Single sourcing versus multiple sourcing

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We show that in contrast to results in the extant literature, single sourcing may not be the optimal strategy of a buyer facing suppliers with strictly convex costs. As we argue, previous findings relied crucially on the joint assumption that, first, there is only a single buyer and that, second, procurement takes place in an auction organized by the buyer. Relaxing these restrictions, we obtain a richer set of results. In particular, we show that even in the original setting, where suppliers bid, committing to single sourcing is only optimal if the respective buyer controls a sufficiently large fraction of the whole procurement market.

1. Introduction

Over the last two decades, a growing literature in economics has emerged that studies, both theoretically and empirically, optimal procurement practices. This article is primarily concerned with a particular element of a firm’s or public agency’s procurement strategy: the choice between single and multiple sourcing.

One of the early seminal contributions to this literature is that by Anton and Yao (1989), who derive the following simple but powerful result: under complete information and if suppliers have strictly convex costs, a single buyer that conducts an auction to procure a fixed volume is strictly better off when committing to single sourcing.1 Anton and Yao (1989), as well as a number of subsequent papers, then go on to develop arguments for why multiple sourcing (or, likewise, split-award contracts) could still be beneficial, for instance as this encourages more bidder participation or ensures more competition in the long run.2

This article revisits the original idea of Anton and Yao (1989) in order to derive a richer set of results. Our main result is the following. We show that if there is more than one buyer, which seems to be characteristic of many though clearly not all settings, then single sourcing may no longer be optimal. In fact, if all buyers demand the same volume, it is now most profitable for each buyer to spread his purchases evenly over suppliers. Single sourcing remains, however, optimal for a buyer who controls a sufficiently large share of the respective procurement market.

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1 The theoretical literature on share or menu auctions, to which Anton and Yao (1989) as well as our article contributes, originates from work by Wilson (1979) and Bernheim and Whinston (1986).

2 For instance, Anton and Yao (1987), Laffont and Tirole (1988), and Riordan and Sappington (1989) show how second sourcing reduces informational rents in a dynamic model. Recently, Biglaiser and Vettas (2005) have taken a different route by analyzing equilibrium purchasing strategies over time if suppliers have fixed capacity.
That several buyers procure at the same market may often be more realistic than assuming that there is only a single buyer. The key distinction between the two cases is the following. If there are more buyers and if one buyer makes more or even all of his purchases at one supplier, then in equilibrium the whole pattern of purchases and supplies will readjust. As a consequence, although one supplier will then account for most or all of the sales to the particular buyer, other suppliers will sell more to different buyers. In contrast, such a reshuffling of purchases and sales is simply not possible if the buyer is a monopsonist. As we bring out in detail below, this simple difference has profound implications for the optimality of single sourcing.

We also extend the analysis by considering the case where it is now suppliers who run an auction to sell their capacity. In marked contrast to the results obtained if suppliers bid at auctions that are run by buyers, single sourcing is now only optimal if a buyer is sufficiently small. What drives this stark reversal of results is the insight that the (strategic) choice of single sourcing plays an entirely different role in the two scenarios. If suppliers bid and have thus all “contracting power,” single sourcing is chosen if it enhances the value of a buyer’s outside option, that is, the value of the buyer’s second-best alternative, namely to procure instead more from other suppliers. In contrast, if buyers bid, then single sourcing is chosen if it erodes the value of suppliers’ outside option.

Overall, we thus obtain a richer set of results, relating the optimality of single versus multiple sourcing to both the size of the respective buyer, relative to that of the total procurement market, and to how procurement is organized. This may help to shed more light on observed variations in the organization of procurement. As documented in Tunca and Wu (2005), companies such as Sun or HP that use online auctions to procure products worth hundreds of millions of dollars frequently opt for multiple sourcing. Moreover, recent studies in operations research and logistics document cases of both single sourcing and multiple sourcing when firms procure via web-based auctions and business-to-business (B2B) platforms (see, e.g., Elmaghraby, 2000). Below we also comment on the implications of our findings for the optimal procurement strategy of public agencies, which are often special in that they account for a substantial fraction of the respective procurement market (e.g., in health care, road construction, or defense).

The rest of this article is organized as follows. Section 2 introduces the basic model where suppliers post bids. Section 3 analyzes two benchmark cases, namely that of a single buyer and that of two symmetric buyers, whereas Section 4 allows for buyers of different size. Section 5 discusses and further extends the results when suppliers bid. In Section 6, we analyze the opposite case where buyers now post bids. Section 7 concludes.

2. The model

The basic model considers a setting with two symmetric suppliers, indexed by \( m = S_1, S_2 \). Suppliers produce a homogeneous good with a strictly convex and twice continuously differentiable cost function \( C(x) \), where \( C(0) = 0 \). At the downstream level, there are at most two buyers, \( n = B_1, B_2 \). In our basic model, we further specify that each buyer wants to purchase a fixed volume \( X_n \). The total size of the procurement market is thus given by \( X := \sum_{n=B_1,B_2} X_n \).

Buyers’ demands and suppliers’ costs are common knowledge. We show how even the most simple framework, namely that of a one-shot interaction under complete information, can already give rise to a rich set of predictions. Note also that buyers do not compete at a downstream market. This is realistic for many procurement markets, in particular those for services such as computer programming or consulting, where \( X_n \) may denote the man days required for a project.

In the procurement game that we study, first each supplier \( m \) submits to each buyer \( n \) a menu \( t_m^*(x) \), specifying the total transfer that the buyer has to make when purchasing the quantity \( x \). We analyze both the case where buyers do not restrict their purchasing strategies and the case where at least one buyer chooses single sourcing. There may be different ways how a commitment to single sourcing can be sustained, depending on the particular application. For instance, a retailer may dedicate only a limited amount of shelf space to a product category, which limits the number
of listed goods. Also, if the purchased good represents some intermediate input, the buyer’s choice of the production process (or, likewise of its warehousing and inventory management) may commit to only purchase from a single supplier.

3. Analysis of two benchmark cases

First benchmark: the case with a single buyer. Suppose first that there is only a single buyer as \( X_{B_1} = X \) and, consequently, \( X_{B_2} = 0 \). The analysis is then analogous to that in Anton and Yao (1989).

We know that there exists a plethora of (Nash) equilibria in which the single buyer ends up purchasing different fractions of \( X \) from the two suppliers. Although we refer to Section 5 for a more formal analysis, it is straightforward to see already now how different equilibria can be supported. For this purpose, suppose that each supplier submits a “quantity-forcing” contract, namely to, first, supply some fixed quantity \( \hat{x}^m \) for some total payment \( \hat{t} = t^m(\hat{x}^m) \) and to, second, make the purchase of any quantity other than \( \hat{x}^m \) prohibitively expensive by demanding a sufficiently high payment \( t^m(x) >> \hat{t} \) for all \( x \neq \hat{x}^m \). If supplier \( m \) follows this strategy, then this makes it optimal for supplier \( m' \neq m \) to follow a similar strategy with \( \hat{x}^{m'} = X - \hat{x}^m \).

