Notes, Comments, and Letters to the Editor

Contractual distortions in a market with frictions ☆

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Abstract

This paper analyses contract design in a decentralized market environment with frictions. While principals (e.g., firms) have all contractual power, their market power is constrained as agents (e.g., workers) can choose to wait and search for better offers. We find that results depend crucially on how market frictions affect agents’ utilities. With type-independent costs of search and waiting, equilibrium contracts are always first-best. If agents are impatient and discount future payoffs, however, distortions vanish only gradually. In the latter case, we also characterize equilibrium offers and show that the market exhibits two types of externalities, both of which are absent in the case of type-independent costs of search.

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1. Introduction

The analysis of contract design by uninformed principals has until recently mainly focused on the two extremes, where a principal either enjoys a monopolistic situation
or where he faces perfect competition. With private values, i.e., if the agent’s private information (his type) does not enter directly the principal’s utility function, perfect competition yields efficient contracts, while a monopolistic principal distorts his offers, trading off surplus maximization with rent extraction.

For many applications of contract theory, e.g., to managerial or labor contracts, neither of the two extremes of perfect competition or monopoly seems appropriate. Individual firms (principals) rarely enjoy a monopolistic position, while even if vacancies outnumber job-seekers (agents), limited transparency in the labor market softens competition between firms. In this paper, we consider a decentralized market with frictions. Our main question is whether and how some of the results obtained in the standard monopolistic case still hold. We find that the answer to this question depends crucially on how market imperfections impose costs on agents who try to elicit new offers.

We consider a model where each period agents attract a random number of principals, who subsequently propose contracts. Even if an agent does not succeed in attracting more than one offer in a given period, the monopolistic power of the respective principal is limited as the agent can always choose to wait. The value of the agent’s option to wait, i.e., the agent’s reservation value, depends on the frictions prevailing in the market. If agents incur fixed costs of search or waiting, we find that principals make efficient offers regardless of the level of frictions prevailing in the market, i.e., all types of agents who enter the market receive first-best contracts. In contrast, if waiting is costly as agents are impatient, an agent who only attracted a single principal will be offered an inefficient menu of contracts. Low types are excluded and contracts are distorted for all but the highest type. These distortions vanish only gradually with lower frictions.

The two different costs of delay, i.e., impatience and constant costs of search and waiting, are known to produce qualitatively similar results in search and matching models if either there is no private information or if contracts specify only transfers. In contrast, results are markedly different in our case. If agents are impatient, the level of frictions determines not only how surplus is split, but also how much surplus is realized in a given match. This result has a straightforward intuition. If frictions affect the value of the option to search and wait differently for agents of different types, as it is the case with discounting, the reservation value function has a different slope than the first-best surplus function. This leaves principals with some scope to extract (information) rent by offering distorted contracts. In contrast, if all types are affected equally by the costs of search or waiting, reservation values fully reflect the differences in first-best surplus.

Besides affecting the nature of equilibrium contracts, the way frictions impose costs on agents has far-reaching implications for market efficiency. If market participants are impatient, we find two types of externalities, which are otherwise absent. First, new principals increase competition, which exerts a positive “contracting externality” on the outcome of all other matches. Second, there may exist multiple Pareto-ranked equilibria as the principals’ decision with whom to contract affects the distribution of agents in the market.
As shown in [14], equilibrium contracts can take on a wide range of different forms if only few restrictions are imposed on the reservation value function. Using the linear preferences studied in [14], we are able to characterize equilibrium offers for the case of discounting even if the market exhibits considerable frictions, implying highly distorted contracts. In this case, the distorted menu exhibits the following three features: (i) low types are excluded; (ii) the participation constraint is binding for the medium segment of types; and (iii) the participation constraint is slack for the top segment of types. As frictions decrease, efficiency is gradually improved as the set of types at which the participation constraint binds is extended both upwards and downwards (to previously excluded types).

Our results on the shape of contracts mirror previous results derived for non-linear price discrimination in a differentiated duopoly. Under some additional assumptions, efficient contracts are obtained in a standard Hotelling framework in Armstrong and Vickers [2] and Rochet and Stole [17]. In Stole [18] distance does not affect all types equally, implying equilibrium contracts similar to those obtained in our model with discounting. We have more to say on this comparison below.

The rest of this paper is organized as follows. Section 2 introduces the model. Sections 3 and 4 analyze the cases with discounting and additive search costs. Section 5 concludes. All proofs are relegated to Appendix A.

2. The model

2.1. Players and payoffs

We consider an anonymous market populated by a continuum of principals and agents, who can generate a surplus if they match pairwise. A contract specifies a transfer \( w \) from the principal to the agent and an additional real-valued variable \( q \geq 0 \). The agent has private information about his type, which is represented by a real-valued variable \( \theta \). We specify that \( \theta \) is drawn from an interval \( [\underline{\theta}, \bar{\theta}] \), where \( 0 < \bar{\theta} < \bar{\theta} < 1 \).

In what follows, we cast our model and our results in the framework of a labor market. We therefore refer to principals as firms and to agents as workers. The variable \( q \) may represent working hours, contractible effort, or output.

The worker’s utility is linear in his type. Such preferences are commonly assumed, e.g., in [14,15,17]. This specification allows to explicitly characterize the (distorted) equilibrium contracts in the case of discounting. The worker’s utility is given by

\[
V(\theta, w, q) := w - v(\theta, q),
\]

where \( v(\theta, q) := q(1 - \theta) \) represents his disutility from achieving \( q \). The firm’s preferences are given by

\[
U(w, q) := u(q) - w,
\]

where

\[
u(q) := q(1 - \theta).
\]

\[1\] See [9,10] for the case with discrete types. Type-dependent reservation values arise also in different economic environments, e.g., if an initial contract is in place as in [11], if the agent is a regulated firm facing competition as in [3,5], or if the agent has the opportunity to make offers at some future point of time as in [8]. Related work on countervailing incentives includes [4,12,13].
\[ q - \frac{1}{2} q^2. \] Total surplus thus equals

\[ S(\theta, q) = q\theta - \frac{1}{2} q^2, \] which is maximized by setting

\[ q^*(\theta) = \theta \] to obtain

\[ S^*(\theta) = \theta^2 / 2. \]

### 2.2. Market environment

We consider an anonymous and stationary market with endogenous entry. Time runs discretely. Each period, the mass one of new workers arrives at the market fringe. New workers are distributed over \([\theta, \bar{\theta}]\) according to the distribution function \(F^0(\theta)\), which has a continuously differentiable density \(f^0(\theta)\), satisfying \(f^0(\theta) > 0\) for all \(\theta \in [\theta, \bar{\theta}]\). We invoke the following standard assumption on the (inverse) hazard rate.

**Assumption 1.** \( \frac{d}{d\theta} \left( \frac{1-F^0(\theta)}{f^0(\theta)} \right) \leq 0. \)

Workers can decide to enter the market or to take up an outside option, which has the type-independent value of zero. We denote the set of types entering the market by \(\Theta^E \subseteq \Theta\). The distribution of types in the market is denoted by \(F(\theta)\) over \(\Theta^E\). The time-invariant stock of workers in the market is denoted by \(m_W\). The stock of firms, which is denoted by \(m_F\), is determined by a zero-profit requirement. Firms have to incur up-front costs \(K > 0\) to open a vacancy. We specify that the maximum surplus that can be produced with the lowest type is just sufficient to cover the costs of opening a vacancy. Using \(S^*(\theta) = \theta^2 / 2\), we thus invoke:

**Assumption 2.** \( \theta = \sqrt{2K}. \)

If the market opens up, we denote the ratio of firms to workers by \(t = m_F / m_W\). The “tightness” \(t\) determines the speed with which firms and workers can match. The speed of matching is important as delay is costly. Below, we consider two different specifications for these costs. In each period, a given worker may attract a random number of firms. A worker who has not encountered a firm must wait another round. Otherwise, he may pick one of the contracts offered by those firms who have chosen this particular worker. If a worker rejects all proposals, he and the respective firms must re-enter the market. If a worker accepts one of the received offers, the worker and the chosen firm leave the market.

