Competitive Strategies when Consumers are Relative Thinkers: Implications for Pricing, Promotions, and Product Choice

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Abstract

How should firms optimally choose prices and promotional strategies and how should they position their products when consumers are "relative thinkers"? We provide answers in a model that extends the seminal contributions of Varian (1980) and Narasimhan (1988) and derive both managerial implications and implications for empirical researchers with regards to promotional frequency and depth as well as observed product heterogeneity in the market.

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1 Introduction

Various contributions in the marketing and economics literature suggest that when comparing different offerings, consumers frequently care more about *relative* differences rather than *absolute* differences. This should apply, in particular, when consumers are faced with new offers or changes in existing offers, as such relative comparison facilitates decision-making. In an environment with frequent promotions, building on Varian (1980) and Narasimhan (1988), we explore the implications of such "relative thinking" for firms' strategies with respect to pricing, the depth and frequency of promotions, and product positioning in terms of quality. In contrast to some of the extant literature, as reviewed below, our focus is on deriving a full market equilibrium, solving also for firms' best replies to the anticipated strategies of their rivals. We are particularly interested in deriving testable implications that are, both qualitatively and quantitatively, different from those when consumers have standard preferences ("absolute thinkers").

At the heart of our analysis are promotional strategies. In fact, the inclusion of promotional strategies is of particular importance for the overall applicability of the model for the following reason: While other attributes of a product, such as quality, should be relatively persistent over time, it is precisely through firms' promotional activities that consumers – or at least those who shop – experience changes and are forced to reassess their perception of a particular product's value, compared to that of other products.[1] From an empirical perspective, promotions thus generate variation to make our implications testable. To thus account for promotional activities, we build on the seminal contribution of Varian (1980), which has found widespread adoption also in strategy and marketing, such as through Narasimhan's (1988) well-known model of competitive promotional strategies. In fact, we take Narasimhan's model as our starting point and enrich it by considering both heterogeneous (and possibly endogenous) product qualities as well as relative thinking.

To clarify the further discussion of our research questions and results, we next describe formally the concept of relative thinking that we employ in this paper. Below we offer support for this modeling choice from different strands of the literature. In our model, consumers who actively shop for the best alternative undertake the following comparison. Take a product $i = 1$ with price p_1 and quality v_1, which a shopper compares to another product $i = 2$ with respective characteristics p_2 and v_2. We suppose that quality is suitably

[1] Promotional discounts can also be seen as "surprises", for which, following the subsequently referenced literature on consumer behavior, observed consumer choice is typically considered to be more likely to deviate from the predictions of standard models.

normalized so that, according to standard choice theory, an absolute thinker compares the net benefits $v_1 - p_1$ and $v_2 - p_2$.[2] For concreteness, let 2 be the higher-quality product, $v_2 > v_1$, so that also $p_2 > p_1$, as otherwise product 1 would be dominated by product 2 in all relevant aspects. Thus, an absolute thinker asks himself whether the absolute difference in quality $v_2 - v_1$ is worth the respective absolute difference in price $p_2 - p_1$ and chooses product 2 only if $v_2 - v_1 \geq p_2 - p_1$. Instead, a relative thinker asks himself whether the *relative* difference in quality, i.e., that the quality of $i = 2$ is $100 \cdot \frac{v_2 - v_1}{v_1}$ *percent* higher, is worth to pay a $100 \cdot \frac{p_2 - p_1}{p_1}$ *percent* higher price. A simple transformation then reveals that a relative thinker compares two products in terms of the respective ratios of quality to price, $\frac{v_1}{p_1}$ and $\frac{v_2}{p_2}$, i.e., in terms of the respective "quality-per-dollar".[3]

In what follows, we provide various empirical and theoretical rationales for such relative thinking. Note also that while all consumers may follow the same decision rule, in our model relative thinking obviously matters only for those consumers who actively compare offers. As we discuss below, the prevalence of shopping in the market has therefore more far-reaching implications as with standard preferences. We show below that the decision rule derived from "relative thinking" applies also when consumers restrict their perception to the most salient attribute of a product, when such salience is obtained again from relative differences; and it applies likewise when consumers maximize with a fixed (and exhausted) budget. Our focus is however more applied, as we explore, amongst others, the following questions:

1. How does the presence of relative thinkers affect differently the strategic behavior of firms offering products with different (lower or higher) quality?

2. For which firms does the presence of relative thinkers affect (more) the choice of the frequency and depth of promotions?

3. How should a manufacturer optimally react (differently) to changes in rivals' strategies, notably also to changes in the quality of rival products?

4. How does the presence of relative thinkers affect (long-term) equilibrium choices of product quality and price? And what implications does this have for overall firm profits?

[2] As this paper is focused on business implications rather than welfare implications, we need not distinguish between a consumer's "true" (or "hedonic") or his "perceived" (or "normed") utility. All that matters is our subsequent specification of how available prices and qualities affect a consumer's choice.

[3] Hence, the relative thinker chooses product 2 if $\frac{v_2 - v_1}{v_1} > \frac{p_2 - p_1}{p_1}$, which becomes $\frac{v_2}{v_1} > \frac{p_2}{p_1}$ and is thus equivalent to $\frac{v_2}{p_2} > \frac{v_1}{p_1}$.

As we already noted, we take as a starting point Narasimhan's (1988) well-known model of competitive promotional strategies, which enriches Varian's (1980) model of sales by allowing for heterogeneity in firms' loyal customer base, which we also do in our model. A long strand of the literature, including Raju et al. (1990) and Agrawal (1994), have built on this to study in particular the frequency and depth of promotions. Our main novelty is to capture relative thinking, which is only relevant for consumers who actively shop and compare. Our model is thus not only meant to add additional realism, to the extent that consumers as relative thinkers indeed follow the described choice model, but it offers new (testable) implications, such as with regards to the impact that shoppers or, more generally, consumers with different degrees of product loyalty, have on firms' strategies. The focus is thereby always on firms' strategic interaction and the resulting equilibrium choices of prices, promotions, and product characteristics. This differs from various contributions, notably in the marketing literature, that focus, instead, on a (single) firm's optimal strategy to design its range of products and prices so as to exploit the fact that consumers assess products contingent on the overall choice context (building on contributions such as Huber et al. (1982) or Simonson (1989)).[4]

We now describe our key results in terms of answers to the questions that we posed above. We derive various implications for managerial strategy and empirical research, which we collect in Section 6. Relative thinking matters only for shoppers. The fraction of shoppers, as well as the size of firms' loyal base of consumers, affect thus not only the degree of competition. Relative thinking makes it less likely that a higher-quality product is promoted and more likely that a lower-quality product is promoted. Furthermore, when consumers are absolute thinkers, firms could remain relatively "inward looking" when determining their optimal product choice and positioning, trading off higher quality with higher marginal costs of production. But this strategy could be seriously wrong with relative thinkers, as now the rival's choice crucially affects consumer perceptions and thus the optimality of choosing either a lower-quality or a higher-quality product (and the corresponding promotional strategy).

We identify circumstances when, albeit only with relative thinkers, firms' product choices represent strategic substitutes and when they represent strategic complements, so that

[4]Bordalo et al. (2014) consider also competition where firms may differ in prices and qualities, albeit there is perfect competition and, in equilibrium, firms choose the same qualities, so that the issue of relative thinking (or salience) does not arise on equilibrium. There is also no scope, in their model, to develop implications for promotions or for how pricing and product strategies are affected by parameters such as the different degree of competition or the prevalence of shopping.

a rival's choice of higher quality renders it less (with strategic substitutes) or more (with strategic complements) profitable to follow suit. In equilibrium, we derive conditions for when with relative thinking, but not so with absolute thinking, firms will thus offer heterogeneous product qualities. We also show how the presence of relative thinkers can give rise to a first-mover advantage, where a firm would wish to first occupy the position of a low-quality, promotion-intense "discounter".

Relative thinking. In the marketing literature, there exists by now a long tradition of models that look "inside consumers' minds" to predict choices. The importance of relative comparisons, such as with respect to a consumer's internal reference point, was recognized early on by Monroe (1973).[5] In this paper we take as a given the assumption that such relative thinkers compare two offerings according to the relative differences in prices and qualities, as described above. We already noted that this is equivalent to stipulating that consumers use a "price-per-quality" metric. For instance, Azar (2011) starts directly with such a specification. The same choice criterion would however also be obtained when consumers derive a constant marginal utility of quality and maximize consumption with respect to a binding fixed budget constraint (which again could be motivated from a theory of mental accounting, e.g., Thaler 1985).[6] In the main part of this paper we provide yet another motivation based on the presumption of consumer limited attention. Precisely, in this case consumers assess a product only on one dimension and choose for this the dimension along which the respective product is relatively most different to others. We can show that a consumer then chooses again the product with the highest ratio $\frac{v_i}{p_i}$. This observation ties into a recent literature that formalizes how consumers attach different weights to different attributes, depending on the set of considered alternative products.[7] In sum, while our contribution is solely to analyze the implications of consumers' relative thinking for firms' competitive strategy, we stress that the underlying assumption that consumers

[5] This was related to Kahneman and Tversky's (1989) Prospect Theory by, e.g., Diamond and Sanyal (1990). Also the term "relative thinking" is clearly not new and shared with, for instance, Azar (2011) and Bushong et al. (2015).

[6] We thank Thomas Otter for pointing this out. See, for instance, equation (3.11) in Chandukala et al. (2007). Again we provide more details in Section 7.

[7] As we discuss later, the considered approach of relative thinking is, at first, reminiscent of that of salient thinking in Bordalo et al. (2013), in case consumers focus only on the most salient attribute. There, however, also the comparison with the outside option of not buying is distorted, even though this clearly can not be compared along the same dimensions, that is price and quality in our setting. Other recent approaches to model the relevance of the choice context include Koszegi and Szeidl (2013) and Bushong et al. (2015). We acknowledge that between these approaches there are important differences when it comes to choices from a larger context.

compare products in terms of relative differences has received various foundations in the literature, and we do not need to take sides as to which one is more appropriate.

Organization of the paper. The rest of the paper is organized as follows. Section 2 introduces the competitive context and consumer behavior. Section 3 considers the (benchmark) case where consumers are absolute thinkers. We then break up into two steps the analysis where consumers are relative thinkers. In Section 4 we derive firms' optimal price and promotional strategies. In Section 5 we then solve for firms' (long-term) product strategies. Section 6 brings together our main implications and Section 7 offers some concluding remarks.

2 Firm Strategies and Consumer Choice

As in Narasimhan (1988), we consider competition between two firms $i = 1, 2$ marketing one product each, which they can produce at constant per-unit cost c_i and which they sell at price p_i. As we discuss in Section 7 (and illustrate in the Appendix), our approach is however not constrained to two firms.

We normalize market size such that there is a total mass 1 of consumers, which are segmented as follows. For each firm i, there is a loyal consumer base of size $\alpha_i \geq 0$ that considers only the respective firm's offer. We suppose that there is always also a positive share of shoppers who are considering all offers, $1 - \alpha_1 - \alpha_2 > 0$. For the purpose of our analysis, we can be agnostic as to the reasons why only this fraction of consumers compares offers, e.g., because they have sufficiently low costs of information acquisition or switching (which are all outside our model). We note however already now that variations in α_i across firms may be key observables available to practitioners and empiricists and will thus play an important role for our implications.

Irrespective of the subsequent specification of consumer behavior, to which we turn below, the structure of pricing equilibria will be similar. We describe this next, that is even before the formal analysis, as it follows much of the extant literature on promotions and allows us to posit more succinctly our subsequent research questions. Each firm in the market will choose a price from a distribution $F_i(p_i)$ with support $[\underline{p}_i, \overline{p}_i]$, where (at most) the highest ("regular") price $\overline{p}_i$ is selected with strictly positive probability γ_i ("mass point"), and where the firm randomizes over various discounts otherwise. The probability $g_i = 1 - \gamma_i$ thus denotes the *frequency* of firm i's promotions, while the difference $d_i = \overline{p}_i - \underline{p}_i$ denotes the maximum *depth* of its respective promotional discount.

Consumers have a reservation value of zero. A consumer who is an absolute thinker has a valuation v_i for firm i's product, and he strictly prefers the offering of firm i over that of firm j if $v_i - p_i > v_j - p_j$. When a consumer is a relative thinker, he strictly prefers firm i's product if $\frac{v_i}{p_i} > \frac{v_j}{p_j}$. Recall from the Introduction that there are various motivations for such "relative thinking", to which we return, with a perspective on future research, in Section 7.

3 The Baseline Case with Absolute Thinkers

The following analysis with absolute thinkers serves mainly as a baseline. Still, as we discuss below, it also extends previous literature. We first focus on firms' price and promotional strategies, taking the choice of quality as given.

3.1 Pricing and Promotions

We start by analyzing for which firm it is more attractive to promote its product. This depends, as we will argue, on both the competitive strength of the product, $v_i - c_i$, and the share of its loyal customers, α_i. Suppose that firm i has the stronger product, $v_i - c_i > v_j - c_j$. When firm i offers consumers the same absolute benefit as firm j, this implies that its respective margin is higher. Ceteris paribus, this makes it more attractive for firm i to compete for shoppers and expand demand. What dampens a firm's incentives to lower prices and compete for shoppers is however a larger share of loyal consumers, α_i, as any discount is also given to loyal consumers without a corresponding benefit for the firm. When we take these two forces together, we find that firm i promotes its product (weakly) more often than its rival j if it holds that

$$\frac{v_i - c_i}{v_j - c_j} \geq \frac{1 - \alpha_j}{1 - \alpha_i}, \tag{1}$$

while if the converse holds, firm j promotes more often. When a firm promotes more often, we also say in what follows that it competes more aggressively for shoppers.

Condition (1) is derived in the proof of Proposition 1. Intuitively, the left-hand side of (1) captures the competitive strength of firm i relative to that of firm j. As this increases, it is indeed more likely that firm i promotes more often than firm j (so that $g_i > g_j$). The right-hand side of (1) depends on the two firms' respective loyal shares: When either the fraction of loyal consumers for firm i decreases (α_i lower) or that for firm j increases (α_j

higher), condition (1) is relaxed and it becomes again more likely that firm i promotes more often than firm j, $g_i > g_j$.

