Tying under Double-Marginalization

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Abstract

In a model of contractual inefficiencies due to double-marginalization, we analyze the practice of tied rebates that incentivizes retailers to purchase multiple products from the same manufacturer. We isolate two opposing effects: a surplus-sharing effect that enhances efficiency and a rent-extraction effect that reduces efficiency. The overall effect is more likely to be negative when the manufacturer has a particularly strong brand for which the retailer's alternatives are much inferior. Foreclosure of a more efficient provider of the manufacturer's weaker product is not a sufficient condition for a welfare loss. Our key positive implication relates to the seemingly inefficient introduction of weaker products by the owners of particularly strong brands.

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1 Introduction

The contractual terms negotiated in intermediary industries are rarely observed by researchers. This applies in particular to those set between manufacturers and retailers, which are often treated as commercial secrets. From what we can learn from official publications in antitrust cases, however, manufacturers that supply multiple products may tie their respective contractual terms. Such tying may be explicit in the form of aggregate rebates, or it may be implicit in that the negotiated contractual terms would be different if the retailer delisted one of the manufacturer's products. A case in point is the antitrust proceeding of the European Commission against The Coca-Cola Company (TCCC),[1] which focused on the complaint that TCCC had made the supply of its strongest "must-stock" products (e.g., carbonated soft drinks like Coca-Cola or Fanta Orange) conditional upon the purchase of other, weak products (e.g., bottled water of the brand Bonaqua).[2]

Competition concerns regarding such practices typically relate to the inefficient foreclosure of stand-alone rivals by dominant firms. Missing from the discussion, however, is a recognition that, at least in the case of intermediary-goods markets, such practices can also generate specific efficiency gains, notably when the tying concerns largely independent products or product categories. Our contribution closes this gap and shows how under a single contractual inefficiency, that of double-marginalization, tying the contractual terms of the supply of even independent products has two opposing effects: a surplus-sharing effect that enhances efficiency and a rent-extraction effect that reduces efficiency and can lead to inefficient foreclosure. Although either effect can dominate, we show that the overall effect is more likely to be negative when the manufacturer has a particularly strong brand for which the retailer's alternatives are much inferior. We also show that even though the negative effect of tying is larger when it leads to the exclusion of a more efficient rival, this is not a sufficient condition for a loss in welfare – due to the potential for efficiency gains.

In light of its importance for our analysis, we briefly reflect on the (real-world) importance of our key assumption, that of inefficient linear contracting leading to a problem of double-marginalization. We first note that linear wholesale prices feature prominently in many contributions in Industrial Organization, notably on price discrimination in inter-

[1] See Case COMP/A.39.116/B2 *Coca-Cola* (2005).

[2] Another well-known case is that of Viacom in the U.S. (Case No. 13 Civ. 1278 *Viacom* (2014)), in which Cablevision complained against Viacom's bundling strategy, which forced Cablevision to distribute Viacom's less popular channels in order to obtain access to Viacom's popular channels. Here, as in the *Coca Cola* case, the salient feature is that weak products are tied to strong products. The effects we identify hold whether the products are independent, substitutes, or even complements in final demand.

mediary markets (e.g., Katz, 1987; DeGraba, 1990; Yoshida, 2000; Valletti, 2003; Inderst and Valletti, 2009; O'Brien, 2014) but also beyond (e.g., on the opportunism problem, see Gaudin, 2019). It is also a key assumption in many structural models of (negotiated) wholesale price determination in empirical research. Empirical research has also unveiled explicit cases where linear contracts prevail, such as between hospitals and medical device suppliers (Grennan, 2013, 2014), between hospitals and insurers (Ho and Lee, 2017), and between book publishers and resellers (Gilbert, 2015). Lastly, the relevance of the double-marginalization problem is confirmed through its explicit recognition in antitrust guidelines as a source of potential efficiency in vertical mergers and agreements. That said, given the admitted greater complexity of many real-world contracts, including those between retailers and manufacturers, and as we ourselves have frequently used models of non-linear contracting in our research, the picture of linear contracting may often amount to a stark simplification. What is needed for our results to hold is that the bilateral contracting between firms does not lead to joint surplus maximization (as there may be insufficient flexibility to disentangle this objective from that of surplus sharing/extraction).

In our model, we distinguish between a manufacturer's strong product and a weak product that he potentially introduces as well. When the two products' terms of supply are not tied, the manufacturer either does not supply the weak product (which occurs when the retailer has a more attractive alternative), or his wholesale price on the weak product just matches that of the retailer's outside option. Similarly, the manufacturer's wholesale price on the strong product also just matches that of the retailer's outside option when the retailer has an inferior, but sufficiently close alternative. In this case (when both the weak and the strong products' wholesale prices are constrained by the retailer's respective outside options), tying the supply of the two products together unambiguously enhances contractual efficiency, with the two wholesale prices following a Ramsey-pricing-like rule that allows surplus to be shared in the most efficient way. The thereby realized efficiency gains also compensate the retailer for foregoing its outside option. Even if this forecloses a more efficient supplier of the weak product, in this case contractual efficiencies dominate.

The welfare effects are different when the manufacturer enjoys with his strong product a sufficiently large advantage (akin to a "must-stock" product). Then, tied contracting allows the manufacturer to extract from the retailer more rent, which the manufacturer must leave the retailer under separate contracting due to the double-marginalization problem. As such, the shift in surplus to the manufacturer leads to higher wholesale and retail prices under double-marginalization, which implies a reduction in output and welfare.

How the two effects of tied contracting balance thus depends crucially on whether the manufacturer owns a sufficiently strong product. The stronger it is (relative to the retailer's alternatives), the more likely the overall effect will be negative. Our analysis thus has two key results. First, we show that, from the same problem of double-marginalization, there are two countervailing effects of tied contracting, effects that can arise even when the practice does not serve to strategically foreclose rivals. An effects-based analysis is thus warranted. Second, we show that a negative overall effect is more likely when the manufacturer ties the supply of a weak product to that of a particularly strong ("must-stock") product. We illustrate these effects with the help of an example with linear demand.

Our contribution ties into a large literature on vertical contracting and channel management. As we have already noted, a distinctive feature of our analysis is that we assume contractual inefficiencies, which we capture with linear contracts. In contrast, O'Brien and Shaffer (2005), building on Shaffer (1991), show that when the manufacturer supplies substitute products, a restriction to separate offers can be inefficient even with non-linear contracts, as the manufacturer then optimally distorts each individual contract so as to negatively affect the retailer's outside options (cf. also Dertwinkel-Kalt and Wey, 2020). To isolate the novel effects of our analysis, we thus focus on the case where the demands for the two products are independent, so that the effects they identify are not present.

While we consider contracting between a manufacturer and a retailer, the restriction to linear pricing relates our results to those obtained for multi-product firms operating in final-goods markets. Our welfare-enhancing effect of improved contractual efficiency and the resulting Ramsey-like prices thus mirror existing results in this literature (cf. notably Armstrong and Vickers, 2018). To this, we add the negative effect of increased rent extraction and analyze the resulting trade-off when comparing separate to tied contracting. Such a comparison in turn relates our contribution to the large literature on bundling and tying, which again focuses mostly on firms operating in final-goods markets. Faced with different consumers, and when first-order price discrimination is not possible, tying and bundling allows manufacturers to price discriminate and extract more consumer surplus (e.g., Adams and Yellen, 1976; McAfee et al., 1989), which may then also have the effect of (credibly) foreclosing a more efficient single-product supplier (e.g., Nalebuff, 2004).