We denote the probability with which a given worker is visited by exactly one firm by \(p_W(1)\), the probability with which he encounters at least two firms by \(p_W(2)\), and the residual probability with which he does not meet any firm by \(p_W(0) = 1 - p_W(1) - p_W(2)\). Clearly, these probabilities will depend on the market conditions. We turn to this further below. The probability with which a firm will find itself in a monopolistic position is denoted by \(p_F(1)\). With probability \(p_F(2)\), the firm will find a worker, but it will have to compete with other firms. With the residual probability \(p_F(0) = 1 - p_F(1) - p_F(2)\), the firm does not meet a worker in a given period. Note that \(p_W(0) > 0\) need not imply \(p_F(0) = 0\) and vice versa. In fact, if the
market is not very efficient, a substantial fraction of both workers and firms may fail to match in a given period. Market imperfections, in particular those relating to coordination failure among firms when contacting job searchers or responding to applicants, will typically also imply that \( p_W(0) > 0 \), \( p_W(1) > 0 \), and \( p_W(2) > 0 \) hold at the same time. That is, some workers are not matched, some are only matched with a single firm, and some attract more than one firm.

One possibility may be that all firms decide randomly which worker to contact. For each worker, the number of firms met in a given period becomes then Poisson \( t \) distributed. In this case, we obtain for workers \( p_W(0) = e^{-t} \), \( p_W(1) = te^{-t} \), and \( p_W(2) = 1 - e^{-t}(1 + t) \). Moreover, for firms it holds that \( p_F(0) = 0 \), \( p_F(1) = p_W(0) \), and \( p_F(2) = 1 - p_W(0) \). Below, we will use this “random assignment” (or “exponential matching”) technology for an illustration of our results.2

We make only few assumptions on the matching technology. Taken together, for low costs of search and waiting, these assumptions would ensure existence of a unique equilibrium if there were no informational problems, i.e., if firms could observe workers’ types. We assume that all probabilities \( p_F(n) \) and \( p_W(n) \), where \( n = 0, 1, 2 \), depend only on the ratio of firms to workers \( t \) and change continuously with \( t \). (For the sake of brevity we chose to omit the argument \( t \), writing, for instance, \( p_F(n) \) instead of \( p_F(n, t) \).) Moreover, as \( t \) increases, the probability that workers encounter more than one firm, \( p_W(2) \), increases and the probability that firms enjoy a monopolistic position, \( p_F(1) \), decreases, one strictly. Finally, we assume that \( p_W(2) \) and \( p_F(1) \) become both strictly positive for some values of \( t \).3

**Assumption 3.** (i) All matching probabilities \( p_F(n) \) and \( p_W(n) \), where \( n = 0, 1, 2 \), depend only on \( t \) and are continuous in \( t \).

(ii) As \( t \) increases, \( p_F(1) \) decreases and \( p_W(2) \) increases, one strictly. Moreover, for some \( t \) it holds that both \( p_F(1) > 0 \) and \( p_W(2) > 0 \).

2.3. Market frictions

We consider two types of frictions.

*Additive search costs*: In this case, we assume that waiting another round comes at the constant (search) cost \( c > 0 \) to firms and workers.

*Impatience*: In this case, we assume that firms and workers discount future payoffs by the factor \( d \in (0, 1) \).

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2 This random assignment technology is popular in the literature and used, for instance, in [16,19].

3 We omit specifying the conditions that any matching technology must satisfy such that “supply” equals “demand”. For example, it must hold that \( m_W \pi_W(1) = m_F \pi_F(1) \), i.e., the measures of firms and workers in “monopolistic matches” must be equal, and that \( m_F \pi_F(2) \geq 2m_W \pi_W(2) \), i.e., the measure of firms in “competitive matches” must be at least twice the measure of workers who can choose between the offers of at least two firms. Moreover, for our purposes, we do not have to fully specify all matching probabilities, e.g., with which probability workers will find three or four firms.
Additive search costs have various interpretations. The worker may incur costs from posting personal advertisements or sending out his CV. He may also have substantial opportunity costs for spending his time on searching for a (better) job, instead of spending it, for instance, on leisure.

2.4. Contracting

Firms offer a menu of deterministic contracts, from which the worker may choose one after acceptance. We require equilibrium strategies to be stationary and sequentially optimal. Given stationarity of the market and of strategies, the expected payoffs of firms and workers are also time-independent. We refer to the payoffs as the reservation values of firms and workers, respectively. Reservation values are denoted by $U^R$ for firms and by $V^R(\theta)$ for workers of type $\theta$. Firms know whether they find themselves in a monopolistic situation, i.e., whether the particular worker has not attracted any other firm, or whether they must compete for the particular worker. If firms have to compete, the following implications are immediate. First, the winning firm can only extract its reservation value $U^R$. Second, contracts are first-best, i.e., they specify $q = q^*(\theta)$ for all $\theta \in \Theta^E$. Suppose next some firm enjoys a monopolistic position. We denote the set of types who receive an acceptable offer in the monopolistic case by $\Theta^M$. The respective contracts are denoted by $(w^M(\theta), q^M(\theta))$ for $\theta \in \Theta^M$. We denote the respective payoffs by $V^M(\theta) = w^M(\theta) - v(\theta, q^M(\theta))$ and $U^M(\theta) = u(q^M(\theta)) - w^M(\theta)$.

Both the set of types $\Theta^M$ and the respective set of contracts $(w^M(\theta), q^M(\theta))$ for $\theta \in \Theta^M$ are chosen optimally by firms. Firms must take into account workers’ participation constraints, i.e., that $V^M(\theta) \geq V^R(\theta)$ holds for all $\theta \in \Theta^M$, and workers’ incentive compatibility constraints, i.e., that $V^M(\theta) \geq w^M(\theta') - v(\theta, q^M(\theta'))$ holds for all $\theta, \theta' \in \Theta^M$. We restrict consideration to equilibria where $\Theta^M$ can be expressed as the union of closed (non-degenerate) intervals in $\Theta$. Intuitively, we find that firms will offer acceptable contracts to all sufficiently high types. In the appendix, we set up the firms’ program more formally when proving our results.

2.5. Market composition

The composition of the market is described by the ratio of firms to workers $t$, the set of entering workers $\Theta^E$, and the distribution of types in the market $F(\theta)$. We assume that entering the market consumes one unit of time, implying by stationarity that the expected payoff from entering the market equals the respective reservation value. Below, we will specify in detail how reservation values are determined under different forms of market frictions.

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4Our results depend neither on the simplification that firms and workers incur the same costs $c$ nor on the assumption that workers incur $c$ also when failing to attract any offer in a given period.