In our main analysis, we want to abstract from this multiplicity. We do so by applying a common refinement to the set of Nash equilibria, which is owing to Bernheim and Whinston (1986). The key notion of a refinement in the present context is to pin down the parts of the schedules \( t^m \) that are only of importance off equilibrium. Bernheim and Whinston (1986) do so by essentially fixing the slope of the payment schedules, requiring that they truthfully reflect the respective supplier’s marginal costs. Put somewhat differently, under the “truthfulness requirement,” each schedule of payments must then satisfy

\[
t^m(x') - t^m(x) = C(x') - C(x),
\]

where we have dropped the subscript \( n = B_1 \) for the supply to the single buyer (given that currently \( X_{B_1} = X \) and, consequently, \( X_{B_2} = 0 \)). Section 5 shows that our main results still hold if we do not impose the truthfulness requirement (1).

With truthful menus, it is immediate that the equilibrium outcome must be efficient given that the schedule of payments offered to the retailer by either supplier reflects the respective supplier’s marginal costs at all quantity levels. By symmetry and strict convexity of costs, efficiency requires that each supplier produces \( X/2 \). Another implication of the truthfulness requirement is that if the buyer rejects one of the two bids and thus ends up purchasing the entire quantity \( X \) from a single supplier, then the incremental price equals the respective supplier’s incremental costs, namely \( C(X) - C(X/2) \). In equilibrium, it holds from optimality that each supplier chooses \( t^m(X/2) \) such that the buyer is just indifferent between acceptance and rejection. Taken together, this implies that \( t^m(X/2) \) is just equal to the incremental costs of procuring instead only from a single supplier, \( C(X) - C(X/2) \). Altogether, the buyer thus ends up paying \( 2t^m(X/2) \), which is equal to \( 2[C(X) - C(X/2)] \).

If the buyer commits to single sourcing, then by symmetry, suppliers compete themselves down to zero profits. Consequently, the buyer has to compensate the winning supplier just for the respective costs of production, \( C(X) \). Single sourcing is then optimal if the respective total payment \( C(X) \) is strictly smaller than the respective payment under multiple sourcing, \( 2[C(X) - C(X/2)] \). After rearranging expressions, this is the case if \( C(X) < 2C(X/2) \), which in turn holds from strict convexity of \( C \).

Second benchmark: the case with two symmetric buyers. Our second benchmark case is that with two symmetric buyers such that \( X_{B_1} = X_{B_2} = X/2 \). We first extend the truthfulness requirement to this setting. (Again, we show below that our main results continue to hold if we do not impose this restriction.) For this, we first denote for a given equilibrium the resulting
allocation, that is, the distribution of sales and purchases over suppliers and buyers, by \( \{ \hat{x}_n \}_{m=1,2} \).

That is, \( \hat{x}_n^m \) denotes the quantity that supplier \( m \) will sell to buyer \( n \) in the respective equilibrium. The respective transfers are denoted by \( t_n^m := t_n^m(\hat{x}_n^m) \). Note also that the supplier’s total level of production is then \( \hat{x}_m := \sum_{n=1,2} \hat{x}_n^m \).

If a menu \( t_n^m \) truthfully reflects the incremental costs of supplier \( m \), then given the supplier’s (rationally anticipated) total production \( \hat{x}_m \) we must have for all \( \Delta_x \), that

\[
t_n^m(\hat{x}_n + \Delta_x) - t_n^m(\hat{x}_n) = C(\hat{x}_n + \Delta_x) - C(\hat{x}_n).
\]

(2)

In words, buyer \( n \) can purchase from \( m \) an incremental quantity \( \Delta_x \) above \( \hat{x}_n^m \) at \( m \)’s incremental costs, where incremental costs are calculated on the basis of the supplier’s rationally anticipated production volume \( \hat{x}_m \) under the respective equilibrium. At this point, it may be useful to note that another way of expressing the truthfulness requirement (2), which then looks more similar to that in (1), is that

\[
t_n^m(x') - t_n^m(x) = C(x' + \hat{x}_n^m) - C(x + \hat{x}_n^m)
\]

must hold for any pair \( x' \) and \( x \) and for \( n' \neq n \).

With (2) it is again intuitive that without further restrictions on the allocation of sales and purchases, an equilibrium must again be efficient. Again, this requires that each supplier produces just one half of the total volume \( X/2 \). On the other hand, as the suppliers’ products are homogeneous, it does not matter for efficiency how each buyer mixes and matches between the products from the two suppliers. Consequently, all allocations \( \{ \hat{x}_n \}_{m=1,2} \) are efficient as long as \( \hat{x}_1 = \hat{x}_2 = X/2 \). Intuitively, we can also support all of these (efficient) allocations as equilibrium outcomes.

At one extreme of the characterized continuum of equilibrium allocations is the case where each buyer purchases exclusively from one supplier. (This clearly uses the symmetry of buyers and suppliers.) At the other extreme is the case where each buyer maximally spreads his purchases over both suppliers, thus purchasing \( X/4 \) from either supplier. Note once again, though, that in either case each supplier produces exactly \( X/2 \).

We next turn to equilibrium transfers. By optimality, the required payment \( t_n^m \) makes buyer \( n \) again indifferent between procuring \( \hat{x}_n^m \) from supplier \( m \) or, instead, increasing his purchases from the alternative supplier \( m' \) by \( \hat{x}_n^{m'} \). Note that from the truthfulness requirement (2), the incremental price that \( n \) would have to pay to the other supplier \( m' \) equals the supplier’s respective incremental costs \( C(X/2 + \hat{x}_n^{m'}) - C(X/2) \), where we use that \( \hat{x}_m = X/2 \) holds by efficiency. Hence, we have that \( \hat{t}_n^m = C(X/2 + \hat{x}_n^m) - C(X/2) \).