Although the theme of increasing surplus by (more) efficient contracting, and that of extracting more surplus from buyers by tying two or more goods, has thus already been explored in the literature, albeit in different contexts, our contribution brings these themes together in a model of double-marginalization in intermediary goods markets, which may be of particular relevance for many real-world antitrust cases involving such practices.

In Section 2, we introduce the model. Section 3 derives the equilibrium under separate and tied contracting, which is used in Section 4 to isolate the two countervailing effects of tied contracting. Section 5 extends the analysis to the case where such tying excludes a more efficient rival. We conclude in Section 6 by pointing to future research avenues.

2 The Model

To make our points in the simplest way possible, we conceive of a single strategic manufacturer offering two different products, indexed by $n = A, B$, which are produced at constant unit costs c_n. The manufacturer must sell his products through a monopolistic retailer. For each product the retailer can also access alternative suppliers. For the moment, we assume that these alternative suppliers produce inferior variants. To be specific, we assume that the respective unit costs of production of these suppliers are (strictly) higher and denoted by c_n^o, where the superscript stands for the retailer's alternative (outside) option.

When the strategic manufacturer's advantage relates to costs (as we assume here), there is a single demand function $D_n(p_n)$ for product n that applies irrespective of where the retailer sources product n. Nevertheless, our results extend naturally to the case where the strategic manufacturer has alternatively an advantage, at least for product A, in terms of quality or consumer loyalty. For instance, with some parameter $d \geq 0$, the retailer's demand under the alternative supplier's variant could be $D_n(p_n + d)$. Note that the other product's price p_m does not feature in the respective demand of product n, as demands for the two products are fully independent. As discussed in the Introduction, this allows us to abstract from other reasons why a manufacturer might want to offer multiple products and make the delivery of one product or its price contingent on the terms of the other.

Before we draw up the contracting game and derive some preliminary results, we note that one of the main issues of interest is why a manufacturer who presently offers only one product, say A, may have an incentive to introduce *and* tie an independent product B. Notably, we ask this question not only for the present case in which the manufacturer may have little (cost) advantage regarding product B, but also in the extensions section where his product may even be at a cost disadvantage. In light of this, we will refer to product A throughout as the manufacturer's strong product and to product B as his weak product.

Throughout our analysis, we will also restrict attention to linear contracts w_n. As discussed in the Introduction, the resulting double-marginalization problem is our key source of contractual inefficiency. There, we also argued that such inefficiencies should be of practical relevance. We distinguish between two types of contracting. Under separate contracting (offers), the manufacturer cannot make the condition for the supply of one product contingent on the retailer's procurement of the other product. In the second scenario of tied contracting (offers), the manufacturer ties together the conditions for the supply of the two products. As will be evident in what follows, we can restrict attention to a tied offer (w_A, w_B) under which the retailer can procure the respective product n at a per-unit price w_n, while the manufacturer makes separate procurement of only product A or B sufficiently unattractive. For instance, compared to when stocking only one product, the offer (w_A, w_B) could come at a sufficiently large (cross-products or tied) rebate.

In our contracting game, the manufacturer makes a take-it-or-leave-it offer to the retailer, for which, depending on the subsequently discussed antitrust regime, he can either offer only separate contracts or both separate and tied contracts. The retailer can accept or reject the manufacturer's offer. In the latter case he procures from his alternative sources. After that, the retailer sets the respective prices p_n and sales are made to final consumers.

Preliminary Analysis. Before proceeding to the equilibrium analysis, we first introduce some additional notation. If the retailer rejects the manufacturer's offer and thus sources under the outside option for product n, he can realize a profit on the option of

$$\pi_n^o = \max_{p_n}[D_n(p_n)(p_n - c_n^o)],$$

which may be zero when c_n^o is above the so-called choke-off price of demand (at which the quantity demanded drops to zero). If instead the retailer accepts the manufacturer's offer and thus buys from the manufacturer, he can realize a profit on product n of

$$\pi_n(w_n) = \max_{p_n}[D_n(p_n)(p_n - w_n)],$$

where we have used the fact that the independence of A and B implies that the pricing strategy of the retailer for one product is independent of the wholesale price of the other product – or whether he even purchases this product (or alternatively from other suppliers).

It is convenient to stipulate that the respective maximization problems are well-behaved in the sense that the objective functions are strictly quasi-concave (where positive). In the latter case, given the manufacturer's offer, we denote the retailer's optimal price by $p_n(w_n)$ and the thereby realized quantity sold by $q_n(w_n)$ (where we sometimes drop the dependency on w_n). The manufacturer's profit is denoted by capital letters and given by

$$\Pi_n(w_n) = q_n(w_n)(w_n - c_n).$$

Assuming the retailer purchases both products from the manufacturer, we abbreviate the retailer's total profits by $\pi = \pi_A + \pi_B$ and the manufacturer's total profits by $\Pi = \Pi_A + \Pi_B$.

3 Separate and Tied Contracting

In this Section, we solve for the equilibrium under both separate and tied contracting. In the process, we will show that the wholesale prices that arise when tying is feasible are generically different than the wholesale prices that arise when it is not, implying that tying is profitable when feasible. Our maintained assumption throughout is that the retailer's alternatives are inferior (i.e., $c_n^o \geq c_n$), so that under both separate and tied contracting, the manufacturer will always be supplying both products. Later, we will relax this assumption.

Separate Contracting. To set the stage, it is useful to begin by considering the case in which the manufacturer can use only separate contracting. In this case, the manufacturer's wholesale prices must be set independently. Under our maintained assumption that the retailer's alternatives are weakly inferior, it is then easy to see that the manufacturer's optimal choice of w_n maximizes $\Pi_n(w_n)$ subject to the retailer's participation constraint

$$\pi_n(w_n) \geq \pi_n^o. \tag{1}$$

To characterize the solution, we first ignore the retailer's respective outside option and denote the manufacturer's unconstrained optimal choice of wholesale price w_n by

$$w_n^m = \arg\max_{w_n} \Pi_n(w_n). \tag{2}$$

Once again, we suppose that the objective is strictly quasi-concave, which implies that w_n^m is unique. It also implies that for all $w_n < w_n^m$, the manufacturer can increase his profit by raising his wholesale price, while for all $w_n > w_n^m$, the resulting reduction in the retailer's indirect demand dominates, and the manufacturer is better off lowering his wholesale price. On the other side, it is straightforward to see that the retailer's profit, $\pi_n(w_n)$, is everywhere decreasing in the manufacturer's wholesale price. We thus have two regimes to consider, depending on whether the retailer's participation constraint binds.

Lemma 1 *Suppose the manufacturer offers both products but is constrained to use separate contracts. Then, there exists a threshold for the cost of the alternative supply, $\hat{c}_n^o$, so that the following case distinction applies for characterizing the manufacturer's offer w_n^s :*

Case 1: If $c_n^o > \hat{c}_n^o$, then the retailer's participation constraint (1) is slack and $w_n^s = w_n^m$.

Case 2: If $c_n^o \leq \hat{c}_n^o$, then the retailer's participation constraint (1) binds and $w_n^s = c_n^o$.