5The assumption that entering the market consumes one unit of time is only made for convenience. None of our results depends on this assumption.
The set $\Theta^E$ is determined by requiring that workers optimally decide whether to enter the market or not. Intuitively, we will find that all sufficiently high types will enter the market in equilibrium. The distribution $F(\theta)$ depends on how successful different types are in receiving an acceptable offer. Recall that only workers of type $\theta \in \Theta^M$ receive an acceptable offer when failing to attract more than one firm. Hence, in a given period workers of type $\theta \in \Theta^M$ are successful with probability $\pi_W(1) + \pi_W(2)$, while workers of type $\theta \in \Theta^E \setminus \Theta^M$ are only successful with probability $\pi_W(2)$. We denote the mass of workers $\theta \in \Theta^E$ in each new cohort by $m^E_w$ and the mass of workers $\theta \in \Theta^M$ in each new cohort by $m^M_w$. By stationarity, the stock of all workers in the market, which we denoted by $m_w$, must then satisfy

$$m_w = \frac{m^M_w}{\pi_W(1) + \pi_W(2)} + \frac{m^E_w - m^M_w}{\pi_W(2)}.$$  \hfill (1)

As the distribution of types in each cohort of new workers $F^0(\theta)$ has a density $f^0(\theta)$, it follows that $F(\theta)$ has a density $f(\theta)$ given by\(^6\)

$$f(\theta) := \begin{cases} \frac{\pi_W(1) + \pi_W(2)}{m_w [\pi_W(1) + \pi_W(2)] - m^M_w \pi_W(1)} f^0(\theta) & \text{for } \theta \in \Theta^E \setminus \Theta^M \\ \frac{\pi_W(2)}{m_w [\pi_W(1) + \pi_W(2)] - m^M_w \pi_W(1)} f^0(\theta) & \text{for } \theta \in \Theta^M \end{cases}.$$  \hfill (2)

2.6. Reservation values

As a final step in the description of our model, we determine the reservation values of firms and workers. Take first the case of workers. Recall that $V^M(\theta)$ is the payoff of a type-$\theta$ worker who accepts the offer of a monopolistic firm. We extend the definition of $V^M$ to all types in the market by setting $V^M(\theta) := V^R(\theta)$ for types who do not receive an acceptable offer, i.e., types $\theta \in \Theta^E \setminus \Theta^M$. To calculate $V^R(\theta)$, recall that firms compete themselves down to their reservation values $U^R$ in case a particular worker attracts more than one firm. In this case, the payoff of a type-$\theta$ worker equals $S^*(\theta) - U^R$. If the worker fails to attract a firm in a given period, he has to wait for another period. In this case, stationarity implies that his expected payoff is equal to his reservation value $V^R(\theta)$. Finally, recall that $U^R = K$ must hold in equilibrium. Substituting these payoffs, we obtain for the case with additive search costs

$$V^R(\theta) = \pi_W(0) V^R(\theta) + \pi_W(1) V^M(\theta) + \pi_W(2) (S^*(\theta) - K) - c$$  \hfill (3)

and for the case with discounting

$$V^R(\theta) = \delta [\pi_W(0) V^R(\theta) + \pi_W(1) V^M(\theta) + \pi_W(2) (S^*(\theta) - K)].$$  \hfill (4)

\(^6\)This follows immediately from (1) after noting that stationarity implies $f(\theta) \pi_W(2) m_w = f^0(\theta)$ for all $\theta \in \Theta^E \setminus \Theta^M$ and $f(\theta) [\pi_W(1) + \pi_W(2)] m_w = f^0(\theta)$ for all $\theta \in \Theta^M$.\n
Analogously, we can next determine the firms’ reservation value $U^R$. In a given period, a firm will enjoy a monopolistic position with probability $\pi_F(1)$. With probability $\pi_F(2)$, the firm will have to compete for a worker. And with the residual probability $\pi_F(0)$, the firm will not encounter any worker at all. Both when competing with other firms and when failing to attract any worker at all, a firm’s payoff will be equal to $U^R$. We then obtain for the case with additive search costs

$$U^R = \pi_F(1) \int_{\theta \in \Theta^e} U^M(\theta) \, dF(\theta) + [1 - \pi_F(1)]U^R - c$$

and for the case with discounting

$$U^R = \delta \left[ \pi_F(1) \int_{\theta \in \Theta^e} U^M(\theta) \, dF(\theta) + [1 - \pi_F(1)]U^R \right].$$

By the zero profit requirement, $U^R = K$ must hold in equilibrium. This requirement together with (5) and (6), respectively, will be used to determine the equilibrium value of $t$, i.e., of the ratio of firms to workers prevailing in the market.

3. Frictions due to costly delay

3.1. Analysis

Assume now that players discount future payoffs. Using (4), we then obtain for workers’ reservation values

$$V^R(\theta) = \frac{\delta}{1 - \delta \pi_W(0)} [\pi_W(1)V^M(\theta) + \pi_W(2)(S^*(\theta) - K)].$$

In the particular case where $V^M(\theta) = V^R(\theta)$ holds for some type $\theta$, we can use $l := \delta \pi_W(2)/(1 - \delta(1 - \pi_W(2)))$ to transform (7) into

$$V^R(\theta) = l(S^*(\theta) - K).$$

Eq. (7) describes how reservation values $V^R(\theta)$ depend on workers’ payoffs from offers made by monopolistic firms $V^M(\theta)$. On the other side, $V^R(\theta)$ represents the participation constraint in the program of monopolistic firms, where contracts $(w^M(\theta), q^M(\theta))$ and thus payoffs $V^M(\theta)$ are determined. This interdependence of $V^R(\theta)$ and $V^M(\theta)$ is key to characterize the equilibrium outcome. In what follows, our focus is on deriving equilibria for high values of $\delta$. The main part in the characterization of equilibria is the derivation of offers made by monopolistic firms. We first state our results before commenting on them in detail.

**Proposition 1.** In the case of discounting, an equilibrium with the following characteristics exists for high $\delta$. All types of workers enter the market. Offers made
by monopolistic firms exhibit the following characteristics:

(i) There exist two threshold types $\theta < \theta^M < \bar{\theta} < \overline{\theta}$ such that only types $\theta \geq \theta^M$ receive an acceptable offer, the participation constraint binds for types $\theta \in [\theta^M, \theta^T]$, and the participation constraint is slack for types $\theta \in (\theta^T, \overline{\theta})$.

(ii) The sorting variable $q^M$ is continuous over $\theta \in [\theta^M, \overline{\theta}]$. For $\theta \in [\theta^M, \theta^T]$ it is determined by $q^M(\theta) = l\theta$ and for $\theta \in (\theta^T, \overline{\theta})$ by

$$q^M(\theta) = \theta - \frac{1 - F^0(\theta)}{f^0(\theta)}.$$  

(iii) As $\delta \to 1$, we have that $\theta^M \to 0$, $\theta^T \to \bar{\theta}$, and $l \to 1$, implying that inefficiencies gradually vanish.

**Proof.** See Appendix A. \qed

To ensure that the zero profit constraint for firms is satisfied, the ratio of firms to workers $t$ must become sufficiently high as $\delta$ increases. This ensures $\pi_W(2) > 0$, while $l = \delta \pi_W(2) / [1 - \delta(1 - \pi_W(2))]$ becomes close to one. If $\pi_W(2) > 0$ holds, Assumption 2 ensures that all types $\theta > \bar{\theta}$ strictly prefer to enter. Note next that the characterization of equilibrium offers in Proposition 1 reveals two types of inefficiencies. First, not all types receive an acceptable offer by monopolistic firms. Second, all contracts but that for the highest type are distorted as $q^M(\theta) < q^*(\theta)$. These distortions vanish only gradually as $\delta \to 1$ and, consequently, $l \to 1$.

Contracts are inefficient as, in equilibrium, the reservation value function $V^R(\theta)$ has a strictly smaller slope than the first-best surplus function $S^*(\theta)$. This makes it profitable for firms to distort their offers in order to extract higher payoffs from higher types. This will be markedly different in the case of additive search costs, which is analyzed below.

We next provide more details on the offers made by monopolistic firms. It is intuitive that firms will make an acceptable offer to all sufficiently high types. Moreover, as they could only extract $K$ from the lowest type $\theta$ by making a first-best offer, a lower segment of types will not receive an acceptable offer. Types $\theta \geq \theta^M$, who receive an acceptable offer, fall into two ranges. For the lower interval $\theta \in [\theta^M, \theta^T]$, the participation constraint $V^M(\theta) \geq V^R(\theta)$ becomes binding. This requires $dV^M/d\theta = dV^R/d\theta$, which by (8) holds if $q^M(\theta) = lq^*(\theta)$, i.e., $q^M(\theta) = l\theta$. For higher types $\theta > \theta^T$, the participation constraint remains slack. In this case, $q^M$ is determined by the binding incentive compatibility constraint. Eq. (9) for $q^M$ is standard. At the threshold $\theta^T$, the two different values for $q^M$ coincide, i.e., $lq^*(\theta^T)$ equals $\theta^T - [1 - F^0(\theta^T)]/f^0(\theta^T)$. Moreover, at the threshold $\theta^M$, which is the lowest acceptable type, the surplus realized by specifying $q^M(\theta^M)$ is just sufficient to cover both $K$ and the worker’s reservation value $l[S^*(\theta^M) - K]$. As $l$ increases, $\theta^M$ decreases and $\theta^T$ increases, while contracts specifying $q^M(\theta) = lq^*(\theta)$ become also more efficient.