When consumers are absolute thinkers, both firms, regardless of their competitive strength, need however the same maximum discount for their promotion to ensure that shoppers are surely attracted, i.e., we find for the depth of the discounts that $d_i = d_j$. To now fully characterize the equilibrium price and promotional strategies, in what follows denote

$$\underline{\underline{p}}_i = c_i + (v_i - c_i)\frac{\alpha_i}{1-\alpha_j}.$$

Proposition 1 *Suppose consumers are absolute thinkers and that, without loss of generality, condition (1) holds. Then there exists a unique pricing equilibrium such that:*
i) Firm i chooses prices $p_i \in [\underline{\underline{p}}_j + v_i - v_j, v_i)$ according to the CDF

$$F_i(p) = 1 - \frac{\alpha_j}{1-\alpha_i-\alpha_j}\left(\frac{v_i - p}{(v_j - c_j) - (v_i - p)}\right),$$

so that the (maximum) depth of promotions is

$$d_i = (v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right)$$

and profits are

$$\pi_i = (v_i - c_i)\alpha_i + (1-\alpha_i-\alpha_j)(v_j - c_j)\left(\frac{v_i - c_i}{v_j - c_j} - \frac{1-\alpha_j}{1-\alpha_i}\right).$$

ii) Firm j chooses prices $p_j \in [\underline{\underline{p}}_j, v_j)$ according to the CDF

$$F_j(p) = 1 - \frac{\alpha_i}{1-\alpha_i-\alpha_j}\left(\frac{v_j - p}{(v_i - c_i) - (v_j - p)}\right) - \frac{(v_j - c_j)\left(\frac{v_i - c_i}{v_j - c_j} - \frac{1-\alpha_j}{1-\alpha_i}\right)}{(v_i - c_i) - (v_j - p)}$$

and the non-discounted price $p_j = v_j$ with probability

$$\gamma_j = 1 - \left(\frac{1-\alpha_j}{1-\alpha_i}\right)\left(\frac{v_j - c_j}{v_i - c_i}\right),$$

while the (maximum) depth of promotions satisfies $d_j = d_i$ and profits are $\pi_j = (v_j - c_j)\alpha_j$. Thus, firm i promotes more often (and strictly so when (1) holds strictly, $g_i > g_j$), while the (maximum) depth of promotions is the same for both firms ($d_i = d_j$).

Interpretation of pricing and promotional strategies. To shed light on the preceding characterization, consider first γ_j: the probability with which firm j chooses the non-discounted price. The respective term that is subtracted from 1 is reminiscent of the term in condition (1). Now, firm j promotes more often when i) product j becomes stronger ($v_j - c_j$ increases), ii) the rival product i becomes weaker ($v_i - c_i$ decreases), iii) firm j's share of loyal consumers α_j decreases, or iv) firm i's share of loyal consumers α_i increases.

Moreover, note that the likelihood with which firm j, which promotes its product less often, still offers a discount to compete for shoppers directly affects the profit of the more aggressive firm i, as can be seen immediately from inspection of π_i. In fact, π_i exceeds the profits that firm i could make with its loyal share exactly by $(v_i - c_i)(1 - \alpha_i - \alpha_j)\gamma_j$, i.e., the product of the firm's margin, of the share of shoppers in the market, and of the likelihood with which the rival firm chooses not to compete through promotions.[8]

Note further that the (maximum) depth of promotions is the same for both firms. Intuitively, both firms must discount the price by the same absolute amount relative to their highest (non-discounted) price so as to ensure that they attract shoppers. For instance, when product i becomes less competitive as its value v_i decreases, then both the non-discounted price $p_i = v_i$ and the lowest price $v_i - d_i$ decrease by the same amount. The depth of promotion depends in an intuitive manner on the composition of the market: i) It increases as the firm's own loyal share α_i decreases and ii) it also increases when the loyal share of its rival α_j decreases. Both changes make it more attractive to discount prices so as to attract (the larger number of) shoppers.

As we noted above, the present analysis with absolute thinkers provides mainly a benchmark. We also noted that the structure of the pricing equilibrium will be similar when consumers are relative thinkers, albeit the determinants of a firm's competitive strength will be different. We thus postpone a full description of managerial implications until we have characterized the equilibrium with relative thinkers.

3.2 Product Choice

The preceding analysis, which focused on pricing and promotions, should apply for the short and medium term, where a product's other attributes are given. But also in the longer

[8]That the profits of the more aggressive firm i strictly exceed its "monopoly profits", that is the maximum profits the firm can realize with its share of loyal consumers, follows immediately from condition (1).

term, firms' choice of product characteristics may be limited by technological constraints. In what follows, we consider instead industries where this is not or much less so, and thus allow firms to also (re-)position their products.

We allow each firm to choose either of two product variants, $v \in \{v_H, v_L\}$, with $v_H > v_L$, and where producing a higher-value product also involves higher per-unit costs $c_H > c_L > 0$. For brevity, we restrict the subsequent exposition to the (generic) case where $v_H - c_H \neq v_L - c_L$ (and, for the purpose of the subsequent analysis, also $\frac{v_H}{c_H} \neq \frac{v_L}{c_L}$).

Proposition 2 *If consumers are absolute thinkers, in the unique product-choice equilibrium, both firms select the product that is "absolutely stronger" as it maximizes the difference $v - c$.*

The result in Proposition 2 is intuitive and follows from the subsequent argument. Suppose, having chosen a product with value v_i, firm i wants to compete with an offer that leaves a consumer with net utility $v_i - p_i$. The respective margin is then $p_i - c_i$. Suppose now that there exists another and competitively stronger product (v_i', c_i'), such that $(v_i' - c_i') - (v_i - c_i) = \Delta > 0$. By choosing instead this product, firm i can offer the same net utility and pocket the difference $\Delta > 0$ for each unit that it sells. For instance, when $v_i = v_H$ and $v_i' = v_L$, the firm would need to lower its price by $v_H - v_L$ to make the new offer equally attractive, but this would be more than compensated by the cost difference $c_H - c_L$ if it holds that $v_L - c_L > v_H - c_H$.[9]

Unless firms are differently constrained, for instance by technology, each firm thus faces the same trade-off, and this will result in the same choice, which in our model is notably independent from its expectation of what product its rival will offer. To find the optimal trade-off between cost and a product's value, a firm can thus be "inward looking".[10] This will be an important difference to the case where consumers are relative thinkers.

4 Price and Promotional Strategy with Relative Thinkers

As we noted above, the general structure of the pricing equilibrium is unaffected by whether consumers are absolute or relative thinkers. However, now a product's "relative strength",

[9] Of course, as product choice precedes pricing and is observable to the rival, we need to analyze profits at the equilibrium pricing strategies. This is done in the proof.

[10] We note that this would be different when the choice of technology involved also fixed costs, F_H and F_L. Clearly, a firm would be in a better position to recoup higher fixed costs when it captured a higher share of the market, which depends on the rival's choice as well.

$\frac{v}{c}$, instead of its "absolute strength", $v - c$, matters. This follows as when shoppers are relative thinkers, they compare products in terms of "value-per-dollar". We already note now however that also a product's "absolute strength" will still matter. This follows as loyal consumers still behave effectively as if they were absolute thinkers.

We now proceed as follows. We first characterize the equilibrium price and promotional strategies with relative thinking. Subsequently, we summarize key implications and notably analyze the different implications when consumers are in fact relative thinkers, compared to when they are absolute thinkers.

4.1 Characterization

Taking also into account possible differences in firms' loyalty bases, in analogy to condition (1) we find that firm i promotes its product (weakly) more frequently than its rival j under relative thinkers when

$$\frac{1-\frac{c_i}{v_i}}{1-\frac{c_j}{v_j}} \geq \frac{1-\alpha_j}{1-\alpha_i}, \tag{2}$$

while firm j promotes more frequently when the converse holds.

Hence, the likelihood that condition (2) holds increases when firm i's product becomes stronger in relative terms ($\frac{v_i}{c_i}$ increases), when that of its rival becomes weaker in relative terms ($\frac{v_j}{c_j}$ decreases), and again when the loyal base of firm i, α_i, decreases and that of firm j, α_j, increases. Note moreover again that when products are equally strong (now in relative terms), such that the left-hand side of condition (2) equals one, the identity of the more aggressive firm, which promotes more often, is determined solely by the (lower) share of loyal consumers.

Proposition 3 *Suppose consumers are relative thinkers and suppose also that, without loss of generality, condition (2) holds. Then there exists a unique pricing equilibrium such that:*
i) Firm i chooses prices $p_i \in [\underline{p}_j \frac{v_i}{v_j}, v_i)$ according to the CDF

$$F_i(p) = 1 - \left(\frac{\alpha_j}{1-\alpha_i-\alpha_j}\right)\left(\frac{1-\frac{p}{v_i}}{\frac{p}{v_i}-\frac{c_j}{v_j}}\right),$$

so that the (maximum) depth of promotions is

$$d_i = \frac{v_i}{v_j}(v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right)$$

and profits are

$$\pi_i = (v_i - c_i)\alpha_i + (1-\alpha_i-\alpha_j)(v_j-c_j)\frac{v_i}{v_j}\left(\frac{1-\frac{c_i}{v_i}}{1-\frac{c_j}{v_j}} - \frac{1-\alpha_j}{1-\alpha_i}\right).$$

ii) Firm j chooses prices $p_j \in [\underline{p}_j, v_j)$ according to the CDF

$$F_j(p) := 1 - \left(\frac{\alpha_i}{1-\alpha_i-\alpha_j}\right)\left(\frac{1-\frac{p}{v_j}}{\frac{p}{v_j}-\frac{c_i}{v_i}}\right) - \frac{\left(1-\frac{c_j}{v_j}\right)\left(\frac{1-\frac{c_i}{v_i}}{1-\frac{c_j}{v_j}} - \frac{1-\alpha_j}{1-\alpha_i}\right)}{\frac{p}{v_j}-\frac{c_i}{v_i}}$$

and the non-discounted price $p_j = v_j$ with probability

$$\gamma_j = 1 - \left(\frac{1-\alpha_j}{1-\alpha_i}\right)\left(\frac{1-\frac{c_j}{v_j}}{1-\frac{c_i}{v_i}}\right),$$

while the (maximum) depth of promotions is

$$d_j = (v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right)$$

and profits are $\pi_j = (v_j - c_j)\alpha_j$.
Thus, firm i promotes more often (and strictly so when (2) holds strictly, $g_i > g_j$). The (maximum) depth of promotions is such that it is the same for both firms only in relation to the values of the two products, $\frac{d_i}{v_i} = \frac{d_j}{v_j}$.

Interpretation of pricing and promotional strategies. It is first useful to note that the two characterizations in Proposition 1 and Proposition 3 indeed fully coincide when the two products offer the same gross value to consumers, $v_i = v_j$. Otherwise, that is when $v_i \neq v_j$, also the characterization differs. For shoppers who are relative thinkers, it is now no longer the absolute value of a product that is decisive but the respective relative "value-per-dollar". Proposition 3 and condition (2) make the relevance of this difference transparent. Notably when one product is absolutely stronger, that is $v_i - c_i > v_j - c_j$, but relatively weaker as $\frac{v_i}{c_i} < \frac{v_j}{c_j}$, a comparison of Proposition 1 and Proposition 3 reveals that the identity of the firm that promotes its product more often will change (provided that firms have a sufficiently homogeneous loyalty base, as otherwise, the difference in loyalty bases may outweigh the difference in competitive strength). We turn to the difference that relative thinking makes and its managerial implications in more detail below.

Observe next that the (maximum) depth of promotions is now no longer the same when consumers are relative thinkers and when $v_i \neq v_j$. What is the same however is the relative discount that is necessary to attract shoppers for sure (which is the definition of the depth of promotions), i.e., the discount relative to the value of the respective product, $\frac{d_i}{v_i} = \frac{d_j}{v_j}$. If i is promoted more often and if this is also the higher-quality product, then the promotion needs to be absolutely deeper than its rival's. A lower discount is however needed when i is the lower-quality product. We explore this in more detail below, where we discuss at greater length how relative thinking disadvantages higher-quality products. We also provide a further discussion of implications in the subsequent section, contrasting also the cases with relative or absolute thinkers.

4.2 Implications for Price and Promotional Strategies

Before we isolate key implications when consumers are relative thinkers, we first summarize some *analogies* between the two cases where consumers are absolute or relative thinkers. The respective results essentially summarize our preceding observations on the general structure of the pricing equilibria in the two cases.

Corollary 1 *Which firm will promote its product more frequently is determined by condition (1) when consumers are absolute thinkers and by condition (2) when consumers are relative thinkers. In both cases, the impact of loyal shares on each firm's promotional frequency g_i and depth d_i follows the insights in Narasimhan (1988), that is:*
i) When firm i's base of loyal consumers α_i increases, then it becomes (weakly) less likely that firm i promotes its product (g_i decreases), while the (maximum) depth of promotion decreases.
ii) When the rival firm j's base of loyal consumers α_j increases, then it becomes (weakly) more likely that firm i promotes its product (g_i increases), while the (maximum) depth of promotion again decreases.

The intuition for Corollary 1 follows immediately from the preceding discussion. In terms of managerial implications, it is reassuring that the reported results apply (qualitatively) irrespective of whether consumers are absolute or relative thinkers. As we analyze next, it still makes a difference whether consumers are relative thinkers – and firms should adjust their strategies accordingly. Fortunately, we can provide guidance that does *not* depend on fine details but only on whether a firm can provide consumers a higher *absolute* value or not.[11]

[11] For ease of exposition, we have omitted in all statements in Corollary 2 a qualifier for when the respective results hold only weakly.

Corollary 2 *Suppose that firm i has a product that offers consumers an absolutely higher value than that of its rival, $v_i > v_j$. Then comparing the two unique equilibrium outcomes with absolute and with relative thinkers (as characterized in Propositions 1 and 3), the following differences emerge: With relative thinkers,*
i) the higher-value product i is promoted less often and the lower-value product j more often (precisely, g_i decreases and g_j decreases);
ii) the maximum depth of promotion of the higher-value product i increases and that of the lower-value product j decreases (precisely, d_i increases and d_j decreases);
iii) the profits of firm i with a higher-value product decrease and those of firm j with a lower-value product increase (precisely, π_i decreases and π_j increases).

Some of the implications of Corollary 2, such as those on profits, will prove important also later when we consider firms' choice of product attributes. Note further that the implications for firms' strategies are already derived in equilibrium, so that both firms' reactions have been taken into account.

Discussion. Altogether, firms must be aware that when consumers are relative thinkers, they will assess differences in value not absolutely but relative to the respective prices. As products that have a higher (intrinsic) value are sold at higher prices to cover higher costs of production, this dilutes their value in the eyes of relative thinkers. Note again, however, that this applies only to shoppers, as only shoppers compare different products, while loyal consumers decide only between whether to buy from their respective firm or not. The fraction of shoppers will thus be decisive not only as it intensifies competition, but also as it makes relative in contrast to absolute differences in offerings more important. Notably firms with higher-quality products, as captured by a higher v, should thus be wary not to invest too much into frequent promotions when consumers are relative thinkers: Their optimal promotional strategy, once they realize that consumers are relative thinkers, involves less promotions (assertion i). However, when a promotion shall be successful and attract shoppers for sure, the discount must be larger (assertion ii).

