value \( \eta_A \), while it undertakes activity \( A \) for high as well as for some lower range of values \( \eta_A \). Instead, if I select in Case (ii) and for \( \eta^*_{A1} < \eta_A < \eta^*_{A2} \) the equilibrium where the initially more risky bank also undertakes activity \( A \), then for each bank the predictions change monotonically in \( \eta_A \): The respective bank will undertake activity \( A \) if and only if \( \eta_A \) lies above a threshold value.

Proposition 7. Suppose banks have already initially different risk profiles with bank \( i = 2 \) being more risky as \( s_1 > s_2 \). Then as competition increases, the initially more risky bank \( i = 2 \) becomes more likely to undertake, in addition, the risky activity \( A \), while its initially less risky rival \( i = 1 \) is less likely to do so.

8. Efficiency

The discussion so far focused on making positive predictions. I thus did not analyze whether the chosen strategies were also optimal from a social perspective.\(^{23}\) What complicates the analysis of efficiency is the following observation. To compare total welfare, I have to consider two different sources of inefficiency that may arise from banks’ decision to undertake activity \( A \). First, this decision involves a trade-off between the present-value from this activity (if positive) and the risk of early withdrawal of funds and thus inefficient liquidation. Second, if banks end up with asymmetric profiles, then there are also inefficiencies in the loan market, given that the borrower may no longer obtain its loan at the “closest” bank, which would be the case only if \( \bar{x} = 1/2 \). In what follows, I take these two effects into account in two steps. Furthermore, it proves to be helpful to discuss the risk-taking incentives of the two banks in sequence, supposing first that only one bank considers to “gamble” and taking subsequently into account the incentives of the second bank.

(Efficient) Risk-taking incentives of “First Bank” \((i = 1)\). Suppose one bank, \( i = 2 \), is expected to stay prudent. If the other bank chooses strategy \( A \), instead, then this generates the net present value of \( \eta_A \) from the risky activity, while given early withdrawal the efficiency from a newly made loan is decreased by \((1-p)q/(q-\beta)\). The crux is now, however, that by undertaking the risky activity, the bank’s likelihood of being successful in the loan market will also be reduced. Precisely, starting from \( \bar{x} = 1/2 \) in the symmetric case, we have from (8), where \( p_1 = p \) and \( p_2 = 1 \), that subsequently the likelihood of making a loan (or “market share”) is reduced to

\[
\bar{x} = \frac{1}{2} - \frac{1}{2\tau} (1-p) \frac{1}{3} (q-\beta) y.
\]

(16)

I take this into account and focus first, as noted above, solely on the benefits and costs generated by the risky activity of bank \( i = 1 \). Then, that bank \( i = 1 \) takes on the additional risk is also efficient if

\[
\eta_A \geq \eta^*_{Eff} := (1-p)q/(q-\beta) \bar{x}.
\]

(17)

where \( \bar{x} \) is given by (16). After some transformations one can show that \( \eta^*_{Eff} > \eta_A \) holds if and only if \( \tau \) is sufficiently large. In contrast, if competition is sufficiently intense, such that \( \tau \) is relatively small, the opposite holds: \( \eta^*_{Eff} < \eta_A \) (cf. the proof of

\(^{23}\) This paper, as well as much of the literature that analyzes banks’ risk-taking incentives, abstracts from other reasons why regulators may want to ensure a more prudent behavior of banks, e.g., the risk of contagion if one bank fails.
Proposition 8. In words, if competition is sufficiently relaxed, then the bank will choose “too early” to gamble. Precisely, given \( \eta_A < \eta_{Eff} \) this holds if \( \eta_A^o < \eta_A < \eta_{Eff}^o \), in which case the expected inefficiency from possibly having to recall the loan still outweighs the created value \( \eta_A \). However, if competition is intense, then one can show that \( \eta_A^o > \eta_{Eff}^o \). Hence, for values \( \eta_{Eff} < \eta_A < \eta_{Eff}^o \) bank \( i = 1 \) should undertake the risky activity when we compare \( \eta_A \) with \( (1-p)y(q-\beta)x \), but it finds it privately profitable to stay prudent, instead.

