

# A Corporate Finance Perspective on Environmental Policy\*

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## Abstract

This paper examines optimal environmental policy when external financing is costly for firms. We introduce emission externalities and industry equilibrium in the Holmström and Tirole (1997) model of corporate finance. While a cap-and-trade system optimally governs both firms' abatement activities (internal emission margin) and industry size (external emission margin) when firms have sufficient internal funds, external financing constraints introduce a wedge between these two objectives. When a sector is financially constrained in the aggregate, the optimal cap is strictly above the Pigouvian benchmark and emission allowances should be allocated below market prices. When a sector is not financially constrained in the aggregate, a cap that is below the Pigouvian benchmark optimally shifts market share to less polluting firms and, moreover, there should be no "grandfathering" of emission allowances. With financial constraints and heterogeneity across firms or sectors, a uniform policy, such as a single cap-and-trade system, is typically not optimal.

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# 1 Introduction

Across the globe, governments step up their efforts to combat climate change.<sup>1</sup> While many would argue that what is done is still insufficient, certainly industries will have to undergo radical transformations towards a greener, possibly even decarbonized economy. Imposing prices on firms that truly reflect the social cost of emissions is a major instrument in this endeavour.<sup>2</sup> This will have considerable financial implications. Firms will have to invest in abatement activities and to adopt new sustainable technologies, and they need to shoulder the direct costs of emission taxes or of more expensive emission allowances. Moreover, capital needs to be reshuffled from browner to greener firms and sectors.<sup>3</sup>

To scholars of corporate finance, raising considerable amounts of new capital as well as such reshuffling of resources between firms and industries do not come without frictions. These may arise from informational asymmetries between firm insiders and outside investors or from dilution of insiders' stakes and with this a misalignment of incentives.<sup>4</sup> Such frictions also impose limits on the amount of external financing that a firm can raise. This not only impedes the investment of an individual firm or of a specific industry, but it presents an obstacle also to the reallocation of capital. Such a corporate finance perspective on the green transformation seems, however, to have been overlooked so far - not only in the public debate, but also in scholarly work. There, the implications and optimality of different environmental instruments, such as emission taxes or cap-and-trade regimes, are typically derived without consideration of the associated investment requirements and costly external financing. This observation defines the objective of our research: We set out to examine the interaction of financing frictions, emission externalities, and environmental policy.

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<sup>1</sup>In Europe, the Commission has committed itself to such transformation under the so-called "Green Deal". See also the Glasgow summit's Climate Pact [https://unfccc.int/sites/default/files/resource/cop26\\_auv\\_2f\\_cover\\_decision.pdf](https://unfccc.int/sites/default/files/resource/cop26_auv_2f_cover_decision.pdf)

<sup>2</sup>According to various studies, such a social cost of carbon is several times higher than, for instance, that indicated by the European cap-and-trade policy. While in 2021 that price has reached 30 € per ton of emitted CO<sub>2</sub>, it was constantly below 10 € in all the years leading up to 2018. Based on an extensive review of estimates in the literature, the High-Level Commission on Carbon Prices, led by Joseph Stiglitz and Nicholas Stern, concluded that a range of 40–80 US \$ per ton of CO<sub>2</sub> in 2020, rising to 50–100 US \$ per ton of CO<sub>2</sub> by 2030, would be needed to achieve the objective of the Paris Agreement, i.e., to keep average global warming to below 2 degrees (Stern and Stiglitz 2017, 2021).

<sup>3</sup>Investors recognize these costs for firms and demand a carbon premium (Bolton and Kacperczyk 2021a, 2021b). The immense financing needs, and the question of international burden sharing, are also recognized in the various international treaties and statements, as in the Glasgow Pact of September 2021 ("Adaptation Financing").

<sup>4</sup>See the seminal work by Jensen and Meckling (1976) and Myers and Majluf (1984).

For this we employ the by now standard workhorse model of Holmström and Tirole (1997). There, firms' financing capacity is endogenously limited by the availability of internal funds. To derive welfare implications, we model both the supply and the demand side. All firms are price takers, so that equilibrium prices are determined by either a zero marginal return condition on investment or by binding financial constraints. A cap-and-trade system governs firms' abatement activities, which we refer to as the internal margin, and the size of the industry, which we refer to as the external margin of emissions.<sup>5</sup>

When financing constraints do not bind, which is our benchmark, the first-best outcome is Pigouvian, so that the price of emission allowances is equal to their marginal social costs.<sup>6</sup> When external financing constraints bind at the industry level, the two objectives, that is the efficient choice of abatement activities and of industry size, become conflicting. In our baseline case with a single sector and homogeneous firms, this unambiguously leads to a higher cap on emissions and a corresponding lower price for emissions, strictly below the Pigouvian level. The tension between the two objectives can be mitigated when emission allowances are initially allocated at a price below market value. The recognition of financing constraints, which should be realistic given the enormous challenges and costs that will be imposed on industries, leads thus, already in our baseline model, to two implications that are in stark contrast to much of the received wisdom (cf. however below for a more nuanced picture). First, the optimal emission price is no longer equal to its social cost. Second, how the proceeds of such taxes or of the sale of emission allowances are used is not inconsequential.

When the cap-and-trade system imposes a single price for heterogeneous firms and sectors, this leads to further deviations from standard results. Then, when a sector is not financially constrained in the aggregate, but individual firms are, a cap that is tighter than the Pigouvian benchmark becomes optimal, as this shifts investment and production to less polluting firms. The key mechanism is that a reduction of total output and a resulting increase in its market price alleviate financial constraints for less polluting firms. With heterogeneity, a commonly applied rule of "grandfathering", whereby rights are

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<sup>5</sup>The EU Emissions Trading System (EU ETS) imposes such a cap on CO<sub>2</sub> emissions. While there is no global CO<sub>2</sub> cap in place in the US, allowance trading systems apply to SO<sub>2</sub> and NO<sub>x</sub> under the Acid Rain Programme (<https://www.epa.gov/acidrain/acid-rain-program>). The SO<sub>2</sub> allowance-trading programme, established under the 1990 Clean Air Act Amendments, was the world's first large-scale pollutant cap-and-trade system.

<sup>6</sup>Throughout our analysis we abstract from well-known leakage problems, which lead to a potentially compensating increase of pollution in other jurisdictions. See Harstad (2012) for a proposal that overcomes leakage (namely of an international coalition that generates tradable rights to exploit fossil-fuel deposits).

allocated according to past pollution, is inefficient. Again, under financial constraints the initial allocation of emission rights is thus not innocuous for efficiency, even when there is trading in emission rights, as it now affects the third margin, that governing the allocation of investment and production between low- and high-polluting firms.

We can ultimately combine our insights in a cross-sectoral perspective. A cap-and-trade mechanism that imposes a single price on emissions is typically not efficient under financial constraints. A more targeted policy would impose a regime that is stricter than the Pigouvian benchmark in industries that are not financially constrained in the aggregate, e.g., as the marginal investment is determined by capital-rich "brown" firms. Instead, a regime that leads to an emission price below the Pigouvian benchmark should be imposed in industries that are predominantly populated by financially constrained "green" newcomers.

To the best of our knowledge, the literature on environmental and resource economics has largely ignored the issue that is at the core of our analysis, that is firms' financing constraints.<sup>7</sup> From this perspective, most closely related are Hoffmann et al. (2017), Tirole (2010), and Oehmke and Opp (2020), as they all consider environmental concerns, or externalities more generally, in a framework where firms have limited resources. The analysis of Tirole (2010) focuses on the implications of liability, which is constrained by available resources.<sup>8</sup> Oehmke and Opp (2020) share with us the use of the workhorse model of Holmström and Tirole (1997). Their focus lies, however, not on an analysis of environmental regulation, as they examine how the presence of "green investors" may (optimally) alleviate financing constraints for green investment. Hoffmann et al. (2017) do not rely on the Holmström and Tirole model, but instead on an effort-based agency model. An increase in outside financing consequently affects effort and thus the likelihood of success, but not investment and thus not industry size, which, because of the associated price effect, is one of the key margins that we analyze.

Setting an emission tax equal to marginal social cost or a corresponding cap is the textbook Pigou (1932) solution.<sup>9</sup> While the optimality of such policy has been retrieved

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<sup>7</sup>Capital market imperfections, such as potential short-sightedness of investors, have however been recognized as obstacles to change (e.g., Stern and Stiglitz 2021).

<sup>8</sup>Predecessors of this work are Boyer and Laffont (1997) and Hiriart and Martimort (2006). This relates also to a larger literature on "judgement proofness" in the presence of limited liability (Pitchford 1995).