Summing up over the payments made to the two suppliers, buyer \( n \) will thus end up paying the total price of

\[
\sum_{m=1,2} \left[ C(X/2 + \hat{x}_n^m) - C(X/2) \right].
\]

(3)

We analyze next how (3) changes as we move between different (efficient) allocations. Precisely, we are interested in how (3) changes as a buyer’s purchases become more or less concentrated on one supplier. For this purpose, we specify now without loss of generality that a given buyer \( n \) purchases (weakly) more from some supplier \( m \) such that \( \hat{x}_n^m \geq \hat{x}_n^{m'} \). That is, we can refer to supplier \( m \) as the larger supplier to buyer \( n \). Differentiating then (3) with respect to \( \hat{x}_n^m \), while using that the buyer’s total purchases remain constant at \( \sum_{m=1,2} \hat{x}_n^m = X/2 \), we thus have

\[\sum_{m=1,2} C(x_n^m + x_n^{m'}) \]

where \( \sum_{m=1,2} x_n^m = X/2 \) for both \( n = 1, 2 \), is strictly concave. Note next that by (2), it must hold for any \((x, x')\) that \( t_n^m(x') - t_n^m(x) = C(x' + \hat{x}_n^m) - C(x + \hat{x}_n^m) \). Consequently, holding all other \( \hat{x}_n^{m'} \) fixed, a buyer’s choice of \( \hat{x}_n^m \) must also satisfy the first-order condition of the preceding program.
that shifting more of the buyer’s purchases to his larger supplier increases total purchasing costs if

\[ C'(X/2 + \hat{x}_n^m) > C'(X/2 + \hat{x}_n^m'). \]  

From strict convexity of \( C(\cdot) \), we have that condition (4) holds indeed whenever \( \hat{x}_n^m > \hat{x}_n^m' \) holds strictly.

We can rephrase this result as follows. With two symmetric buyers, a buyer’s total purchasing costs, as given by (3), are minimized if the buyer spreads his purchases evenly over the two suppliers. At the other extreme, purchasing costs are highest if the buyer’s purchases are concentrated on only a single supplier.

□ **Comparison.** With two symmetric buyers, the outcome under single sourcing represents thus the worst possible outcome for either buyer. The crucial difference to the case with a single supplier is the following. With two buyers, if one buyer purchases more (or even all) from one supplier, then the other supplier simply sells more to the second buyer. In contrast, if there is only a single buyer and if this buyer commits to single sourcing, then one of the two suppliers will end up producing nothing. In the latter case, single sourcing therefore leads to “all-out” competition between suppliers, whereas this is not the case if there are two symmetric buyers and suppliers can consequently perfectly reallocate sales.

To put this differently, note first that what sustains a buyer’s payoff in the auction above zero is the possibility to purchase the respective quantity instead from the other supplier. Under the truthtelling requirement, the buyer would then have to compensate the other supplier for the respective incremental costs of production. (See, however, Section 5 on a generalization of our results to all Nash equilibria of the auction.) By spreading purchases equally over the two suppliers, a buyer purchasing \( X/2 \) in total and thus \( X/4 \) at each supplier minimizes the “average dependency” on either supplier, where dependency is measured by the costs it takes to replace the respective volume of purchases. More formally, note that a buyer obtains a more attractive bid from a given supplier if the incremental purchasing costs that he would incur when procuring instead exclusively from the other supplier are low. If a large fraction of a buyer’s purchases is concentrated on one supplier, while both suppliers still produce exactly \( X/2 \) in equilibrium, then this increases the incremental costs per unit when switching away from this supplier. Though on the other side it is then also less costly to replace the smaller volume that the buyer purchases from the other supplier, given strict convexity of suppliers’ costs the average costs of substitution are still strictly higher the more unevenly the buyer’s purchases are distributed over suppliers.

We summarize our results for the two benchmark cases as follows.

**Proposition 1.** A single buyer who conducts an auction strictly prefers to commit to single sourcing. In contrast, if there are two symmetric buyers, then either buyer strictly prefers to spread his purchases evenly over both suppliers. More generally, in the latter case, a buyer’s costs of purchasing increase the more his purchases are concentrated on a single supplier, making single sourcing the worst outcome.

### 4. Analysis with asymmetric buyers

In the preceding section, we compared the cases where a buyer either accounted for the whole of the respective procurement market or for only one half of it. The comparison of the two cases suggests, more generally, that single sourcing is more likely to be optimal, at least in the current case where buyers run auctions, if a buyer accounts for a larger fraction of the total procurement market.4

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4 Importantly, we will show that what matters is the relative size, that is, relative to the size of the total procurement market, rather than a buyer’s absolute size.
Suppose thus that without loss of generality, buyer \( n = B_1 \) is always the (weakly) larger buyer as \( X_{B_1} \geq X/2 \). Even with asymmetric buyers it is still intuitive that without a commitment to single sourcing, all efficient outcomes can still be supported as equilibrium outcomes, that is, any allocation \( \{x^m_{1^n=1} x^S_{1^n=2} \} \) where \( \hat{x}_{B_1} = \hat{x}_{B_2} = X/2 \) is an equilibrium outcome. Moreover, under the truthfulness requirement (2) also the converse holds again, namely that any equilibrium must give rise to an efficient allocation. With these observations, we can thus fully characterize any equilibrium allocation by the quantity \( \hat{x}_{B_2} \geq X_{B_2}/2 \) that buyer \( B_2 \), who is also the smaller of the two buyers, purchases from his larger supplier. We can then simply trace out the continuum of all equilibria satisfying the truthfulness requirement (2) by varying the respective purchases \( \hat{x}_{B_2} \) between \( X_{B_2}/2 \) and \( X_{B_2} \). For a given choice of \( \hat{x}_{B_2} \), all other purchases are then uniquely pinned down by the requirement that equilibrium allocations are efficient such that \( \hat{x}_{B_2} = X_{B_2} - \hat{x}_{B_2} \) as well as \( \hat{x}_{B_1} = X/2 - \hat{x}_{B_2} \) and \( \hat{x}_{B_1} = X/2 - \hat{x}_{B_2} \). Of course, as the purchases of buyer \( B_2 \) become more concentrated, this is also the case for buyer \( B_1 \) and vice versa.

We turn next to single sourcing. Suppose first that only buyer \( B_2 \) would commit to single sourcing. Intuitively, as this is the smaller buyer, the equilibrium outcome will still be efficient. The supplier \( m \) from whom buyer \( B_2 \) exclusively purchases will in addition sell the quantity \( X/2 - X_{B_2} \) to the other buyer, \( B_1 \). This is clearly different if the larger buyer commits to single sourcing. In this case, one supplier will end up producing more than the efficient share of total supply. Moreover, as \( B_1 \) accounts for a larger share of total purchases given that \( X_{B_1}/X \) increases, the difference \( X_{B_1} - X_{B_2} \) between the two suppliers’ sales becomes larger. Importantly, as \( X_{B_2} \) becomes smaller, the supplier \( m \) who sells only to buyer \( B_2 \) would be willing to supply additionally to \( B_1 \) at a lower price, reflecting his lower average incremental costs that he would have to incur. This essentially reduces the rent that the exclusive supplier can extract from \( B_1 \). In fact, we already know that in the extreme case where \( X_{B_1}/X = 1 \), the respective supplier makes zero profits.

These observations suggest more generally that the large buyer will prefer single sourcing only if his purchases account for a sufficiently large fraction of the total procurement market. It is now convenient to state this result first only for the case where without commitment to single sourcing, purchases would be allocated symmetrically over both suppliers.