Proof of Lemma 1. We first note that $d\pi_n/dw_n = -q_n < 0$, and that likewise $d\pi_n^o/dc_n^o = -q_n^o < 0$, where $q_n^o = D_n(p_n^o)$ with p_n^o denoting the unique optimal price when the retailer chooses an alternative supplier of product n. We next note that when $\pi_n(w_n^m) \geq \pi_n^o$ holds at the unique solution to the unconstrained problem, the manufacturer's unique optimal offer is $w_n^s = w_n^m$ (Case 1). Given that $\pi_n(w_n^m) = \pi_n^o$ when $c_n^o = w_n^m$, and the fact that $d\pi_n^o/dc_n^o = -q_n^o < 0$, it follows that Case 1 holds if and only if $c_n^o \geq w_n^m$. Last, we note that if instead $\pi_n(w_n^m) < \pi_n^o$, then it must be that $c_n^o < w_n^m$ (Case 2). In this case, by the strict quasi-concavity of $\Pi_n(w_n)$ and the strict monotonicity of π_n, $w_n^s = c_n^o$ is uniquely optimal. The asserted threshold separating the two cases is thus $\hat{c}_n^o = w_n^m$. **Q.E.D.**

In what follows, we focus on the case where product A represents the manufacturer's strong product. In this sense, we always suppose that the manufacturer is constrained when offering product B, so that Case 2 applies (with $c_B^o < \hat{c}_B^o$). For the strong product A, we allow for a case distinction, so that either of the two cases in Lemma 1 can apply.

Tied Contracting. Recall that under a tied offer (w_A, w_B), the manufacturer either commits not to supply only one product, or he makes the choice of only one product sufficiently unattractive (in which case the tied offer must be sufficiently rebated). Either way, there is thus only a single, joint participation constraint to consider for the retailer:

$$\pi_A(w_A) + \pi_B(w_B) \geq \pi_A^o + \pi_B^o. \tag{3}$$

This collapsing of the individual participation constraints into one joint participation constraint has the effect of relaxing the manufacturer's overall maximization problem (across the two products) and potentially incentivizes him to choose a different pair of wholesale prices than he would have chosen under separate contracting. Since the latter is still feasible, if he does choose a different pair, then he must be better off. As we will now show, the manufacturer's optimal wholesale prices will indeed be generically different under tying.

To see this, note that under tied contracting, the manufacturer's problem is to maximize his total profit Π subject to the joint participation constraint (3). If (3) does not bind, then the optimal tied offer, which we denote by w_n^t, satisfies $w_n^t = w_n^m$ for both A and B, as given by (2), an outcome that does not arise under separate contracting given our assumption that Case 2 applies for product B. More interesting is what happens if (3) binds, because then even though demands are independent, the optimal choice of wholesale prices will be interrelated through the binding constraint. Setting up the Lagrangian

$$\mathcal{L} = \Pi_A + \Pi_B + \lambda \left(\pi_A + \pi_B - \pi_A^o - \pi_B^o \right), \tag{4}$$

the first-order requirement reads

$$\frac{d\Pi_A/dw_A}{d\pi_A/dw_A} = \frac{d\Pi_B/dw_B}{d\pi_B/dw_B}. \tag{5}$$

Or, in other words, the marginal rate of substitution between the manufacturer's and the retailer's profit must be the same for the two products in equilibrium. Substituting $d\Pi_n/dw_n = q_n + (w_n - c_n)dq_n/dw_n$ and $d\pi_n/dw_n = -q_n$ into the above expression yields

$$\frac{dq_A}{dw_A}\frac{(w_A - c_A)}{q_A} = \frac{dq_B}{dw_B}\frac{(w_B - c_B)}{q_B}.$$

Noting that q_n represents the derived demand function and $\eta_n = (dq_n/dw_n)(w_n/q_n)$ its elasticity, we ultimately have that under the manufacturer's optimal offer it must be that

$$\frac{(w_A - c_A)/w_A}{(w_B - c_B)/w_B} = \frac{\eta_B}{\eta_A}. \tag{6}$$

Here, the ratio of the respective (percentage) margins equals the inverse ratio of the respective elasticities of the derived demand functions. Intuitively, when the elasticity of the derived demand is higher for one product, then under the optimal tied offer the manufacturer accepts a lower (percentage) margin on this product. The manufacturer thus follows a Ramsey-pricing-like rule (also called more descriptively an "inverse elasticity rule").[3] This observation formalizes the insight that when (3) binds, the manufacturer optimally extracts surplus from the retailer in the most efficient way (in terms of industry profit), which mirrors the use of Ramsey prices in many other contexts (such as optimal taxation).

Which of the two cases, i.e., that with a binding joint participation constraint (3) or that without, arises depends on the attractiveness of the retailer's alternatives. We will make this dependency more explicit in the next section, where we analyze the profit and welfare implications of tying. For now, we can summarize the preceding findings as follows:

Lemma 2 *Suppose the manufacturer offers both products and can tie the respective terms of supply. If (3) is slack, the manufacturer sets $w_n^t = w_n^m$ (the unconstrained wholesale price) for both products. Otherwise, the choice of w_n^t follows the Ramsey pricing rule (6).*

[3] The inverse elasticity rule is a special case of the Ramsey pricing rule and only applies when cross-price elasticities are zero.

Before continuing on to the next section, it is useful to point out that the mapping between the cases in Lemma 1 and the possibilities in Lemma 2 can take one of three forms. If the retailer's participation constraint on product A (the manufacturer's strong product) binds under separate contracting (Case 2 holds), then the joint participation constraint (3) will also necessarily bind under tied contracting.[4] This is one possibility. However, if the retailer's participation constraint on product A does not bind under separate contracting (Case 1 holds), then the joint participation constraint (3) may bind under tied contracting (a second possibility) or may not bind (the third possibility).[5] From these possibilities, we can identify two distinct effects of tied contracting, which we will isolate and discuss next.

4 (In-)Efficiencies of Tied Contracting

In this Section, we isolate two effects that arise from the transition to tied contracting and show that they have opposite implications in terms of (consumer) welfare. We then analyze the interaction of these two effects and identify settings in which each is dominant.

4.1 Isolating the Effects

We begin by isolating the two effects of tying. Both effects can be seen, albeit indirectly, from the outcomes in Lemma 2, where we found that the wholesale prices under tying either followed the Ramsey-pricing-like rule (6) or were unrestrained. The first effect we analyze is that of more efficient surplus sharing, which tends to increase welfare. The second effect we analyze is that of increased rent extraction, which, due to double-marginalization, works in the opposite direction. In what follows, we let total industry profit be the sum of the retailer's and the manufacturer's profit, i.e., the sum of $PS_n = (p_n - c_n)q_n$ for $n = A, B$, and we denote the consumer surplus realized with product n as $CS_n = \int_0^{q_n} [P_n(s) - p_n]ds$. This then allows us to define total welfare W_n as the sum of $PS_n + CS_n$ for $n = A, B$.

Improved Contractual Efficiency under Tied Contracting. To isolate the first effect of tying, we consider what the effects of Ramsey pricing would be in the absence of any added rent extraction. Specifically, we fix the retailer's total profit to be the same as

[4] If this were not true, then, from Lemma 2, the manufacturer would optimally set $w_n^t = w_n^m$ on both products and a slack participation constraint (3) would imply that $\pi_A(w_A^m) + \pi_B(w_B^m) > \pi_A^o + \pi_B^o$. But this would mean that either $\pi_A(w_A^m) > \pi_A^o$, $\pi_B(w_B^m) > \pi_B^o$, or both, which contradicts the supposition.

[5] Our assumption that product B is weak excludes the case where the manufacturer is unconstrained on both products. If this were to hold, so that Case 1 of Lemma 1 applied for both of the manufacturer's products, then the joint participation constraint would also not bind and there would be no effect of tying.

it would be under separate contracting and denote it by $\pi^s = \pi_A(w_A^s) + \pi_B(w_B^s)$. We then consider an auxiliary problem in which the manufacturer maximizes his profit Π subject to promising the retailer a profit of at least π^s. In this case, it is easy to see that the manufacturer's optimal wholesale prices satisfy the Ramsey pricing rule (6), and that they generically differ from his optimal offer under separate contracting (which generally does not satisfy (6)). The fact that they differ, even though he could have chosen the same prices, implies that the manufacturer will be strictly better off under the auxiliary problem with tying than he would be under separate contracting. Given that the retailer's profit is the same in both cases, it also implies that total industry profits will be higher as well.