<sup>9</sup>For evidence of how a carbon tax on fuel reduces energy consumption, emissions, and economic activity see Martin et al. (2014) and Andersson (2019). Känzig (2021) examines the impact of carbon-price changes in EU ETS on broader economic consequences. The practical challenge of a carbon tax lies in determining (or agreeing on) the social costs. Their measurement hinges not only on the respective (climate) modeling assumptions, but crucially on the used social rate of discounting (see, for instance, Inderst et al. 2021 for

also under complex (general equilibrium) scenarios (e.g., Golosov et al. 2014), we acknowledge that the literature has also identified rationales other than financial constraints for when the socially optimal price of emissions differs from their social cost and when a single instrument is not sufficient. One strand of research has focused on the industrial organization of the respective industry. When oligopolistic firms exert market power, so that there is a deadweight loss, the optimal emission tax will typically differ from the Pigouvian benchmark, depending on the industry-wide cost pass-on (Buchanan 1969, Barnett 1980). Carlton and Loury (1980) have shown that when production technologies are non-linear and there is free entry, the single instrument of a Pigouvian tax does not secure the social optimum.<sup>10</sup> Another strand of the literature has focused on interactions with other distortionary taxes (Sandmo 1975) and the non-separability of externalities in consumption (Diamond 1973).<sup>11</sup>

As we have noted above, also for realism we consider a cap-and-trade system. With financial constraints the initial allocation of the respective pollution permits and their initial price are not inconsequential. This finding is different from other reasons discussed in the literature for why the allocation of emission allowances impacts the performance of a cap-and-trade system, such as transaction costs or market power in the permit trading system (Hahn and Stavins 2011). A simple cap-and-trade mechanism also implies a single price of emissions. Recently, Nicholas Stern and Joseph Stiglitz have forcefully argued for an "alternative approach", which notably includes sector-specific responses as well as alternative instruments so as to deal with many real-world complexities, including distributional issues, imperfect markets, or risk and uncertainty.<sup>12</sup> Our contribution speaks also to this broader agenda.

The rest of this paper is organized as follows. In Section 2, we describe our model. In Section 3, we examine the case of one industry sector with homogeneous firms. In Section 4, we introduce heterogeneity. Section 5 concludes.

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a short overview).

<sup>10</sup>As noted above, this is different in our setting without financial constraints, as for any given output externality damages are independent of the scale of individual firms.

<sup>11</sup>On the interference with other taxes see also the overview in Bovenberg and Goulder (2002). With non-separability there is an indirect effect from the change in consumption of the externality-inducing good. We note that the recent public financing and taxation literature has focussed on non-linear taxes (e.g., in a mixed-taxation problem with respect to income in Cremer et al. 1998 and with respect to the taxation of externalities in Hoffmann et al. 2017). Our focus on cap-and-trade systems excludes such non-linearities.

<sup>12</sup>Stiglitz (2019) and Stern and Stiglitz (2021). In the most simple framework, instead, a single price is a prerequisite for efficiency (Diamond and Mirrlees 1971).

## 2 The model with homogenous firms

We presently consider a single sector of the economy. This consists of a mass one of firms, each endowed with initial funds of size  $A$ . For our analysis it is inconsequential whether these are on-going concerns with internal funds or whether these are potential entrepreneurs with respective own wealth. Also, it is inconsequential for our present results that these potential owner-managers are all endowed with the same size of funds, as we explain below. Each firm, indexed by  $i$ , has access to the same scalable investment technology. We denote the investment size of firm  $i$  by  $I_i$ , so that total investment is  $I = \int_0^1 I_i di$ .

Owner-managers can raise additional funds from households for whom the (marginal) alternative is to store their respective funds. This normalizes the required return to zero. To receive external funds (the size of which we still need to specify), the firm promises a repayment of  $D_i$ . The firm will only be able to honor its obligation in case of success.<sup>13</sup> It is here that we bring in an agency problem of external financing, thereby adopting the workhorse model of Holmström and Tirole (1997), as follows:

The owner-manager needs to monitor the investment technology so as to increase the likelihood of success. To simplify the subsequently derived expressions we suppose that in case of such monitoring, the venture succeeds with probability one. If the owner-manager shirks on monitoring, which is unobservable to outsiders, she obtains a private benefit per unit of investment  $b > 0$ , but with probability  $q > 0$  the technology fails and returns no output. Output in terms of produced quantity is denoted by  $x_i$ , where for simplicity we set  $x_i = I_i$  in case of success, while  $x_i = 0$  holds in case of failure. The firm's output  $x_i$  is sold in the market, where the price depends on all firms' output,  $x = \int_0^1 x_i di$ , and is given by the inverse demand  $P(x)$  with  $P' < 0$ . Aggregation across all firms is possible as we presently assume that they all operate in the same sector. As each firm has mass zero, its own output does not affect the market price, so that a firm has no strategic incentives to withhold output from the market. In the characterized equilibrium, where each owner-manager has sufficient incentives for monitoring and investments are thus successful, we have  $x = I$ . We also note that while each firm's investment will always be capped by external financing constraints, even when owner-managers' funds are large in the aggregate, the size of investment in the sector will be constrained by the downward

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<sup>13</sup>It is well-known (and straightforward to show) that we can without loss of generality restrict attention to the case where the owner-manager co-finances the investment with all her funds  $A$ , so that, protected by limited liability, she can only repay out of the firm's proceeds (rather than retaining a fraction of  $A$  from which repayments can be made as well).

sloping inverse demand  $P(x)$ .

Output generates negative externalities on society, which firms can reduce through costly investments in abatement activities. If no abatement activities are undertaken, emissions generated per-unit of output are  $y > 0$  (measured in the respective unit, such as tons of CO2 equivalent). A firm can reduce this by  $s_i$ , which incurs the per-unit cost  $c(s_i)$ , where  $c(0) = c'(0) = 0$  and  $c''(s_i) > 0$  with  $c'(y) = \infty$ . Given output of  $x_i$  and abatement activity of  $s_i$ , a firm thus produces total emissions  $e_i = (y - s_i)x_i$  and incurs abatement costs  $c(s_i)x_i$ . We suppose that social costs of emissions can be monetized and are given by  $v > 0$ . Aggregate social cost of emissions are thus  $ve$  with  $e = \int_i e_i di$ . They are sufficiently diffused so that an individual owner-manager has no private incentives to reduce own externalities.

**A cap-and-trade mechanism.** Our focus lies on the immense financial burden generated by the green transformation. Here, the decarbonization of the economy is at the forefront, and with it the reduction of CO2 emissions. We consider as an environmental instrument a cap-and-trade system, such as the EU Emissions Trading System (EU ETS). There, a cap is set on the total amount that can be emitted by installations covered by the system in a given year. Within the cap, companies receive or buy emission allowances, which they can trade with one another as needed.<sup>14</sup> We thus let the social planner choose a cap  $K$  on total emissions, so that  $e \leq K$ , which will give rise to a market clearing price  $\tau$  per emission allowance. For the moment we specify that emission allowances will be sold to firms, and that the respective proceeds are distributed to households in a way so that there is an at most negligible flow-back of such funds to entrepreneurs in the considered sector. We later discuss other ways to initially allocate emission rights. We note that in our setting the choice of a cap-and-trading system is equivalent to that of a uniform emission tax (equivalent to the clearing price  $\tau$ ).<sup>15</sup>

We conclude the exposition of the model by summing up firms' total financing needs. Firm  $i$  needs funds for its productive investment  $I_i$  and, depending on output and resulting emissions, it must spend, in addition,  $\tau(y - s_i)x_i$  on pollution rights and incurs abatement

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<sup>14</sup>Under this system they can also buy limited amounts of international credits from emission-saving projects around the world.

<sup>15</sup>Such an equivalence was first shown formally in Montgomery (1972). It is however also well known that such an equivalence does not hold generally e.g., when a social planner faces uncertainty about damages and firms' emission control costs (Weitzman 1974). Recently, the "prices vs. quantity" debate has shifted to questions of implementability under (voluntary) international agreements (e.g., Mideksa and Weitzman 2018; Harstad et al. 2021).

costs  $c(s_i)x_i$ . If the owner-manager monitors, so that  $x_i = I_i$ , this amounts to total funding needs of  $F_i = I_i[1 + z(s_i)]$  with

$$z(s_i) = \tau(y - s_i) + c(s_i),$$

of which the fraction  $F_i - A$  needs to be raised externally. We note that  $z(s_i)$  captures, depending on abatement activity  $s_i$ , the firm's private marginal cost of emissions (per unit of output). Finally, we invoke the following parameter restrictions. With

$$P(0) > 1 + \min_s [v(y - s) + c(s)] \tag{1}$$

we ensure that it is socially optimal that production takes place, as the marginal consumer surplus for the first unit of investment and quantity exceeds marginal social costs. Furthermore, a sufficient condition for that monitoring is always uniquely optimal, irrespective of the imposed environmental restriction, is that

$$b < q^2. \tag{2}$$

### 3 The case of homogenous firms

#### 3.1 First-best benchmark

We start our analysis by considering as a benchmark the outcome of a social planner's problem. The planner's objective is the (unweighted) sum of households' welfare. She can control individual investment and monitoring decisions, and for the respective investment she can also freely (re-)allocate funds within the economy. Note first that given the linearity of investment technology, we need not specify how the planner allocates investment across firms: All that matters is aggregate productive investment  $I$ .<sup>16</sup> Also, the planner needs to determine only a single level of abatement activity  $s$ .