**Proposition 2.** With two buyers of different size, the smaller buyer never prefers single sourcing. In contrast, if without commitment to single sourcing, purchases are evenly distributed over both suppliers as \( \hat{x}_{B_1} = \hat{x}_{B_2} = X/2 \), then the large buyer prefers single sourcing if and only if his purchases account for a sufficiently large fraction of the total procurement market as \( X_{B_1}/X \) is sufficiently large.

**Proof.** See the Appendix.

Proposition 2 only considers the case where without single sourcing, buyers’ purchases would be evenly distributed over both suppliers. To extend the result we now proceed as follows. As we increase the large buyer’s share of the total procurement market, \( X_{B_1}/X \), we want to keep unchanged the degree to which purchases are concentrated without single sourcing. We do this by holding constant for the smaller buyer \( B_2 \) the fraction \( \beta \geq 1/2 \) that he purchases from his larger supplier. (Note that for \( \beta = 1/2 \) we are back to the case of Proposition 2.)

**Proposition 3.** Generalizing Proposition 2, even if without single sourcing a buyer’s purchases are not evenly distributed over both suppliers, the large buyer still prefers single sourcing if and only if he controls a sufficiently large fraction of the total procurement market. More formally, this is the case if \( X_{B_1}/X \geq \gamma \), where the respective threshold \( 1/2 < \gamma < 1 \) is strictly lower if without single sourcing, purchases are also more concentrated (given that \( \beta \) is higher).

**Proof.** See the Appendix.

Before proceeding with the analysis, we briefly comment on the issue of multiplicity of equilibria. One way to narrow down the set of equilibrium allocations is to introduce some
heterogeneity in suppliers’ products or services. This would, for instance, seem appropriate if buyers are retailers who can stock one or two goods in a particular category. It is then straightforward to show that if goods are not perfect substitutes, then without commitment to single sourcing, each buyer purchases $X/2$ from either supplier. This is the outcome that we chose for comparison with single sourcing in Proposition 2.

5. Discussion

- **Relaxing the truthfulness requirement.** Our results so far were obtained under the truthfulness requirement, which ensured that all equilibrium allocations are efficient and that for a given allocation, transfers are uniquely pinned down. We show now that this requirement, though it narrows down the set of equilibrium allocations, is not crucial to obtain our results. Precisely, we show that even if we can support a larger set of outcomes, some of them even inefficient, then commitment to single sourcing is still optimal for the single large buyer but not so for the two symmetric smaller buyers.

Take again first the case with a single large buyer. We now replace the truthfulness requirement by the weaker requirement that neither supplier is willing to sell more than the equilibrium quantity $\hat{x}_m$ at an incremental price that is strictly below incremental costs. Formally, for given $\{\hat{x}_m\}_{m=S_1,S_2}$ we require that for all $\Delta x \geq 0$,

$$t_m(\hat{x}_m + \Delta x) - t_m(\hat{x}_m) \geq C(\hat{x}_m + \Delta x) - C(\hat{x}_m). \tag{5}$$

The main implication of (5) is that for a given allocation it fully pins down the respective equilibrium transfers. Namely, each supplier will again extract a transfer that is exactly equal to the respective incremental costs at the other supplier. In difference to the truthfulness requirement, however, under the weaker condition (5), we can now support any allocation where $0 \leq \hat{x}_m \leq X$ and thus no longer only the efficient allocation with $\hat{x}_m = X/2$. Intuitively, requirement (5) still allows supplier $m$ to make it extremely unattractive for the buyer to purchase any quantity lower than $\hat{x}_m$ from him, say by specifying an exorbitantly high transfer $t_m(x)$ for all $x < \hat{x}_m$.

For any allocation with $0 \leq \hat{x}_m \leq X$, the single buyer will then pay the total price of

$$\sum_{m=S_1,S_2} [C(X) - C(\hat{x}_m)], \tag{6}$$

which just sums up the incremental costs at the respective alternative supplier. Differentiating (6) with respect to $\hat{x}_m$ and noting that $\hat{x}'_m = X - \hat{x}_m$ for $m' \neq m$ shows that the total price is still minimized at the two corners where $\hat{x}_{S_1} = X$ or $\hat{x}_{S_2} = X$.

With two buyers, we have in analogy to (5) the requirement that for all $\Delta x \geq 0$,

$$t_m(\hat{x}_m + \Delta x) - t_m(\hat{x}_m) \geq C(\hat{x}_m + \Delta x) - C(\hat{x}_m). \tag{7}$$

With (7) instead of (2), we can again support a wider range of equilibrium allocations. However, buyers’ competition on the procurement market now limits the market share that any given supplier can obtain. (See Corollary 1 below.) We are first interested in whether our result from Proposition 1, namely that each of the two small buyers strictly prefers multiple sourcing, still holds. This is indeed the case.

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5 This requires also to extend the truthfulness requirement (2) accordingly.

6 This relatively weak requirement is akin to the standard requirement that if sellers compete for the sale of a single unit, then none offers the good below costs even if he is certain that the buyer will purchase instead the other seller’s (potentially superior) good. (That is, none uses weakly dominated strategies.) Incidentally, when replacing (5) by the seemingly only slightly stricter requirement that it applies also to $\Delta x < 0$, we obtain again only efficient equilibria.

7 For instance, we could imagine that the contract of supplier $m$ offers a steep all-unit discount at $\hat{x}_m$. 

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Proposition 4. The results from Proposition 1 carry over if we replace the stronger truthfulness requirement by the weaker requirement that incremental quantity is not offered below incremental costs. That is:

(i) If there is a single large buyer, then single sourcing is still uniquely optimal.
(ii) If there are two buyers, then both buyers are worse off under single sourcing.

Proof. See the Appendix.

As a byproduct of the proof of Proposition 4, we have the following characterization of all equilibria, including those that are inefficient.

Corollary 1. Replacing the truthfulness requirement by (5) and (7), respectively, the following equilibrium allocations can now be supported:

(i) If there is a single large buyer, then the market share of either supplier can range from zero to 100%.
(ii) Instead, if there are two symmetric buyers, then either supplier can only have a share between 1/3 to 2/3 of the total procurement volume.

Proof. See the Appendix.

Corollary 1 is interesting in itself. Put differently, it says that if a given procurement volume is distributed over more than one buyer, as in assertion (ii), then this provides tighter bounds on the inefficiencies in production that can arise in equilibrium as some suppliers sell more than others.

Flexible adjustment of purchases. So far we have stipulated that buyers will always purchase exactly the same quantity, namely $X$ for the large buyer and $X/2$ for the two smaller buyers. In particular, this was the case both on equilibrium, that is, if all offers were accepted, and off equilibrium, that is, after only one supplier's offer was accepted. A fixed purchase volume may sometimes be realistic, for example, if this is just one input for a Leontieff-type production function and if the buyer has already purchased the right amount of all other inputs. More generally, however, a buyer may have some flexibility in adjusting his optimal purchase volume.