The intuition behind this somewhat surprising result is the following. With intense competition the more risky (and thus in the eyes of the potential borrower inferior) bank will be at a substantial disadvantage, in terms of the equilibrium likelihood of making a loan. For low \( \tau \) this makes bank \( i = 1 \) too conservative in terms of its incentives to undertake \( A \).

The preceding observations ignore, however, the additional welfare implications that arise from the fact that with \( \tilde{x} < 1/2 \) there is a further loss in efficiency, given that a borrower with \( x \in (\tilde{x}, 1/2) \) will incur higher “shoe leather” costs than in a symmetric equilibrium. With less intense competition this reinforces the previous findings, given that we already observed that bank \( i = 1 \) had too high incentives to take on activity \( A \). Take thus the case of more intense competition and \( \eta_{Eff}^o < \eta_A^o \). As I show in the proof of Proposition 8, once we take into account welfare implications, then regardless of the intensity of competition we have for the newly defined threshold \( \tilde{\eta}_{Eff} \) that \( \eta_A^o < \tilde{\eta}_{Eff} \). That is, holding the “prudent” choice of bank \( i = 2 \) fixed, bank \( i = 1 \) has always too high incentives to choose the risky strategy, once total welfare is taken into account.

(Efficient) Risk-taking incentives of “Second Bank” \( (i = 2) \). Suppose now that bank \( i = 1 \) is expected to undertake \( A \). But bank \( i = 2 \) undertakes the risky activity, from a welfare perspective we again have to first trade-off again the realization of \( \eta_A \) with the fact that the loan may be inefficiently recalled. Taking the threshold \( \tilde{x} \) from (16), the latter inefficiency is equal to \( 1 - \tilde{x} \) times \( (1-p)y(q-\beta) \). From this I obtain the criterion

\[ \eta_A \geq \tilde{\eta}_{Eff} = (1-p)y(q-\beta)(1-\tilde{x}), \]

where \( \tilde{\eta}_{Eff} \) is clearly strictly higher than the threshold \( \eta_{Eff} \) in (17). Moreover, I now find that \( \eta_A^o < \tilde{\eta}_{Eff} \) always holds. That is, when only trading off \( \eta_A \) with the possibility of an inefficient termination of a loan, then the transition to a symmetric equilibrium where both banks undertake the risky activity is always too “early”. However, when one considers total welfare, i.e., again including the “shoe leather” costs of the borrower, then as competition becomes sufficiently relaxed, results are reversed: Bank \( i = 2 \) has insufficient incentives to take on additional risk.

Summary. The following Proposition summarizes results and provides a comparison between the efficiency benchmark and the equilibrium outcome. For brevity I restrict the statement of the Proposition to the full-welfare benchmark.

**Proposition 8.** Take as a benchmark that of total welfare, including the NPV of strategy \( A \), inefficiencies from early liquidation of a funded project, and possibly inefficiently high (“shoe leather”) costs in the market for borrowers. Then, there are two thresholds \( \tilde{\eta}_{Eff} < \tilde{\eta}_{Eff}^o \) such that no bank should undertake the risky activity for \( \eta_A \leq \tilde{\eta}_{Eff} \), both banks for \( \eta_A > \tilde{\eta}_{Eff}^o \), and only one bank for \( \tilde{\eta}_{Eff} < \eta_A < \tilde{\eta}_{Eff}^o \). These thresholds compare with the equilibrium outcome in Proposition 3 as follows: (i) It always holds that \( \eta_A^o < \tilde{\eta}_{Eff}^o \). The “first” bank that chooses the risky strategy will always do so “too early”, i.e., for values of \( \eta_A \) that are still too low from a welfare perspective. (ii) In contrast, \( \eta_A^o < \tilde{\eta}_{Eff}^o \) holds only if competition is not too intense (low \( \tau \)), while otherwise (high \( \tau \)) it holds that \( \eta_A^o > \tilde{\eta}_{Eff}^o \). It thus depends on competition whether for interim values of \( \eta_A \) there is too much risk taking as both banks choose \( A \), but should not, or too little risk taking as both banks should choose \( A \), but only one bank does so.