The simple form of equilibrium contracts, i.e., the “two-regime” nature, where the participation constraint binds only for a lower segment of types, is due to the way
how \( V^R \) and \( V^M \) interact in equilibrium. While \( V^M \) feeds back into the reservation value \( V^R \), \( V^R \) represents the participation constraint in the monopolistic program, where \( V^M \) is determined. This interdependence implies, together with the choice of preferences, that, in case the participation constraint binds at some type \( \theta' \), it must also bind at all lower types \( \theta < \theta' \), irrespective of the choice of \( \delta \) and thus of \( l \). (For details, see the proof of Proposition 1.) While this result no longer holds generally for other preferences, we have shown in a previous version that the monopolistic offer must also be of the “two-regime” nature for quite general preferences in case \( l \) becomes close to one (i.e., \( \delta \) becomes relatively high).

In the proof of Proposition 1, we show that the “two-regime” offer is the unique equilibrium offer in case \( l \) is above some threshold \( \bar{l} \approx 0.536 \). If we choose the exponential matching technology, which was introduced above, and set \( d = 0.95 \), the condition \( l \geq 0.536 \) is satisfied whenever the ratio of firms to workers \( t \) satisfies \( t \geq 0.397 \). With \( d = 0.95 \), we can further show that, in this case, an equilibrium exists where \( t = 1.015 \).

Finally, note that Proposition 1 mirrors results in Stole [16], who considers two firms competing in non-linear prices. Firms are differentiated by distance. Consumers with a higher valuation are assumed to put also more value on closeness. This assumption has the same effect as discounting in our model.

### 3.2. Externalities among firms

Under discounting, we find two types of externalities, which will be absent in case of additive search costs.

#### 3.2.1. Positive pool externality

In Proposition 1, we made no claims to uniqueness. In fact, it is an interesting feature of the case with discounting that there may exist multiple (Pareto-ranked) equilibria. To see this, recall how firms’ profits from entering the market are determined. If a firm must compete with other firms for a given worker, its expected payoff equals \( K \). This is also its expected payoff if it fails to encounter any worker in a given period. As \( U^M(\theta) \) equals its payoff from type \( \theta \) in the monopolistic case, the participation constraint can only bind at the lowest type receiving an acceptable offer \( \theta^M \). As types \( \theta < \theta^M \) circulate strictly longer than types \( \theta \geq \theta^M \), the market density \( f \) jumps downwards at \( \theta = \theta^M \). But then it follows from standard arguments that it cannot be optimal for a given firm to choose the threshold \( \theta^M \). This observation is reminiscent of the non-existence of single-wage equilibria in [1].

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7 On the other hand, Proposition 1 is silent on the case with low \( \delta \) and, consequently, low \( l \). In this case, an equilibrium where monopolistic firms use symmetric and pure strategies may fail to exist. We can show for low \( l \) that the participation constraint can only bind at the lowest type receiving an acceptable offer \( \theta^M \).

8 At the threshold \( \bar{l} \), it holds that \( \bar{\theta}^T = \theta^M \), where \( lq^*(\bar{\theta}^T) \) equals \( \bar{\theta}^T - [1 - f^0(\theta^T)]/f^0(\bar{\theta}^T) \) and where \( S(\theta^M, lq^*(\theta^M)) = K + l[S^*(\theta^M) - K] \). For the given specifications, this yields \( \bar{\theta}^T = \bar{\theta}/(2 - \bar{\theta}) \) and \( \theta^M = \sqrt{2K}/\bar{\theta} \). Substituting \( \sqrt{2K} = \bar{\theta} \), we obtain the requirement \( \bar{\theta}/\bar{\theta} = (2 - \bar{\theta})/\sqrt{\bar{\theta}} \). For \( \bar{\theta} = 0.8 \) and \( \bar{\theta} = 0.4 \), this yields the stated result.
the firm’s payoff from entering the market (or, likewise, from starting to search anew) equals
\[
\delta \left[ \pi_F(1) \int_{\theta \in \Theta^E} U^M(\theta) \, dF(\theta) + [1 - \pi_F(1)]K \right].
\]  
(10)

By the zero profit constraint, (10) must be equal to $K$ in equilibrium. We argue now that this can be the case for different values of $t$ and thus for different matching probabilities for firms and workers.

Given Assumption 3 on the matching probabilities, the first intuition would be that the payoff in (10) is non-increasing in $t$. After all, when increasing the ratio of firms to workers, it becomes more likely for workers to encounter more than one firm at a time and less likely for firms to avoid competition.

But these considerations miss one essential feature of our model. The distribution of types in the market is determined endogenously. Recall that $y_M$ is determined by the requirement that $S(y_M, q_M(y_M))$ equals the sum of $K$ and of the worker’s reservation value $l[S^*(y_M) - K]$. Substitution of $q_M(y_M)$ yields $y_M = \sqrt{2K}/l$.

For an illustration, take a uniform distribution over the interval between $\bar{\theta} = 0.65$ and $\bar{\theta} = 0.8$ and the exponential matching technology. Moreover, set $d = 0.995$. In this case, we have established existence of two equilibria, one where $t = 0.311$ holds and one where $t = 0.415$ holds. In the first case, where $t$ is lower, we obtain the threshold type $\theta^M = 0.690$, whereas in the second case, where $t$ is higher, we obtain $\theta^M = 0.674$. (Note that the difference between the two thresholds is larger than 10% of the support length $\bar{\theta} - \bar{\theta} = 0.15$.) When switching between the two equilibria, the density of the market distribution at all sufficiently high types $\theta \geq 0.690$ increases from $f(\theta) = 2.611$ to $f(\theta) = 3.968$.

3.2.2. Contracting externality

In our model, the prevailing ratio of firms to workers $t$ is determined by a zero profit requirement. It is well known in the search and matching literature that $t$ may be inefficient as individual firms do not internalize their impact on the matching probabilities of other market participants, i.e., both of other firms and of workers

9 Recall that $\theta^M$ is determined by the requirement that $S(\theta^M, q^M(\theta^M))$ equals the sum of $K$ and of the worker’s reservation value $l[S^*(\theta^M) - K]$. Substitution of $q^M(\theta^M)$ yields $\theta^M = \sqrt{2K}/l$.

10 More formally, the effect on $f$ can be seen from inspecting (2) and substituting $\theta^E = \Theta$ and $\theta^M = [\theta^M, \bar{\theta}]$.

11 The derivation of these results can be obtained from the author upon request.
(see [7]). In our model, where contracts include a sorting variable, a new type of externality arises, which has not been discussed in the search or matching literature. A higher value of l implies by Proposition 1 that contracting becomes more efficient. One way to achieve a higher value of l is to increase the measure of firms in the market, i.e., t. Hence, more competition among firms not only transfers more of the available surplus to workers, but it also increases surplus creation in the market. As this “contracting externality” is, again, not internalized by individual entrants, a Pareto improvement may be feasible by subsidizing the entry of firms.

4. The case of additive search costs

If workers incur type-independent costs of search and waiting, the analysis is remarkably different to that with impatience and discounting. In particular, we can show that, in case the market opens up, a unique equilibrium exists where all contracts are efficient.