Of course, Corollary 2 does not imply that firms with higher-quality products should always promote their products less often than firms with lower-quality products. We already know that this depends also on firms' loyal customer base. In addition, products can clearly be stronger in both absolute and relative terms. A particularly interesting case is, however, the one in which this ordering is reversed. In fact, when we endogenize product choice below, this will be the only case where firms offer heterogeneous products.[12] If one abstracts from differences in firms' loyalty base, the presence of relative thinkers then fully overturns firms' competitive position. It should then be the lower-quality product that is more often promoted under relative thinking. The higher-quality firm's absolute strength, which matters for its loyal customers, does not extend to shoppers. For the high-quality firm it is then best to (partially) retreat from outright competition, while the low-quality firm should become more aggressive. This is thus a particularly clear-cut case where assertion ii) of Corollary 2 applies. And the respective implications for profits are equally clear-cut (assertion iii).[13]

5 Product Choice with Relative Thinkers

Cases with unambiguously stronger products. We now extend the analysis to firms' product choice. Recall first that with absolute thinkers, provided that both firms could freely choose product attributes (though these are associated with different costs), both firms resolved the (quality-cost) trade-off in the same way and irrespective of the other firm's choice. This result only extends under specific circumstances to the case where consumers are relative thinkers, namely when the same product is both absolutely and relatively stronger. Then also the equilibrium with relative thinkers does not involve product heterogeneity:

Proposition 4 *If one product is both absolutely and relatively stronger, then also when consumers are relative thinkers there is a unique product-choice equilibrium where both firms choose this product, that is:*
i) when $v_L - c_L > v_H - c_H$ (which implies $\frac{v_L}{c_L} > \frac{v_H}{c_H}$), then $v_1 = v_2 = v_L$;
ii) when $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} > \frac{v_L}{c_L}$, then $v_1 = v_2 = v_H$.

Targeting shoppers or loyal consumers. In what follows, we thus consider the (residual) case where $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} < \frac{v_L}{c_L}$. That is, one product variant is stronger when consumers are absolute thinkers and one is stronger when consumers are relative thinkers. Recall that here and in what follows, we assume that consumers are relative

[12] That is, the converse to this is mathematically not feasible.

[13] Precisely, when we simplify $\alpha_i = \alpha_j = \alpha$, so that we abstract from possible differences in loyalty shares, and assume for now that $v_i > v_j$, $v_i - c_i > v_j - c_j$, and $\frac{v_j}{c_j} > \frac{v_i}{c_i}$, we have the following: First, with absolute thinkers we have $\pi_j = (v_j - c_j)\alpha$ and $\pi_i = (v_i - c_i)\alpha + (1 - 2\alpha)\left[(v_i - c_i) - (v_j - c_j)\right]$; second, with relative thinkers we have $\pi_i = (v_i - c_i)\alpha$ and $\pi_j = (v_j - c_j)\alpha + (1 - 2\alpha)v_j(\frac{c_i}{v_i} - \frac{c_j}{v_j})$.

thinkers. The reason why firms will nevertheless not always choose the relatively stronger product $v_i = v_L$ is that relative thinking *effectively* applies only to consumers who indeed shop and compare, and thus not to firms' loyal customer base. Instead, given that they do not compare offerings, loyal customers behave as if they were absolute thinkers.

5.1 Best Responses

Considering firms' product choice, we are interested in two questions. First, we are interested in a firm's optimal choice for a given (anticipated) product choice of its rival. Second, we are interested in the equilibrium that obtains when both firms make their optimal choices.

While the first step is clearly a prerequisite to derive the (Nash) equilibrium and answer the second question, it is also of independent interest, as sometimes only one firm may be in a position to choose or change the positioning of its product, while this does not apply to its rival. For instance, the rival may face technical limitations, and only one firm may have the knowledge or patents that allow to "upgrade" its product's quality. For these reasons, we also devote some space to first deriving a firm's "best response". We subsequently derive the equilibrium when both firms have the same flexibility in choosing product quality.[14]

Optimal product choice when rival chooses low quality.

Lemma 1 *Suppose firm i anticipates that its rival chooses low quality, $v_j = v_L$. Then firm i's optimal response is described as follows: First, for any given loyal share of its rival α_j, firm i also chooses low quality only if its own loyal share α_i is sufficiently small; second, now for given own loyal share α_i, firm i may choose low quality only if its rival's loyal share takes on intermediate values, that it is when it lies between a lower and an upper boundary.*

Formally, $v_i = v_L$ is strictly more profitable if:
i) Either $0 < \alpha_j \leq \underline{\alpha}_{j,L}$ and $\alpha_i < \tilde{\alpha}_{i,L}(\alpha_j) = 1/2 - \sqrt{1/4 - \alpha_j(1-\alpha_j)\frac{v_L - c_L}{v_H - c_H}}$, where

$$\underline{\alpha}_{j,L} = \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L) + \frac{v_L}{v_H}(c_H v_L - c_L v_H)};$$

[14] Recall again that in what follows, we consider only the case where $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} < \frac{v_L}{c_L}$, as otherwise Proposition 4 applies.

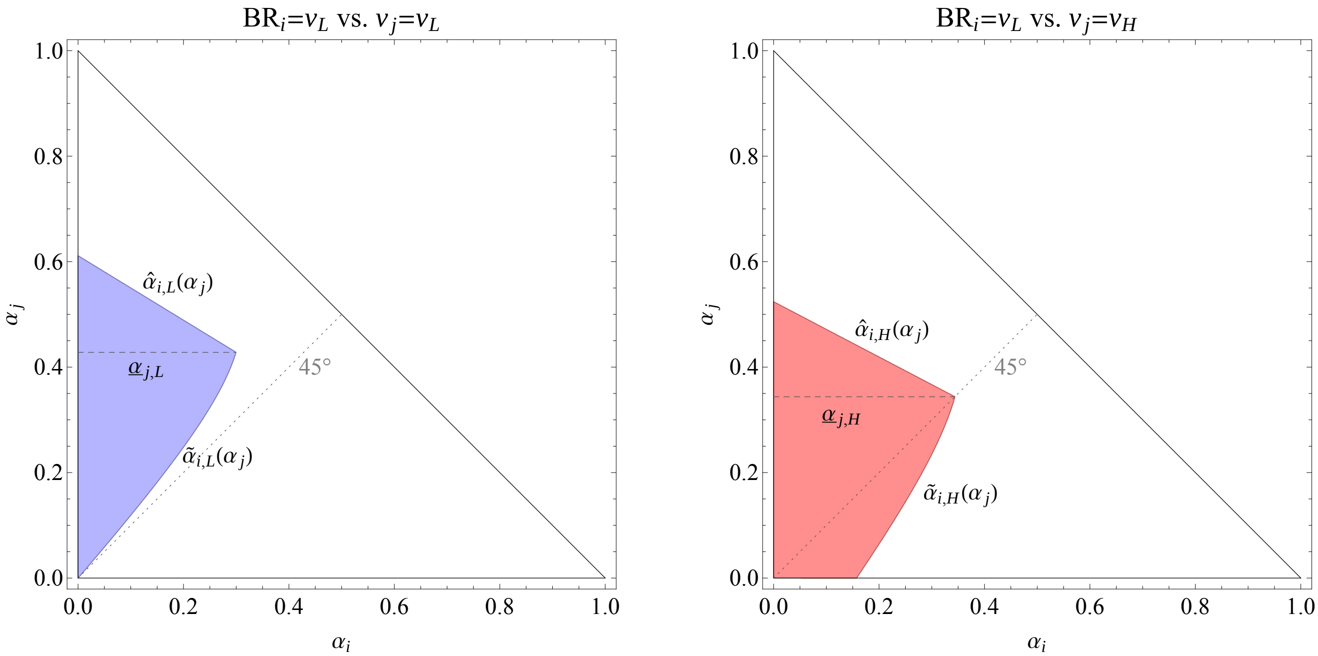

Figure 1: Illustration of regions in (α_i, α_j)-space in which firm i finds it optimal to choose v_L, given $v_j = v_L$ (left panel) and $v_j = v_H$ (right panel). The parameters used are $v_H = 1$, $c_H = 0.65$, $v_L = 0.7$, $c_L = 0.4$.

ii) or $\underline{\alpha}_{j,L} < \alpha_j < \overline{\alpha}_{j,L} < 1$ *and* $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j) = 1 - \alpha_j \left(\frac{(v_H - v_L)(v_L - c_L)}{c_H v_L - c_L v_H}\right)$, *where*

$$\overline{\alpha}_{j,L} = \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L)}.$$

Otherwise, it is more profitable to choose $v_i = v_H$.

The characterization of Lemma 1 is illustrated in the left-hand panel of Figure 1. The shaded area depicts the parameter range for which the best response of firm i is to choose low quality as well. We next provide intuition for this characterization.

Here, the role of firm i's own loyal share is particularly intuitive. Loyal consumers, even when they are relative thinkers, effectively only consider the absolute value of the firm's product, simply as they do not compare different offers. From $v_H - c_H > v_L - c_L$ a firm can thus extract more value from loyal consumers when it offers the high-quality product. The low-quality product is however the better choice when the firm wants to attract shoppers. While this is not explicitly stated in Lemma 1, it is thus only profitable for firm i to choose the low-quality product when, at the same time, it promotes its product more often. Precisely, when the firm chooses the low-quality product, we know that it promotes its product more often than its rival. Choosing the low-quality product and then promoting its product more aggressively is clearly only profitable when the firm has sufficiently few loyal customers, i.e., when α_i is sufficiently small. In Figure 1 this can be seen as, for a given choice of its rival's loyal share α_j (i.e., moving on a horizontal line), the shaded area lies to the left of the respective curve. Somewhat less immediate is the impact that a change of the rival's loyal share has on firm i's product choice. From Lemma 1 (and Figure 1) we know that α_j has a non-monotonic impact on firm i's best response, and we turn next to the rationale for this.

When α_j is (very) high, this also means that there are, ceteris paribus, few shoppers in the market. By the preceding observations, it is then not profitable for firm i to choose low quality.[15] When α_j is instead low, while there may then be sufficient shoppers to make the low quality attractive for firm i, this also means that firm j will itself aggressively pursue shoppers. This in turn renders shoppers, relative to loyal consumers, overall less attractive also for firm i. The interplay of the two described forces leads to the outcome described in Lemma 1: For a given own share of loyal consumers, provided that it is then indeed sometimes profitable to choose the low-quality product, this is only the case when the loyal share α_j of the firm's rival takes on intermediate values. Graphically, holding now fixed a value of α_i, we see in Figure 1 (now by remaining on a vertical line) that the respective range of values α_j in the shaded area is indeed an interior interval.

Note at this point also that $\alpha_i < \alpha_j$ needs to hold to make the low-quality product optimal for firm i when $v_j = v_L$. In other words, it is never optimal for firm i to challenge its rival and choose as well the low-quality product to compete for shoppers when its rival has fewer loyal consumers and thus remains more aggressive.

Optimal product choice when rival chooses high quality. It turns out that, while following the same logic, the explicit characterization of the best response of firm i when $v_j = v_H$ (instead of $v_j = v_L$) is slightly more involved.

Lemma 2 *Suppose firm i anticipates that its rival chooses high quality, $v_j = v_H$. Then firm i's optimal response is qualitatively similar to that characterized in Lemma 1 (when $v_j = v_L$) and depicted in the right-hand panel of Figure 1.*

Formally, $v_i = v_L$ is now strictly more profitable if:

[15] Note that we presently do not ask whether, in this case, the choice of $v_j = v_L$ is optimal for firm j. We turn to a characterization of the equilibrium below. Recall however that either firm may be restricted (e.g., for technological reasons) to a particular product variant.

i) Either $0 \le \alpha_j \le \underline{\alpha}_{j,H}$ *and* $\alpha_i < \tilde{\alpha}_{i,H}(\alpha_j)$, *where*

$$\underline{\alpha}_{j,H} = \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_H - c_H) + c_H v_L - c_L v_H},$$

$$\tilde{\alpha}_{i,H}(\alpha_j) = \frac{1 + (1-\alpha_j)\left(\frac{c_H \frac{v_L}{v_H} - c_L}{v_H - c_H}\right)}{2} - \sqrt{\left[\frac{1 - (1-\alpha_j)\left(\frac{c_H \frac{v_L}{v_H} - c_L}{v_H - c_H}\right)}{2}\right]^2 - (1-\alpha_j)\alpha_j \frac{v_L}{v_H}};$$

ii) or $\underline{\alpha}_{j,H} < \alpha_j < \overline{\alpha}_{j,H} < 1$ *and* $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j) = 1 - \alpha_j \left(\frac{(v_H - v_L)(v_H - c_H)}{c_H v_L - c_L v_H}\right)$, *where*

$$\overline{\alpha}_{j,H} = \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_H - c_H)}.$$

Otherwise, it is more profitable to choose $v_i = v_H$.

Again, choosing $v_i = v_L$ is only optimal when firm i's loyal share α_i is sufficiently low. Note now however that in difference to Lemma 1, when $v_j = v_H$ instead of $v_j = v_L$, it may be profitable for firm i to choose low quality even though it has more loyal customers than its rival, $\alpha_i > \alpha_j$. This is so as even when the rival has fewer loyal customers, the fact that only firm i has then the relatively stronger product may still make firm i more aggressive and thus induces firm i to promote more and firm j to promote less, which is a prerequisite to make $v_i = v_L$ optimal.

Figure 2 compares the two best responses for v_i when the rival has either low or high quality. There, the two light shaded areas (blue and red) depict the areas where the two best responses $v_i = v_L$ to $v_j \in \{v_L, v_H\}$ do not overlap. The darker shaded area (purple) depicts the region where $v_i = v_L$ is a best response to either rival product. We can thus observe that the rival's choice of low quality $v_j = v_L$ makes it *relatively* more profitable that also firm i chooses low quality, compared to when $v_j = v_H$, when the rival's share of loyal consumers α_j is high. Instead, when α_j is low, then firm i finds it *relatively* less profitable to choose low quality when this is chosen by its rival. Before commenting on this result, we now make this more precise by referring to the concepts of strategic substitutes and complements.

Consider the difference in firm i's profit with low quality instead of high quality. Then firms' strategies are strategic complements when this difference is higher in case also firm j chooses low quality.[16] Instead, firms' strategies are strategic substitutes when this dif-

[16] In light of the preceding discussion we find it more instructive to express the difference in this way, rather than by the possibly more standard subtraction of low-quality profits from high-quality profits. Note also that for ease of exposition we do not distinguish between weak and strong strategic substitutes (and subsequently complements), where the respective difference in differences is strictly positive (or negative).