We can formally state this "improved contractual efficiency" effect of tying as follows:

Proposition 1 *Consider an auxiliary problem in which the manufacturer maximizes his profit Π subject to the retailer earning a profit of at least π^s. Then, total industry profits are generically strictly higher (and always so if under separate contracting the manufacturer is unconstrained on its strong product) under tied contracting than under separate contracting.*

Proof of Proposition 1. Under the auxiliary problem, the manufacturer maximizes Π subject to $\pi \geq \pi^s$. Arguing to a contradiction, we first show that in the auxiliary problem, the retailer's participation constraint must bind, so that the manufacturer's respective wholesale prices must satisfy condition (6). In fact, if the retailer's (auxiliary) participation constraint did not bind, the manufacturer would set $w_n^t = w_n^m$ for both products. But then, from $w_B^s < w_B^m$, the retailer's profits would be strictly lower than under separate contracting, a contradiction. Suppose now that pricing for product A is not constrained under separate contracting. Then, the first-order condition for w_A^s requires that $(w_A - c_A)/w_A = 1/\eta_A$, while $w_B^s < w_B^m$ implies that $(w_B - c_B)/w_B \neq 1/\eta_B$, which does not satisfy condition (6). In this case, the solutions do not coincide.[6] If under separate contracting the manufacturer is constrained for both products, then $w_n^s = c_n^o$. In this case, the solutions coincide only if the alternative options satisfy $c_n^o = w_n^t$ (satisfying condition (6)). This shows the manufacturer's wholesale prices will generically be different under the auxiliary problem than under separate contracting, and thus we know that the manufacturer's profit (and therefore total industry profits) will be strictly higher. **Q.E.D.**

[6] A more intuitive way of understanding why the solutions do not coincide is to notice that in this case, from the envelope theorem, a marginal reduction of the wholesale price for product A, where $w_A^s = w_A^m$, does not have a first-order impact on the manufacturer's profit, but it strictly increases the retailer's profit and thereby relaxes his participation constraint. This in turn allows the manufacturer to marginally increase the wholesale price of the weak product, which from $w_B^s = c_B^o < w_B^m$, strictly increases his profit.

The intuition is simply that by tying together the terms for the two products, the manufacturer is able to satisfy more efficiently the retailer's fixed participation constraint. Specifically, tying benefits the manufacturer in this case because it gives him more flexibility in choosing his wholesale prices. It allows him, for example, to potentially increase his profit by lowering his wholesale price on the product on which he is initially less constrained and raising his wholesale price on the product on which he is initially more constrained, all the while continuing to keep the retailer's profit the same. Having two contractual instruments at his disposal, the manufacturer can thus better reconcile the conflicting objectives of maximizing overall profit and extracting the respective surplus.

Note that Proposition 1 does not make any prediction as to which wholesale price will increase under tying and which will decrease. To shed light on this, recall from (5) that the marginal rate of substitution between profits must be equal for the two products whenever the joint participation constraint binds under tying. In some instances, it is easy to see how the wholesale prices must adjust to make this happen. In other instances, however, it is less straightforward. Consider first the case of $w_A^s = w_A^m$ and $w_B^s < w_B^m$. Then, by optimality, $d\Pi_A/dw_A = 0$ (implying that $d\Pi_A/d\pi_A = 0$ under separate contracting) and $d\Pi_B/dw_B > 0$ (implying that $d\Pi_B/d\pi_B < 0$ under separate contracting). Since in this case transferring marginally higher profit to the retailer by lowering the price on product A has a zero first-order effect on the manufacturer's profit, while transferring marginally higher profit to the retailer by lowering the price on product B has a negative first-order effect on the manufacturer's profit, it should be clear that the optimal tied contract must specify a lower wholesale price on product A and a higher wholesale price on product B.[7]

Things are more nuanced, however, in the case of $w_A^s < w_A^m$ and $w_B^s < w_B^m$, as then transferring marginally higher profit to the retailer via a lower wholesale price always results in a first-order loss to the manufacturer. Writing out explicitly the marginal rate of substitution between the manufacturer's and the retailer's profit for product n, we have

$$\frac{d\Pi_n}{d\pi_n} = -1 - \frac{w_n - c_n}{q_n}\frac{dq_n}{dw_n},$$

which is equal to minus one at $w_n = c_n$, zero at $w_n = w_n^m$, and bounded between minus one and zero for all other values of w_n between c_n and w_n^m. If we further assume, as seems reasonable, that $d\Pi_n/d\pi_n$ is (at least weakly) increasing (and in absolute terms decreasing) in w_n, and if this property holds for both products, then the prediction as

[7] If the opposite were true, the manufacturer would see his profit on both products decrease, which would make him unambiguously worse off under tying than under separate contracting, a contradiction.

to how wholesale prices would change in the auxiliary problem can be found simply by comparing the respective marginal rates of substitution at the wholesale prices prevailing under separate contracting: To equalize the marginal rates of substitution under tied contracting, the wholesale price of the product with a marginal rate of substitution that was previously closer to zero must decrease and that of the other product must increase.[8]

The case of $w_A^s < w_A^m$ and $w_B^s < w_B^m$ is of especial interest, because when it holds, the joint participation constraint under tied contracting must be binding (cf. footnote 4). The auxiliary problem in Proposition 1 then exactly coincides with the manufacturer's true maximization problem, and the only effect of tying in this case would be the one that we have identified here — that of improved contractual efficiency. The outcome can potentially be very different, however, when the manufacturer's strong product is sufficiently strong that the retailer's outside option for it does not constrain the manufacturer under separate contracting. When this is the case, whether or not the joint participation constraint (3) would be binding under tying, there is a second effect at work, which we analyze next.

Increased Rent Extraction under Tied Contracting. To illustrate the second effect of tying, that of increased rent extraction, we focus on the case where under separate contracting, the manufacturer leaves the retailer with a rent on the strong product. This would be the case, for example, whenever Case 1 of Lemma 1 applies. Leaving rent with the retailer when Case 1 applies is optimal, if not ideal, for the manufacturer, because although he could reduce the retailer's profit by increasing his wholesale price above $w_A^s = w_A^m$ in this case without infringing on the retailer's participation constraint, he would also be reducing his own profit, which is not in his interest to do. Given this, the retailer's profit, π^s, will be strictly higher than the sum of his respective two outside options, $\pi_A^o + \pi_B^o$.

Tying can help in this instance. By tying his two offers together, the manufacturer can "tap into" the respective rent that the retailer would realize under separate contracting. This can be seen by marginally adjusting upwards the separate wholesale price for product B, starting from $w_B^s = c_B^o$. Under separate contracting, the retailer would not accept such a higher wholesale price for product B. But when the two offers are tied together, the retailer accepts it, because otherwise he would lose the rent that he earns from product A.

We can formally state this "increased rent extraction" effect of tying as follows:

[8] There is a technical reason to suppose that this property of a (at least weakly) decreasing marginal rate of substitution holds generally for the considered demand functions. If this were not the case, there would be a region where Π_n as a function of π_n would cease to be (at least weakly) concave. Then profits could be transferred more efficiently between the two firms by involving lotteries over wholesale contracts. In technical terms, this would convexify the (efficiency) boundary of the set of feasible pairs (π_n, Π_n).