**Proof.** See Appendix.

A key observation from Proposition 8, as well as from our preceding discussion, is that even in the absence of (additional) risk-taking incentives from leverage, banks may not have first-best incentives to take on risk. However, as our discussion and Proposition 8 reveal, risk-taking incentives can be too high or too low, depending on how a bank’s concern for its reputation with potential corporate borrowers interacts with competition. Here, in line with our preceding observations on risk-taking incentives, from Proposition 8 we obtain ambiguous results if competition becomes sufficiently intense. In this case, there may be either too much (namely, for low values of \( \eta_A \)) or too little (namely, for high values of \( \eta_A \)) risk taking in the market.24

Admittedly, Proposition 8 does not generate clear-cut prescriptions for regulation and supervision. Also, recall that our welfare analysis neglects any externalities that arise from risk-taking and the resulting possible failure of a bank. Still, however, Proposition 8 and the preceding analysis point to the key beneficial role that competition can play to mitigate risk-taking incentives. In particular, when one bank is expected to engage in additional risk taking, with intense competition a rival bank has much to gain when it stays “prudent” and, thereby, enhances its attractiveness in the eyes of commercial borrowers. This mechanism is, however, only at work when competition prevails.

9. Conclusion

This paper analyzes how a bank’s reputation with potential corporate borrowers interacts with its incentives to take on (additional) risk, e.g., through proprietary trading, additional loan commitments, guarantees, or other contingent obligations. Corporate borrowers, in particular those who find it difficult to borrow from the market or from other

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24 That applies, more precisely, for values of \( \eta_A \) that are low but not too low, and values of \( \eta_A \) that are high but not too high—as, otherwise, no bank or both banks would and should take on additional risk.
lenders at short notice, care about the bank’s overall financial strength, as this will determine their future access to credit. Banks that face a short-fall of profits or that are hit by high payment obligations will cease to roll over outstanding loans or they will call back existing credit facilities.

This paper investigates primarily the interaction between risk-taking incentives and competition. Though much has already been said on this topic (cf. the Introduction for an account of the literature), a key distinctive feature of our analysis is that it generates an asymmetric response of banks’ risk-taking incentives to competition. In particular, I found that banks that are already more riskier have an increasing appetite to take on more risk as competition intensifies, while their safer rivals are more likely to stay prudent. As noted in the Introduction, this may shed further light on some conflicting and ambiguous findings in the literature.

In addition, our findings provide some guidance for bank supervisors and regulators. Supervisors’ response to increased competition should be asymmetric, by targeting already riskier banks, while relaxing oversight of presently more prudent institutions (provided that the objective is to identify and potentially limit (additional) risk taking). Furthermore, regulators and supervisors should view competition not as being detrimental to financial stability. Instead, when banks’ risk positions are transparent to commercial borrowers, I argue that competition may stifle banks’ risk appetite, as it turns “prudence” into a key competitive variable.

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Appendix A

Proof of Proposition 3. Using (10), if both banks undertake activity A, then each bank realizes the profit \( \eta_A + \tau/2 \). A deviating bank obtains, instead, the profit

\[
\frac{1}{2}\tau + \frac{(qy - y)(1-\beta)}{3}. 
\]

Comparing the two profit levels shows that we can support a symmetric equilibrium with risk taking if and only if \( \eta_A \geq \eta_A^* \), as characterized in (12). If both banks are prudent, they realize profits of \( \tau/2 \). If one bank deviates and undertakes activity A instead, its profit equals

\[
\eta_A^* + \frac{1}{2}\tau - \frac{(qy - y)(1-\beta)}{3}. 
\]

Comparing the two profit levels shows now that we can support a symmetric equilibrium without risk taking if and only if \( \eta_A \leq \eta_A^* \), as characterized in (11).