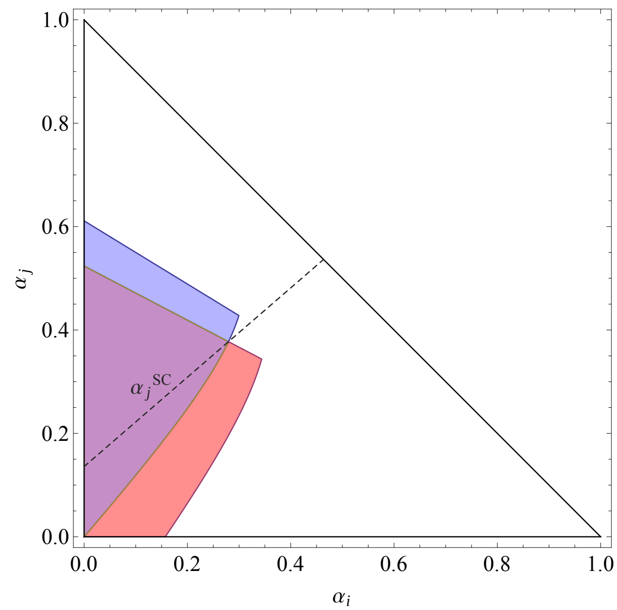

Figure 2: Comparison of firm i's best-response regions in (α_i, α_j)-space. The parameters used are $v_H = 1$, $c_H = 0.65$, $v_L = 0.7$, $c_L = 0.4$.

ference is higher in case firm j chooses high quality. Now the dotted line in Figure 2 precisely separates the areas where firms' product strategies are strategic complements and substitutes, both for ease of exposition considered only from the perspective of some firm i:

Lemma 3 *Define*

$$\alpha_j^{SC}(\alpha_i) = \left[\alpha_i + \frac{c_H \frac{v_L}{v_H} - c_L}{v_H - c_H}\right] / \left[1 + \frac{c_H \frac{v_L}{v_H} - c_L}{v_H - c_H}\right] > \alpha_i.$$

Then, product strategies represent for firm i strategic complements if $\alpha_j \geq \alpha_j^{SC}(\alpha_i)$ and strategic substitutes if $\alpha_j \leq \alpha_j^{SC}(\alpha_i)$.

The observation in Lemma 3 needs to be put into a wider perspective. Recall that when consumers are absolute thinkers, a firm's optimal product choice was independent of the anticipated choice of its rival. Hence, with absolute thinkers the firm could make the optimal choice even when completely ignoring the rival's choice, thereby just trading off higher quality with the corresponding higher production costs. But this is no longer the case when consumers are relative thinkers. Then, the firm's optimal product choice does depend on the product strategy of the firm's rival. With relative thinkers, the firm can thus no longer remain "inward looking" without risking to make the wrong product choice. When α_j is low relative to α_i, so that firm j is likely to aggressively pursue shoppers, choosing low quality is less likely to be optimal for firm i when firm j has also low quality. As put in Lemma 3, firms' product strategies are then strategic substitutes. They are however strategic complements when α_j is high relative to α_i: Firm i is then more likely to choose low quality as well when firm j chooses low quality, as it otherwise risks losing its competitive advantage vis-á-vis shoppers.

5.2 Equilibrium Product Strategies

Note again that so far we have taken one firm's (that is, firm j's) product choice as given. As discussed above, this may sometimes represent a realistic scenario. Now, however, we suppose that both firms can choose product quality. We are thus interested in characterizing all pure-strategy (Nash) equilibria, where $v_i \in \{v_L, v_H\}$. To save space, without loss of generality we now restrict consideration to the case where $\alpha_1 \geq \alpha_2$. In Figure 3 this is captured by the fact that we only need to consider parameter values (α_1, α_2) "below" the line where $\alpha_1 = \alpha_2$.

Proposition 5 *Suppose that both firms can choose whether to offer low or high quality. Suppose also without loss of generality that $\alpha_1 \geq \alpha_2$. Then the equilibrium product strategies, as depicted in Figure 3, are characterized as follows:*
*i) **Area I**: If there are altogether few shoppers as the (appropriately weighted) sum of α_1 and α_2 is high, both firms choose high quality ($v_1 = v_2 = v_H$). Formally, this is the case if $\alpha_2 > \hat{\alpha}_{2,H}(\alpha_1)$.*
*ii) **Area II**: If there are sufficiently many shoppers as the (appropriately weighted) sum of α_1 and α_2 is not too high, and if the loyal share of firm 2 is sufficiently smaller than that of firm 1, only firm 2 chooses low quality ($v_1 = v_H, v_2 = v_L$). Formally, this is the case if $\alpha_2 < \hat{\alpha}_{2,H}(\alpha_1)$, and $\alpha_2 < \tilde{\alpha}_{2,L}(\alpha_1)$ or $\alpha_1 > \tilde{\alpha}_{1,H}(\alpha_2)$ (or both).*
*iii) **Area III**: Otherwise, i.e., if there are sufficiently many shoppers and the two firms' loyal shares are sufficiently similar, there exist two equilibria, in which one firm chooses high and the other firm low quality ($v_1 = v_H, v_2 = v_L$ or $v_1 = v_L, v_2 = v_H$). Formally, this is the case if $\alpha_2 \geq \tilde{\alpha}_{2,L}(\alpha_1)$ and $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$.*

The characterization of Areas I and II is particularly intuitive and relies on our preceding discussion of the role of shoppers vs. that of a firm's share of loyal customers in determining the optimality of either the (absolutely stronger) high-quality product or the

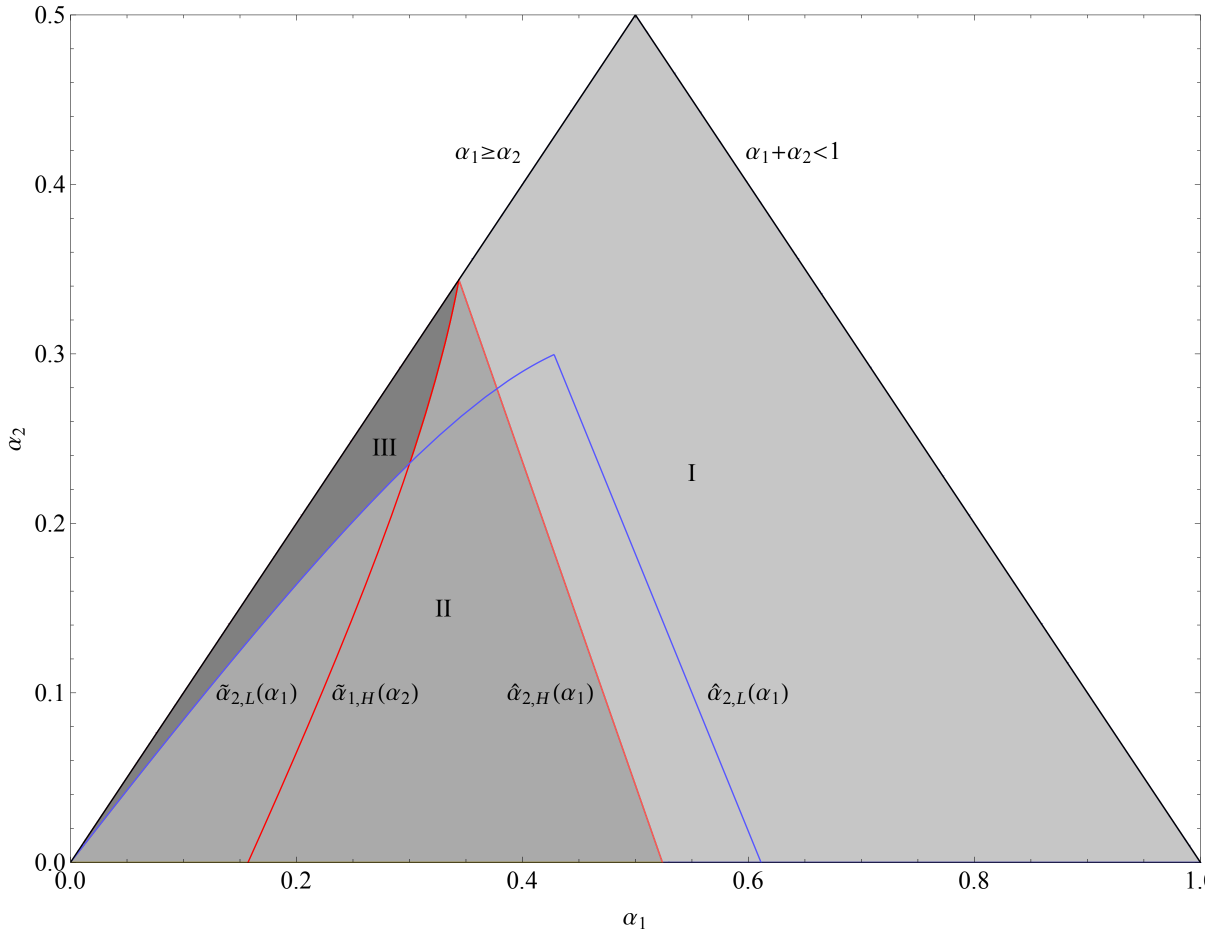

Figure 3: Illustration of product-choice equilibrium regions for $\alpha_1 \geq \alpha_2$. The parameters used are $v_H = 1$, $c_H = 0.65$, $v_L = 0.7$, $c_L = 0.4$.

(relatively stronger) low-quality product. The characterization of Area III, where there exist multiple equilibria, in turn follows from the observation in Lemma 3 that firms' product strategies can be strategic substitutes. That is, in the considered parameter region, where firms' loyal shares are not too different, each firm would choose low quality if and only if its rival chooses high quality. The firm that then ends up with the low-quality product will promote its product more frequently, thereby capturing (on average) a larger share of shoppers. As this is more profitable than choosing instead the high-quality product, while then the rival promotes (its low-quality product) more frequently, in this case a firm strictly profits from occupying (or "preempting") the low-quality ("discount") space.

Corollary 3 *In Area III of Figure 3 there are multiple equilibria, where either one of the firms chooses the high quality and the other firm the low quality. There, a firm would strictly benefit if it could move first and preempt its rival by choosing the low-quality product and competing more aggressively for shoppers.*

Both in Area II and in Area III we thus have equilibria where firms choose different products. Recall that this case never applies in our model when consumers are absolute thinkers, as then both firms choose the same product.

Apart from the equilibrium in Area III where firm 1 (with $\alpha_1 > \alpha_2$) preempts the other, when firms have different qualities, it is the firm with the larger loyal share of consumers that should have the higher quality. It is important to emphasize that this does not follow from an assumption that a firm with a higher quality can command over greater consumer loyalty. The causality is instead reversed. In fact, taking now the firm with a lower loyal share, it pays for this firm to choose the low-quality product as this provides *relatively* more value for its cost and is thus more profitable when competing for shoppers who are relative thinkers.

Corollary 4 *Under relative thinking, but not so under absolute thinking, firms with a lower share of loyal consumers are more likely to choose low quality and firms with a higher share more likely to choose high quality. Precisely, supposing still without loss of generality that $\alpha_1 \geq \alpha_2$, we have: i) when there exists an equilibrium where $v_1 = v_L$ then there also exists an equilibrium where $v_2 = v_L$ (while the converse does not hold) and ii) when there exists an equilibrium where $v_2 = v_H$ then there also exists an equilibrium with $v_1 = v_H$ (while the converse does not hold).*

In the following section we now collect the various managerial and empirical implications.

6 Implications

We organize our various results in a way that makes them directly applicable to practitioners and empirical researchers. We consequently split our implications in two parts, turning first to managerial implications and then to guidance for empirical researchers.

Managerial implications. We restrict consideration to the implications that the presence of relative thinkers should have for managerial strategies. Following the preceding analysis, we first hold the positioning of firms' products constant.

Implication 1 (Price and promotional strategy). *Firms that realize that their customers are relative thinkers should adjust their price and promotion strategies as follows:*

i) Firms that offer a relatively high quality, compared to competitors, should optimally decrease their promotional frequency, but, provided that they choose to promote their product to shoppers, they should increase promotional depth.

ii) Firms that offer a relatively low quality, compared to competitors, should optimally increase their promotional frequency, but they can decrease promotional depth.

Implication 1 is on purpose framed in terms of a change of promotional strategy, once firms realize the importance of relative thinking. Alternatively, we could envisage a firm that newly enters a market with a product of given quality, so that we can again consider in isolation the optimal choice of a firm's price and promotional strategy. A firm that is aware of the fact that consumers are prone to relative thinking will then, depending on the quality of its product relative to that of its competitors, adjust its promotional strategy accordingly, that is as described in parts i) and ii) of Implication 1.

Implication 2 (Product strategy I). *If consumers are relative thinkers, then firms' product strategy should, in general, take into account the following:*

i) Providing high quality is typically less profitable than when consumers are absolute thinkers.

ii) Firms need to be less "inward looking" and, instead, observe more closely how their rivals choose and position their products. Notably, when a rival offers low quality, this should make choosing low quality as well less profitable when the rival is likely to promote its product aggressively (which is the case when the rival has few loyal customers).

iii) Firms should watch out for a first-mover advantage in occupying the "discounter" space, which preempts a similar move of a rival and ensures that the firm, and not its rival, subsequently promotes more often and obtains a larger share of shoppers.

We have already commented above in detail on all parts of Implication 2 and thus move on to the following summary of the preceding comparative analysis in firms' loyal shares:

Implication 3 (Product strategy II). *When consumers are relative thinkers, the share of loyal consumers that both the firm and its rivals have should affect a firm's product choice as follows:*

i) If a firm's own loyal share is relatively low, this should make choosing a low quality (and subsequently more frequent promotions) relatively more profitable. The opposite is the case when the firm's own loyal share is relatively high.

ii) The profitability of choosing low vs. high quality depends more generally on the fraction of shoppers in the market. A higher fraction of shoppers makes choosing a lower-quality product relatively more attractive, unless also rivals choose low quality and are then more likely to promote it aggressively, as they have a smaller fraction of loyal consumers.

Empirical implications. Recall first that in Section 4.2 we have already collected implications and predictions for firms' pricing and promotional strategies that hold invariably whether consumers are absolute or relative thinkers, notably thereby extending the work of Narasimhan (1988). As we already noted, for an empirical researcher it may be difficult to establish whether or which fraction of consumers in a given market are, in a given context, relative thinkers instead of absolute thinkers. The available data may, however, allow to establish, for instance, what fraction of consumers actively shop and compare offerings and also which product has higher quality.[17]

We now collect some of our findings from the perspective of what difference relative thinking should make when setting up hypotheses or interpreting empirical findings. Part i) of Implication 4 relies on our equilibrium characterization for product choice, while part ii) mirrors the managerial implications for pricing and promotional strategies in Implication 1.