We assume first that the planner wants to induce monitoring. Under this assumption, given her choices of  $I$  and  $s$ , total welfare is given by

$$\Omega = \int_0^I P(\hat{I})d\hat{I} - I[1 + v(y - s) + c(s)], \tag{3}$$

which equals consumer surplus in the first term, representing the "triangle" under inverse demand from zero to the total output  $x = I$ , minus total social costs of production, which

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<sup>16</sup>This would be different if there were fixed costs involved either in productive investment or in the abatement technology.

include investment costs and costs related to emissions and their avoidance (using again that  $x = I$  under monitoring). Maximization of  $\Omega$  yields the first-best level of abatement activity  $s_{FB}$  from  $c'(s_{FB}) = v$ . The first-best scale of productive investment depends on consumers' willingness-to-pay, as captured by inverse demand, and externalities. When interior, it solves  $P(I_{FB}) = 1 + v(y - s_{FB}) + c(s_{FB})$ . From (1) such an interior solution exists.

Summarizing, when the social planner wants monitoring to be undertaken and when she wants the considered sector to operate, the first-best outcome sets marginal abatement costs equal to the costs of the externality,  $c'(s_{FB}) = v$ , and it sets industry size so that consumers' marginal willingness-to-pay at the realized output,  $P(I_{FB})$ , equals marginal costs, which comprise costs of investment and social costs of externalities (including those spent on their reduction),  $1 + v(y - s_{FB}) + c(s_{FB})$ . We show in the proof that from (2) monitoring is always socially optimal.

**Lemma 1** *An unconstrained social planner maximizing aggregate welfare would implement monitoring, an abatement activity  $s_{FB}$  as given by  $c'(s_{FB}) = v$ , and industry size  $I_{FB} > 0$  as given by  $P(I_{FB}) = 1 + v(y - s_{FB}) + c(s_{FB})$ .*

### 3.2 Firms' problem

To derive the market equilibrium, we first set up the contracting problem between a firm's owner-manager and outside investors. While most of this is standard, the precise timing of the financing of abatement costs and emission rights needs to be tied down. More specifically, we need to stipulate when the respective environmental costs are incurred. We find it realistic that the respective costs need to be incurred before output is sold on the market, though they are only realized when the venture is successful and output is indeed realized. When the owner-manager monitors, the venture is successful with probability one,  $x_i = I_i$ . Consequently, the additional financing that the firm must raise to cover pollution taxes and abatement costs is then used up for sure. It is only off-equilibrium, when the owner-manager shirks, that with positive probability these funds are not used. We suppose for this case that investors can secure repayment of these funds, so they cannot be diverted privately by the owner-manager.

In what follows, we first suppose that the financial contract with investors needs to prevent the owner-manager from shirking. In case of success and thus with  $x_i = I_i$ , the

owner-manager realizes  $I_i P(I)$  minus the promised repayment  $D_i$ .<sup>17</sup> This uses that costs related to emissions and their avoidance have already been paid out of raised funds. In case of failure, no output is realized and the owner-manager receives no monetary return. Recall that when shirking, failure occurs with probability  $q$  and that the owner-manager realizes private benefits  $bI_i$ . To ensure that the owner-manager monitors the technology, comparing the respective payoff to that under shirking, the following incentive constraint must thus be satisfied:

$$I_i P(I) - D_i \geq (1 - q)[I_i P(I) - D_i] + bI_i,$$

which can be transformed to

$$D_i \leq I_i [P(I) - b/q]. \quad (4)$$

Condition (4) places restrictions on the external financing capacity of an individual firm. This is limited by the agency problem, as expressed by the subtraction of  $b/q$  from the product price. We return to an interpretation below.

We turn now to investors' break-even constraint. Recall that we assumed that households have abundant funds and otherwise only access to a storage technology with zero return. If the incentive constraint is satisfied, so that the owner-manager monitors, investors anticipate to be repaid for sure. Hence,  $D_i$  must at least equal the raised external funds  $F_i - A$ :  $D_i \geq F_i - A$ . Substituting for total funding requirement  $F_i = I_i[1 + z(s_i)]$ , we obtain the break-even constraint

$$D_i \geq I_i[1 + z(s_i)] - A. \quad (5)$$

As the owner-manager is the residual claimant, by optimality the break-even constraint binds. After substitution for  $D_i$  from the thus binding constraint (5), the owner-manager's objective becomes

$$U_i = I_i[P(I) - 1 - z(s_i)], \quad (6)$$

which is just a formal restatement of her position as the residual claimant: Per unit of investment she realizes the full net marginal return  $P(I) - 1 - z(s_i)$ .

We now return to the incentive constraint (4). There, we substitute from the binding break-even constraint  $D_i = I_i[1 + z(s_i)] - A$ , which allows to rewrite (4) as

$$I_i \leq \frac{1}{1 + b/q - P(I) + z(s_i)} A. \quad (7)$$

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<sup>17</sup>Note again that from the perspective of each individual firm (of mass zero), the per-unit price of output  $P(I)$  is unaffected by its own decisions.

Hence, if  $1 + b/q - P(I) + z(s_i) > 0$ , the maximally feasible size of the productive investment is constrained: It is equal to a given factor times the owner-manager's own funds  $A$ . This is the gist of the agency problem in the Holmström-Tirole workhorse model: It generates binding funding constraints in a tractable way, with a particularly simple (linear) relationship between a firm's own (internal) funds and total feasible investment size. It is useful to note that in equilibrium, each individual firm will always be financially constrained, i.e., that  $1 + b/q - P(I) + z(s_i) > 0$ , even when the industry in the aggregate is not financially constrained. This follows as in our model the marginal return depends on aggregate investment and is strictly decreasing in  $P(I)$ .

The owner-manager chooses the two controls,  $s_i$  and  $I_i$ , to maximize her payoff  $U_i$  in (6). Minimization of  $z(s_i)$  yields the first-order condition  $c'(s_i) = \tau$ . In contrast to the planner's problem, this equates marginal abatement costs to private costs of emissions, as represented by the price of emission rights. As the optimal  $s_i$  does not depend on other parameters and is thus also homogeneous across firms, we denote it by  $s^*$  (so that  $c'(s^*) = \tau$ ). With this the constant marginal return from investment, as obtained from differentiating  $U_i$  with respect to  $I_i$ , is thus  $P(I) - 1 - z(s^*)$ . The optimal choice of  $I_i$  depends on the sign of this expression. Notably when  $P(I) - 1 - z(s^*) > 0$ , the owner-manager would like to scale up as much as possible, i.e., until the incentive constraint (7) holds with equality.

**Lemma 2** *Suppose that the financial contract with outside investors needs to ensure that the owner-manager does not shirk. Then, the owner-manager optimally chooses the abatement level  $c'(s^*) = \tau$ , where  $\tau$  equals the price of emission rights, and an investment size  $I_i$  such that  $I_i = 0$  when  $P(I) - 1 - z(s^*) < 0$  and*

$$I_i = \frac{1}{1 + b/q - P(I) + z(s^*)} A \quad (8)$$

*when  $P(I) - 1 - z(s^*) > 0$ . When  $P(I) - 1 - z(s^*) = 0$ , the owner-manager is indifferent between any feasible size from zero to (8).*

### 3.3 Market equilibrium

We first still work under the assumption that all owner-investors' contracts incentivize monitoring. In the proof of the subsequent Proposition 1 we show that from (2) monitoring is indeed incentivized in equilibrium.

Recall that  $I = \int_0^1 I_i di$ . In what follows we need to distinguish between the case where financing constraints bind in the aggregate and when this is not the case. When all firms

are levered up maximally, using (8) and that there is a mass one of owner-managers, the equilibrium investment level solves the following fixed point problem. For this we denote the respective financially constrained outcome by  $I_{con}$ , which from (8) solves

$$I_{con} = \frac{1}{1 + b/q - P(I_{con}) + z(s^*)} A. \quad (9)$$

We note that this has a unique solution  $I_{con} > 0$  as long as  $1 + b/q - P(0) + z(s^*) > 0$ . Otherwise, it is unbounded, and for this case we set  $I_{con} = \infty$  (which will however not arise in equilibrium). When from the declining  $P(I)$  the marginal return to investment would, however, be negative at such a high level of investment and output  $I = I_{con}$ , the industry is in fact not financially constrained in the aggregate, and the equilibrium level of investment is pinned down by the standard condition that the marginal net return is just zero. We denote the unconstrained equilibrium investment level by  $I_{uc}$ , solving

$$P(I_{uc}) - 1 - z(s^*) = 0. \quad (10)$$

This resembles the condition of the social planner, with the difference, that firms only take into account private costs of emissions rather than social costs.