Proposition 2 *Suppose the manufacturer's strong product A is sufficiently strong that the retailer's outside option for it does not bind under separate contracting. Then, under tied contracting, the retailer's profit strictly decreases and the manufacturer's profit increases.*

Proof of Proposition 2. Take the characterization of tied contracting in Lemma 2, where two cases are distinguished. When the joint participation constraint of the retailer binds, the retailer's profits under tied contracting are necessarily strictly lower than his profits under separate contracting, as there $\pi^s > \pi_A^o + \pi_B^o$. In the second case, the manufacturer offers the two unconstrained optimal contracts, $w_n^t = w_n^m$, so that the assertion follows immediately from $w_B^s < w_B^m$ under separating contracting (while $w_A^s = w_A^t = w_A^m$). **Q.E.D.**

This increased-rent-extraction effect of tying is necessarily harmful for welfare as it implies an increase in at least one (if not both) of the manufacturer's wholesale prices without the other decreasing. Consider, for example, the case in which neither the retailer's participation constraint on product A under separate contracting nor the joint participation constraint under tied contracting binds. Then, the optimal wholesale prices under separate contracting are $w_A^s = w_A^m$ and $w_B^s = c_B^o$, respectively, and the optimal wholesale prices under tied contracting are $w_A^t = w_A^m$ and $w_B^t = w_B^m$, respectively, implying that there will be an increase in the wholesale price of product B with no change in the wholesale price of product A. In this case, the only effect of tying is the increased-rent-extraction effect.

The more interesting case is when Case 1 of Lemma 1 holds for product A, but the joint participation constraint under tied contracting binds. In this instance, both effects of tying will be operating. Starting from the initial setting of $w_A^s = w_A^m$ and $w_B^s = c_B^o$, one can then decompose the overall change that would arise in the wholesale prices under tied contracting into the change in the wholesale prices that would arise in moving to Ramsey pricing while fixing the retailer's profit at π^s (the "improved-contractual-efficiency" effect) plus the change in the wholesale prices that would arise from reducing the retailer's profit from π^s to $\pi_A^o + \pi_B^o$ while continuing to satisfy the Ramsey-pricing-like rule (6) (the "increased-rent-extraction" effect). Whether the overall change in this case would lead to an increase or decrease in welfare would then depend on the strength of the two effects.

4.2 Balancing of Effects

The two effects of tying that we have described can have opposite implications for welfare. When the manufacturer's strong product is sufficiently strong that the joint participation constraint does not bind under tying, then only the increased-rent-extraction effect of tying is present, and welfare is unambiguously worse off (as the wholesale and hence retail price of product B will then be higher with no reduction in the strong product's prices). If instead all participation constraints are binding, both pre and post tying, then only the improved-contractual-efficiency effect of tying is present, and the presumption is that welfare will improve (the Ramsey pricing that arises increases industry profit (which we have shown in general) without necessarily making consumers in aggregate any worse off).[9]

In this section, we illustrate the model using an example with linear demands, and show that in this case the improved-contractual-efficiency effect of tying does indeed increase total welfare. We then use it to explore the case in which both effects are jointly present. The main result is that either effect can dominate, with tying more likely to decrease welfare the stronger is the manufacturer's strong product relative to the retailer's alternatives.

To keep the example as simple and as illustrative as possible, we let $D(p_n) = a - bp_n$, which is kept symmetric for both products. We also stipulate symmetry in the manufacturer's unit costs, $c_A = c_B$, which are set to zero. This allows us to focus attention on the key parameters c_n^o, which determine the attractiveness of the retailer's alternatives and thereby the strength of the strategic manufacturer's own products. Recall also that we have assumed that $c_B^o < \hat{c}_B^o$ for the weaker product B (Case 2 of Lemma 1 always holds for product B). With symmetry in all other aspects, product A is thus (weakly) stronger if and only if the alternative supply option for product A is less attractive (i.e., $c_A^o \geq c_B^o$).

The solution and derivations of all relevant expressions are given in the Appendix. Figure 1 below is representative. It depicts wholesale prices in the special case of $a = b = 1$, $c_n = 0$, and $c_B^o = 1/3$. Wholesale prices are on the vertical axis, and the attractiveness of the retailer's outside option for product A, as measured by c_A^o, is on the horizontal axis.

What we find is that when $c_A^o = c_B^o$, tied contracting has no effect at all: it neither enhances efficiency or increases rent extraction. Instead, tied contracting is only effective when there is some degree of asymmetry, as when the cost of the retailer's alternative for product A begins to increase. When this is sufficiently small, so that $c_A^o - c_B^o$ is sufficiently small, the retailer's participation constraints will continue to bind under separate contracting (and thus also under tied contracting) and only the positive efficiency effect of tying is present. In the case of Figure 1, it can be seen that under separate contracting, w_B^s is constrained to equal $c_B^o = 1/3$, whereas for all $c_A^o \leq \hat{c}_A^o$, w_A^s is constrained to equal c_A^o, which is increasing along the horizontal axis. In contrast, under tied contracting, Ramsey

[9] The Ramsey-pricing-like formula relies on the elasticities of derived demand, which reflect the respective properties of final demand. It works to increase the manufacturer's profits by better managing the allocative distortions that arise across products which are caused by his wholesale price mark-ups.

pricing requires that both wholesale prices w_n^t be the same (given that demands and the manufacturer's unit costs are symmetric). With linear demands, this necessarily results in an increase in both industry profits and total welfare. Compared to separate contracting, the wholesale price of product A decreases while that of the weaker product B increases.

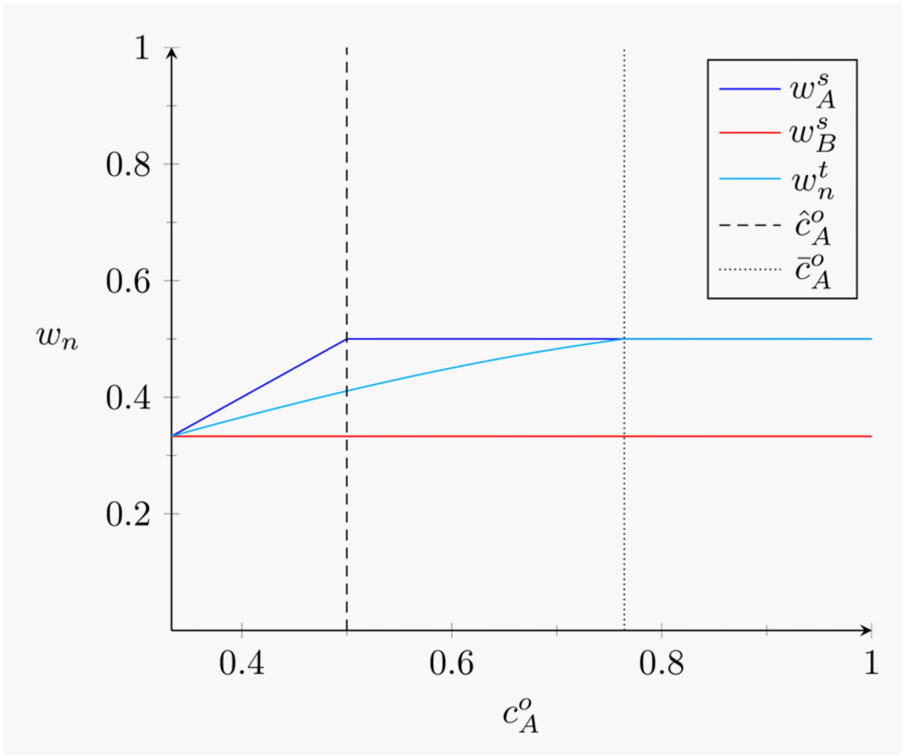

Figure 1: Wholesale prices under linear demand (with $a = b = 1$, $c_n = 0$, $c_B^o = 1/3$).