[17]Such a ranking is sometimes straightforward with some "Fast Moving Consumer Goods" (FMCGs), where one can measure, for instance, the cocoa content in chocolate, the fruit content in jam, or even the reported quality grade for fresh produce.

Implication 4 (Empirical predictions relating to the presence of relative thinkers in the market).

i) <u>Product choice:</u> *The difference that relative thinking, compared to absolute thinking, makes with respect to firms' product choice and positioning should be particularly pronounced when there is a sizable fraction of both shoppers, who indeed actively compare offerings and are thus prone to relative thinking, and loyal consumers, who thereby effectively act as if they were absolute thinkers. Then, when consumers are relative thinkers, we should generally expect greater product heterogeneity (in terms of quality) than otherwise.*

ii) <u>*Pricing and promotions:*</u> *When consumers are relative thinkers, compared to the benchmark when they are absolute thinkers, high-quality firms should promote their products less frequently and low-quality firms more frequently.*

7 Concluding Remarks

The focus of this paper is on the strategic implications when consumers are relative thinkers. For this purpose, we have taken the characterization of consumers' behavior as given, focusing instead fully on the derivation of firms' optimal price, promotion, and product strategies and the respective market equilibrium. In the Introduction, however, we have provided various motivations for our chosen specification of "relative thinking". We now return to this issue.

In the Introduction we referred to various motivations for our specification of consumer choice that all effectively result in a comparison of the respective ratios $\frac{v_1}{p_1}$ and $\frac{v_2}{p_2}$. To show this now formally, we assume again without loss of generality that $v_2 > v_1$, so that also $p_2 > p_1$ (as otherwise product 2 would unambiguously be the preferred choice). As noted in the Introduction, we may suppose directly that consumers evaluate each product in terms of "price-per-quality" or "quality-per-price", which immediately implies that 2 is chosen if $\frac{v_2}{p_2} > \frac{v_1}{p_1}$, but not if the converse holds strictly. Such an approach was used and motivated by Azar (2011). As a second approach, a relative thinker may ask himself whether, when comparing two products, the relative (that is, percentage) difference in quality is worth to pay the respective relative (that is, percentage) difference in price. Then he chooses product 2 if $\frac{v_2-v_1}{v_1} > \frac{p_2-p_1}{p_1}$, which becomes $\frac{v_2}{v_1}-1 > \frac{p_2}{p_1}-1$ and is thus equivalent to $\frac{v_2}{p_2} > \frac{v_1}{p_1}$. This consideration ties together intuitively the first approach with the subsequent (third) approach that expresses consumers' choice in terms of salience. Following the concept of salience in Bordalo et al. (2013) and considering product 2, the higher quality is salient if, given the average quality $v_\varnothing = \frac{1}{2}(v_1 + v_2)$ and the average price $p_\varnothing = \frac{1}{2}(p_1 + p_2)$ in the market, it holds that $\frac{v_2}{v_\varnothing} > \frac{p_2}{p_\varnothing}$, while when the converse holds strictly, then the higher price of product 2 is salient. Note first that when quality is salient for product 2, then it is also salient for product 1; and this holds analogously for price. When we now stipulate that consumers only compare two products based on the salient attribute, we retrieve again the same choice logic. Finally, it is also obtained when consumers derive constant marginal utility and maximize consumption with respect to a fixed budget constraint. To see this, suppose that consumers choose quantities $x_i \geq 0$ so as to maximize $\sum\limits_{i \in I} x_i(v_i - p_i)$ subject to the (binding) resource constraint $\sum\limits_{i \in I} x_i p_i \leq E$, where E could be motivated from a theory of mental accounting (cf. Thaler 1985). When the constraint binds, we are indeed back to our specification of relative thinking.[18]

We acknowledge that the appropriate specification of consumers' choice and decision rule ultimately remains an empirical question, albeit, as the literature reviewed in the Introduction also suggests, this may be dependent on the specific context. We explicitly model a context with frequent promotions, in which consumers are forced to make new decisions. In markets where promotions are less common and consumers repeatedly face the same set of offerings, relative thinking may be less relevant. The importance of relative thinking, in our model, derives however also from the degree to which consumers shop, as only shoppers compare (new) offers, while loyal consumers are supposed to simply frequent, say, the same retailer. A side insight of our model is thus that the (strategic) relevance of relative thinking, or possibly also of other behavioral traits, depends crucially on other intermediating factors, such as consumers' shopping habits.

[18]We note however that the multi-unit demand case is considerably more complex. While there the optimal choice is still (generically) a corner solution, with $x_i > 0$ only for the product where the respective "bang for the buck" $\frac{v_i}{p_i}$ is highest, in this case x_i depends on p_i.

8 References

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9 Appendix A: Proofs

Proof of Proposition 1. The subsequent proof is relatively short as we can rely on well-known arguments from, notably, Narasimhan (1988). We first confirm the characterized equilibrium. Recall that absolute thinkers strictly prefer firm i's offer over that of firm j if and only if $v_i - p_i > v_j - p_j$, that is, $p_j > p_i + (v_j - v_i)$. Hence, given $F_j(\cdot)$, for some price $p_i = p$ firm i's expected profit is

$$\pi_i(p; F_j(\cdot)) = (p - c_i)\left[\alpha_i + (1 - \alpha_i - \alpha_j)\left(1 - F_j(p + v_j - v_i)\right)\right]. \tag{3}$$

By construction of $F_j(\cdot)$ this is indeed equal to π_i over the respective support. (Recall that firm i prices up to but not including v_i.) Clearly, pricing below $\underline{p}_i$ or at or above v_i can not be optimal, so that we have confirmed optimality for firm i. Turn now to firm j, where we have again from construction of now $F_i(\cdot)$ that

$$\pi_j(p; F_i(\cdot)) = (p - c_j)\left[\alpha_j + (1 - \alpha_i - \alpha_j)\left(1 - F_i(p + v_i - v_j)\right)\right] \tag{4}$$

is equal to π_j over the full support, but strictly less below $\underline{p}_j$ or above v_j. Note finally that both $F_i(p)$ and $F_j(p)$ are well-behaved, that is strictly increasing over the respective supports with $F_i(\underline{\underline{p}}_j + v_i - v_j) = 0$, $F_i(v_i) = 1$, $F_j(\underline{\underline{p}}_j) = 0$, $\lim_{p \to v_j} F_j(p) = \left(\frac{1-\alpha_j}{1-\alpha_i}\right)\frac{v_j - c_j}{v_i - c_i}$, and firm j's mass point at v_j of $\gamma_j = 1 - \left(\frac{1-\alpha_j}{1-\alpha_i}\right)\frac{v_j - c_j}{v_i - c_i} > 0$, where the strict inequality follows from condition (1).

For the characterized equilibrium, the respective claims for profits π_i and π_j, the frequency of promotions $g_i = 1 - \gamma_i$, and finally the depth of promotions d_i and d_j follow immediately by definition and simple calculations (notably, that $\underline{p}_i - \underline{p}_j = v_i - v_j$).

Finally, for the arguments that support uniqueness we can next directly refer to Narasimhan (1988), who proved for symmetric qualities $q_i = q_j$ that both firms must randomize over convex supports ("no gaps") and that there can be at most one mass point - and if so, only for one firm and then at the upper boundary of its support.[19] These necessary characteristics of any equilibrium then imply that the respective mixed strategies must satisfy (3) and (4). **Q.E.D.**

Proof of Proposition 2. We will show this for the case $v_H - c_H > v_L - c_L$, noting that the proof for $v_H - c_H < v_L - c_L$ proceeds completely analogous. To see that (v_H, v_H) constitutes an equilibrium, note first that under symmetric quality levels, the firm with

[19] An application of these arguments to the presently considered case, as well as to the case with relative thinkers in the proof of Proposition 3, can be obtained from the authors upon request.

more loyal consumers, say firm j, will be less aggressive, i.e., it promotes with probability one and earns its "monopoly profit": $\pi_j = (v_H - c_H)\alpha_j$, while then also

$$\begin{aligned}\pi_i &= (v_H - c_H)\alpha_i + (1-\alpha_i-\alpha_j)(v_H - c_H)\left(\frac{v_H - c_H}{v_H - c_H} - \frac{1-\alpha_j}{1-\alpha_i}\right)\\ &= (v_H - c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}\end{aligned}$$

(cf. Proposition 1 for $v_i = v_j = v_H$, $c_i = c_j = c_H$). Now, if firm j were to deviate to v_L, firm i would continue to be more aggressive, giving firm j an expected deviation profit of $(v_L - c_L)\alpha_j < \pi_j$. If instead firm i were to deviate to v_L, there are two cases. If doing so rendered firm j more aggressive, firm i's deviation profit would be given by $(v_L - c_L)\alpha_i < \pi_i$. If firm i were to stay more aggressive, it would make a deviation profit of $(v_L - c_L)\alpha_i + (1-\alpha_i-\alpha_j)(v_H - c_H)\left(\frac{v_L - c_L}{v_H - c_H} - \frac{1-\alpha_j}{1-\alpha_i}\right) < \pi_i$, as is easy to check. Hence, no firm has a profitable deviation.

To see that (v_H, v_H) is the unique equilibrium, we have to rule out that (v_L, v_L), (v_L, v_H), and (v_H, v_L) can constitute an equilibrium. Clearly, (v_L, v_L) cannot constitute an equilibrium, as firm i would be more aggressive, giving firm j an expected profit of $(v_L - c_L)\alpha_j$, which falls short of its min-max profit of $(v_H - c_H)\alpha_j$. That (v_L, v_H) and (v_H, v_L) cannot constitute an equilibrium follows from the existence proof above. **Q.E.D.**

Proof of Proposition 3. The subsequent proof proceeds in analogy to that in Proposition 1. As discussed in the main text, the single difference in the construction of the equilibrium is that as relative thinkers, shoppers strictly prefer firm i's offer over firm j's if and only if $\frac{v_i}{p_i} > \frac{v_j}{p_j}$, that is, $p_j > p_i \frac{v_j}{v_i}$. Consequently, the respective firm profits (3) and (4) become

$$\pi_i(p; F_j(\cdot)) = (p - c_i)\left[\alpha_i + (1-\alpha_i-\alpha_j)\left(1 - F_j(p\frac{v_j}{v_i})\right)\right] \tag{5}$$

and

$$\pi_j(p; F_i(\cdot)) = (p - c_j)\left[\alpha_j + (1-\alpha_i-\alpha_j)\left(1 - F_i(p\frac{v_i}{v_j})\right)\right]. \tag{6}$$

Given the respective definition of $F_j(\cdot)$ (for (5)) and $F_i(\cdot)$ (for 6)), we can again confirm, first, that firms realize the same profit, π_i and π_j, for all prices in the respective supports; second, that profits are strictly lower for prices outside the respective supports; and, third, that the distribution functions $F_i(\cdot)$ and $F_j(\cdot)$ are indeed well behaved. Note here, in particular, that firm j's mass point at v_j is also well-defined, since $\gamma_j > 0$ follows from the assumption that condition (2) holds. Again, the claims for profits, frequency of promotions, and depth of promotions follow immediately from substitution and simple transformations.

Finally, as in the proof of Proposition 1, uniqueness follows again from the arguments in Narasimhan (1988). **Q.E.D.**

Proof of Corollary 1. With respect to g_i, note first that this only changes in case firm i does not always promote. Suppose thus that $g_i = 1 - \gamma_i < 1$. (To avoid confusion, note that in Proposition 1 and Proposition 3 we have used the indices i and j generically, so that clearly now, with $g_i < 1$, firm i is the less aggressive firm.) Both with absolute and relative thinkers, now with firm i being less aggressive, the respective value γ_i is strictly decreasing in $\frac{1-\alpha_i}{1-\alpha_j}$, so that g_i is indeed strictly decreasing in α_i and strictly decreasing in α_j (as claimed in assertions i) and ii)).

Take next the depth of promotion, d_i. With absolute thinkers, this is independent of whether firm i is more or less aggressive and it is strictly decreasing in $\frac{\alpha_j}{1-\alpha_i}$, so that it is indeed strictly decreasing in firm i's own loyal share α_i as well as in that of its rival, α_j. Though the depth of promotion differs between firms when consumers are relative thinkers, again d_i is strictly decreasing in $\frac{\alpha_j}{1-\alpha_i}$, regardless of whether firm i is more or less aggressive, so that d_i is again strictly decreasing in both α_i and α_j. **Q.E.D.**

Proof of Corollary 2. Recall that $v_i > v_j$. There are three possible cases (as the fourth is mathematically not possible):

Case (A): Conditions (1) and (2) hold, such that i is more aggressive with absolute and relative thinkers.
Case (B): The converse of (1) and (2) hold, such that j is more aggressive with absolute and relative thinkers.
Case (C): Condition (1) holds but not (2), such that i is more aggressive with absolute thinkers and j more aggressive with relative thinkers.

We now discuss the three cases in turn, drawing always on the characterization of the unique equilibria with absolute and with relative thinkers in Propositions 1 and 3.

Case (A): Take the high-quality firm i, which thus promotes with probability one with absolute and relative thinkers. Inspecting the respective expressions for profits, these are higher with absolute thinkers if

$$\frac{v_i - c_i}{v_j - c_j} - \frac{1-\alpha_j}{1-\alpha_i} > \frac{v_i}{v_j}\left(\frac{1-\frac{c_i}{v_i}}{1-\frac{c_j}{v_j}} - \frac{1-\alpha_j}{1-\alpha_i}\right), \tag{7}$$

which transforms to $\frac{v_i}{v_j} > 1$. Further, the depth of promotions increases if

$$\frac{v_i}{v_j}(v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right) > (v_j - c_j)\left(1 - \frac{\alpha_j}{1-\alpha_i}\right), \tag{8}$$

which follows again from $\frac{v_i}{v_j} > 1$. Turning to firm j, we have that profits are unchanged, as is the depth of promotions. The likelihood of promotions is strictly higher with relative thinkers if the respective value γ_j is strictly lower, i.e., if

$$1 - \left(\frac{1-\alpha_j}{1-\alpha_i}\right)\left(\frac{1-\frac{c_j}{v_j}}{1-\frac{c_i}{v_i}}\right) < 1 - \left(\frac{1-\alpha_j}{1-\alpha_i}\right)\left(\frac{v_j - c_j}{v_i - c_i}\right), \tag{9}$$

which again transforms to the requirement that $\frac{v_i}{v_j} > 1$.