Turning finally to asymmetric pure-strategy equilibria, note first that the respective profits of the prudent and the gambling bank are then given by (18) and (19), respectively. Comparing these profits to the profits \( \tau/2 \) and \( \eta_A + \tau/2 \) under a symmetric choice, i.e., if one bank deviates, we obtain that the bank undertaking activity A obtains, instead, the profit \( Z \). Comparing these profits to the profits \( Z \), we obtain that the bank undertaking activity A will not deviate if \( \eta_A \geq \eta_A^* \), while the prudent bank will stay so only if \( \eta_A \geq \eta_A^* \). Observe finally that \( \eta_A^* > \eta_A^* \). □

Proof of Proposition 5. Note first that there are two cases to be distinguished: that with \( \beta y - f + f < D < C + f - F \) and that with \( C + f - F < D < C \).

The Case with \( \beta y - F + f < D < C + f - F \): Using (3) for \( \hat{x} \), the expected payoff to equityholders of bank \( i = 1 \) is given by

\[
\hat{x}(p_1 q(R_1 + f_1 - D_1)) + (1-\hat{x})(p_1 (C + f_1 - D_1) + (1-p_1)(C + f_1 - F - D_1)),
\]

where \( p_1 = \beta \) and \( f_1 = f \) if the bank chooses activity A, while otherwise \( p_1 = 1 \) and \( f_1 = 0 \). Bank \( i = 1 \) chooses \( R_1 \) to maximize (20). For bank \( i = 2 \) we obtain likewise for equityholders’ payoff

\[
(1-\hat{x})(p_2 q(R_2 + f_2 - D_2)) + \hat{x}(p_2 (C + f_2 - D_2) + (1-p_2)(C + f_2 - F - D_2)).
\]

From the respective first-order conditions for (20) and (21) we obtain after some transformations (and in case of an interior solution for \( \hat{x} \)) that

\[
R p_1 q = \tau + C - (D-f_1)(1-qp_1) - F(1-p_1) + \frac{(f_1-f_1)(1-qp_1) + (p_1-p_1)(q(y-D) + qf_1-F)}{3},
\]

which after substitution into (20) and (21), respectively, yields the expected payoff to equityholders of bank \( i \)

\[
f_1 - F(1-p_1) - D + \tau + \frac{(f_1-f_1)(1-qp_1) + (p_1-p_1)(q(y-D) - F + qf_1)}{3}.
\]
If both banks undertake activity $A$, then each bank’s equityholders’ payoff is calculated by inserting $p_i = p_j = p$ and $f_i = f_j = f$ into (22), yielding
\[
\frac{\tau}{2} - D + \eta A.
\]
A deviating bank's equityholders' payoff is, instead, after substitution of $p_i = 1$, $p_j = p$, $f_i = 0$, and $f_j = f$, equal to
\[
\frac{1}{2\tau} \left( \tau + \frac{pf(1-q) + (1-p)(q(y-D) + f - F)}{3} \right)^2 - D.
\]
Comparing this to (23) shows that we can support a symmetric equilibrium with risk taking if and only if
\[
\eta A \geq \tilde{\eta} A := \frac{1}{2\tau} \left( \tau + \frac{pf(1-q) + (1-p)(q(y-D) + f - F)}{3} \right)^2 - \frac{\tau}{2}.
\]
If both banks are prudent ($p_i = p_j = 1$, $f_i = f_j = 0$), their equityholders’ payoff is $\tau/2 - D$. If one bank deviates and undertakes activity $A$ instead ($p_i = p$, $p_j = 1$, and $f_i = f$, $f_j = 0$), its equityholders’ payoff equals
\[
\eta A - D + \frac{1}{2\tau} \left( \tau - \frac{(1-q)(p(1-q) + (1-p)(q(y-D) + f - F))}{3} \right)^2.
\]
Comparing the two payoffs shows now that we can support a symmetric equilibrium without risk taking if and only if
\[
\eta A \leq \tilde{\eta} A := \frac{1}{2\tau} \left( \tau - \frac{(1-q)(p(1-q) + (1-p)(q(y-D) + f - F))}{3} \right)^2.
\]
Turning to asymmetric pure-strategy equilibria, note first that the payoffs of the equityholders of the prudent and the risk taking bank are then given by (24) and (25), respectively. Comparing these to $(\tau/2) - D$ and $(\tau/2) - D + \eta A$ under a symmetric choice, we obtain an asymmetric equilibrium if $\tilde{\eta} A \leq \eta A \leq \eta A''$, where indeed $\tilde{\eta} A > \eta A''$.