We show now that our key insights still hold if we allow for this additional flexibility. For this, we stipulate that if there are two buyers, then each derives the payoff (or revenues) $r(x)$ from purchasing the total quantity $x$ of the input. (Recall that suppliers' goods are homogeneous.) We assume that $r(x)$ is continuously differentiable and strictly concave with $r'(0) > C'(0)$. We again denote the unique efficient level of total output by $X$ such that $X/2 = \arg\max_x [r(x) - C(x)]$. If there is a single large buyer, then this buyer simply controls both of these two firms. Note that in this case, given symmetry and strict concavity, the large buyer's (gross) payoff from purchasing the total quantity $x$ is equal to

$$R(x) := 2r\left(\frac{x}{2}\right).$$

Take now first the case with a single large buyer. In equilibrium, the buyer purchases $X/2$ from either supplier. Compared to the case with fixed purchasing quantities, what changes now are the purchases off-equilibrium, that is, if one bid from the two suppliers is rejected. Given the truthfulness requirement, when rejecting the bid of some supplier $m$, the buyer will now purchase from the other supplier the total quantity

$$X' = \arg\max [R(x) - C(x)],$$

which satisfies $X/2 < X' < X$, at a total price equal to the sum of $\hat{s}_n'$ and the respective incremental costs, $C(X') - C(X/2)$. By optimality, equilibrium transfers are again chosen so as
to make the buyer just indifferent between accepting or rejecting the respective offer. As there are two symmetric suppliers, this yields

$$\hat{t}_m = [R(X) - R(X')] + [C(X') - C(X/2)].$$  \hspace{1cm} (9)$$

If the single large buyer resorts to single sourcing, then suppliers again compete themselves down to zero profits. The outcome will be constrained efficient such that the buyer purchases the quantity $X'$ as defined in (8) from the winning supplier. Total purchasing costs are just equal to the respective supplier’s costs of production $C(X')$.

The procedure to characterize an equilibrium allocation if there are two symmetric buyers is analogous. Again, off-equilibrium, a buyer will optimally adjust the respective incremental purchases. Proposition 5 now confirms that our previous results from Proposition 1 extend to the currently considered case with flexible quantities. Though the algebra is somewhat more involved, the intuition is fully analogous.

**Proposition 5.** Proposition 1 extends to the case where buyers will optimally adjust their purchased quantities according to their own revenue function $r(x)$. Precisely, also in this case, a single large buyer strictly prefers single sourcing, whereas two smaller buyers strictly prefer to spread their purchases equally over the two suppliers.

**Proof.** See the Appendix.

6. Buyers competing at suppliers’ auctions

**Analysis.** As noted in the Introduction, we also want to compare the results from the case where suppliers compete to that where it is now buyers that make bids. Hence, we now stipulate that buyers submit menus $t_m^n(x)$ to suppliers. Clearly, if there is only a single large buyer, then the analysis is trivial: the equilibrium outcome is efficient and the single buyer extracts all profits.

Turning thus to the case with two buyers, for brevity we restrict consideration to the case where buyers are again symmetric.\(^8\) However, to ensure that suppliers who reject a buyer’s bid have always profitable alternative options, namely to supply more to the other buyer, we still specify that each of the two buyers can realize the payoff $r(x)$ when purchasing the quantity $x$. We invoke now again the truthfulness requirement. That is, this time the respective menu $t_m^n(x)$ must now truthfully reflect a buyer’s marginal revenues $r'(x)$. To formalize this, we again have to choose a particular equilibrium allocation with respective values of $\hat{x}_m^n$. Given the respective (rationally anticipated) quantity $\hat{x}_n := \sum_{m=1,2} \hat{x}_m^n$ that buyer $n$ will purchase, truthfulness for the menu $t_m^n(x)$ thus requires for all $\Delta_x$ that

$$t_m^n(\hat{x}_n + \Delta_x) - t_m^n(\hat{x}_n) = r(\hat{x}_n + \Delta_x) - r(\hat{x}_n).$$  \hspace{1cm} (10)$$

It is immediate that given (10), the set of supported allocations is again equal to that of all efficient allocations. Moreover, it is also straightforward to show that for any given allocation, both buyers now pay strictly less than if suppliers make bids. In other words, the right to make offers is clearly valuable. What is at first somewhat surprising, however, is that the ranking of the different outcomes from the perspective of both buyers is now exactly the opposite to that in the previous case, where suppliers made bids.

**Proposition 6.** Suppose now that buyers bid in auctions organized by suppliers and that the truthfulness requirement still applies. Then the ranking of equilibrium allocations is reversed compared to that in Proposition 1: both buyers are strictly better off the more a buyer’s purchases are concentrated on a particular supplier. On the other side, a single buyer who can post bids to suppliers will always strictly prefer multiple sourcing.

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\(^8\) The case with fixed quantities and asymmetric buyers, which is symmetric to the analysis in Proposition 2, is solved in the working paper version.
Proof. See the Appendix.

If buyers make bids, then there are now two reasons for why average purchase prices are lowest under single sourcing. The first reason is analogous to that underlying Proposition 1, though now it applies symmetrically to suppliers instead of buyers. That is, whereas concentrating purchases more on one supplier reduces the total value of a buyer’s alternative options across the two suppliers if suppliers make bids, if buyers make bids then it now also reduces the total value of suppliers’ alternative options, namely to sell more to another buyer. This is now profitable for the buyer if he has all “contracting power” as he makes the bid in the respective auction.

If buyers make bids, there is also a second reason for why average purchase prices are now lower the more a competing buyer purchases from one supplier. If we ignore for a moment a supplier’s option to sell more to another buyer, then a buyer’s bid would just have to cover a supplier’s respective incremental costs. If a buyer purchases all from one supplier, then he has to compensate the supplier for the respective costs \( C(X/2) \). Instead, if he purchases \( X/4 \) from either supplier, then he must compensate each of them for the incremental costs \( C(X/2) - C(X/4) \), given that the respective supplier then also sells \( X/4 \) to the other buyer. With strictly convex costs, the respective incremental costs in the latter case, namely two times \( C(X/2) - C(X/4) \), are strictly higher than \( C(X/2) \). The latter effect has already been recognized in Chipty and Snyder (1999).

\[ \square \]

Corollary 2. Summarizing results, a buyer should choose single sourcing

(i) if he is either sufficiently large and invites bids from suppliers,
(ii) or if he is sufficiently small and submits bids to suppliers.

If we interpret the choice between the two (auction) formats as one between different distributions of contracting power, we can rephrase Corollary 2 as follows: a buyer should then be more likely to choose single sourcing if (i) contracting power resides more with suppliers and the buyer accounts for a sufficiently large fraction of the procurement market or if (ii) contracting power is more on the side of buyers but the buyer is relatively small compared to the overall size of the respective procurement market.