For increases in c_A^o beyond $\hat{c}_A^o$ (i.e., the threshold at which the manufacturer is no longer constrained on its pricing of product A under separate contracting), the second effect of tying kicks in, with a resulting negative effect on efficiency. When the retailer's respective rent under separate contracting is however still small, this negative effect remains muted, so that the positive effect of enhanced contractual efficiency is still dominant. At some point, though, the increased-rent-extraction effect of tying begins to gain the upper hand, which occurs when the retailer's alternative option for product A is sufficiently unattractive. In the case of Figure 1, this occurs when under separate contracting, $w_B^s = 1/3$ and $w_A^s = w_A^m = 1/2$ is now at its unconstrained upper bound. Although it is still the case that Ramsey pricing requires that both wholesale prices w_n^t be the same in this region, the decrease in w_n^t relative to w_A^s flattens as one moves from left to right, even as the increase in w_n^t relative to w_B^s widens. Further increases in c_A^o beyond a second cutoff, $\bar{c}_A^o$, cause even the joint participation constraint under tying to be slack, in which case the manufacturer optimally sets $w_n^t = w_n^m$ on both products. Welfare in this case is unambiguously lower.

Summarizing our findings, we have the following results with linear demand:

Proposition 3 *Consider the case of linear and symmetric demand, for which we calculate the difference in welfare between tied contracting and separate contracting:* $\Delta W = W^t - W^s$*. Starting from* $c_A^o = c_B^o$*, as* c_A^o *increases, thereby making the manufacturer's product* A *stronger, there exists* $c_W > \hat{c}_A^o$ *such that* $\Delta W > 0$ *if* $c_A^o < c_W$ *and* $\Delta W < 0$ *if* $c_A^o > c_W$*.*

Proposition 3 trades off the two effects of tying. Depending on c_A^o, there are clear implications for the overall welfare effect and thus for antitrust policy. When a manufacturer's competitive advantage on his strong product A is not too large, the improved-contractual-efficiency effect of tying dominates. Tying the terms of the weak product B to those of the strong product A should then be permitted. The opposite recommendation applies when the manufacturer has a sufficiently large competitive advantage on his strong product (formally, when the retailer's alternative is sufficiently worse at $c_A^o > c_W$). Then, the increased-rent-extraction effect dominates, which has negative welfare implications due to double-marginalization. This again highlights the role of the underlying contractual inefficiency, as this is the source of both the positive and the negative effects of tying. Figure 2 illustrates the results of Proposition 3, where it can be seen that the change in welfare is positive and increasing in the leftmost region, decreasing and eventually turning negative in the middle region, and then remaining negative and leveling off in the rightmost region.

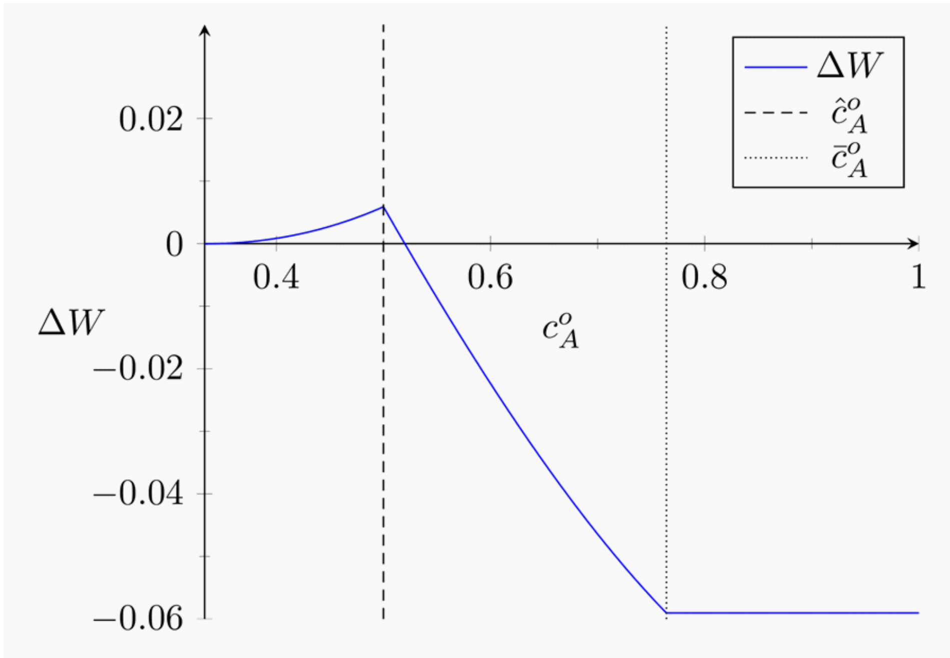

Figure 2: Difference in welfare under linear demand (with $a = b = 1$, $c_n = 0$, $c^o_B = 1/3$).

We have seen that there is no scope in our model thus far for tying to have strategic effects, e.g., to foreclose competitors or to transfer rents from them to the contracting manufacturer and retailer. As we discuss next, this is so even when a more efficient supplier is excluded.

5 (In-)Efficient Exclusion

Our focus up to now has been on cases in which the manufacturer can supply both products at least as efficiently as his competitors. This was useful for two reasons. First, it meant that we did not need to check whether supplying both products was indeed optimal for the manufacturer, and second, it meant that we could focus the welfare analysis on an analysis of contractual efficiency. We now extend the analysis to consider the case where $c_B > c^o_B$, and ask, first, whether and when the introduction of such a weak product is optimal for the manufacturer, and second, what the welfare implications are if he does introduce it.

Strategic Incentives for the Introduction of a Weak Product. When the manufacturer can supply product B as efficiently as his competitors, so that $c_B = c^o_B$, we know that he strictly prefers to offer both products as long as he can tie his weak product B to his strong product A. We now imagine an increase in c_B that makes the manufacturer less efficient at producing product B than other suppliers. Then, clearly, under separate contracting the manufacturer would strictly prefer not to supply product B at the same terms as the others, as this would result in losses. Hence, his maximum profit under separate contracting, Π^s, will equal his profit on product A alone, Π_A^s, and any further increases in c_B will have no impact. This is different when the manufacturer supplies both products under tied contracting, as there his profits, which we denote by Π^t, strictly decrease with c_B. Together with the continuity of profits in costs, we thus have the following result:

Proposition 4 *Suppose the manufacturer can no longer supply product B as efficiently as his competitors. Then, he will never supply product B under separate contracting, but may, depending on how inefficient he is, supply product B under tied contracting. In particular, there exists a threshold c_{Ex} that strictly exceeds the alternative suppliers' costs, c_B^o, such that the manufacturer will supply product B under tied contracting as long as $c_B < c_{Ex}$.*

Proof of Proposition 4. Denote the maximum manufacturer profit under tied contracting by Π^t, where Π^t is obtained by maximizing $\Pi = \Pi_A + \Pi_B$ subject to (3). Because the solution to this program is assumed to be unique, we have that Π^t is continuous and strictly decreasing in c_B as long as $q_B > 0$. Denote the maximum manufacturer profit from supplying only product A under separate contracting by Π^s, where Π^s is obtained by maximizing Π_A subject to (1) for product A. As the program for Π^t constrains the manufacturer to satisfy the joint participation constraint of the retailer (whether or not product B is offered), and as $q_B = 0$ for sufficiently high c_B, it follows that $\Pi^t < \Pi^s$ for such c_B when $\pi_B^o > 0$. To obtain a unique cutoff $c_{Ex} > c_B$ where $\Pi^t = \Pi^s$, it thus remains to show that the converse holds at $c_B = c_B^o$. As the manufacturer's profit from offering product B when $w_B = c_B = c_B^o$ is zero, the claim follows immediately when the optimal tied wholesale prices differ from $(w_A^s, w_B = c_B^o)$, which follows from Lemma 2. **Q.E.D.**

Proposition 4 implies that the manufacturer may have an incentive to expand his product offerings and supply a weak product even when he cannot supply it as efficiently as his competitors. For products that are independent in demand (as we have assumed here), this might mean the introduction of a weak product in a different product category, i.e., other than the manufacturer's core category (of product A). But even within a given category, demand for different products can often be independent, which may be the case with the offering of Coca Cola and Bonaqua, as discussed in the Introduction.[10]

[10]In fact, in many categories the different products supplied by leading brand manufacturers are not strong substitutes, e.g., the different milk-based products in the dairy category or various tinned products.