Each firm takes the price of emission rights  $\tau$  as a given: By the atomistic structure, no firm can affect the market price. Now, the social planner sets a fixed quantity  $K$  that can be traded. Recall that we presently stipulate that firms must acquire the respective rights. At the equilibrium price  $\tau$ , demand is equal to the fixed supply  $K$ . In the subsequent proof we establish that, for given  $K$ , there is indeed a unique such equilibrium price.

**Proposition 1** *Let the social planner choose a cap  $K > 0$  on emissions. Unless  $K$  is so high that it does not bind, there exists a unique combined equilibrium in the productive sector and the emissions trading market. Emission rights trade at a unique price  $\tau = \rho(K)$ , which is a continuous and strictly decreasing function of  $K$ . Firms, who act as price takers both in the goods market and the emissions trading market, choose a unique abatement level  $s^*$ , as given by  $c'(s^*) = \tau$ . When the industry's internal funds are sufficiently large with*

$$A \geq \frac{b}{q} I_{uc}, \quad (11)$$

*where  $I_{uc}$  uniquely solves (10), financial constraints do not bind in the aggregate and  $I^* = I_{uc}$ . Otherwise, the size of industry investment and output is restricted by financial constraints, with  $I^* = I_{con}$ , solving (9).*

We briefly discuss condition (11). Obviously, firms' inside funds  $A$  are a key determinant of whether the industry as a whole is financially constrained. Also, we may interpret  $b/q$  as a combined parameter capturing the severity of the incentive problem. The unconstrained industry size  $I_{uc}$ , as given by (10), only depends on the return without shirking. This is lower as the cap decreases, so that the price for emissions  $\tau$  and with it  $z(s^*)$  increase.

### 3.4 Optimal environmental policy

We return to the problem of a social planner, but now, in difference to the first-best scenario, we constrain the available instruments. Specifically, we presently allow the planner to determine only the cap  $K$ . Again, we make the social planner a utilitarian, so that, given the thereby induced market outcome  $(s^*, I^*)$ , the planner's program is

$$\max_{\tau} \Omega = \int_0^{I^*} P(\hat{I}) d\hat{I} - I^*[1 + v(y - s^*) + c(s^*)]. \quad (12)$$

We note that the induced emissions trading price does not enter the welfare function directly as it merely constitutes a transfer.

**Starting from the laissez-faire benchmark.** Consider first a laissez-faire policy where there is no cap (or  $K$  is chosen so high that it does not bind), so that  $\tau = 0$ . Evidently, from  $s^* = 0 < s_{FB}$  this leads to inefficiently low abatement. To assess the size of the polluting industry we have to change the benchmark, as the first-best size  $I_{FB}$  is calculated based on higher abatement activity  $s_{FB}$  and thus lower per-unit emissions. For given  $s^*$  and thus emissions  $y - s^*$  per unit of output, the (second-best) efficient industry size is denoted by  $I_{SB}(s^*)$  and solves

$$P(I_{SB}(s^*)) = 1 + v(y - s^*) + c(s^*).$$

At  $\tau = 0$  and thus  $s^* = 0$ , compared to this second-best benchmark, industry size becomes inefficiently large when  $I^*(\tau = 0) > I_{SB}(s^* = 0)$ , which transforms to  $P(I^*) - 1 < vy$ : The industry is inefficiently large when the marginal market return, calculated from marginal consumer welfare minus investment cost, falls below marginal social cost. This is evidently always the case when the size of the industry is not determined by financial constraints, as then, with  $I^* = I_{uc}$ ,  $P(I^*) - 1 = 0 < vy$ . In this case, under the laissez-faire benchmark, the industry not only pollutes too much per unit of output, but it is inefficiently large. Emissions are thus too large on both the internal and the external

margin, and this is why starting to impose a binding cap on emissions improves welfare on both accounts, through raising  $s^*$  and lowering  $I^*$ .

This is still the case when  $I^*$  is determined by financial constraints as long as  $I_{con}(\tau = 0) > I_{SB}(s^* = 0)$  or, likewise,  $P(I_{con}) - 1 < vy$ . Instead, when the industry is severely financially constrained, its size may be below the second-best efficient level even when there is no binding cap on emissions. In this case, while the social planner would like to increase abatement activity,  $s^*$ , the ensuing reduction in productive investment and output reduces welfare. Hence, there is a conflict along the two margins. Such a conflict may eventually arise even when at the laissez-faire benchmark the industry is still inefficiently large. As we tighten emission capacity, this makes abatement more efficient, but from a certain point onwards industry size may become (second-best) inefficiently low. From that point onwards, the two objectives of inducing the efficient abatement level and the efficient industry size become again conflicting.

**Deriving the optimal policy.** These observations suggest that to derive the optimal environmental policy, we need to distinguish between different cases. In one case, financing constraints are ultimately not of relevance. Intuitively, this case applies when financing constraints do not bind at the Pigouvian level, i.e., when the cap  $K$  is chosen so that the price of emissions is equal to social costs,  $\tau = v$ . It can be immediately confirmed that this policy then leads to the first-best outcome, with  $s^* = s_{FB}$  and  $I^* = I_{uc}(\tau = v) = I_{FB}$ . In this case the single tool of a cap on emissions is sufficient to implement efficient abatement activities and the efficient size of the industry. But when financial constraints become relevant also in the aggregate, at the Pigouvian benchmark the level of productive investment and output is too low. The optimal environmental policy is then less stringent and lies below the Pigouvian benchmark.

**Proposition 2** *We determine the optimal cap on emissions  $K > 0$  imposed by a social planner who has only this instrument at her disposal and maximizes aggregate welfare. When the industry's internal funds are sufficiently large so that (11) holds for  $I_{uc}$  determined at the Pigouvian level ( $\tau = v$ ), the social planner tightens  $K$  sufficiently to achieve the first-best outcome. Otherwise, the social planner faces a trade-off between efficient abatement activity and industry size. The optimal cap  $K$  is then strictly higher, and the implied price of emissions is below the Pigouvian level,  $\tau < v$ .*

**Discussion.** We briefly comment on the key role of financial constraints in Proposition 2 (the proof of which is contained in the Appendix). For this we note that there could also be other reasons for why, even without environmental regulation, the social return on output would be strictly positive in equilibrium. For instance, installed capacity could be fixed at least in the short and medium term, e.g., due to large lump-sum investment requirements. If we decreased the cap  $K$  in this case, firms would adjust abatement activities sufficiently so as to still fully utilize their limited productive capacity and stay within the cap.<sup>18</sup> In other words, there would not be a trade-off along the internal and the external margins. The case with financial constraints is however different. There, abatement uses up financial slack and thus reduces productive investment and output. When firms' internal funds are low, the single instrument is then not sufficient to obtain the desired first-best outcome of efficient abatement and an efficient size of the polluting industry. We show below that when this is the case, the outcome can be improved when emission rights are allocated below market price (or for free), albeit then the cap is still below the Pigouvian level.

When we consider ongoing concerns, internal funds  $A$  may represent current or (saved) past cash flows, or (internal) collateral from previous investments. While our essentially static set-up abstracts from this, internal funds and such collateral may already be encumbered by outstanding external (debt) financing. What ultimately determines the extent of financial constraints is then the respective net position.<sup>19</sup> Whether individual firms or the industry as a whole are financially constrained may be learnt from business insights, but also from the various indicators developed in an extensive empirical literature in corporate finance (e.g., Kaplan and Zingales 1997; Farre-Mensa and Ljungqvist 2016).

Recall now that the economy is made up by firm owners and other households whose savings could either be allocated to the considered sector or saved in a storage technology. We did not make precise the total available funds in the economy, but supposed that these were sufficient to cover the always limited equilibrium investment size  $I^*$ . If this was feasible and if the social planner cared only about aggregate welfare (and not about its distribution), she could tax households' investment in the storage technology and allocate these funds to the financially constrained sector. Together with setting the Pigouvian cap this would allow to achieve the first-best outcome in terms of total welfare. In what follows, we do not consider such far-reaching transfers of wealth between households. As a

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<sup>18</sup>More precisely, over the considered range,  $\tau$  would adjust to induce a sufficiently high response in  $s^*$ .