Considering public procurement, civil servants may often lack the appropriate (financial) incentives in negotiations. What is more, the fear of corruption or the requirement to increase accountability by making the procurement process more transparent may even dictate a particular format such as an open tender. Moreover, in some markets such as those of health services or certain segments of the construction industry, public agencies may indeed be the major (local) buyers. To the extent that our key assumption applies, namely that of increasing marginal costs (at least over the relevant range), Corollary 2 would thus prescribe that officials should try to design large lots and rely on one or only few suppliers as much as possible. By increasing competition for the one big lot, the procurement agency basically compensates for its lack of bargaining power. In contrast, in markets where the public body is less dominant, it should secure lower purchase prices by relying on (strategic) second sourcing.

7. Conclusion

This article takes as a starting point the seminal analysis on optimal procurement strategies by Anton and Yao (1989). As in their basic setting, we explore a parsimonious static model with

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9 For instance, to foster the creation of an internal market in the European Union, all public tenders satisfying certain thresholds must be subject to open tender or comparable processes.
symmetric information. Our departure from Anton and Yao (1989) is that we allow for more than one buyer and that, in the second part of the article, we also consider auctions that are organized by suppliers instead of buyers. Interpreting the difference in procurement formats more generally as a difference in contracting power, our analysis provides, despite the simplicity of the setting, a rich set of results on when single sourcing as opposed to multiple sourcing may be an optimal strategy.

One key insight is the following. Whether single sourcing is optimal or not depends on a buyer’s relative size, more precisely, on the fraction of the total procurement market that the buyer accounts for in equilibrium. Only sufficiently large buyers can substantially change the total allocation of production among suppliers if they commit to single sourcing. In contrast, single sourcing by a small buyer will merely lead to a reshuffling of purchases and sales without affecting any supplier’s overall production. A second and related observation is that single sourcing serves different purposes under the two considered procurement formats. If suppliers post bids, then for a sufficiently large buyer, single sourcing makes competing bids more attractive. Instead, for smaller buyers, single sourcing makes their alternative options less attractive and thus allows suppliers to extract a higher price. If buyers post bids, committing to single sourcing affects now in a systematic way the attractiveness of suppliers’ alternative options, namely to supply more to another buyer.

Appendix

Proof of Proposition 2. For buyer n we have from (3), which still applies, that in case \( x'_n = \frac{2}{\gamma} \), the total purchasing price equals \( 2[C(X/2 + X_n/2) - C(X/2)] \). With single sourcing, the buyer pays likewise the incremental costs \( C(X) - C(\alpha) \), where \( \alpha \neq n \). Note that in equilibrium the other supplier will sell \( X_n \) to buyer \( n \).\(^{10}\) Comparing total purchasing costs, single sourcing is then (weakly) optimal for buyer \( n \) if

\[
2[C(X/2 + X_n/2) - C(X/2)] - [C(X) - C(\alpha)] \geq 0. \tag{11}
\]

Note first that condition (11) holds strictly at \( X_n = X \), whereas the converse holds strictly at \( X_n = X' = X/2 \). To prove the assertion for the large buyer, it remains to show that (11) is strictly increasing in \( X_n = X_B \). Differentiating the left-hand side of (11) w.r.t. \( X_B \) while using that \( x_n = X_B = X - X_B \), the derivative is strictly positive if and only if \( C'(X/2 + X_B/2) > C'(X_B/2) \). This holds as \( X_B < X/2 < X_B/2 \).

For the small buyer, note that with \( X_B = X_B \) and \( X_n = \alpha \), the derivative of the left-hand side of (11) w.r.t. \( X_B/2 \) is now negative if and only if \( C'(X/2 + X_B/2) > C'(X_B/2) \) and \( X_B/2 < X/3 \). As we also know that (11) does not hold at \( X_B = X/2 \) and as the value of the left-hand side of (11) is clearly zero at \( X_B = 0 \), (11) does not hold for the smaller buyer. Q.E.D.

Proof of Proposition 3. We thus suppose now moreover that \( x_n^w = \beta X_B \) and \( x_n^w = (1 - \beta) X_B \) hold without single sourcing for some \( 1/2 < \beta < 1 \). Using that \( x_n = X/2 - x_n^w \) and that \( x_n^w = X/2 - x_n^w \), together with (3), the total price of the large buyer is equal to

\[
[C(X/2 + (X/2 - \beta X_B)) - C(X/2)] + [C(X/2 + (X/2 - (1 - \beta) X_B)) - C(X/2)].
\]

implying that the large buyer prefers single sourcing if

\[
[C(X - \beta X_B) + C(X - (1 - \beta) X_B) - 2C(X/2)] - [C(X) - C(X_B)] \geq 0. \tag{12}
\]

We establish first that also for each \( \beta > 1/2 \) there is a unique \( 0 < \gamma < 1 \) at which (12) is satisfied with equality. To see this, note again first that at \( X_B = 0 \) and thus \( \gamma = 1 \) the left-hand side in (12) is strictly positive, whereas at \( \gamma = 1/2 \) and thus \( X_B = X/2 \) it is strictly negative. Note next that the derivative of the left-hand side of (12) with respect to \( X_B \) equals

\[
-\beta C'(X - \beta X_B) - (1 - \beta) C'(X - (1 - \beta) X_B) + C'(X_B). \tag{13}
\]

Using that \( X - \beta X_B > X_B > X_B \) and \( X - (1 - \beta) X_B > X_B > X_B \), it holds that \( C'(X - \beta X_B) > C'(X_B) \) and that \( C'(X - (1 - \beta) X_B) > C'(X_B) \), by which (13) is strictly negative.

Having thus established existence of a threshold \( \gamma \) for each \( \beta \), we obtain next

\[
\frac{d\gamma}{d\beta} = \frac{1}{X} \left[ C'(X - (1 - \beta) X_B) - C'(X - \beta X_B) \right] \leq 0,
\]

where we used that \( \beta > 1/2 \). Q.E.D.

\(^{10}\) We use again that the other supplier stands ready to supply incremental quantities at marginal costs.
Proof of Proposition 4. Assertion (i) follows from the arguments in the main text. Turn thus to assertion (ii). We argue first that if one buyer single sources, then ignoring the identity of the two suppliers, the outcome is unique as each supplier sells to exactly one buyer. That is, we can rule out cases where for some \((n, m)\) it holds that \(\hat{x}^n_m = X/2\) while also \(x^m_n > 0\). We argue to a contradiction, in which case we have the following requirements. First, from optimality for the supplier and from (7) we have

\[
\hat{t}^n_m = C(X/2) - C\left(\hat{x}^n_m\right),
\]

where we also use that by assumption \(m'\) only sells to \(n'\). Second, to make supplying also to buyer \(n'\) profitable for supplier \(m\), it must hold that

\[
\hat{t}^n_m \geq C\left(X/2 + \hat{x}^n_m\right) - C(X/2).
\]

Taken together, (14) and (15) thus jointly require that

\[
2C(X/2) \geq C\left(X/2 + \hat{x}^n_m\right) + C\left(\hat{t}^n_m\right).
\]

As this is just satisfied with equality in case \(\hat{x}^n_m = 0\) and thus \(\hat{x}^n_m = X/2\), while the right-hand side of (16) is strictly increasing in \(\hat{x}^n_m\), using also that \(\hat{x}^n_m = X/2 - \hat{t}^n_m\), we have thus arrived at a contradiction.