Moreover, as we noted in the Introduction, the two effects of tying that we have identified do not depend on demands being independent, but would be expected to hold even with substitution. It then follows from the same logic that manufacturers of strong brands may have incentives to extend their product line (line extensions) to weak, substitute products, even if their new offerings would not otherwise be competitive on a stand-alone basis.

We now show that the manufacturer's incentive for such a seemingly over-extension of his offerings/product line is especially high the stronger is his core product A. This holds for two reasons. First, as the distance between his product and his competitors' products increases, via an increase in c_A^o, a constrained manufacturer's optimal stand-alone wholesale price w_A^s under separate contracting increases, and with it so does the double-marginalization inefficiency. This can be mitigated by balancing the wholesale price of the strong product A with that of the weaker product B. Second, in settings in which product A is particularly strong, the manufacturer cannot fully exploit his advantage relative to his competitors under linear contracting, and the retailer necessarily realizes a rent. The introduction of even an inefficiently supplied weak product then helps by generating a second instrument to extract this rent. Together, the two reasons imply the following:

Proposition 5 *Ceteris paribus, the threshold c_{Ex} up to which the manufacturer will supply product B despite an increasing cost disadvantage is strictly increasing in the strength of product A, as given by the distance to competitors (i.e., c_{Ex} is strictly increasing in c_A^o).*

Proof of Proposition 5. Recall from the proof of Proposition 4 that $\Pi^t - \Pi^s = 0$ at $c_B = c_{Ex}$, where Π^s is the maximum profit that can be obtained by supplying only product A. We note that Π^s is independent of c_B. From implicit differentiation we have that

$$\frac{dc_{Ex}}{dc_A^o} = - \left. \frac{d\Pi^t/dc_A^o - d\Pi^s/dc_A^o}{d\Pi^t/dc_B} \right|_{c_B = c_{Ex}} .$$

Because we have already established that $d\Pi^t/dc_B < 0$ when $c_B < c_{Ex}$ (as then $q_B > 0$), it remains only to prove that $d\Pi^t/dc_A^o > d\Pi^s/dc_A^o$. To this end, we note that at $c_B = c_{Ex}$ the joint participation constraint (3) must bind under tied contracting (because otherwise $\Pi^t > \Pi^s$). This implies that $d\Pi^t/dc_A^o > 0$. The claim then follows immediately when the participation constraint does not bind under separate contracting for the strong product A. This captures formally the second of the two rationales described in the main text.

We now consider the case where the participation constraint for product A does bind. In this case, the respective comparison of outcomes coincides with the discussion of the auxiliary problem in Proposition 1. Consider first $d\Pi^s/dc_A^o$. It is useful to rewrite this as $[d\Pi_A/d\pi_A^o]/[d\pi_A^o/dc_A^o]$. As the effect on the participation constraint is the same under tied contracting, $d\Pi^t/dc_A^o > d\Pi^s/dc_A^o$ holds whenever $d\Pi^t/d\pi^o > d\Pi_A/d\pi_A^o$ (both strictly negative), where $\pi^o = \pi_A^o + \pi_B^o$. This captures formally the first rationale described in the text. For this it is sufficient to suppose that as a response to an increase in π^o the manufacturer only adjusts w_A also under tied contracting. Then the result follows when $d\Pi_A/d\pi_A$ is lower in absolute terms with the wholesale price under tied contracting, which, given assumed monotonicity of the marginal rate of substitution, holds if $w_A^t > w_A^s$ is lower under tied contracting than under separate contracting. But this follows immediately as we analyze the outcome at $\Pi^t - \Pi^s = 0$, and thus at $c_B = c_{Ex} > c_B^o$, as otherwise tying the inefficiently provided product would not have been optimal. **Q.E.D.**

Proposition 5 implies that the stronger is the manufacturer's core product relative to the retailer's outside options, the greater will be his incentive to tie it to a weak product, even one that is inefficiently supplied. The desirability of introducing an inefficiently supplied product into the market, from a welfare perspective, is the topic we turn to next.

Welfare Implications. The introduction of an inefficiently supplied product naturally raises the concern that welfare may be reduced, all else being equal, by the higher costs of production. Focusing only on the inefficient production, however, would be misleading as it would ignore the positive effects of tying on contractual efficiency. There is thus a tradeoff to consider, and all else will not in general be equal. In fact, as we will show next, as long as the manufacturer's core product A is not too strong, the manufacturer will have an incentive to introduce his inefficiently supplied weak product B only when it increases total industry profits (and when demand is linear, also consumer welfare). In this case, contrary to what one may have thought, the exclusion of the more efficient suppliers of product B can actually lead to higher overall efficiency. But, as we will also show, this conclusion need not hold when there is also rent shifting going on. In that case, the manufacturer has another incentive to introduce the weak product (beyond contractual efficiency), which only serves the purpose of transferring (but not increasing) joint surplus.

Proposition 6 *Suppose the manufacturer cannot supply his weak product as efficiently as his competitors. If under separate contracting the manufacturer would be constrained on product A, then the manufacturer will introduce the weak product B only when this increases industry profit (and, when demand is linear, also total welfare). This need not be the case, however, when the introduction of the weak product B also serves to extract rent* *from the retailer. In that case, the manufacturer's incentive to introduce the weak product B is too high under tied contracting compared to the social optimum (formally, $W^t < W^s$ at $c_B = c_{Ex}$), and inefficient exclusion can prevail, at least when c_B is sufficiently high.*

Proof of Proposition 6. We know that the retailer's profit under tying (given that the manufacturer is maximizing $\Pi = \Pi_A + \Pi_B$ subject to (3)) will be the same as his profit under separate contracting when the participation constraint for product A is binding. In both cases, the retailer will earn a profit of $\pi_A^o + \pi_B^o$. We also know that consumer surplus will be unaffected in this case if demand is linear (per our findings in Section 4). Together with $\Pi^t = \Pi^s$ at $c_B = c_{Ex}$, this establishes the first claim. To show the second claim, note that if the participation constraint for product A does not bind under separate contracting, then the retailer's profit under separate contracting (and also consumer surplus if demand is linear) will be strictly greater than his profit under tying (because of the increased-rent-extraction effect). Again together with $\Pi^t = \Pi^s$ at $c_B = c_{Ex}$, this establishes that $W^t < W^s$ at $c_B = c_{Ex}$. **Q.E.D.**

To understand the intuition for this, note that when only the improved-contractual-efficiency effect of tying is present, the manufacturer fully internalizes the higher costs of the inefficient production when deciding whether to introduce his weak product B. This is because the retailer is indifferent in both cases. In the absence of tying, the retailer buys from the alternative suppliers of product B at cost $c_B^o < c_B$, but pays a relatively high per-unit markup to obtain product A from the manufacturer, whereas in the optimal offer when the manufacturer introduces his inefficient product and ties it to product A, the retailer is required to pay a higher per-unit price on product B, but then realizes an offsetting lower per-unit price on product A. What is lost on the former is gained on the latter. In effect, the retailer (and society) benefits from the existence of the efficient alternative suppliers of product B, whether or not the retailer actually buys from them.