Case (B): Now, starting again with firm i, π_i and d_i are not affected, while the likelihood of promotions is strictly lower with relative thinkers if the respective value γ_i is lower. Taking care of the respective indices, now the converse of (9) must hold strictly with i and j interchanged, which follows again from $\frac{v_i}{v_j} > 1$. Turning to firm j, which promotes with probability one with absolute and relative thinkers, profits are strictly higher with relative thinkers if the converse of (7) holds strictly, once i and j are interchanged, which holds again from $\frac{v_i}{v_j} > 1$. The same logic applies with respect to (8), so that now d_j is strictly higher with relative thinking.

Case (C): As now (1) holds but not (2), it is immediate that the likelihood of promotions is higher for the high-quality firm i with absolute thinkers (that is, equal to one) than with relative thinkers (that is, strictly below one), while the converse holds for the low-quality firm j. Likewise, profits strictly exceed the "monopoly" (or min-max) profits for firm i only with absolute thinkers and for firm j only with relative thinkers. While this also follows from a direct comparison of the respective expressions d_i and d_j, the assertion for promotional depth is also immediate from the respective reduction in profit for firm i and the respective increase in profit for firm j, as at the lowest possible price each firm attracts all shoppers and must realize the respective equilibrium profits. **Q.E.D.**

Proof of Proposition 4. The proof that the respective characterization of an equilibrium (in product choice) is unique is complicated by the various cases that we need to consider.

Case (A): $v_H - c_H < v_L - c_L$. To see that (v_L, v_L) constitutes an equilibrium, note first that under symmetric quality levels, the firm with more loyal consumers, say firm j, is less aggressive: $\pi_j = \pi_{j,L} = (v_L - c_L)\alpha_j$ and

$$\begin{aligned}\pi_i &= \pi_{i,L} = (v_L - c_L)\alpha_i + (1-\alpha_i-\alpha_j)(v_L - c_L)\frac{v_L}{v_L}\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_L}{v_L}} - \frac{1-\alpha_j}{1-\alpha_i}\right) \\ &= (v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}\end{aligned}$$

(cf. with Proposition 3 for $v_i = v_j = v_L$, $c_i = c_j = c_L$). Now, if firm j were to deviate to v_H, firm i would continue to be more aggressive, giving j an expected deviation profit of $(v_H - c_H)\alpha_j < \pi_{j,L}$. If instead firm i were to deviate to v_H, there are two cases. If doing so rendered firm j more aggressive, firm i's deviation profit would be given by $(v_H - c_H)\alpha_i < \pi_{i,L}$. If firm i were to stay more aggressive, it would make a deviation profit of

$$\begin{aligned}&(v_H - c_H)\alpha_i + (1-\alpha_i-\alpha_j)(v_L - c_L)\frac{v_H}{v_L}\left(\frac{1-\frac{c_H}{v_H}}{1-\frac{c_L}{v_L}} - \frac{1-\alpha_j}{1-\alpha_i}\right) \\ &= (v_H - c_H)(1-\alpha_j) - (1-\alpha_i-\alpha_j)\left(\frac{1-\alpha_j}{1-\alpha_i}\right)\frac{v_H}{v_L}(v_L - c_L) \\ &< (v_L - c_L)(1-\alpha_j) - (1-\alpha_i-\alpha_j)\left(\frac{1-\alpha_j}{1-\alpha_i}\right)\frac{v_H}{v_L}(v_L - c_L) \\ &= (v_L - c_L)(1-\alpha_j)\left[1-\left(\frac{1-\alpha_i-\alpha_j}{1-\alpha_i}\right)\frac{v_H}{v_L}\right] < \pi_{j,L},\end{aligned}$$

where the last inequality is straightforward to verify. Hence, no firm has a profitable deviation.

We show next that (v_L, v_L) is the unique equilibrium. Clearly, (v_H, v_H) cannot be an equilibrium, as firm j would be less aggressive, giving j an expected profit of $(v_H - c_H)\alpha_j < (v_L - c_L)\alpha_j$, i.e., less than what firm j would get when choosing v_L and subsequently $p = v_L$ (firm j's min-max profit). The combinations (v_H, v_L) and (v_L, v_H) can be eliminated by the preceding existence proof of (v_L, v_L), as we established that neither firm finds it profitable to deviate to v_H.

Case (B): $v_H - c_H > v_L - c_L$, with $\frac{v_H}{c_H} > \frac{v_L}{c_L}$. We again first establish existence of an equilibrium with (v_H, v_H). Then, the firm with more loyal consumers, say firm j, is less aggressive, so that profits are $\pi_j = \pi_{j,H} = (v_H - c_H)\alpha_j$ and $\pi_i = \pi_{i,H} = (v_H - c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$ (see Case (i) above). For firm i, deviating to v_L yields higher profits if

$$(v_L - c_L)\alpha_i + (1-\alpha_i-\alpha_j)(v_H - c_H)\frac{v_L}{v_H}\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}} - \frac{1-\alpha_j}{1-\alpha_i}\right) > (v_H - c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}. \quad (10)$$

Since $\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i} > \alpha_i$ (as is easy to check), condition (10) can only be satisfied if firm i is indeed more aggressive when choosing v_L. Hence, only condition (10) needs to be assessed in order to check whether firm i has a profitable deviation. Rearranging and simplifying (10), this is the case if and only if

$$\left(c_L - c_H \frac{v_L}{v_H}\right)(1-\alpha_i) + \alpha_j(v_H - v_L)\frac{v_H - c_H}{v_H} < 0,$$

which cannot be satisfied, since $c_L - c_H \frac{v_L}{v_H} > 0$ as follows from $\frac{v_H}{c_H} > \frac{v_L}{c_L}$. Finally, via manipulation of (10) we can show that firm j can only have a profitable deviation (to v_L) if firm i has one, as the corresponding profitability condition for firm j is stricter. This concludes the existence proof.

In order to prove that (v_H, v_H) constitutes the unique equilibrium, note first that (v_L, v_L) can be eliminated immediately, as firm i would then be more aggressive, giving firm j an expected profit of $(v_L - c_L)\alpha_j$, which falls short of its min-max profit of $(v_H - c_H)\alpha_j$. The combinations (v_L, v_H) and (v_H, v_L) can again be ruled out directly by the existence proof of (v_H, v_H), which showed that neither firm finds it profitable to deviate to v_L. **Q.E.D.**

Proof of Lemma 1. Recall first that presently it is assumed that $v_j = v_L$. When $\alpha_i \geq \alpha_j$, $v_i = v_L$ would thus generate for firm i profits of $\pi_i(v_L) = (v_L - c_L)\alpha_i < (v_H - c_H)\alpha_i \leq \pi_i(v_H)$, where we have now made explicit the dependency on firm i's product choice. Consider next the case $\alpha_i < \alpha_j$, where we know from the preceding transformations that $\pi_i(v_L) = (v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$. If i chooses v_H, there are two cases to consider. First, consider the case where i is then more aggressive, which holds when $\frac{1-\frac{c_H}{v_H}}{1-\frac{c_L}{v_L}} > \frac{1-\alpha_j}{1-\alpha_i}$ and thus when

$$\alpha_i < 1 - (1-\alpha_j)\left(\frac{v_L - c_L}{v_H - c_H}\right)\frac{v_H}{v_L} := \check{\alpha}_{i,L}(\alpha_j) < \alpha_j.$$

Then, we we have profits of

$$\pi_i(v_H) : (v_H - c_H)(1-\alpha_j) - (1-\alpha_i-\alpha_j)\left(v_H - c_L\frac{v_H}{v_L}\right)\frac{1-\alpha_j}{1-\alpha_i}.$$

Second, if $\alpha_i > \check{\alpha}_{i,L}(\alpha_j)$, we have $\pi_i(v_H) = (v_H - c_H)\alpha_i$. We treat both cases in turn.

Case (A): $\alpha_i < \check{\alpha}_{i,L}(\alpha_j)$. Then i strictly prefers v_L over v_H if and only if

$$(v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i} > (v_H - c_H)(1-\alpha_j) - (1-\alpha_i-\alpha_j)\left(v_H - c_L\frac{v_H}{v_L}\right)\frac{1-\alpha_j}{1-\alpha_i},$$

which is equivalent to

$$\alpha_i < 1 - \alpha_j \left(\frac{(v_H - v_L)(v_L - c_L)}{c_H v_L - c_L v_H} \right) = \hat{\alpha}_{i,L}(\alpha_j).$$

Hence, if both $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j)$ and $\alpha_i < \check{\alpha}_{i,L}(\alpha_j)$, firm i strictly prefers v_L. Comparing these two constraints, the first is stricter if

$$\alpha_j > \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L) + \frac{v_L}{v_H}(c_H v_L - c_L v_H)} = \underline{\alpha}_{j,L}.$$

Hence, case (A) can be split up into two subcases. First, if $\alpha_j > \underline{\alpha}_{j,L}$, firm i strictly prefers v_L over v_H if and only if $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j)$. Second, if $\alpha_j \leq \underline{\alpha}_{j,L}$, firm i strictly prefers v_L over v_H if and only $\alpha_i < \check{\alpha}_{i,L}(\alpha_j)$. Note finally that for $\alpha_j \geq \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L)} = \overline{\alpha}_{j,L} \in (\underline{\alpha}_{j,L}, 1)$, the inequality $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j)$ requires that $\alpha_i < 0$, which can never be satisfied.

Case (B): $\alpha_i \geq \check{\alpha}_{i,L}(\alpha_j)$. Then i strictly prefers v_L over v_H if and only if

$$(v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i} > (v_H - c_H)\alpha_i,$$

or

$$g(\alpha_i) := \alpha_i^2 - \alpha_i + \left(\frac{v_L - c_L}{v_H - c_H} \right) \alpha_j(1-\alpha_j) > 0.$$

Note that $g(\alpha_i)$ is strictly convex in α_i, with $g(0) > 0$, $g(\alpha_j) < 0$, and $g(1) > 0$. Hence, the critical α_i below which choosing v_L becomes profitable is given by the lower root to $g(\alpha_i)$, which equals

$$1/2 - \sqrt{1/4 - \alpha_j(1-\alpha_j)\frac{v_L - c_L}{v_H - c_H}} = \tilde{\alpha}_{i,L}(\alpha_j) < \alpha_j.$$

We thus have that for $\alpha_i \in [\check{\alpha}_{i,L}(\alpha_j), \tilde{\alpha}_{i,L}(\alpha_j))$ firm i strictly prefers v_L over v_H in case (B). This interval is only non-empty if $\tilde{\alpha}_{i,L}(\alpha_j) > \check{\alpha}_{i,L}(\alpha_j)$, or, after inserting and rearranging,

$$\sqrt{1/4 - \alpha_j(1-\alpha_j)\frac{v_L - c_L}{v_H - c_H}} < (1-\alpha_j)\left(\frac{v_L - c_L}{v_H - c_H} \right) \frac{v_H}{v_L} - 1/2. \tag{11}$$

It is easy to see that the expression under the root is strictly positive, so the LHS is well-defined and strictly positive. If the RHS is not positive, which is true if $\alpha_j \geq 1 - 1/2 \left(\frac{v_H - c_H}{v_L - c_L} \right) \frac{v_L}{v_H}$, the inequality cannot be satisfied. Hence, this obtains the requirement that

$$\alpha_j < 1 - 1/2 \left(\frac{v_H - c_H}{v_L - c_L} \right) \frac{v_L}{v_H}. \tag{12}$$

If this is true, such that the RHS of inequality (11) is strictly positive, after simplifying expressions, the inequality transforms to

$$\alpha_j < \frac{c_H v_L - c_L v_H}{(v_H - v_L)(v_L - c_L) + \frac{v_L}{v_H}(c_H v_L - c_L v_H)} = \underline{\alpha}_{j,L}. \tag{13}$$

One can see that (13) is stricter than (12), which implies that $\alpha_j < \underline{\alpha}_{j,L}$ is necessary in order for i to strictly prefer v_L in the considered case (B). (Note that this is the same critical value of α_j as in case (A) above.) To sum up, in case (B), firm i strictly prefers to choose v_L over v_H if and only if $\alpha_j < \underline{\alpha}_{j,L}$ and $\alpha_i \in [\check{\alpha}_{i,L}(\alpha_j), \tilde{\alpha}_{i,L}(\alpha_j))$.

Finally, we can combine cases (A) and (B). If $\alpha_j < \underline{\alpha}_{j,L}$, firm i strictly prefers v_L if $\alpha_i < \check{\alpha}_{i,L}(\alpha_j)$ (case A) or if $\alpha_i \in [\check{\alpha}_{i,L}(\alpha_j), \tilde{\alpha}_{i,L}(\alpha_j))$ (case B), i.e., if and only if $\alpha_i < \tilde{\alpha}_{i,L}(\alpha_j)$. If $\alpha_j \in [\underline{\alpha}_{j,L}, \overline{\alpha}_{j,L})$, firm i never finds it optimal to choose v_L in case (B), while it finds it strictly optimal to do so in case (A) if and only if $\alpha_i < \hat{\alpha}_{i,L}(\alpha_j) < \alpha_j$. Lastly, if $\alpha_j \geq \overline{\alpha}_{j,L}$, firm i never finds it optimal to choose v_L.

Having characterized the respective parameter regions for which $v_i = v_L$ or $v_i = v_H$ is optimal, we turn to the comparative analysis in firms' loyalty share. Note first that the assertion for α_i follows immediately from the preceding characterization. With respect to the comparative analysis in α_j, note first that for $\alpha_j = 0$, i's best response is always to choose v_H. Next, note that for $\alpha_j \geq \underline{\alpha}_{j,L}$, the boundary $\hat{\alpha}_{i,L}(\alpha_j)$ is a linearly decreasing function in α_j, with $\hat{\alpha}_{i,L}(\overline{\alpha}_{j,L}) = 0$ and $\hat{\alpha}_{i,L}(\underline{\alpha}_{j,L}) = \tilde{\alpha}_{i,L}(\underline{\alpha}_{j,L})$ (as is easy to check). To show that, now for given α_i, the respective set of values α_j for which $v_i = v_L$ is optimal is indeed convex, it is sufficient to show that the boundary $\tilde{\alpha}_{i,L}(\alpha_j)$ is strictly quasi-concave (cf. also Figure 1). (Note that it need not be strictly monotonic, which would only be the case when $\alpha_j < 1/2$, which is however not implied by $\alpha_j < \underline{\alpha}_{j,L}$.) To simplify expressions, let $k = \frac{v_L - c_L}{v_H - c_H} \in (0,1)$, such that $\tilde{\alpha}_{i,L}(\alpha_j) = 1/2 - [1/4 - \alpha_j(1-\alpha_j)k]^{1/2}$. Then it is straightforward to establish that $\tilde{\alpha}''_{i,L}(\alpha_j)$ has the same sign as

$$\frac{k(1-2\alpha_j)^2}{1/2 - 2\alpha_j(1-\alpha_j)k} - 2,$$

which can be further simplified to

$$\frac{k-1}{1/2 - 2\alpha_j(1-\alpha_j)k} < 0,$$

where the inequality follows from $k < 1$ and $\alpha_j(1-\alpha_j) \leq 1/4$. Hence, $\tilde{\alpha}_{i,L}(\alpha_j)$ is strictly concave in α_j, which completes the proof. **Q.E.D.**

Proof of Lemma 2. We first consider two separate cases: (A) $\alpha_i < \alpha_j$ and (B) $\alpha_i \geq \alpha_j$.