The crucial difference to the case with impatience is that now frictions affect all types equally. In matches where firms enjoy a monopolistic position, this fully erodes the scope for rent extraction by offering distorted contracts. To see this, suppose that, indeed, all firms offer only first-best contracts, while specifying wages such that the participation constraint binds for all types. By (3), the reservation value for some type y is then given by

$$V^R(y) = \frac{S^r(y)}{C_3(y)} - s - c/[1 - \pi_w(2)].$$

(Recall that c measures the constant costs of search and waiting.) Clearly, $V^R(\theta)$ has the same slope as $S^r$. Compare this to the case with discounting. If the participation constraint was binding for all types and if all firms offered first-best contracts, the reservation value function would be given by $V^R(\theta) = l[S^r(\theta) - K]$, which has a slope strictly smaller than $S^r(\theta)$. In contrast to the case with additive search costs, it is now optimal for a single firm to deviate and offer a menu of distorted contracts. This allows to extract more from high types, who lose more due to impatience.

With additive search costs, it is clearly no longer optimal for all types to enter the market. In fact, we find that all types exceeding a threshold $\theta^E$ enter the market. The threshold type is just indifferent, i.e., by (11) it holds that $S^r(\theta^E) - K = c/[1 - \pi_w(2)]$. Clearly, to ensure that the market opens up, c must not be too high.

Proposition 2. Under additive search costs, there exists a unique equilibrium if c is sufficiently small. Only sufficiently high types $\theta \geq \theta^E$ enter the market, and firms offer all workers efficient contracts, i.e., $\theta^M = \theta^E$ and $q^M(\theta) = q^*(\theta)$ for all $\theta \geq \theta^M$.

Proof. See Appendix A. □

Proposition 2 mirrors recent results in Armstrong and Vickers (2001) and Rochet and Stole (2002), who, amongst other things, consider a Hotelling duopoly where
firms compete in non-linear prices. All consumer types put the same value on
closeness, i.e., they have the same “transport costs”, irrespective of their valuations. Hence, if one firm offers an efficient menu of contracts, i.e., a cost-plus-fixed-fee menu, the reservation value function of consumers at a particular location has the same slope as the first-best surplus function.\textsuperscript{12}

5. Concluding remarks

This paper considers contract design by uninformed principals (firms) in a
decentralized environment with frictions. The presence of other principals, whom
agents (workers) can meet if they spend time or resources on waiting and searching, restricts the monopolistic power of each individual principal.

We find that results depend much on how frictions affect the value of the agents’ option to wait. With additive costs of search or waiting, which affect all types equally, contracts are always efficient and there exists a unique equilibrium. This holds regardless of the level of frictions (as long as the market opens up), i.e., even if it becomes very costly for agents to search for new matches. In contrast, if waiting is costly as agents discount future payoffs, contracts become distorted. Moreover, in this case there may exist multiple (Pareto-ranked) equilibria, while subsidizing the entry of principals to create more competition can increase surplus. In the case of discounting, we are also able to characterize the menu of distorted contracts for a common class of (linear) preferences. We find that (i) low types do not receive an acceptable offer, that (ii) the participation constraint binds for medium types, and that (iii) the participation constraint is slack for high types.

In this paper, we have assumed that all agents use the same discount factor or that they incur the same constant costs of search or waiting. Exploring the case of multidimensional heterogeneity may represent a fruitful avenue for further research. Agents may then differ in their valuations and in their costs of search and waiting, both of which are their private information.

Appendix A. Proofs

A.1. Proof of Proposition 1

The proof of Proposition 1 proceeds in several steps.

We start with some preliminary results. In particular, we show that, in equilibrium, the set $\Theta^M$ must be convex and that it must extend to the highest

\textsuperscript{12} However, it is crucial that the whole market is covered, which will only hold if transport costs are sufficiently low and the valuation of the lowest type is sufficiently high. In contrast, in the search model the distribution of types in the market is endogenous. It should also be noted that consumers must be distributed independently over locations and types to obtain the efficient outcome in the Hotelling models. Finally, these papers do not claim uniqueness. As an individual firm has no impact on market characteristics or reservation values in our model, we can also show uniqueness.
type \( \tilde{\theta} \), i.e., that \( \Theta^M = [\theta^M, \tilde{\theta}] \) holds for some threshold type \( \theta^M \). Subsequently, we set up more formally the optimization program of monopolistic firms. Using the derived optimality conditions for the schedule \( q^M \), we then show that, in equilibrium, the offer can only take on two possible forms. Either the participation constraint binds only at the lowest type \( \theta^M \) or it binds only at a lower interval of types. In a further step, we can then rule out for high values of \( l \) the case where the participation constraint binds only at \( \theta^M \). Moreover, we can show that \( l \) must be close to one in any equilibrium in case \( \delta \) is sufficiently high. Having thus shown that any equilibrium must satisfy the characteristics of Proposition 1 for high \( \delta \), we subsequently prove existence. Finally, we use the characterization of the equilibrium to show how distortions gradually vanish as \( \delta \) increases.

**Lemma A.1.** In any equilibrium with discounting, it must hold that: (i) \( V^R(\theta) \) is continuous and strictly increasing; (ii) \( \Theta^E = \Theta \); and (iii) \( \Theta^M = [\theta^M, \tilde{\theta}] \), where \( \theta^M > \theta \).

**Proof.** By incentive compatibility of the offer \((w^M(\theta), q^M(\theta))\), \( V^M(\theta) \) is continuous and non-decreasing.\(^\text{13} \) As \( S^*(\theta) \) is continuous and strictly increasing, \( V^R(\theta) \) is by (7) also continuous and strictly increasing.\(^\text{14} \) Assertion (ii) follows then immediately. Before proving assertion (iii), note that incentive compatibility also implies that \( q^M(\theta) \) is non-decreasing.

By optimality and \( U^R = K \), we have \( U^M(\theta) \geq K \) for all \( \theta \in \Theta^M \). As a monopolistic firm must realize strictly more than \( K \) to ensure \( U^R = K \), it also follows from the last implication and from Assumption 2 that the monopolistic offer is not extended to all sufficiently low types. To prove that \( \Theta^M \) is convex and extends to \( \tilde{\theta} \), we use next that \( \Theta^M = \bigcup_{i \in I} [\theta_i, \tilde{\theta}_i] \), where \( I \) denotes an arbitrary index set and where the intervals \([\theta_i, \tilde{\theta}_i] \subseteq \Theta \) are non-overlapping. We argue to a contradiction, which implies existence of two adjacent intervals \( i, j \in I \) such that \( \tilde{\theta}_i < \theta_j \). (That is, there does not exist some \( k \in I \) satisfying \( \theta_k \leq \theta_j \).) Denote \( \hat{\theta} = \tilde{\theta}_i \) and \( \tilde{\theta} = \theta_j \). Incentive compatibility implies \( V^M(\theta) = V^R(\theta) \) and thus, by (8), \( V^R(\theta) = l[S^*(\theta) - K] \) at both \( \theta = \hat{\theta} \) and \( \theta = \tilde{\theta} \). Additionally, incentive compatibility implies \( dV^M/d\theta \leq V^R/d\theta \) at \( \theta = \hat{\theta} \) and \( dV^M/d\theta \geq V^R/d\theta \) at \( \theta = \tilde{\theta} \).\(^\text{15} \) Using (8), this yields \( q^M(\hat{\theta}) \leq lq^*(\hat{\theta}) \) and \( q^M(\tilde{\theta}) \geq lq^*(\tilde{\theta}) \). We argue now that a given firm would be strictly better off by extending its offer to types \( \theta \in (\hat{\theta}, \tilde{\theta}) \), while specifying \( q^M(\theta) = lq^*(\theta) \) and choosing \( w^M(\theta) \) to ensure that \( V^M(\theta) = V^R(\theta) \). Note first that this extension to the firm’s offer leaves the menu incentive compatible. (In particular, \( q^M(\theta) \) is still non-decreasing over the set \( \Theta^M \cup (\hat{\theta}, \tilde{\theta}) \).) The firm is then strictly better off if \( U^M(\theta) > K \) holds for all

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\( ^{13} \)Recall that we have extended the definition of \( V^M(\theta) \) by setting \( V^M(\theta) = V^R(\theta) \) for all \( \theta \notin \Theta^M \).