To prove assertion (ii) we thus have to show that the now unique total price paid under single sourcing, which is

\[
C(X) - C(X/2),
\]

is strictly above the total price paid under \(any\) other equilibrium allocation. Suppose without loss of generality that \(m\) is the larger supplier with \(\hat{x}^m_n = X/2 + a\) such that \(\hat{x}^n_m = X/2 - a\). From Proposition 1 we can restrict consideration to values \(a > 0\). In a first step, we now derive the range of values \(a\) that can be supported in equilibrium. For future reference, it is helpful to prove a stricter assertion, providing a full characterization of all equilibria, and to state this as a separate result.

Claim 1. Any equilibrium allocation for the case without the truthfulness requirement is fully characterized by two parameters, namely \((a, \hat{x}^n_m)\), where \(a \in [0, X/6]\) and \(\hat{x}^n_m \in [(X/2 + a)/2, X/2 - a]\). All other supplies are then obtained by the requirements that \(\hat{x}^m_n = X/2 + a\) and that \(\hat{t}^n_m = \hat{x}_n = X/2\).

Proof. Consider some transfer \(\hat{t}^n_m\). Also with the weaker requirement (7), the maximum (and consequently equilibrium) transfer that supplier \(m\) can demand is given by the respective incremental costs of the other supplier \(m'\), namely

\[
C\left(X/2 - a + \hat{x}^n_m\right) - C(X/2 - a).
\]

This can only be supported as an equilibrium outcome if it also covers the own incremental costs of supplier \(m\), which equal

\[
C(X/2 + a) - C\left(X/2 + a - \hat{x}^n_m\right).
\]

Hence, for both \(n\) it must hold under an equilibrium allocation that (18) is not smaller than (19), which transforms further to the requirement that

\[
C\left(X/2 - a + \hat{x}^n_m\right) + C\left(X/2 + a - \hat{x}^n_m\right) \geq C(X/2 + a) + C(X/2 - a).
\]

Condition (20) is only satisfied if we have for both \(n\) that \(\hat{x}^n_m \geq 2a\). If \(n'\) is the buyer that purchases (weakly) less from \(m\), that is, if \(\hat{x}^n_m \leq \hat{x}^n_{n'}\), then setting \(\hat{x}^n_m = 2a\) and thus \(\hat{x}^n_m = X/2 - a\) yields finally the restriction that \(a \leq X/6\).

Although we have thus shown that the conditions in Claim 1 are necessary to support an allocation as an equilibrium, that these conditions are also sufficient follows immediately, as under the weaker requirement (7) we can choose all \(t^m_n\) \((x)\) for \(x < \hat{x}^n_m\) arbitrarily high. Q.E.D.

Using now the characterization of Claim 1 and that in equilibrium transfers are equal to the respective incremental costs at the other supplier, the total price that \(n\) pays is strictly lower than (17) under single sourcing if

\[
\left[C\left(X/2 + a + \hat{x}^n_m\right) - C(X/2 + a)\right] + \left[C\left(X/2 - a + \hat{x}^n_m\right) - C(X/2 - a)\right] < C(X) - C(X/2).
\]

Extending the expression in the second line of (21) by writing

\[
\left[C(X) - C\left(X - \hat{x}^n_m\right)\right] - \left[C\left(X - \hat{x}^n_m\right) - C(X/2)\right]
\]

and substituting \(\hat{x}^n_m = X/2 - \hat{t}^n_m\) into the first line, after rearranging expressions we have the requirement that

\[
\left[C\left(X + a - \hat{x}^n_m\right) - C(X/2 + a)\right] - \left[C\left(X - \hat{x}^n_m\right) - C(X/2)\right] < \left[C(X) - C\left(X - \hat{x}^n_m\right)\right] - \left[C\left(X/2 - a + \hat{x}^n_m\right) - C(X/2 - a)\right].
\]
Note next for the expressions in the first line of (22) that \((X + a - \hat{x}_m^u) - (X/2 + a) = X/2 - \hat{x}_m^u\) and \((X - \hat{x}_m^u) - (X/2) = X/2 - \hat{x}_m^u\). With these observations, we can then transform the first line from (22) into

\[
\int_{X/2}^{X/2 + a} \int_0^{X/2 - \hat{x}_m^u} C'' \, dxdy.
\]

Likewise, after noting for the expressions in the second line of (22) that \((X) - (X - \hat{x}_m^u) = \hat{x}_m^u\) and \((X/2 - a + \hat{x}_m^u) - (X/2 - a) = \hat{x}_m^u\), this transforms to

\[
\int_{X/2 - a}^{X/2 - \hat{x}_m^u} \int_0^{\hat{x}_m^u} C'' \, dxdy.
\]

Hence, (22) becomes

\[
\int_{X/2}^{X/2 + a} \int_0^{X/2 - \hat{x}_m^u} C'' \, dxdy < \int_{X/2 - a}^{X/2} \int_0^{\hat{x}_m^u} C'' \, dxdy. \tag{23}
\]

We now distinguish between two cases. In the first case, buyer \(n\) purchases at least \(X/4\) from the large firm such that \(\hat{x}_m^u \geq X/2 - \hat{x}_m^u\). Note next that \(X - \hat{x}_m^u \geq X/2 + a\). To see this, substitute the highest possible value for \(\hat{x}_m^u\), which by the arguments from the proof of Claim 1 equals \(\hat{x}_m^u = X/2 - a\), in which case this condition holds with equality. Hence, we can make (23) (weakly) stricter by replacing the boundary \(X - \hat{x}_m^u\) in the integral on the right-hand side by the boundary \(X/2 + a\). That the thereby “relaxed” condition (23) holds then still strictly follows as \(X/2 > X/2 - a\) due to \(a > 0\), as \(\hat{x}_m^u \geq X/2 - \hat{x}_m^u\) by assumption of the current case, and as \(C'' > 0\). In words, we have thus shown that if a buyer purchases at least \(X/4\) from the larger supplier, then he pays overall always less than under single sourcing.