The Case with $C + f - F < D < C$: If $C + f - F < D < C$, using (3) the expected payoff of the equityholders of bank $i = 1$ is given by
\[
\hat{x}(p, q(R_1 + f_i, D - f_i)) + (1 - \hat{x})(p_1(C + f, 1 - D)).
\]
Likewise, the optimal choice of $R_2$ for the rival bank $i = 2$ maximizes
\[
(1 - \hat{x})(p_2 q(R_2 + f_2, 1 - D)) + \hat{x}(p_2 C + f_2, 1 - D)).
\]
From the respective first-order conditions for (26) and (27) we have that
\[
p_i q R_i = \tau + C p_i - p_i(1-q)(D - f_i) + \frac{(p_i - p_j)(qy - C) + (1-q)(p_i D - f_i) - p_j(D - f_j))}{3}.
\]
Using expressions (26) and (28), we can calculate equityholders’ equilibrium payoffs:
\[
\frac{1}{2\tau} \left( \tau + \frac{(p_i - p_j)(qy - C) + (1-q)(p_i D - f_i) - p_j(D - f_j))}{3} \right)^2. - (1-p_i)C - p_i(D - f_i).
\]
If both banks undertake activity $A$, then each bank’s equityholders’ payoff is calculated by inserting $p_i = p_j = p$ and $f_i = f_j = f$ into (29), which yields
\[
\frac{\tau}{2} + \eta A + (F - f)(1-p) - C(1-p) - pD.
\]
In case of a deviation, such that $p_i = 1$, $p_j = p$, $f_i = 0$, and $f_j = f$, we have that
\[
\frac{1}{2\tau} \left( \tau + \frac{(1-p)(qy - C) + (D(1-p) + fp)(1-q)}{3} \right)^2 - D.
\]
Comparing the two payoffs shows that we can support a symmetric equilibrium with risk taking if and only if
\[
\eta A \geq \tilde{\eta} A := (1-p)(-F + f + C - D) + \tau \frac{1}{2\tau} \left( \tau + \frac{pf(1-q) + (1-p)(D(1-q) + qy - C)}{3} \right)^2.
\]
If both banks are prudent ($p_i = p_j = 1$, and $f_i = f_j = 0$), their equityholders’ payoff is $(\tau/2) - D$. If one bank deviates and undertakes activity $A$ instead, such that $p_i = p$, $p_j = 1$, $f_i = f$, and $f_j = 0$, its equityholders’ payoff equals
\[
\frac{1}{2\tau} \left( \tau - \frac{(1-p)(qy - C) + (D(1-p) + fp)(1-q)}{3} \right)^2 - C(1-p) - pD.
\]
Comparing the two payoffs shows now that we can support a symmetric equilibrium without risk taking if and only if
\[
\eta A \leq \tilde{\eta} A := (1-p)(-F + f + C - D) + \tau \frac{1}{2\tau} \left( \tau - \frac{pf(1-q) + (1-p)(D(1-q) + qy - C)}{3} \right)^2.
\]
Turning to asymmetric pure-strategy equilibria, note first that the payoffs of the equityholders’ of the prudent and the gambling bank are then given by (31) and (32), respectively. Comparing these to $(\tau/2) - D$ and $(\tau/2) + \eta A + (F - f)$
Comparing these two profits, we have that bank initially a different risk profile, as expressed by the probabilities 1 \(- p\) under a symmetric choice, we can support an asymmetric outcome if \( \hat{\eta}'_A \leq \eta_A \leq \hat{\eta}''_A \), where we use that \( \hat{\eta}''_A \gg \hat{\eta}'_A \). □