\(^{14} \)Precisely, this holds whenever \( \pi_w(2) > 0 \). For matching technologies where this is not the case for all values \( t > 0 \), this is now assumed to hold. As argued further below, for high \( \delta \) this must necessarily hold to satisfy firms’ zero profit requirement.

\(^{15} \)We use that \( V^M \) is differentiable a.e. over \([\theta_i, \tilde{\theta}_i] \) and \([\theta_j, \tilde{\theta}_j] \).
types $\theta \in (\tilde{\theta}, \tilde{\theta})$. This follows finally from $U^M(\theta) \geq K$ at $\theta = \tilde{\theta}$ and as
\[
\frac{d}{d\theta} [S(\theta, q^M(\theta)) - V^R(\theta)] = \frac{dq^M}{d\theta} \frac{dS(\theta, q^M(\theta))}{dq}
\]
is strictly positive by the specification $q^M(\theta) = lq^*(\theta)$ for $\theta \in (\tilde{\theta}, \tilde{\theta})$ and by concavity of $S$ in $q$.

Having shown that $\Theta^M$ is convex and that $\theta^M = \min \Theta^M > \tilde{\theta}$, it remains to prove that $\max \Theta^M = \overline{\theta}$. This follows by the same arguments as in the proof that $\Theta^M$ is convex, i.e., it would, otherwise, be strictly profitable for firms to extend the offer to types $\theta \in (\max \Theta^M, \overline{\theta})$. □

Suppose now all firms offer an acceptable contract only to types in some interval $[\tilde{\theta}^M, \overline{\theta}]$. Denote the truncated distribution by $F^M(\theta) := F(\theta)/[1 - F(\tilde{\theta}^M)]$ with respective density $f^M(\theta)$. Note that the hazard rate of $F^M$ is identical to that of $F$.

We characterize next the optimal menu $(q^M, w^M)$, which must satisfy the incentive compatibility and participation constraints of all types $\theta \in [\tilde{\theta}^M, \overline{\theta}]$. Incentive compatibility holds if and only if $dV^M/d\theta = q^M(\theta)$ and $q^M(\theta)$ is non-increasing. In what follows, we neglect the monotonicity constraint. When checking for existence of an equilibrium below, we make sure that it is satisfied. Define $\Delta(\theta) := V^M(\theta) - V^R(\theta)$. Let now $q^M$ be the control variable and $V^M(\theta)$ the state variable of the problem.\(^{16}\) The problem of a monopolistic firm is then to choose $q^M(\theta)$ and $V^M(\theta)$ to maximize
\[
\int_{\theta \in [\tilde{\theta}^M, \overline{\theta}]} [S(\theta, q^M(\theta)) - V^M(\theta)] f^M(\theta) \, d\theta,
\]
subject to $dV^M/d\theta = q^M(\theta)$ and $\Delta(\theta) \geq 0$. Letting $\mu(\theta)$ become the costate variable and $\tau$ the multiplier of the participation constraint, the Hamiltonian is given by
\[
H(V^M, q^M, \mu, \theta) = [S(\theta, q^M(\theta)) - V^M(\theta)] f^M(\theta) + \mu(\theta) q^M(\theta),
\]
and the Lagrangian is given by $L = H + \tau \Delta$. The first-order condition becomes then
\[
\frac{dS(\theta, q^M(\theta))}{dq} f^M(\theta) + \mu(\theta) = 0,
\]which, by concavity of $S$, is also sufficient.\(^{17}\) We look for a solution where the costate variable is continuous, which excludes jumps in $q^M$. As such a solution exists, this is without restrictions. As $H(\cdot)$ is concave in the state variable $V^M$, the remaining

\(^{16}\)In an abuse of notation, we use for the control variable the notation that was previously introduced for the equilibrium menu of offers. Note also that setting up the control problem in this way is standard (see, e.g., [6]).

\(^{17}\)We neglect the requirement that $q \geq 0$, which will be satisfied given the (equilibrium) choice of $\theta^M$. 
sufficient conditions are:

State equation: \( dV^M/d\theta = q^M(\theta) \).

Costate equation: \( d\mu/d\theta = -\partial L/\partial V^M = f^M(\theta) - \tau(\theta) \).

Complementary slackness: \( \Delta(\theta) \geq 0; \ \tau(\theta) \geq 0; \ \tau(\theta)\Delta(\theta) = 0 \).

Transversality conditions: \( \mu(\theta^M)\Delta(\theta^M) = 0; \ \mu(\theta^M) \leq 0; \ \mu(\overline{\theta})\Delta(\overline{\theta}) = 0; \ \mu(\overline{\theta}) \geq 0 \).

Integration of the costate equation yields \( m(\theta) = F^M(\theta) - T(\theta) - z \), where \( T(\theta) \) is the integral of \( \tau(\theta) \) and \( z \) is the constant of integration. Substituting into the first-order condition (A.1), \( q^M(\theta) \) finally solves

\[
\theta - q^M(\theta) = -\frac{\mu(\theta)}{f^M(\theta)}. \tag{A.2}
\]

We use next the derived optimality conditions together with the definition of \( V^R \) to prove that the monopolistic offer must take on relatively simple forms in equilibrium.

**Lemma A.2.** In any equilibrium with discounting, the offer of monopolistic firms must have the following characteristics. Either the participation constraint binds only at \( \theta^M \) or the participation constraint binds only at a lower subset of types \([\theta^M, \theta^T]\), where \( \theta^T < \overline{\theta} \).

**Proof.** By the definition of reservation values in (7), we obtain:

\[
\frac{dV^M}{d\theta} < \frac{dV^R}{d\theta} \iff q^M(\theta) < q^*(\theta). \tag{A.3}
\]

The proof of Lemma 2 proceeds now in two steps.

**Claim A.1.** In equilibrium, the monopolistic offer must satisfy: (i) \( V^M(\overline{\theta}) > V^R(\overline{\theta}) \); (ii) \( q^M(\overline{\theta}) = q^*(\overline{\theta}) \) and \( q^M(\theta) < q^*(\theta) \) for all \( \theta^M \leq \theta < \overline{\theta} \); and (iii) \( V^M(\theta^M) = V^R(\theta^M) \).

**Proof.** Consider first assertion (i). We argue to a contradiction and assume that \( V^M(\overline{\theta}) = V^R(\overline{\theta}) \). To ensure that \( V^M(\theta) \geq V^R(\theta) \) holds in the neighborhood of \( \overline{\theta} \), it must then hold by (A.3) that \( q^M(\overline{\theta}) < q^*(\overline{\theta}) \). By (A.2) this implies \( \mu(\overline{\theta}) < 0 \), which contradicts the transversality condition \( \mu(\overline{\theta}) \geq 0 \). We have thus shown that \( \Lambda(\overline{\theta}) > 0 \) must hold, which by the transversality condition \( \mu(\overline{\theta})\Delta(\overline{\theta}) = 0 \) implies \( \mu(\overline{\theta}) = 0 \) and thus, using (A.2), \( q^M(\overline{\theta}) = q^*(\overline{\theta}) \). We prove next that \( \mu(\theta) < 0 \) holds for all \( \theta < \overline{\theta} \). Assume, instead, \( \mu(\theta') \geq 0 \) for some \( \theta' < \overline{\theta} \), which by (A.2) implies \( q^M(\theta') \geq q^*(\theta') \). At the (right-side) neighborhood \( \theta > \theta' \) it must thus hold by (A.3) that \( \Lambda(\theta) > 0 \), implying
\( \tau(\theta) = 0 \) by the complementary slackness condition, which yields \( \mu(\theta) > \mu(\theta') \). As 
\( \mu(\theta') \geq 0 \) holds by assumption, we obtain \( \mu(\theta) > 0 \). We can now proceed like this up to \( \overline{\theta} \), where we finally obtain \( \mu(\overline{\theta}) > 0 \), which contradicts assertion (i). By (A.2), \( \mu(\theta) < 0 \) implies \( q^M(\theta) < q^*(\theta) \) for all \( \theta < \overline{\theta} \), which concludes the proof of assertion (ii). We can finally use this result to prove also assertion (iii). In case \( A(\theta^M) > 0 \) holds, the transversality condition \( \mu(\theta^M)A(\theta^M) = 0 \) implies \( \mu(\theta^M) = 0 \), which yields a contradiction to assertion (ii). \( \square \)

**Claim A.2.** \( V^R(\theta') > V^M(\theta') \) implies \( V^R(\theta) > V^M(\theta) \) for all \( \theta \geq \theta' \).