Case (A): $\alpha_i < \alpha_j$. In this case, firm i will be more aggressive irrespective of whether it chooses v_H or v_L. It thus strictly prefers v_L over v_H if and only if

$$(v_L - c_L)\alpha_i + (1-\alpha_i-\alpha_j)\left(v_L - c_H\frac{v_L}{v_H}\right)\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}} - \frac{1-\alpha_j}{1-\alpha_i}\right) > (v_H - c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i},$$

which is equivalent to

$$\alpha_i < 1 - \alpha_j\left(\frac{(v_H-v_L)(v_H-c_H)}{c_H v_L - c_L v_H}\right) = \hat{\alpha}_{i,H}(\alpha_j).$$

Hence, if the above inequality holds (together with $\alpha_i < \alpha_j$), then firm i strictly prefers v_L over v_H. Put differently, i strictly prefers v_L if $\alpha_i < \min\{\alpha_j, \hat{\alpha}_{i,H}(\alpha_j)\}$. Solving $\alpha_j < \hat{\alpha}_{i,H}(\alpha_j)$ for α_j gives

$$\alpha_j < \frac{c_H v_L - c_L v_H}{(v_H-v_L)(v_H-c_H) + c_H v_L - c_L v_H} = \underline{\alpha}_{j,H} \in (0, 1/2).$$

Thus we can split the case $\alpha_i \leq \alpha_j$ into two subcases. First, if $\alpha_j \leq \underline{\alpha}_{j,H}$, the stricter constraint is given by $\alpha_i < \alpha_j$, such that firm i always strictly prefers v_L over v_H (given that $\alpha_i < \alpha_j$). Second, if $\alpha_j > \underline{\alpha}_{j,H}$, the stricter constraint is given by $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j)$, such that i strictly prefers v_L if and only if $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j)$ (given that $\alpha_i < \alpha_j$). Note moreover that for $\alpha_j \geq \frac{c_H v_L - c_L v_H}{(v_H-v_L)(v_H-c_H)} = \overline{\alpha}_{j,H} \in (\underline{\alpha}_{j,H}, 1)$, the inequality $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j)$ requires that $\alpha_i < 0$, which can never be satisfied.

Case (B): $\alpha_i \geq \alpha_j$. We know that a necessary condition that v_L is preferred over v_H is that firm i becomes more aggressive if it chooses v_L (as otherwise, $\pi_i(v_H) = (v_H - c_H)\alpha_i > \pi_i(v_L) = (v_L - c_L)\alpha_i$). Suppose that this is the case. Then i strictly prefers v_L over v_H if and only if

$$(v_L - c_L)\alpha_i + (1-\alpha_i-\alpha_j)\left(v_L - c_H\frac{v_L}{v_H}\right)\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}} - \frac{1-\alpha_j}{1-\alpha_i}\right) > (v_H - c_H)\alpha_i, \tag{14}$$

which we further transform to

$$(1-\alpha_j)\left(c_H\frac{v_L}{v_H} - c_L\right) + \left(v_L - c_H\frac{v_L}{v_H}\right)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i} > (v_H - c_H)\alpha_i \tag{15}$$

and finally

$$f(\alpha_i) := \alpha_i^2 - \alpha_i\left[1 + (1-\alpha_j)\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H}\right] + (1-\alpha_j)\left[\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H} + \alpha_j\frac{v_L}{v_H}\right] > 0.$$

Note that $f(\alpha_i)$ is strictly convex in α_i, with $f(0) > 0$ and $f(1) > 0$. Moreover, at the critical α_i below which firm i becomes more aggressive when choosing v_L, $\check{\alpha}_{i,H}(\alpha_j) = 1 - \frac{1-\frac{c_H}{v_H}}{1-\frac{c_L}{v_L}}(1-\alpha_j)$, it clearly holds that $f(\check{\alpha}_{i,H}(\alpha_j)) < 0$ (compare with inequality (14)). Hence, the critical α_i below which choosing v_L becomes profitable is given by the lower root to $f(\alpha_i)$,

$$\alpha_i < \frac{1+(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)}{2} - \sqrt{\left[\frac{1+(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)}{2}\right]^2 - (1-\alpha_j)\left[\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}+\alpha_j\frac{v_L}{v_H}\right]},$$

which simplifies to

$$\alpha_i < \frac{1+(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)}{2} - \sqrt{\left[\frac{1-(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)}{2}\right]^2 - (1-\alpha_j)\alpha_j\frac{v_L}{v_H}} = \tilde{\alpha}_{i,H}(\alpha_j).$$

To sum up, with $\alpha_i \geq \alpha_j$, firm i strictly prefers v_L over v_H if and only if $\alpha_i \in [\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$. This range of α_i's is only non-empty if $\tilde{\alpha}_{i,H}(\alpha_j) > \alpha_j$. Let now

$$z = \frac{1+(1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)}{2},$$

so that substituting z in the requirement $\tilde{\alpha}_{i,H}(\alpha_j) > \alpha_j$ transforms this to

$$z - \sqrt{(1-z)^2 - (1-\alpha_j)\alpha_j\frac{v_L}{v_H}} > \alpha_j,$$

or

$$z - \alpha_j > \sqrt{(1-z)^2 - (1-\alpha_j)\alpha_j\frac{v_L}{v_H}}.$$

Since clearly $z > 1/2$ and $\alpha_j < 1/2$ due to $\alpha_i + \alpha_j < 1$ and $\alpha_i \geq \alpha_j$, both sides are strictly positive[20], and we may square both sides of the inequality. It thus has to hold that

$$z^2 - 2z\alpha_j + \alpha_j^2 > (1-z)^2 - (1-\alpha_j)\alpha_j\frac{v_L}{v_H},$$

which, after expanding $(1-z)^2$ and simplifying, becomes

$$2z(1-\alpha_j) > 1 - \alpha_j^2 - (1-\alpha_j)\alpha_j\frac{v_L}{v_H}.$$

[20] To see that the expression under the root, $D := (1-z)^2 - (1-\alpha_j)\alpha_j\frac{v_L}{v_H}$, is non-negative (such that the root is indeed well-defined), observe first that D is strictly increasing in c_L. Since we have assumed throughout that $v_H - c_H > v_L - c_L$, at worst it holds that $c_L = v_L - (v_H - c_H)$. Substituting $v_L - (v_H - c_H)$ for c_L in D and simplifying yields $D \geq \left(\frac{\alpha_j - (1-\alpha_j)\frac{v_L}{v_H}}{2}\right)^2 \geq 0$.

Dividing both sides by $1-\alpha_j$ and noting that $1-\alpha_j^2 = (1-\alpha_j)(1+\alpha_j)$, the condition boils down to

$$2z > 1 + \alpha_j - \alpha_j \frac{v_L}{v_H}.$$

Substituting back z and simplifying yields

$$(1-\alpha_j)\left(\frac{c_H \frac{v_L}{v_H} - c_L}{v_H - c_H}\right) > \alpha_j \left(1 - \frac{v_L}{v_H}\right),$$

which is linear in α_j. Solving the above inequality for α_j subsequently reveals that the interval $[\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$ is non-empty if and only if

$$\alpha_j < \underline{\alpha}_{j,H},$$

i.e., the same critical value of α_j as in case (A) above. Summarizing the above results for case (B), if $\alpha_i \geq \alpha_j$, i finds it strictly optimal to choose v_L if and only if $\alpha_j < \underline{\alpha}_{j,H}$ and $\alpha_i \in [\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$.

Finally, we can combine cases (A) and (B). If $\alpha_j < \underline{\alpha}_{j,H}$, firm i finds it either optimal to choose v_L if $\alpha_i < \alpha_j$ (case A), or if $\alpha_i \in [\alpha_j, \tilde{\alpha}_{i,H}(\alpha_j))$ (case B). Thus, firm i strictly prefers v_L if and only if $\alpha_i < \tilde{\alpha}_{i,H}(\alpha_j)$. If $\alpha_j \in [\underline{\alpha}_{j,H}, \overline{\alpha}_{j,H})$, firm i never finds it optimal to choose v_L in case (B), while it finds it strictly optimal to do so in case (A) if and only if $\alpha_i < \hat{\alpha}_{i,H}(\alpha_j) < \alpha_j$. Lastly, if $\alpha_j \geq \overline{\alpha}_{j,H}$, firm i never finds it optimal to choose v_L.

Having characterized the respective parameter regions for which $v_i = v_L$ or $v_i = v_H$ is optimal, we turn to the comparative analysis in firms' loyalty share. Note first that the assertion for α_i follows immediately from the preceding characterization. With respect to the comparative analysis in α_j, note first that for $\alpha_j \geq \underline{\alpha}_{j,H}$ the boundary $\hat{\alpha}_{i,H}(\alpha_j)$ is a linearly decreasing function in α_j, with $\hat{\alpha}_{i,H}(\overline{\alpha}_{j,H}) = 0$. Since it also holds that $\tilde{\alpha}_{i,H}(\underline{\alpha}_{j,H}) = \hat{\alpha}_{i,H}(\underline{\alpha}_{j,H})$, in order to show that the respective set of values α_j for which $v_i = v_L$ is optimal is indeed convex, it is sufficient to show that the boundary $\tilde{\alpha}_{i,L}(\alpha_j)$ is strictly quasiconcave (cf. also Figure 1). (Note again that it need not be strictly monotonic.).

To simplify expressions, let $m := (c_H \frac{v_L}{v_H} - c_L)/(v_H - c_H)$, such that

$$\tilde{\alpha}_{i,H}(\alpha_j) = \frac{1 + (1-\alpha)m}{2} - \sqrt{\frac{(1 - m + \alpha_j m)^2}{4} - (1-\alpha_j)\alpha_j \frac{v_L}{v_H}}.$$

Note further that $m \in (0, 1 - \frac{v_L}{v_H})$, as follows from the requirements that $v_H - c_H > v_L - c_L$ and $\frac{v_H}{c_H} < \frac{v_L}{c_L}$. Then, it is straightforward to establish that $\tilde{\alpha}''_{i,H}(\alpha_j)$ has the same sign as

$$\frac{\left[\alpha_j\left(\frac{1}{2}m^2+2\frac{v_L}{v_H}\right)+\frac{1}{2}m(1-m)-\frac{v_L}{v_H}\right]^2}{(1-m+\alpha_j m)^2-4\alpha_j(1-\alpha_j)\frac{v_L}{v_H}}-\frac{1}{2}\left(\frac{1}{2}m^2+2\frac{v_L}{v_H}\right).$$

A tedious calculation reveals that the above expression is equal to

$$\frac{v_L}{v_H^2}\left(\frac{v_L-v_H(1-m)}{(1-m+\alpha_j m)^2-4\alpha_j(1-\alpha_j)\frac{v_L}{v_H}}\right).$$

The nominator of the fraction in brackets is clearly negative, since $m<1-\frac{v_L}{v_H}$. We thus want to show that the denominator of this fraction is strictly positive. For this, note first that the denominator is strictly decreasing in m. Since $m<1-\frac{v_L}{v_H}$, the denominator is bounded from below by

$$\left[\frac{v_L}{v_H}(1-\alpha_j)+\alpha_j\right]^2-4\alpha_j(1-\alpha_j)\frac{v_L}{v_H}=\left[\alpha_j-(1-\alpha_j)\frac{v_L}{v_H}\right]^2\geq 0.$$

Hence, $\tilde{\alpha}_{i,H}(\alpha_j)$ is indeed concave in α_j, which completes the proof. **Q.E.D.**

Proof of Lemma 3. We denote the respective profits, depending on the choice of products, by $\pi_i(v_i,v_j)$. By definition, from firm i's perspective, product quality is a (weak) strategic complement if and only if

$$\pi_i(v_H,v_H)-\pi_i(v_L,v_H)\geq\pi_i(v_H,v_L)-\pi_i(v_L,v_L). \tag{16}$$

(That is, we consider here the more "standard" expression where we subtract profits when $v_i=v_H$ (high quality) from profits when $v_i=v_L$ (low quality).) We consider the following four cases, which together comprise all possibilities:

Case (A) $\alpha_i\geq\check{\alpha}_{i,H}(\alpha_j)=1-(1-\alpha_j)\frac{1-\frac{c_H}{v_H}}{1-\frac{c_L}{v_L}}>\alpha_j$. In this subregion, α_i is so large relative to α_j that firm i is always less aggressive, irrespective of firm i's and j's product choice. Hence, we have that $\pi_i(v_H,v_H)=\pi_i(v_H,v_L)=(v_H-c_H)\alpha_i$ and $\pi_i(v_L,v_H)=\pi_i(v_L,v_L)=(v_L-c_L)\alpha_i$, from which it trivially follows that the converse of (16) holds weakly.

Case (B) $\alpha_i\in[\alpha_j,\check{\alpha}_{i,H}(\alpha_j))$. In this subregion, α_i is moderately large, such that firm i is more aggressive if and only if i chooses v_L while j chooses v_H. Hence,

$$\pi_i(v_H,v_H)=(v_H-c_H)\alpha_i,\pi_i(v_L,v_H)=(v_L-c_L)\alpha_i+(1-\alpha_i-\alpha_j)(v_H-c_H)\frac{v_L}{v_H}\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}}-\frac{1-\alpha_j}{1-\alpha_i}\right),$$

and $\pi_i(v_H,v_L)=\pi_i(v_L,v_L)=(v_L-c_L)\alpha_i$, so that the converse of (16) holds weakly when $\pi_i(v_L,v_H)\geq\pi_i(v_L,v_L)$. This follows immediately from $\alpha_i<\check{\alpha}_{i,H}(\alpha_j)$.