We turn next to the case where \(\hat{x}_m^u < X/4\). We proceed in analogy to the previous case, though this time we extend the second line in (21) in a different way, namely by writing instead

\[
[C(X) - C(X/2 + \hat{x}_m^u)] - [C(X/2 + \hat{x}_m^u) - C(X/2)].
\]

With this, (21) transforms to the requirement

\[
[C(X/2 + \hat{x}_m^u) - C(X/2)] - [C(X/2 - a + \hat{x}_m^u) - C(X/2 - a)] < [C(X) - C(X/2 + \hat{x}_m^u)] - [C(X + a - \hat{x}_m^u) - C(X/2 + a)]
\]

and finally to

\[
\int_{X/2}^{X/2 + a} \int_0^{\hat{x}_m^u} C'' \, dxdy < \int_{X/2 - a}^{X/2} \int_0^{\hat{x}_m^u} C'' \, dxdy. \tag{24}
\]

Condition (24) now holds surely as \(\hat{x}_m^u < X/2 - \hat{x}_m^u\) holds due to \(\hat{x}_m^u < X/4\) and as

\[
(X/2 + \hat{x}_m^u) - (X/2 + a) = \hat{x}_m^u - a \geq (X/2) - (X/2 - a) = a
\]

holds from \(\hat{x}_m^u \geq 2a\), which was shown in the proof of Claim 1. \(Q.E.D.\)

**Proof of Corollary 1.** The proof follows almost directly from Proposition 3. There, we have shown that if \(m\) is the larger supplier with \(\hat{x}_m^u = X/2 + a\), then it must hold that \(a \leq X/6\). Consequently, we have the upper boundary \(X_{B1} \leq X/2 + X/6 = 2X/3\). \(Q.E.D.\)

**Proof of Proposition 5.** Take first the case with a single large buyer. From (9), the buyer’s payoff without single sourcing equals

\[
R(X) - 2[[R(X) - R(X')] + [C(X') - C(X/2)]
\]

compared to \(R(X') - C(X')\) under single sourcing, where \(X'\) was defined in (8). Single sourcing is thus strictly more profitable whenever

\[
R(X) - 2C(X/2) > R(X') - C(X')
\]

holds as \(X\) maximizes \(R(x) - 2C(x/2)\) and as \(2C(x/2) < C(x)\).

Consider next the case with two buyers. Observe first more formally that given strict convexity of \(C\) and strict concavity of \(r\), the program to maximize total surplus is indeed strictly concave. It is also again straightforward that under the truthfulness requirement, an allocation is supported as an equilibrium if and only if it is efficient. Off-equilibrium, if buyer \(n\) rejects the bid of supplier \(m\), his payoff is

\[
\max_{\Delta} \left[ r(\hat{x}_m^u + \Delta) - [C(X/2 + \Delta) - C(X/2)] - \hat{x}_m^u \right].
\]
As \( \hat{x}_n \) makes the buyer again indifferent, in equilibrium buyer \( n \), when purchasing \( \hat{x}_n \) from the respective suppliers, will realize the payoff

\[
r(X/2) - \sum_{m=1,2} \left[ \frac{r(X/2) - \max_{\Delta_i} \left[ r\left( \hat{x}_m + \Delta_i \right) - \left( C(X/2 + \Delta_i) - C(X/2) \right) \right]}{\Delta_i} \right]
\]

(25)

We consider now a shift that makes a buyer's purchases more concentrated on a given supplier. For this it is convenient to denote

\[
\Omega(x) := \max_{\Delta_i} \left[ r(x + \Delta_i) - C(X/2 + \Delta_i) \right]
\]

and to also define for the respective choice of \( \Delta_i \), the quantity \( \hat{x}(x) := x + \Delta_i \). As we increase \( \hat{x}_m \) marginally while thereby reducing \( \hat{x}_n = X/2 - \hat{x}_m \), (25) increases strictly if

\[
\Omega\left( \hat{x}_n \right) > \Omega\left( X/2 - \hat{x}_m \right)
\]

which for \( \hat{x}_n \geq X/2 \) holds if \( \Omega\left( \hat{x} \right) \) is strictly concave. In the remainder of the proof, we show that this is indeed the case. Using from the envelope theorem that \( \Omega'(x) = r'(\hat{x}) \), we have that \( \Omega'(x) = r'(\hat{x}) \frac{dC}{dC} \) holds whenever \( \frac{d\Delta}{dx} > 0 \). To see that this is finally the case, note that from implicitly differentiating the respective first-order condition \( \frac{d\Omega}{d\Delta} = 0 \) for (26), we have that

\[
\frac{d\hat{x}}{dx} = \frac{C'(X/2 + \Delta_i)}{r'(\hat{x})} > 0.
\]

Q.E.D.

Proof of Proposition 6. Suppose supplier \( m \) rejects the bid of buyer \( n \). In this case, the supplier's payoff is

\[
\max_{\Delta_i} \left[ \hat{x}_m + \left( r(X/2 + \Delta_i) - r(X/2) \right) - C\left( \hat{x}_m + \Delta_i \right) \right].
\]

As the supplier's payoff on equilibrium is just \( i_n^m + i_n^m - C(X/2) \), we have

\[
i_n^m = \max_{\Delta_i} [r(X/2 + \Delta_i) - r(X/2)] - \left[ C\left( \hat{x}_m + \Delta_i \right) - C(X/2) \right].
\]

Consequently, a given buyer \( n \)’s payoff, which is \( r(X/2) - i_n^m - i_n^m \), becomes now

\[
r(X/2) - \sum_{m=1,2} \max_{\Delta_i} [r(X/2 + \Delta_i) - r(X/2)] - \left[ C\left( \hat{x}_m + \Delta_i \right) - C(X/2) \right].
\]

(27)

Define now in analogy to the proof of Proposition 5 the function

\[
\hat{\Omega}(x) := \max_{\Delta_i} \left[ r(x + \Delta_i) - C(x + \Delta_i) \right]
\]

and denote for the respective choice of \( \Delta_i \), the quantity \( \hat{x}(x) := \hat{x}_m + \Delta_i \), which is the quantity produced optimally by supplier \( m \) after \( n \) rejected his offer. As we increase \( \hat{x}_m \) marginally while thereby reducing \( \hat{x}_n = X/2 - \hat{x}_m \), the payoff of buyer \( n \) in (27) now decreases if

\[
\hat{\Omega}\left( \hat{x}_n \right) > \hat{\Omega}\left( X/2 - \hat{x}_n \right),
\]

which for \( \hat{x}_n \geq X/2 \) is again the case if \( \hat{\Omega}(x) \) is strictly concave. The argument for why \( \hat{\Omega} \) is strictly concave is now the same as that for \( \Omega \) in the proof of Proposition 5.

Finally, note that the argument also holds if buyers always purchase \( X/2 \). In this case, we have \( \Delta_i = 0 \) in all the preceding expressions such that \( \hat{\Omega}(x) := r(X/2) - C(x) \), which is again strictly concave in \( x \) by strict convexity of \( C \). Q.E.D.

References