**Proof.** The assertion holds if the participation constraint stays slack for all higher types once it has become slack. Suppose thus that \( V^R = V^M \) and \( dV^R/d\theta > dV^R/d\theta \) hold at some type \( \theta' \). Claim A.2 then follows if we can show that this implies \( V^M(\theta') > V^R(\theta) \) for all \( \theta \geq \theta' \). By (A.3) it is then sufficient that \( q^M(\theta')/q^*(\theta') \geq 1 \) holds for all \( \theta > \theta' \). By \( q^M(\theta')/q^*(\theta') \geq 1 \) this holds if \( q^M(\theta)/q^*(\theta) \) is non-decreasing, i.e., if \( dq^M/d\theta - [q^M/q^*]dq^*/d\theta \geq 0 \). By \( q^M(\theta')/q^*(\theta') \leq 1 \), which holds by Claim A.1, it thus remains to show that \( dq^M/d\theta \geq dq^*/d\theta \). Substitution of \( q^M \) from (A.1) and of \( q^* \) reveals that this is the case if \( \mu/f^M \) is non-decreasing. Note next that the sign of the derivative of \( \mu/f^M \) is determined by \( (f^M)^2 - \mu(df^M/d\theta) \). As \( \mu \leq 0 \) holds for all \( \theta \) by the arguments in Claim A.1, it remains to consider the case where \( df^M/d\theta < 0 \) holds, in which case \( \mu/f^M \) is non-decreasing if \( (f^M)^2 \geq \mu(df^M/d\theta) \). To see that this holds, recall first that the hazard rate is invariant to left-side truncations, implying by Assumption 1 that \( (f^M)^2 \geq -(1 - F^M)/(df^M/d\theta) \). Hence, given the definition of \( \mu \), the requirement \( (f^M)^2 \geq \mu(df^M/d\theta) \) holds if \( z + T \leq 1 \) is true for all \( \theta \). As \( T \) is non-decreasing, this follows from \( z + T(\overline{\theta}) = 1 \), which holds by the arguments in Claim A.1. \( \square \)

The assertions in Lemma A.2 follow now immediately from Claims A.1 and A.2. \( \square \)

Consider next in more detail the case where the participation constraint binds at some lower interval. The choice of \( q^M \) for the lower interval follows then immediately from the requirement \( dV^M/d\theta = dV^R/d\theta \), which yields \( q^M(\theta) = lq^*(\theta) \). For the upper interval, where the participation constraint is slack, we obtain \( q^M \) from (A.1) by substituting \( z + T(\theta) = 1 \). (See Lemma A.1.) This yields (9). (Note that the hazard rate is invariant to left-side truncations.) By continuity of \( q^M \), we obtain next for \( \theta^T \) the requirement

\[
\theta^T(1 - l) = \frac{1 - F^0(\theta^T)}{f^0(\theta^T)}. \tag{A.4}
\]
For all \( l \) close to one, (A.4) has a unique solution with \( \theta^T > \bar{\theta} \). We determine next the optimal threshold \( \theta^M \). As the participation constraint is binding over the interval \([\theta^M, \theta^T] \), a marginal adjustment in \( \theta^M \) does not affect the surplus extracted from any other type \( \theta > \theta^M \). The (net) payoff realized with type \( \theta^M \) equals

\[
S(\theta^M, q^M(\theta^M)) - [K + l(S^*(\theta^M) - K)].
\]

(A.5)

Substituting \( q^M(\theta^M) \) into (A.5) reveals that it is strictly increasing in \( \theta^M \). The optimal choice of \( \theta^M \) is then determined by the requirement that (A.5) equals zero, which holds if

\[
\theta^M = \frac{\theta}{l}.
\]

(A.6)

For \( l \) close to one, (A.6) has a unique solution with \( \theta^M < \bar{\theta} \). Moreover, comparing (A.4) with (A.6), we obtain that \( \theta^M < \theta^T \) holds for all \( l \) close to one. Precisely, it holds for all \( l > \bar{l} \), where \( \bar{l} < 1 \) is determined by the requirement

\[
\frac{1 - \bar{l}}{\sqrt{\bar{l}}} = \frac{1}{\bar{l}} \left[ 1 - F^0(\theta/\bar{l}) \right]
\]

We can next prove the following result.

**Lemma A.3.** In any equilibrium with discounting where \( l > \bar{l} \), the following holds:

(i) The monopolistic offer is uniquely determined and of the “two-regime” type, characterized above.

(ii) If all other firms make the “two-regime” offer, this is also uniquely optimal for an individual firm.

**Proof.** We consider first assertion (i). By Lemmas A.1 and A.2, we only have to rule out one alternative, i.e., where only sufficiently high types \( \theta \in [\theta^M, \bar{\theta}] \) receive an offer and where the participation constraint binds only at \( \theta^M \). If this is the case, it holds by (A.1) that \( q^M \) is everywhere determined by (9). (Note that \( q^M \) is strictly increasing by Assumption 1, such that incentive compatibility is satisfied.) Moreover, at \( \theta^M \) it must hold that \( dV^M/d\theta > dV^R/d\theta \). We already know that this implies \( \theta^M > \theta/l \) by (A.3). But in this case it would, by the arguments in Lemma A.1, be strictly optimal for an individual firm to extend the offer to lower types in case \( l > \bar{l} \).

Consider now the remaining equilibrium candidate where monopolistic firms make the “two-regime” offer. Suppose all other firms make the respective offer, which was fully characterized before Lemma A.3. This generates for all types \( \theta \in \Theta \) a respective reservation value \( V^R(\theta) \) according to (7). It now remains to show that, given this choice of \( V^R \), the offer is indeed uniquely optimal. To see this, observe first that \( q^M \), as characterized in the “two-regime” offer, is strictly increasing in \( \theta \). For \( \theta < \theta^T \) this follows from the strict monotonicity of \( q^*(\theta) \), while for \( \theta \geq \theta^T \) we can use

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18 This is different to the (more standard) case where the participation constraint binds only at the lowest type.
Assumption 1. Hence, the characterized offer is incentive compatible. Optimality and uniqueness follows by checking the necessary and sufficient conditions derived above and using the arguments from Lemmas A.1 and A.2. Precisely, we specify for $\theta > \theta^T$ that $\tau(\theta) = 0$, which gives (9). For $\theta < \theta^T$, we choose $\tau(\theta)$ such that

$$\theta - lq^* (\theta) = \frac{z + T(\theta) - F^M(\theta)}{f^M(\theta)},$$

where we substituted $q^M(\theta) = lq^* (\theta)$. The left-hand side of (A.7) is strictly increasing in $\theta$, while we know from Lemma 2 that the right side would be decreasing in case $\tau(\theta) = 0$. We thus obtain $\tau(\theta) > 0$ for $\theta < \theta^T$.

We finally prove existence for high $\delta$ and that distortions vanish as $\delta \to 1$.