<sup>19</sup>When firms are heterogeneous in their financial slack, but also in their emissions (see below), mergers may prove socially beneficial also from an environmental policy perspective (cf. Inderst and Müller 2003 for a model of firm mergers with endogenous financial constraints).

defense for this restriction, households may veto such a policy. We acknowledge, however, that such a defense opens up a discussion of the appropriate social choice function, which we consider to be beyond the scope of this contribution.<sup>20</sup>

### 3.5 Allocating emission rights

So far we have assumed that polluters must buy emission rights at the market price that results from the imposed cap. We stipulated that the respective proceeds are used by the social planner in a way that the impact on firms in the polluting sector are negligible. We now suppose instead that the respective rights are initially allocated for free. If this is based on a measure of previous pollution, this is frequently referred to as "grandfathering". As we will see, this has implications for investment and welfare, but also for the optimal environmental policy, though only when otherwise financial constraints bind in the aggregate.

We thus suppose now that emission rights equalling  $K$  are initially allocated uniformly and for free. We can still determine a (shadow) price  $\tau$ , at which each firm could acquire or sell emission rights. Intuitively, when firms do not need to pay for the initial allocation of emission rights, their pollution-related expenditures comprise only their abatement costs  $c(s^*)$ . When this replaces  $z(s^*)$  in expression (9), we obtain, in the aggregate,  $I_{con,free}$  from the fixed point problem

$$I_{con,free} = \frac{1}{1 + b/q - P(I_{con,free}) + c(s^*)} A. \quad (13)$$

We relegate a more detailed derivation to the proof of Proposition 3. It is easily checked that  $I_{con,free} > I_{con}$ .<sup>21</sup> Allocating emission rights for free thus alleviates the financing constraint and, when this binds, increases industry size. In our model, such a policy of "grandfathering" is equivalent to the imposition of an emission tax, the proceeds of which are then redistributed uniformly to the polluting industry (i.e., in particular not contingent on individual firms' investment or emissions).<sup>22</sup>

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<sup>20</sup>We note however that in the considered economy, despite risk-neutral preferences, utility is not generally transferable (i.e., without changing aggregate welfare).

<sup>21</sup>To see this formally, we rewrite this (with  $I^* = I_{con}$ ) as

$$I^*[1 + b/q - P(I^*) + z(s^*)] = A + (y - s^*)\tau I^*,$$

where the left-hand side is strictly increasing in  $I^*$  and the left-hand side is, for each  $I^*$ , higher from the additional term  $(y - s^*)\tau I^*$  (provided that the cap binds with  $\tau > 0$ ).

<sup>22</sup>To see this, denote such a transfer by  $T_i$ . Even when this is obtained only after taxes have been collected, the firm could sell these (certain) claims up-front, and we can suppose conveniently that its

We know that industry size may be (second-best) inefficiently high or low. Whether allocating emission rights for free increases efficiency is thus a priori unclear. However, as the financing constraint is relaxed, the social planner optimally reduces the cap.

To characterize the (optimal) outcome, we also need to consider the unconstrained industry level. Though emission rights are now allocated for free, the unconstrained size of the industry is still determined by the true costs of a marginal expansion, i.e., including the (shadow) price of emissions, such that still  $P(I_{uc}) - 1 - z(s^*) = 0$ , as in (10). When financing constraints do not bind in the aggregate, the (shadow) price of emissions is in fact unaffected by whether the initial rights are allocated for free or not. But this is different when financial constraints bind in the aggregate, as stated next (and proved in the Appendix):

**Proposition 3** *For a given cap  $K$ , when the industry is not financially constrained, a free allocation of emissions leaves unchanged the (shadow) price of emission rights  $\tau$  as well as the industry's size  $I^*$  and the abatement level  $s^*$ . When the industry is financially constrained, a free allocation of emission rights instead leads, for a given cap  $K$ , to a strictly higher (shadow) price of emission rights, a strictly higher abatement level  $s^*$ , and a strictly larger industry size  $I^*$ . If the industry is financially constrained when emission rights are initially sold at the market price, a free allocation, together with a possibly reduced cap, strictly increase welfare.*

In our model, the initial allocation of emission rights has real consequences for output and efficiency, as well as for the socially optimal cap, but only when the industry is financially constrained. While we have only compared the two scenarios where emission rights are initially allocated to polluters for free or at market prices, this insight clearly holds more generally. For instance, emission rights could be initially owned by other parties in the economy, or the initial allocation could be at fraction of the market price. In our model, if the cap  $K$  is optimally adjusted, welfare is always highest when polluters do not have to pay for the initial allocation. We note, however, that this abstracts from distributional implications as well as from positive benefits that could result from an

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initial funds are thus  $A + T_i$ . Extending (8) and aggregating over the industry obtains

$$I^* = \int I_i di = \frac{1}{1 + b/q - P(I^*) + z(s^*)} (A + \int T_i di),$$

where we can substitute  $\int T_i di = (y - s^*)\tau \int I_i di = (y - s^*)\tau I^*$ . Solving for  $I^*$  obtains (13).

alternative use of the proceeds from selling emission rights (e.g., as raising such financing by other taxes would lead to different distortions).<sup>23</sup>

**Reconsidering the social planner’s problem.** We ask now the question whether the social planner could do better when she directly controlled industry size and abatement activity, while, however, being also constrained, first, by owner-managers’ incentive problem and, second, the aggregate financing constraint within the industry. The answer is no.

The intuition is as follows. When the social planner dictates abatement activities  $s$ , this pins down the necessary per-unit abatement costs  $c(s)$  and with it the maximum size of the industry. Formally, suppose the social planner would directly implement for each firm the level  $s_{dir}$ . When financially constrained, by our preceding observations each firm could thus lever up until

$$I_i = \frac{1}{1 + b/q - P(I_{dir}) + c(s_{dir})} A.$$

Aggregating over all firms thus indeed retrieves the fixed point problem (13). The only additional degree of freedom is thus to cap the size of the industry, which is, however, of no relevance as at the optimal choice of the cap the industry is never (second-best) too large (as otherwise a marginal reduction of  $K$  would improve welfare). We have thus arrived at the following result.

**Proposition 4** *Suppose the social planner could determine directly both abatement activities and investment,  $(s_i, I_i)$ , but that she is constrained by the incentive problem of owner-managers and outside investors’ break-even constraint. Then welfare is not higher than with the hitherto considered instrument of a cap on emissions.*

**Sharing the costs and benefits of stricter environmental policy.** A reduction of the cap induces a higher price for emission rights. When emission rights need to be purchased at market prices initially, this represents a direct financial burden to the industry. When initial rights are allocated for free, firms still incur higher costs for abatement, and also then output shrinks and with it (gross) consumer surplus. Consumers may, however, be burdened further if stricter environmental policy, combined with financial constraints, increase firms’ market power. We explore this next.

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<sup>23</sup>On the other hand, we do not consider other reasons for why a free allocation or the refunding of environmental taxes could improve efficiency (e.g., to counterweight deadweight losses due to imperfect competition in the product market; cf. Gersbach and Requate 2004).

We take the case where firms need to pay for emission rights, so that industry profits are  $I^*[P(I^*) - 1 - z(s^*)]$ .<sup>24</sup> When financial constraints are not binding, we have  $P(I^*) - 1 - z(s^*) = 0$ , and firms make zero profits. Consumers are, however, affected both by the reduction in gross consumer surplus when  $I^*$  decreases, which is  $dI^* \cdot P'(I^*)$ , and by the increase in the product price,  $dP \cdot I^*$ . The latter represents the full pass-through of firms' environmental costs, including their purchases of emission rights  $K \cdot \tau$ . When financial constraints bind, firms make positive profits, given that then  $P(I^*) - 1 - z(s^*) > 0$ . This represents an additional detriment to consumers. In particular, when financial constraints bind only after the introduction of environmental policy, firms strictly benefit, as otherwise perfect competition erodes any positive margin. In this case, stricter environmental policy, together with financial constraints, allow firms to jointly reduce output and to generate positive profits.

## 4 Firm heterogeneity as an additional margin of optimal environmental policy

We now introduce heterogeneity in firms' pollution technologies. For reason of greater transparency, we consider thus two types  $y_l < y_h$ , with  $\mu$  denoting the fraction of low-polluting firms  $y_l$ . The first-best allocation would entrust production only to such firms. We still restrict consideration to the case where the social planner has access only to a limited set of instruments. At first, we only consider a cap on total emission, which gives rise to a single market price for emissions.

In the main text, we further restrict attention to the case where the industry is not financially constrained in the aggregate, which is where we find qualitative differences to the preceding analysis with homogeneous firms. As long as  $\tau > 0$ , high-polluting firms then represent the external margin of the industry, with  $P(I^*) - 1 - z(y_h, s^*) = 0$ , where again  $c'(s^*) = \tau$  and where the function  $z$  now makes explicit the dependency also on a firm's type. From  $y_l < y_h$ , the per-unit net profit of investment is then strictly positive for low-polluting firms:  $P(I^*) - 1 - z(y_l, s^*) = \tau \Delta_y$  with  $\Delta_y = y_h - y_l$ .