Case (C) $\alpha_i < \breve{\alpha}_{i,L}(\alpha_j) = 1 - (1-\alpha_j)\left(\frac{v_L - c_L}{v_H - c_H}\right)\frac{v_H}{v_L} < \alpha_j$. In this subregion, α_i is so low relative to α_j that firm i is always more aggressive, irrespective of firm i's and j's product choice. Hence, we have that $\pi_i(v_H, v_H) = (v_H - c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$,

$$\begin{aligned}
\pi_i(v_L, v_H) &= (v_L - c_L)\alpha_i + (1-\alpha_i-\alpha_j)(v_H - c_H)\frac{v_L}{v_H}\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}} - \frac{1-\alpha_j}{1-\alpha_i}\right) \\
&= (1-\alpha_j)\left(c_H\frac{v_L}{v_H} - c_L\right) + \frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}\left(v_L - c_H\frac{v_L}{v_H}\right), \\
\pi_i(v_H, v_L) &= (v_H - c_H)\alpha_i + (1-\alpha_i-\alpha_j)(v_L - c_L)\frac{v_H}{v_L}\left(\frac{1-\frac{c_H}{v_H}}{1-\frac{c_L}{v_L}} - \frac{1-\alpha_j}{1-\alpha_i}\right) \\
&= (1-\alpha_j)\left(c_L\frac{v_H}{v_L} - c_H\right) + \frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}\left(v_H - c_L\frac{v_H}{v_L}\right),
\end{aligned}$$

and $\pi_i(v_L, v_L) = (v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$. We now rewrite (16) as $\pi_i(v_H, v_H) + \pi_i(v_L, v_L) \geq \pi_i(v_H, v_L) + \pi_i(v_L, v_H)$, or

$$\begin{aligned}
&\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}\left[(v_H - c_H) + (v_L - c_L)\right] \geq (1-\alpha_j)\left[c_H\frac{v_L}{v_H} - c_L + c_L\frac{v_H}{v_L} - c_H\right] \\
&+\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}\left[v_L - c_H\frac{v_L}{v_H} + v_H - c_L\frac{v_H}{v_L}\right].
\end{aligned}$$

After collecting terms and canceling out $(1-\alpha_j)$, this becomes

$$\frac{\alpha_j}{1-\alpha_i}\left[c_H\frac{v_L}{v_H} - c_L + c_L\frac{v_H}{v_L} - c_H\right] \geq c_H\frac{v_L}{v_H} - c_L + c_L\frac{v_H}{v_L} - c_H.$$

Since $c_H\frac{v_L}{v_H} - c_L + c_L\frac{v_H}{v_L} - c_H < 0$ due to $\frac{v_H}{c_H} < \frac{v_L}{c_L}$, this is equivalent to $\alpha_j \leq 1-\alpha_i$, which is indeed satisfied.

Case (D) $\alpha_i \in [\breve{\alpha}_{i,L}(\alpha_j), \alpha_j)$. In this last remaining subregion, α_i is moderately low, such that firm i is less aggressive if and only if i chooses v_H, while j chooses v_L. Hence, $\pi_i(v_H, v_H) = (v_H - c_H)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$,

$$\pi_i(v_L, v_H) = (1-\alpha_j)\left(c_H\frac{v_L}{v_H} - c_L\right) + \frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}\left(v_L - c_H\frac{v_L}{v_H}\right),$$

$\pi_i(v_H, v_L) = (v_H - c_H)\alpha_i$, and $\pi_i(v_L, v_L) = (v_L - c_L)\frac{\alpha_j(1-\alpha_j)}{1-\alpha_i}$. Now rewriting (16) as $\pi_i(v_H, v_H) + \pi_i(v_L, v_L) \geq \pi_i(v_H, v_L) + \pi_i(v_L, v_H)$, inserting the above profit expressions, multiplying by $1-\alpha_i$, and collecting terms, this holds if and only if

$$h(\alpha_i) := \alpha_i^2 - \alpha_i\left[1 - (1-\alpha_j)\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H}\right] - (1-\alpha_j)^2\frac{c_H\frac{v_L}{v_H} - c_L}{v_H - c_H} + (1-\alpha_j)\alpha_j \geq 0,$$

i.e., α_i must lie (weakly) outside the roots of the quadratic equation $h(\alpha_i) = 0$. The lower root of $h(\alpha_i)$ is given by $\alpha_j - (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right) \in (\check{\alpha}_{i,L}(\alpha_j), \alpha_j)$, while the upper root is given by $1-\alpha_j$. To sum up, (16) holds if and only if $\alpha_i \leq \alpha_j - (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)$, since $\alpha_i \geq 1-\alpha_j$ falls outside the permissible parameter space. Thus, region (iv) can be split into two further subregions as follows : First, if $\alpha_i \in [\check{\alpha}_{i,L}(\alpha_j), \alpha_j - (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)]$, where $\alpha_i \leq \alpha_j - (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right)$ is equivalent to $\alpha_j \geq \frac{\alpha_i + \frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}}{1+\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}} = \alpha_j^{SC}(\alpha_i)$, then for firm i, product quality is a strategic complement; second, if instead $\alpha_i \in [\alpha_j - (1-\alpha_j)\left(\frac{c_H\frac{v_L}{v_H}-c_L}{v_H-c_H}\right), \alpha_j)$, product quality is a strategic substitute.

Combining cases (A)-(D), we have that from firm i's perspective product quality is a (weak) strategic complement (substitute) if and only if $\alpha_j \geq \alpha_j^{SC}(\alpha_i)$ ($\alpha_j \leq \alpha_j^{SC}(\alpha_i)$). **Q.E.D.**

Proof of Proposition 5. Note first that $v_1 = v_2 = v_L$ can never be an equilibrium as then firm 2's profits, $(v_L - c_L)\alpha_1$ would fall short of its min-max profit of $(v_H - c_H)\alpha_1$. Take next a candidate equilibrium with choices $v_1 = v_2 = v_H$. Then, firm 1 makes an expected profit of $\pi_{1,HH} = (v_H - c_H)\alpha_1$, while firm 2 is more aggressive and makes an expected profit of $\pi_{2,HH} = (v_H - c_H)\frac{\alpha_1(1-\alpha_1)}{1-\alpha_2}$. If firm 2 deviates to v_L, it remains aggressive and makes an expected profit of

$$\begin{aligned}\pi_{2,LH} &= (v_L - c_L)\alpha_2 + (1-\alpha_1-\alpha_2)(v_H - c_H)\frac{v_L}{v_H}\left(\frac{1-\frac{c_L}{v_L}}{1-\frac{c_H}{v_H}} - \frac{1-\alpha_1}{1-\alpha_2}\right) \\ &= (v_L - c_L)(1-\alpha_1) - (1-\alpha_1-\alpha_2)\frac{1-\alpha_1}{1-\alpha_2}\left(v_L - c_H\frac{v_L}{v_H}\right),\end{aligned}$$

so that $\pi_{2,HH} \geq \pi_{2,LH}$ if and only if

$$\alpha_2 \geq 1 - \alpha_1\left(\frac{(v_H - v_L)(v_H - c_H)}{c_H v_L - c_L v_H}\right) = \hat{\alpha}_{2,H}(\alpha_1).$$

Turning to firm 1, a deviation to v_L is profitable only if

$$\pi_{1,LH} = (v_L - c_L)(1-\alpha_2) - (1-\alpha_1-\alpha_2)\frac{1-\alpha_2}{1-\alpha_1}\left(v_L - c_H\frac{v_L}{v_H}\right) > (v_H - c_H)\alpha_1,$$

which, after multiplying both sides with $\frac{1-\alpha_1}{1-\alpha_2}$, is equivalent to

$$(v_H - c_H)\frac{\alpha_1(1-\alpha_1)}{1-\alpha_2} < (v_L - c_L)(1-\alpha_1) - (1-\alpha_1-\alpha_2)\left(v_L - c_H\frac{v_L}{v_H}\right).$$

That this requirement is stricter than that for firm 2 follows as the term on the LHS is the same as $\pi_{2,HH}$ and the term on the RHS is not larger than $\pi_{2,LH}$.

Take next the candidate equilibrium with $v_1 = v_H$ and $v_2 = v_L$. Since we assumed (without loss of generality) that $\alpha_1 \geq \alpha_2$, we already know that firm 1 can not profitably deviate to v_H. And we have also already established that firm 2 finds it profitable to choose v_L if and only if $\alpha_2 \leq \hat{\alpha}_{2,H}(\alpha_1)$. Taken together, we can support this equilibrium if and only if $\alpha_2 \leq \hat{\alpha}_{2,H}(\alpha_1)$.

Finally, take $v_1 = v_L$ and $v_2 = v_H$. Applying Lemma 2 for $i = 1$ and $j = 2$, thus noting that $\alpha_i \geq \alpha_j$, we know that firm 1 finds it profitable to choose v_L in response to $v_2 = v_H$ if and only if $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$ and $\alpha_2 \leq \underline{\alpha}_{2,H}$. And applying Lemma 1 for $i = 2$ and $j = 1$, now with $\alpha_i \leq \alpha_j$, we know that firm 2 finds it profitable to choose v_H in response to $v_1 = v_L$ if and only if $\alpha_2 \geq \tilde{\alpha}_{2,L}(\alpha_1)$ and $\alpha_1 \in (0, \underline{\alpha}_{1,L}]$, or $\alpha_2 \geq \hat{\alpha}_{2,L}(\alpha_1)$ and $\alpha_1 \in (\underline{\alpha}_{1,L}, \overline{\alpha}_{1,L}]$. From above we also know that the condition $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$ (that is, that firm 1 finds it profitable to choose v_L, given $v_2 = v_H$) is stricter than the condition $\alpha_2 \leq \hat{\alpha}_{2,H}(\alpha_1)$ (that is, firm 2 finds it profitable to choose v_L, given $v_1 = v_H$). In turn, the latter inequality is stricter than $\alpha_2 \leq \hat{\alpha}_{2,L}(\alpha_1)$, as is easy to see. Hence, the constraint $\alpha_2 \geq \hat{\alpha}_{2,L}(\alpha_1)$ for $\alpha_1 \in (\underline{\alpha}_{1,L}, \overline{\alpha}_{1,L}]$ is irrelevant and we can can support the considered equilibrium if and only if $\alpha_1 \leq \tilde{\alpha}_{1,H}(\alpha_2)$ and $\alpha_2 \geq \tilde{\alpha}_{2,L}(\alpha_1)$. **Q.E.D.**

10 Appendix B: Extension to More Firms (and Products)

In this Appendix, we show how the equilibrium characterization (of prices and promotions) extends to more than two firms. We thus take now $I > 2$ firms. Recall next that with two firms only, we had to distinguish between two different cases, depending on which firm was more aggressive (i.e., promoted its product more often, as described by condition (2)). As we noted in the main text, however, depending on firms' loyal shares and the absolute as well as relative strength of their products, with more firms the number of possible cases substantially increases. Still, the characterization always follows the same logic, which we now illustrate for a particular case. We choose symmetric shares $\alpha_i = \alpha < 1/I$. Without loss of generality, we now suppose that firms are ordered such that the respective ratio $\frac{v_i}{c_i}$ is increasing.

Assertion: *With $I > 2$ firms with symmetric loyalty shares, the following constitutes a pricing and promotion equilibrium.*

Case A: *Suppose $\frac{v_I}{c_I} > \frac{v_{I-1}}{c_{I-1}}$ holds strictly. Then i) firms 1 to $I-2$ choose $p_i = v_i$ with probability one, so that they do not promote at all; ii) firm I promotes with probability one and chooses $p \in [\underline{p}_I, v_I)$ according to the CDF*

$$F_I(p) = 1 - \left(\frac{\alpha}{1 - I\alpha}\right)\left(\frac{1 - \frac{p}{v_I}}{\frac{p}{v_I} - \frac{c_{I-1}}{v_{I-1}}}\right);$$

and iii) firm $I-1$ promotes with probability strictly less than one, as it charges the non-discounted price with probability

$$\gamma_{I-1} = 1 - \left(\frac{1 - \frac{c_{I-1}}{v_{I-1}}}{1 - \frac{c_I}{v_I}}\right),$$

and chooses $p \in [\underline{p}_{I-1}, v_{I-1})$ according to the CDF

$$F_{I-1}(p) = 1 - \left(\frac{\alpha}{1 - I\alpha}\right)\left(\frac{1 - \frac{p}{v_{I-1}}}{\frac{p}{v_{I-1}} - \frac{c_I}{v_I}}\right) - \frac{\frac{c_{I-1}}{v_{I-1}} - \frac{c_I}{v_I}}{\frac{p}{v_{I-1}} - \frac{c_I}{v_I}}.$$

Case B: *Suppose that $\frac{v_I}{c_I} = \frac{v_{I-1}}{c_{I-1}}$. Then i) firms 1 to $I-2$ choose $p_i = v_i$ with probability one, so that they do not promote at all and ii) firms I and $I-1$ promote with probability one and choose promotions as follows: Each $i \in \{I-1, I\}$ chooses prices $p \in [\underline{p}_i, v_i]$ according to the CDF*

$$F_i(p) = 1 - \left(\frac{\alpha}{1 - I\alpha}\right)\left(\frac{v_i - p}{p - c_i}\right).$$

Proof. The existence proof extends from the characterization with only two firms as follows. Consider first firms $I-1$ and I. Given that firms 1 to $I-2$ are supposed to choose v_i with probability one and thus do not compete for shoppers, the game between firms $I-1$ and I is essentially that considered with only two firms, albeit there are now only $1-\sum_{i=1}^{I}\alpha_i = 1-I\alpha$ shoppers in the market. The characterization for both cases A and B follows from this observation.

We are thus left with assertion i) for firms 1 to $I-2$, where we need to argue that choosing a strictly lower price is not more profitable. To prove this, note that it is sufficient to show that any firm $k \in \{1, ..., I-2\}$ would not find it profitable to choose a lower price than v_k even if it only needed to beat firm I's price in order to attract the shoppers (ignoring that firm k might still lose vs. firm $I-1$). Then, an upper bound for firm k's deviating profit (for some p) is given by

$$(p-c_k)\left[\alpha+(1-I\alpha)\left(1-F_I\left(p\frac{v_I}{v_k}\right)\right)\right],$$

which, after substituting $F_I(p\frac{v_I}{v_k})$ and simplifying, can be rewritten as

$$\pi_k(p) = \left(\frac{p-c_k}{p\frac{v_{I-1}}{v_k}-c_{I-1}}\right)\alpha(v_{I-1}-c_{I-1}).$$

It is now straightforward to show that the derivative of $\pi_k(p)$ has the same sign as

$$-c_{I-1}+c_k\frac{v_{I-1}}{v_k},$$

which is always non-negative given the way we have ordered firms, i.e., so that the respective ratio $\frac{v_i}{c_i}$ is increasing. Hence, even if firm k only needed to beat firm I in order to attract the shoppers, it would still find it optimal to charge the highest possible price v_k. Clearly, this implies that firm k cannot find it profitable to charge a price lower than v_k if it has to compete against both I and $I-1$, as is the case in the constructed equilibrium. **Q.E.D.**