**Lemma A.4.** In the case of discounting, an equilibrium exists for all sufficiently high values of $\delta$, and any equilibrium satisfies $l > 1$. Moreover, as $\delta \to 1$, it must hold along any sequence of equilibria that $l \to 1$.

**Proof.** We first prove that, along any sequence of equilibria where $\delta \to 1$, it must also hold that $l \to 1$. If this was not the case, we could find a sequence of equilibria where $\delta \to 1$ and the respective values $l$ converged to some boundary strictly smaller than one. By definition of $l$, this implies $\pi_W(2) \to 0$ and thus, by Assumption 3, that $\pi_F(1)$ remains bounded away from zero. As a consequence, a firm’s payoff from entering the market becomes arbitrarily close to what the firm can get in the monopolistic case as $\delta \to 1$. Given that $l$ remains bounded away from one and given the definition of workers’ reservation values, it is then immediate that the firms’ zero profit requirement cannot be satisfied.\(^{19}\)

We turn next to existence. By Lemma 3, it is sufficient to show that for high $\delta$ the zero profit requirement for firms is satisfied for some value $t$ such that $l > 1$. Using the binding incentive compatibility constraint for types $\theta \geq \theta^T$ to express $V^M$ in a standard way, we obtain by (5) that

$$U^R = \delta K [1 - \pi_F(1)] + \pi_F(1) F(\theta^M)$$

$$+ \delta \pi_F(1) \int_{\theta^M}^{\theta^T} \left[ S(\theta, q^M(\theta)) - V^R(\theta) \right] f(\theta) \, d\theta$$

$$+ \delta \pi_F(1) \int_{\theta^T}^{\theta^T} \left[ S(\theta, q^M(\theta)) - \frac{1 - F(\theta)}{f(\theta)} q^M(\theta) - V^R(\theta^T) \right] f(\theta) \, d\theta.$$  

(A.8)

In equilibrium, $t$ must be chosen such that $U^R = K$. The right-hand side of (A.8) is continuous in $t$. To see this, recall first that by Assumption 3 all matching probabilities are continuous in $t$. By construction, this also implies continuity of the

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\(^{19}\)This argument can be easily formalized. By Lemma A.2, we can restrict attention to only two potential equilibrium candidates, for which we can write down firms’ profits in the monopolistic case.
thresholds $\theta^T$ and $\theta^M$, of the contractual variable $q^M$, and of the reservation values $V^R(\theta)$ for $\theta \leq \theta^T$. Finally, inspection of (2) reveals that the mass $F(\theta^M)$ is continuous in $l$, while for all types who receive an acceptable offer in the monopolistic case $f$ is also continuous in $l$. As $t \to 0$, it must hold that $\pi_W(2) \to 0$. By previous arguments, the right-hand side of (A.8) must then exceed $K$ for all sufficiently high $\delta$ and $t \to 0$. On the other side, as $t \to -\infty$, it must hold that $\pi_F(1) \to 0$, implying for fixed $\delta$ that the right-hand side of (A.8) falls below $K$. By continuity in $t$ and the results for $t \to 0$ and $t \to -\infty$, there must exist for all sufficiently high $\delta$ some finite value $t > 0$ such that $U^R = K$. Finally, at this choice of $t$ it must also hold for high $\delta$ that $l > \beta$. This follows from our previous observation that $l \to 1$ must hold along any sequence of equilibria as, otherwise, $U^R = K$ cannot hold.

The assertions in Proposition 1 follow now from Lemmas A.1–A.4 and after observing that for $l \to 1$ it holds that $\theta^T \to \beta$, where $\theta^T$ was defined in (A.4), $\theta^M \to \delta$, where $\theta^M$ was defined in (A.6), and $q^M(\theta) \rightarrow q^*(\theta)$ for all $\theta \in [\theta^M, \theta^T]$.

A.2. Proof of Proposition 2

The proof of Proposition 2 is kept short as we use similar arguments as for Proposition 1.

Lemma A.5. In any equilibrium with additive search costs, it must hold that: (i) $V^R(\theta)$ is continuous and strictly increasing; (ii) $\Theta^E = [\theta^E, \beta]$, where $\theta^E > \delta$; and (iii) $\Theta^M = [\theta^M, \beta]$, where $\theta^M = \theta^E$.

Proof. Assertion (i) is immediate from Lemma A.1, while assertion (ii) is immediate from the strict monotonicity of $V^R$ and Assumption 2. Note next that, if some type $\theta$ does not get an acceptable offer in the monopolistic case, his reservation value equals

$$V^R(\theta) = S^*(\theta) - K - c/[1 - \pi_W(2)].$$

(A.9)

We show now that $\Theta^M$ must be convex. The argument is similar to that in Lemma A.1. If firms make no offer to types in some interval $(\hat{\theta}, \hat{\theta})$, while making offers to types in some interval up to $\hat{\theta}$ and to types in some interval starting from $\hat{\theta}$, incentive compatibility dictates that $dV^M/d\theta \leq dV^R/d\theta$ holds at $\theta = \hat{\theta}$ and that $dV^M/d\theta \geq dV^R/d\theta$ holds at $\theta = \hat{\theta}$. By (A.9), this yields $q^M(\hat{\theta}) \leq q^*(\hat{\theta})$ and $q^M(\hat{\theta}) \geq q^*(\hat{\theta})$. It is then feasible to extend the offer to types $\theta \in (\hat{\theta}, \theta)$ by specifying first-best contracts and extracting the payoff $K + c/[1 - \pi_W(2)]$, which strictly exceeds $K$. The same logic can now be applied to show that $\min \Theta^M = \theta^M$ and $\max \Theta^M = \beta$. □
We use next that, in equilibrium, it must hold from (7) that
\[
\frac{dV^M}{d\theta} < \frac{dV^R}{d\theta} \iff q^M(\theta) < q^*(\theta). \tag{A.10}
\]

**Lemma A.6.** In any equilibrium with additive search costs, the following holds:

(i) Only first-best contracts are offered in the monopolistic case, while the participation constraint binds for all types.

(ii) If all other firms make such an offer, this is also uniquely optimal for an individual firm.

**Proof.** Take first assertion (i). If the participation constraint becomes slack to the right of some type \(\theta^0\), \(q^M(\theta^0) > q^*(\theta^0)\) must hold by (A.10). This implies by (A.2) that \(\mu(\theta) > 0\). Using now Assumption 1, we can use the arguments in Claim A.2 of Lemma A.1 to show that both \(\mu(\theta) > 0\) and \(V^M(\theta) > V^R(\theta)\) must hold for all \(\theta\) up to \(\theta = \bar{\theta}\), which contradicts the transversality condition \(\Delta(\bar{\theta})\mu(\bar{\theta}) = 0\) at \(\theta = \bar{\theta}\). As we have shown that the participation constraint binds everywhere, \(q^M = q^*\) follows from (A.10). Finally, assertion (ii) is immediate. \(\Box\)

By Lemma A.6, we can obtain the threshold \(\theta^E\) from the condition
\[
S^*(\theta^E) - K - c/[1 - \pi_W(2)] = 0. \tag{A.11}
\]
Moreover, as firms realize \(K + c/[1 - \pi_W(2)]\) from every type \(\theta\) in the monopolistic case, the zero profit requirement holds by (5) if and only if
\[
\pi_F(1) = 1 - \pi_W(2) \tag{A.12}
\]
As \(\pi_F(1) \to 0\) holds for \(t \to \infty\) and \(\pi_W(2) \to 0\) holds for \(t \to 0\), (A.12) yields by Assumption 3 a unique solution \(t\), at which both \(\pi_F(1) > 0\) and \(\pi_W(2) > 0\). (Note that this solution is also independent of \(c\)). For low \(c\), we then obtain \(\theta^E > \theta^0\) from (A.11). This concludes the specification of the unique equilibrium, which exists for low \(c\).

**References**