With heterogeneity, the social planner's objective becomes

$$\Omega = \int_0^{I^*} P(\hat{I}) d\hat{I} - I^*[1 + v(y_h - s^*) + c(s^*)] + I_l^* v \Delta_y, \quad (14)$$

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<sup>24</sup>Firms' profits are strictly higher when they do not have to pay for the initial allocation of emissions. In this case, they are given by  $I^*[P(I^*) - 1 - c(s^*)]$ .

where the last term "corrects" for the lower emissions of the low-polluting firms. This makes transparent the additional margin that now affects welfare, namely how total production  $q^* = I^*$  is split up between low-polluting firms,  $I_l^*$ , and high-polluting firms,  $I_h^* = I^* - I_l^*$ .

Given their advantage when  $\tau > 0$ , low-polluting firms lever up maximally:

$$I_l = \frac{1}{1 + b/q - P(I^*) + z(y_l, s^*)} A. \quad (15)$$

Aggregating over all low-polluting firms (of mass  $\mu$ ) and substituting  $P(I^*) - 1 - z(y_l, s^*) = \tau \Delta_y$  in (15), we obtain

$$I_l^* = \mu \frac{1}{b/q - \tau \Delta_y} A. \quad (16)$$

We recall that expression (16) is obtained under our present assumption that the industry is not financially constrained in the aggregate, which is why high-polluting firms make a zero return on investment (and  $I_h^*$  is consequently pinned down as the residuum). As the cap on total emissions decreases and the market price for emissions increases, total industry output decreases, but investment and output of low-polluting firms increase from (16). While all firms need to pay a higher price per emission, which affects negatively their capacity for investment, for low-polluting firms this is more than compensated by the higher product price  $P(I^*)$ , which results from the reduction in total output.

Observe now that when we marginally reduce the cap at the Pigouvian level, by definition the first-order effect of the reduction in both consumer welfare and abatement activity is zero. As this shifts investment and output to low-polluting firms by (16), the overall effect on social welfare is strictly positive.

**Proposition 5** *Consider still a single sector composed of low- and high-polluting firms,  $y_l < y_h$ . Then, welfare depends also on how investment and output are distributed between the two types of firms. When the industry is not financially constrained in the aggregate, the social planner can shift investment and output to low-polluting firms by tightening  $K$  and thereby raising the implied price of emissions. This makes it optimal to set  $K$  below the Pigouvian benchmark and with it  $\tau > v$ .*

We recognize that Proposition 5 together with Proposition 3 give different guidance. In both cases the social optimum is different from the Pigouvian benchmark, but one result advocates for a softer and one for a stricter environmental policy regime. These two results are, however, not contradictory. As we show also in the proof of Proposition 5, when

financial constraints are sufficiently severe, then the cap should optimally be set above the Pigouvian benchmark even with heterogeneous firms. The guidance thus depends still on whether financial constraints are severe in the aggregate, but now, in addition, also on potential heterogeneity within a given sector. We discuss below that circumstances may indeed differ considerably between sectors, e.g., as one sector may be composed largely of capital-rich "brown" incumbents, while an innovative sector may consist mainly, if not exclusively, of green newcomers.

Before we turn to such a cross-sectoral analysis, we again take up the issue of how emission rights should be allocated initially. With homogeneous firms and when financial constraints do not bind in the aggregate, we recovered the standard wisdom that the initial allocation of rights does not affect efficiency and with it the optimal choice of the (Pigouvian) cap on emissions. This is different with firm heterogeneity. We show this by considering the common, but as we show inefficient, practice of "grandfathering", whereby emissions rights are allocated based on past emissions. In our static model, we capture this as follows. There is still a cap  $K$  on total emissions and with it a single price of emissions  $\tau$ . Within each group of low- and high-polluting firms, the social planner now essentially channels back the respective expenditures. In equilibrium, none of the low- or high-polluting firms thus needs to spend resources on the acquisition of emission rights, but (off-equilibrium) each individual firm would have to acquire additional rights at the market price when it pollutes more or it could sell emission rights at the market price when it pollutes less. Such a policy of "grandfathering" thus overproportionally reduces the burden for high-polluting firms, so that their capacity to raise financing for investment increases relatively to that of low-polluting firms. As a consequence, the outcome deteriorates along the new margin of how investment and output are allocated between low- and high-polluting firms, as is proven in Proposition 6.

**Proposition 6** *Consider again a single sector with heterogeneous polluters, which is not financially constrained in the aggregate. A policy of "grandfathering" emission rights is then strictly inferior, as, for given cap  $K$ , it shifts investment and production to high-polluting firms.*

**Sector-Specific Environmental Policy.** As discussed in the Introduction, one insight of the most basic models of environmental regulation is that a single emission price (equal to the social cost) is optimal. Such a single price, as arising under a single cap-and-trade mechanism, should govern all choices. This insight however no longer holds under financial

constraints. This follows immediately from our preceding analysis.

Specifically, suppose that one sector was populated by capital-rich "brown" incumbents ( $y_h$ ) together with "green" newcomers ( $y_l$ ), but that another sector represented more innovative production by (relatively) homogeneous new firms with little access to own funds and collateral. Our preceding results would then suggest to apply a cap and resulting emission price that are stricter than the Pigouvian benchmark to the first sector, but a more lenient environmental policy and with it an emission price below the Pigouvian benchmark to the second sector. This can not be achieved by a single cap-and-trading mechanism. Also, we noted that binding financial constraints make it efficient to allocate emission rights for free, but with heterogeneous polluters a corresponding "grandfathering" approach would be inefficient. We summarize this insight as follows:

**Corollary 1** *Financial constraints render it efficient to take a sector-specific approach for the determination of caps and resulting emission prices, as well as for the initial allocation of emission rights. Using (only) a single cap-and-trade mechanism is instead not generally efficient.*

## 5 Conclusion

As we discussed in the Introduction, the financial impact of currently undertaken or planned environmental policies will be considerable, and with it firms' need to raise additional financing. In a frictionless market, this would only impact investment to the extent that industry size efficiently readjusts, reflecting the true social benefits and costs of firms' activities. But with financial frictions, the additional financial burden impacts also on productive investment. This paper endogenizes the costs of external financing and thereby takes a corporate financing perspective on optimal environmental policy. Financial constraints are important at the firm level, as they constrain the amount of feasible investment. And they are important at the industry level, as they constrain the size of the industry and with it that of output, which impacts on equilibrium prices and consumer surplus.

Employing the corporate finance workhorse model of Holmström and Tirole (1997), we develop a tractable model that embeds a social planner's choice of a cap-and-trade system for emissions into an economy with financial constraints on investments. In the benchmark with no such financial constraints, adopting the Pigouvian level for the cap and the resulting price of emissions leads to the efficient outcome along all margins: Firms

choose efficient abatement activities, industry size is efficient, and with heterogeneous firms only the least-polluting firms operate. Financial constraints, however, generate a wedge between these objectives. Taking first a single-sector perspective, we show how the optimal policy under financial constraints leads to a price of emissions either above or below the Pigouvian level, depending on the severity of financial constraints and on firms' heterogeneity in pollution. While a higher cap on emissions mitigates the loss of consumer surplus following from an excessive reduction in industry size, a lower cap on emissions shifts investment and production to less polluting firms. In this context we also analyze the role of the initial allocation of emission rights. While this would not affect efficiency in the absence of financial constraints, under financial constraints this is no longer the case despite the operation of a cap-and-trade mechanism. A free initial allocation of emission allowances mitigates the trade-off between incentivizing efficient abatement and ensuring an efficient size of the industry. The common practice of "grandfathering", however, leads to an inefficient shift in investment and production to higher-polluting firms, again despite the operation of a trading scheme.

Our results thus also advocate for a more flexible environmental policy, rather than operating (only) a single cap-and-trade mechanism. Specifically, we discussed the optimality of a sector-specific policy. In our analysis we did, however, not extend the instruments available to the social planner. In fact, if efficiency was the only objective, the social planner could improve the outcome by lowering less-polluting firms' financial costs, while at the same time lowering the cap on emissions. This could be accomplished by, for instance, rendering it less profitable for households to invest in the storage technology. Specific taxes on such alternative investments, but also on identified more polluting firms and sectors, could then be used to subsidize green investment.<sup>25</sup> Such policies would however have potentially severe distributional implications, and we would thus need to question the social planner's objective function. We leave this to future research.

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<sup>25</sup>This would tie into the literature on directed technological change (Acemoglu et al. 2012).