**Proof of Proposition 6.** From the respective first-order conditions we obtain the following results.\(^{25}\) If banks have already initially a different risk profile, as expressed by the probabilities 1 \(- p\) with which they will have to recall a loan in \( t = 2 \) even without strategy A, then their expected profits from the loan market are given by

\[
\pi_t = \frac{1}{2\tau} \left( \tau + \frac{y(p_1 - p_2)(q - \beta)}{3} \right)^2 - S(1 - s_1). \tag{33}
\]

Note next that I stipulate without loss of generality that \( s_1 > s_2 \). Suppose now that one bank is anticipated to undertake activity A. If the other bank, say now \( i = 1 \), also undertakes activity A, then from (33) the respective profits equal

\[
\eta_A + \frac{1}{2\tau} \left( \tau + \frac{y(p_1 - s_2)(q - \beta)}{3} \right)^2 - S(1 - s_1),
\]

while if bank \( i = 1 \) stays prudent profits are

\[
\frac{1}{2\tau} \left( \tau + \frac{s_1 - s_2}{3} \right)^2 - S(1 - s_1). \]

Comparing these two profits, we have that if bank \( i = 1 \) expects its rival to undertake A, then it prefers to do so as well if

\[
\eta_A \geq \eta'_{A_1} = \frac{1}{6\tau} y(q - \beta)s_1(1 - p) \left( \frac{y(q - \beta)}{3} (s_1(1 + p) - 2ps_2) + 2\tau \right).
\]

Proceeding likewise for bank \( i = 2 \), we obtain the threshold

\[
\eta_A \geq \eta'_{A_2} = \frac{1}{6\tau} y(q - \beta)s_2(1 - p) \left( \frac{y(q - \beta)}{3} (s_2(1 + p) - 2ps_1) + 2\tau \right).
\]

Note that when subtracting \( \eta'_{A_2} \) from \( \eta'_{A_1} \), we obtain

\[
\frac{1}{3} y(q - \beta)(s_1 - s_2)(1 - p) > 0.
\]

Suppose next that a bank’s rival is anticipated not to undertake activity A. If the other bank, say again first \( i = 1 \), now undertakes activity A alone, then from (33) the respective profits equal

\[
\eta_A + \frac{1}{2\tau} \left( \tau + \frac{y(p_1 - s_2)(q - \beta)}{3} \right)^2 - S(1 - s_1),
\]

while if bank \( i = 1 \) also stays prudent profits are

\[
\frac{1}{2\tau} \left( \tau + \frac{y(s_1 - s_2)}{3} \right)^2 - S(1 - s_1). \]

Comparing these two profits, we have that bank \( i = 1 \) prefers to also undertake A if

\[
\eta_A \geq \eta''_{A_1} = \frac{1}{6\tau} y(q - \beta)s_1(1 - p) \left( \frac{y(q - \beta)}{3} (s_1(1 + p) - 2s_2) + 2\tau \right),
\]

which satisfies \( \eta''_{A_1} < \eta''_A \). Proceeding likewise for bank \( i = 2 \), we obtain the threshold

\[
\eta_A \geq \eta''_{A_2} = \frac{1}{6\tau} y(q - \beta)s_2(1 - p) \left( \frac{y(q - \beta)}{3} (s_2(1 + p) - 2s_1) + 2\tau \right).
\]