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## Appendix: Proofs

**Proof of Lemma 1.** We only need to tackle the case without monitoring. When the planner does not implement monitoring, the expression for welfare must be adjusted for the reduced likelihood of success and the realized private benefits for owner-managers. We note that also with stochastic (iid) success, it is irrelevant whether the planner operates one firm or a mass of firms. The respective welfare with non-monitoring is then

$$\Omega_{n-mon} = bI + (1 - q) \left[ \int_0^I P(\hat{I})d\hat{I} - I[1 + v(y - s) + c(s)] \right] - qI. \quad (17)$$

Note that we use here that investment costs are always incurred (hence, the final "adjustment"  $qI$ ), while emission costs, including for abatement activities, are only incurred when the technology is successful and output is actually produced. We see that the first-best choice of abatement activity remains unchanged at  $s_{FB}$ . Instead, the now optimal size of the industry (in case of success) is different and solves

$$P(I_{n-mon}) = [1 + v(y - s_{FB}) + c(s_{FB})] + \frac{q - b}{1 - q}.$$

Monitoring is thus efficient when  $\Omega(I_{FB}, s_{FB}) > \Omega_{n-mon}(I_{n-mon}, s_{FB})$ . To see that  $q > b$  (provided that (1) holds) is sufficient, we only need to inspect (17). More formally, we generate the auxiliary problem to maximize, for given parameters  $(b, q)$ ,

$$\Omega_{aux}(I, s; b, q) = bI + (1 - q) \left[ \int_0^I P(\hat{I})d\hat{I} - I[1 + v(y - s) + c(s)] \right] - qI,$$

which is just a rewriting of (17). Total differentiation with respect to  $b$  and  $q$ , evaluated at the respective optimal choices of  $I$  and  $s$  (and thereby using the envelope theorem), yields

$$d\Omega_{aux} = I \cdot (db - dq) - dq \left[ \int_0^I P(\hat{I})d\hat{I} - I[1 + v(y - s_{FB}) + c(s_{FB})] \right].$$

As obviously the term in rectangular brackets is always positive at the optimal  $I$ , we have that compared to the case with  $b = q = 0$ , the level of  $\Omega_{aux}$  is always strictly lower at  $b > 0$  and  $q > 0$  when  $q > b$ . **Q.E.D.**

**Proof of Proposition 1.** The proof is organized as follows. We first take  $\tau$  as given, confirming the case distinction under monitoring and that monitoring is indeed the unique equilibrium outcome. We then solve for the combined equilibrium including the market for permission rights.

Turning thus first to the case distinction in the Proposition, note that despite financial constraints at the firm level, the market outcome can indeed reach  $I_{uc}$  when, at this level, it holds that

$$\frac{1}{1 + b/q - P(I_{uc}) + z(s^*)} A \geq I_{uc}.$$

Substituting  $P(I_{uc}) - 1 - z(s^*) = 0$ , we can indeed transform this to  $A \geq \frac{b}{q} I_{uc}$ .

We show next that in the characterized equilibrium, an individual firm can not profitably deviate to non-monitoring. This is immediate when the financing constraint does not bind. Suppose thus that in the aggregate financial constraints bind. Recall that when there is no-monitoring, then the firm can, in principle, lever up as much as it likes (provided that the marginal return is non-negative). Note that now the (binding) break-even constraint of investors becomes

$$D_i(1 - q) = I_i + (1 - q)I_i z(s_i) - A,$$

where the left-hand side expression takes into account that there is repayment only with probability  $1 - q$ , while for the right-hand side we use that the costs of pollution are only incurred in case of success. Substitution into the owner-manager's expected payoff yields

$$U_i = (1 - q)I_i[P(I^*) - 1 - z(s_i)] - qI_i + bI_i.$$

To ensure that a deviation to non-monitoring without a limitation of scale is not profitable, it must hold that  $dU_i/dI_i \leq 0$ , which holds if

$$P(I^*) - 1 - z(s^*) \leq \frac{q - b}{1 - q}. \quad (18)$$

Recall now that we presently need only consider the case where the financing constraint binds in the aggregate, for which a sufficient condition is that  $1 + b/q - P(I^*) + z(s^*) > 0$ . This implies (18) if  $\frac{b}{q} < \frac{q-b}{1-q}$ , which transforms to (2). Note also that from this observation we can rule out an equilibrium with non-monitoring. There, given the scalability of the investment, the marginal return to investment must be zero, i.e., now with  $dU_i/dI_i = 0$ ,  $P(I^*) - 1 - z(s^*) = \frac{q-b}{1-q}$ . But then the marginal return under monitoring is strictly positive, namely equal to  $\frac{q-b}{1-q} > 0$ .

We conclude the characterization by also incorporating the market for emission rights. For this, note first that each firm is a price-taker and that, for given  $\tau$ , we have characterized a unique pair  $(s^*, I^*)$ , which gives rise to total demand for emissions  $e^* = I^*(y - s^*)$ . We write this as  $e^* = \psi(\tau)$ . We show now that  $\psi$  is continuous and strictly decreasing. For

this note first that this holds for  $s^*$  by the properties of  $c(s)$ . Also  $z(s^*)$  is then, using the envelope theorem, continuous and strictly increasing (where  $s^* > 0$ ) in  $\tau$ . If financial constraints are not binding in the aggregate, so that  $P(I^*) - 1 - z(s^*) = 0$ , together with strict monotonicity and continuity of  $P(I)$  this establishes the required properties of  $\psi(\tau)$ . Turn now to the case where financial constraints are binding in the aggregate. Then, the properties follow from (9), again together with the observation that both  $z$  and  $P$  are continuous and monotonic. Using continuity and monotonicity of  $I^*$ , define now  $\bar{\tau} > 0$  where  $I^* = 0$  and thus also  $e^* = 0$ . We note that while  $s^* = 0$  for all  $\tau \leq 0$ ,  $I^*$  still increases monotonically over negative values  $\tau < 0$ . Still, for the following characterization it is sufficient to consider values  $\tau \geq 0$ , and we define  $\bar{e} = \psi(\tau = 0)$ .

Consider thus a cap  $0 < K \leq \bar{e}$ . Given  $K$ , an equilibrium in the market for emission rights is characterized by a price  $\tau$  at which demand equals supply:  $\psi(\tau) = K$ . Existence and uniqueness follow from the derived properties of  $\psi$ . We finally define the inverse  $\rho = \psi^{-1}$ . When  $K > 0$ , then  $\tau = \rho(K)$  ensures that, for the respective realizations of  $s^*$  and  $z(s^*)$ , (1) implies that  $P(0) > 1 + z(s^*)$ , so that we can rule out  $I^* = 0$ . **Q.E.D.**

**Proof of Proposition 2.** It is convenient to express the social planner's problem in terms of the induced price of emission rights  $\tau$  (using the one-to-one relationship  $\psi(\tau) = K$ ). Then, differentiation of the social planner's objective yields

$$\begin{aligned} \frac{d\Omega}{d\tau} &= \frac{dI^*}{d\tau} [P(I^*) - [1 + v(y - s^*) + c(s^*)] + I^* \frac{ds^*}{d\tau} [v - c'(s^*)]] \\ &= \frac{dI^*}{d\tau} [(P(I^*) - 1 - z(s^*)) - (y - s^*)(v - \tau)] + I^* \frac{ds^*}{d\tau} [v - \tau], \end{aligned} \quad (19)$$

where the second line follows from a simple expansion of the first term and substitution of the first-order condition for  $s^*$ . We set now  $\tau = v$ , so that the second term becomes zero: At the efficient level of emissions, a marginal change of abatement activities has a first-order effect of zero. The sign of the derivative is then determined by the first term. Using  $I^* = I_{con}$ , the sign of the derivative is thus indeed strictly negative from  $P(I_{con}) - 1 - z(s^*) > 0$  (and substituting  $v - \tau = 0$ ). This proves that at the optimal cap, the resulting price of emissions is strictly below the Pigouvian level if, at that level, the financing constraint binds. **Q.E.D.**

**Proof of Proposition 3.** Given symmetry, we note at the outset that there will not be trade of emission rights in equilibrium. Still, we can determine a (shadow) price, as in Proposition 1, at which demand equals supply. In light of our subsequent analysis, where

firms will be heterogeneous, we do however not short-cut our exposition.

Note that the market price of emissions is deterministic. Given an allocation of rights  $R_i$  and a market price  $\tau$ , the respective value of emissions rights of firm  $i$  is thus  $\tau R_i$ , so that the firm's initial allocation of funds increases to  $A + \tau R_i$ .<sup>26</sup> With this change, our original derivation of firms' optimal choices remains fully unchanged: i)  $s_i = s^*$  solves  $c'(s^*) = \tau$ ; ii) the firm wants to lever up maximally if  $P(I^*) - 1 - z(s^*) > 0$ , where irrespective of the allocation of emission rights we need to account for the true costs  $z(s^*)$ , it does not become active if  $P(I^*) - 1 - z(s^*) < 0$ , and it is indifferent between a scale of zero and the maximum scale if  $P(I^*) - 1 - z(s^*) = 0$ ; and iii) the maximum scale is now given by

$$I_i \leq \frac{1}{1 + b/q - P(I^*) + z(s^*)} (A + \tau R_i). \quad (20)$$

While we have already commented on  $I_{uc}$ , which is still given by  $P(I_{uc}) - 1 - z(s^*) = 0$ , aggregating over  $I_i$  when (20) binds yields

$$I_{con,free} = \frac{1}{1 + b/q - P(I_{con,free}) + z(s^*)} (A + \tau K),$$

where  $K = I_{con,free}(y - s^*)$ . Substituting and noting that  $z(s^*) = c(s^*) + \tau(y - s^*)$ , this solves indeed for (13).

Suppose now that the financial constraint does not bind in the aggregate (for given  $K$  and resulting  $\tau$ ). If firms then maximally lever up according to (20), we ask when it holds that  $\int I_i di \geq I_{uc}$ . Using that  $P(I_{uc}) - 1 - z(s^*) = 0$ , this condition transforms to  $\frac{q}{b}(A + (y - s^*)\tau I_{uc}) \geq I_{uc}$ . Hence, the industry is indeed not financially constrained if

$$A \geq \left[ \frac{b}{q} - (y - s^*)\tau \right] I_{uc}.$$

When the social planner now chooses  $K$  so that the Pigouvian price of emissions  $\tau = v$  prevails, this outcome is indeed achievable without financing constraints when

$$A \geq \left[ \frac{b}{q} - (y - s_{FB})v \right] I_{uc}(\tau = v). \quad (21)$$

This condition is indeed strictly weaker than that when emission rights are not allocated for free, which from (11) is  $A \geq \frac{b}{q} I_{uc}(\tau = v)$ .

With this we turn to the assertions in the Proposition. The first assertion concerns the (shadow) price for emissions that results for a given cap  $K$ . Comparing the two cases

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<sup>26</sup>If the market for such rights opens up only after investment has been undertaken, the firm could still borrow funds on the basis of these rights.

with and without free allocation, note that in both cases the relationship between  $I_{uc}$  and  $s^*$  is the same, as given by  $P(I_{uc}) - 1 - z(s^*) = 0$ . Together with the requirement that  $K = I_{uc}(y - s^*)$  this establishes a unique pair  $(I_{uc}, s^*)$  and a price  $\tau$  solving  $\tau = c'(s^*)$ , irrespective of how rights are initially allocated. Formally, the function  $\psi(\tau) = K$  and its inverse  $\tau = \rho(K)$  are the same. We next turn to the case where the industry is financially constrained in both cases. We claim that the (shadow) cost of emissions is strictly higher when emission rights are allocated for free, while industry size is strictly higher. This follows immediately from the following observation. We note that for given  $s^*$ , as long as  $\tau > 0$ , we have  $z(s^*) > c(s^*)$ , so that, holding first  $s^*$  fixed,  $I_{con} < I_{con,free}$ . To then ensure  $I^*(y - s^*) = K$ ,  $s^*$  must indeed be strictly higher when emission rights are allocated for free. With a higher  $s^*$  and given  $c'(s^*) = \tau$ , also  $\tau$  must be higher. The intermediate case where, for given  $K$ , the industry is only financially constrained without a free allocation follows immediately from these observations.

From these results we also have immediately that even when  $K$  is left unchanged, a free allocation of emission rights strictly improves welfare when the resulting industry size is not inefficiently high (compared to the second-best benchmark). But if this was the case, welfare could be strictly improved by lowering  $K$ . **Q.E.D.**

**Proof of Proposition 5.** When the cap is binding, so that  $\tau > 0$ , investment by a low-polluting firm is strictly more profitable. Together with the linear technology, this limits the consideration of equilibria to three cases: when only low-polluting firms are in the market, when both types of firms are in the market and both are financially constrained, and when both types of firms are in the market and only low-polluting firms are constrained in the aggregate. We characterize first the third case, which is treated in the Proposition.

As in the proof of Proposition 1 we take first as given  $\tau$ . This determines  $s^*$  and with it  $I^*$  from  $P(I^*) - 1 - z(y_h, s^*) = 0$ . Substituting into  $I_l$ , as described in the main text, yields  $I_l^*$  from (16), which finally pins down  $I_h^* = I^* - I_l^*$ . Following still the approach of Proposition 1, this pins down, for given  $\tau$ , total emissions  $e^* = I_l^*(y_l - s^*) + I_h^*(y_h - s^*)$ , and the respective function  $e^* = \psi(\tau)$ , which still satisfies the respective properties. Given the cap  $K$ , the equilibrium price  $\tau$  ensures that  $\psi(\tau) = K$ . This case applies when  $0 < I^* - I_l^* \leq \frac{q}{b}A$ . We note that as  $\tau$  increases,  $I^*$  decreases and  $I_l^*$  increases, so that also  $I^* - I_l^*$  is monotonic. Remaining within this case, differentiation of the objective function

(14) with respect to  $\tau$  and evaluation at  $\tau = v$  and  $s^* = s_{FB}$  yields, as asserted:

$$\begin{aligned}\frac{d\Omega}{d\tau} &= \frac{dI^*}{d\tau}[P(I^*) - [1 + v(y_h - s^*) + c(s^*)]] - I^* \frac{ds^*}{d\tau}[v(y_h - s^*) + c(s^*)] + \frac{dI_l^*}{d\tau}v\Delta_y \\ &= \frac{dI_l^*}{d\tau}v\Delta_y > 0.\end{aligned}$$

For completeness only we discuss also the remaining two cases. In the case where only low-polluting firms enter the market with  $P(I^*) - 1 - z(y_l, s^*) = 0$ , we know from Proposition 3 that the Pigouvian outcome is socially optimal. Finally, when financial constraints are sufficiently severe so that the industry remains constrained in the aggregate, taking again first  $\tau$  as given, the equilibrium is characterized by

$$\begin{aligned}I_l^* &= \mu \frac{1}{1 + b/q - P(I^*) + z(y_l, s^*)} A, \\ I_h^* &= (1 - \mu) \frac{1}{1 + b/q - P(I^*) + z(y_h, s^*)} A, \\ I^* &= I_l^* + I_h^*.\end{aligned}$$

Evaluating now the derivative of (14) at the Pigouvian benchmark, we have

$$\frac{d\Omega}{d\tau} = \frac{dI^*}{d\tau}[P(I^*) - [1 + v(y_h - s^*) + c(s^*)]] + \frac{dI_l^*}{d\tau}v\Delta_y,$$

where now the first term is strictly negative. **Q.E.D.**

**Proof of Proposition 6** As in the proof of Proposition 3, the allocation of rights  $R_i$  increases the initial availability of funds to  $A + \tau R_i$ . A low-polluting firm's maximum scale is then given by

$$I_l = \frac{1}{1 + b/q - P(I^*) + z(y_l, s^*)} (A + \tau R_l).$$

We note again that as the industry is not financially constrained in the aggregate, the marginal investment of a high-polluting firm generates zero return, which includes the costs of emission rights:  $P(I^*) - 1 - z(y_h, s^*) = 0$ . Hence, we have from substitution

$$I_l = \frac{1}{b/q - \Delta_y \tau} (A + \tau R_l). \quad (22)$$

Note now that as the respective rights are allocated on the basis of the (equilibrium) pollution of either low- or high-type firms in the aggregate, we have  $R_l = (y_l - s^*)I_l^*/\mu$ . Substituting and following the derivation as for (13), we obtain

$$I_l^* = \mu \frac{1}{b/q - \tau(y_h - s^*)} A. \quad (23)$$

The size of high-polluters' investment is again the residual  $I_h^* = I^* - I_l^*$ .

Suppose now instead that pollution rights are initially allocated uniformly over all active firms (which is equivalent to redistributing levied taxes on a per-firm basis). To prove the claim it is sufficient to compare the outcome when a given emission price  $\tau$  is implemented in either case. This ensures that in either case the same aggregate investment  $I^*$  and the same abatement activity  $s^*$  prevails (as  $c'(s^*) = \tau$  and  $P(I^*) - 1 - z(y_l, s^*) = 0$ ). To compare welfare it is thus sufficient to compare the resulting values of  $I_l^*$ . Also with a uniform allocation, the starting point is expression (22), and we have that  $I_l^*$  is now indeed larger if and only if  $R_l$  is larger. This follows directly as the pollution per unit of output is  $(y_l - s^*)$  in the group of low-polluting firms alone (i.e., under "grandfathering") but  $[I_l^*(y_l - s^*) + I_h^*(y_h - s^*)]/I^*$  when considered over all firms. **Q.E.D.**