Note again that when subtracting \( \eta''_{A_2} \) from \( \eta''_{A_1} \), we obtain

\[
\frac{1}{3} y(q - \beta)(s_1 - s_2)(1 - p) > 0.
\]

Note finally that \( \eta''_{A_1} > \eta''_{A_2} \) holds in case

\[
s_1 - s_2 > \frac{3s_1s_2(1 - p)}{y(q - \beta)\tau}.
\]

The characterization of the different equilibria for cases (i) and (ii) follows then immediately from the construction of the different thresholds. □

\(^{25}\) Again, to obtain in equilibrium an interior solution \( 0 < \delta < 1 \) it must hold that \( y(q - \beta)(p_1s_2 - p_1s_1) < 3\tau \), which in turn is always satisfied as long as \( y(q - \beta)(s_1 - ps_2) < 3\tau \).
Proof of Proposition 8. It is convenient to define for this proof $B := (1-p)y(q-\beta)$. The threshold (17) then becomes, after substitution from (16),

$$\eta_{\text{Eff}}' = \frac{1}{2} B \left( 1 - \frac{1}{3\tau} B \right).$$

while I can use from (11) that

$$\eta_A' = \frac{1}{2} \left[ \tau - \frac{1}{\tau} \left( \tau - \frac{B}{3} \right)^2 \right].$$

That $\eta_{\text{Eff}}' > \eta_A'$ follows then if

$$B \left( 1 - \frac{1}{3\tau} B \right) > \tau - \frac{1}{\tau} \left( \tau - \frac{B}{3} \right)^2,$$

which finally transforms to $B < 3\tau/2$. (Note that this is not implied by (6), which only requires that $B < 3\tau$.)

I next take also into account the “shoe leather” costs of the borrower:

$$E(t) = C := \tau \left( \int_0^x x \, dx + \int_x^{\tau-x} x \, dx \right) = \tau \left[ \frac{1}{2} \tau (1 - \bar{x}) \right].$$

We have $C = \tau/4$ in a symmetric equilibrium. Given $\bar{x} = \frac{1}{2} - (1/6\tau)B$ for an asymmetric equilibrium, the respective costs become

$$C = \tau \frac{1}{4} \left( 1 + \frac{B^2}{9\tau^2} \right).$$

(34)

The adjusted efficiency threshold then becomes

$$\tilde{\eta}_{\text{Eff}}' := \frac{1}{2} B \left( 1 - \frac{1}{3\tau} B \right) + \tau \frac{1}{4} \left( 1 + \frac{B^2}{9\tau^2} \right),$$

such that $\tilde{\eta}_{\text{Eff}}' > \eta_A'$ holds only if

$$\tau > \left( \frac{B}{3\tau} \right) (B - 2\tau).$$

(35)

Note now that we have from (6) that $\tau > B/3$, implying that (35) always holds if $B - 2\tau < \tau$, which once again becomes $\tau > B/3$.

To compare $\eta''_{\text{Eff}} = B(1-\bar{x})$ with $\eta''_A$, note that $\eta''_{\text{Eff}} > \eta''_A$ holds if

$$B \left( 1 + \frac{1}{3\tau} B \right) > \tau \left( \tau + \frac{B}{3} \right)^2 - \tau,$$

which after some transformations indeed always holds. Using next C from (34) for the asymmetric case, we obtain a new threshold

$$\tilde{\eta}_{\text{Eff}}'' := \frac{1}{2} B \left( 1 + \frac{1}{3\tau} B \right) - \tau \frac{1}{4} \left( 1 + \frac{B^2}{9\tau^2} \right),$$

for which $\tilde{\eta}_{\text{Eff}}'' > \eta''_A$ holds if

$$\tau < \frac{B}{3\tau} (2\tau + B).$$

(36)

Note that this is compatible with $\tau > B/3$ from (6). Substituting $\tau = B/3$, the condition is satisfied up to some threshold for $\tau$, from which on the converse of (6) holds strictly. 

References