In contrast, when both effects of tying are present, or when only the increased-rent-extraction effect of tying is present, the retailer's profit under tying will be strictly less than what he would earn under separate contracting. In these cases, the manufacturer does not have to lower his wholesale price on product A by as much, if at all, in order to get the retailer to accept his tied offer, and as a result, the retailer may be stuck paying more for the two products than would be optimal from his (and society's) perspective.

6 Conclusion

We have analyzed the practice of tying the contractual terms of different products in intermediary industries. This practice essentially constrains a retailer who might otherwise have purchased from different manufacturers to purchase from the same manufacturer. In the process, we isolated two opposing effects of the practice, with both effects originating from the same underlying contractual inefficiency of double-marginalization. On the one hand, we found that such tying of contractual terms may allow surplus to be shared more efficiently (the improved-contractual-efficiency effect of tying), as wholesale prices can then be set according to a Ramsey-pricing-like rule. On the other hand, we found that when one of the manufacturer's products is sufficiently strong, tying it together with the supply of a weaker product allows the manufacturer to extract more rent from the retailer (the increased-rent-extraction effect of tying), which, due to double-marginalization, reduces efficiency. Although either effect can dominate, we found that the overall effect is more likely to be negative when the manufacturer has a particularly strong product, so that the retailer's alternatives are much inferior. Then, the manufacturer can also have excessive incentives to introduce a particularly weak product, which other suppliers can produce at lower costs. However, importantly, we found that such foreclosure of a more efficient rival does not generally result in a loss of welfare, but only if the rent extraction effect dominates the contractual efficiency effect. As we discussed in the Introduction, cases where antitrust authorities have taken an issue with such tied rebates often concern intermediary industries. Our contribution supports an effects-based approach to such practices.

Although the focus of our contribution lies with its normative implications, we conclude with some remarks of how future work could also derive additional positive implications, beyond providing a rationale for why manufacturers with particularly strong brands may wish to extend their product lines by introducing even very weak products. One set of such positive implications could relate to an analysis of cost pass-through even across independent products and product categories. For this, consider, as outlined in the Introduction, the negotiations between TCCC and a retailer over the supply of TCCC's strong brand (Coca-Cola) and a weak product (Bonaqua). The specific cost shock may affect the sugar price or that of another specific ingredient of cola, but not that of the production of bottled water. Standard theory would predict that in this case only the wholesale and retail price of product A would be affected, but not those of product B. However, it is straightforward to see that this is no longer the case under tied contracting. For instance, in the characterized case of Ramsey pricing, holding all else constant, we would expect the wholesale price of cola to increase and that of water to decrease. In a full-fledged analysis, however, we would also expect the increasing cost of sugar to negatively affect the retailer's outside option with other softdrink suppliers. Interestingly, under tied contracting, a change in the retailer's outside option for one product would also have repercussions for the wholesale prices of the other product, even when the two products are independent in final demand.

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Appendix: Derivations for the Case of Linear Demand

The following derivation for the case of linear demand serves two purposes. First, it serves to prove Proposition 3 as well as the latter part of Proposition 6. Second, we thereby obtain the expressions that are needed for the numerical example.

We proceed as follows: For the proof of the two Propositions, we essentially rely only on one particular feature of the case of linear demand, that which relates to the analogous comparative statics of consumer welfare and retailer profits. We turn to this first. Subsequently, we derive the characterizations used in the linear example in the main text.

A Primer on Retailer Profits and Consumer Welfare. With linear symmetric demands, we have $D(p_n) = a - bp_n$, which gives rise to the optimal retail prices and quantities $p_n(w_n) = (a + bw_n)/2b$ and $q_n(w_n) = (a - bw_n)/2$. With this at hand, we have retailer profits of $\pi_n = (a - bw_n)^2/4b$ and consumer surplus of $CS_n = \int_0^{q_n}(a - q)/bdq - p_nq_n$. After some substitutions, the latter transforms to $CS_n = (a - bw_n)^2/8b$, so that $CS_n = \pi_n/2$ and, aggregated over both products, $CS = \pi/2$. This proves that with linear symmetric demands, consumer surplus is a linear function of the retailer's profits. It remains unchanged, increases, or decreases if the same holds for the retailer's profits.

Separate Contracting. If unconstrained, the manufacturer's optimal wholesale price, maximizing $q_n(w_n)(w_n - c_n)$, is $w_n^m = (a + bc_n)/2b$, which simplifies to $w_n^m = w^m = a/2b$ when $c_n = 0$. Profits for the manufacturer and the retailer are then $\pi_n = a^2/16b$ and $\Pi_n = a^2/8b$, respectively. If instead the manufacturer sets $w_n^s = c_n^o$ under separate contracting, profits are $\pi_n = \pi_n^o = (a-bc_n^o)^2/4b$ and $\Pi_n = (a-bc_n^o)c_n^o/2$, respectively, again using $c_n = 0$. It follows that the manufacturer is not constrained on product A if $c_A^o \geq \hat{c}_n^o = a/2b$.

Tied Contracting. We can now make use of the above expressions to set up the more complex program under tied contracting:

$$\max_{w_A, w_B} \quad \frac{(a - bw_A)w_A + (a - bw_B)w_B}{2}$$
$$\text{s.t.} \quad \frac{(a - bw_A)^2 + (a - bw_B)^2}{4b} \geq \frac{(a - bc_A^o)^2 + (a - bc_B^o)^2}{4b}.$$

When the joint participation constraint binds, given the Ramsey pricing rule (6) and symmetric demand, it must hold that $w_A^t = w_B^t$. Plugging this into the binding joint participation constraint, we thus have

$$w_A^t = w_B^t = c_B^o + \frac{2a(c_A^o - c_B^o) - b\left[(c_A^o)^2 - (c_B^o)^2\right]}{2(a - bc_B^o) + \sqrt{2\left[(a - bc_A^o)^2 + (a - bc_B^o)^2\right]}.}$$

The retailer profits in this case are given by the binding participation constraint, $\pi^t = [(a - bc_A^o)^2 + (a - bc_B^o)^2]/4b$. Substitution yields for the manufacturer's profits

$$\Pi^t = \frac{a\sqrt{2\left[(a - bc_A^o)^2 + (a - bc_B^o)^2\right]} - (a - bc_A^o)^2 - (a - bc_B^o)^2}{2b}.$$

Finally, we need to determine which case applies under tied contracting. Using the respective retailer profits, we need to solve for the cutoff $\bar{c}_A^o$ at which the retailer's profit under the assumption that the joint participation constraint binds equals its profit under the assumption that it does not bind: $\left[(a - b\bar{c}_A^o)^2 + (a - bc_B^o)^2\right]/4b = a^2/8b$. This yields

$$\bar{c}_A^o = \frac{a}{b} - \sqrt{\frac{(bc_B^o)^2 - (a - 2bc_B^o)^2}{2b^2}}.$$

Industry Profits and Welfare under Separate Contracting. When the manufacturer is constrained for both products under separate contracting, the retailer realizes a profit of $\pi^s = [(a - bc_A^o)^2 + (a - bc_B^o)^2]/4b$ and the manufacturer realizes a profit of $\Pi^s = [((a - bc_A^o)c_A^o + (a - bc_B^o)c_B^o]/2$, so that industry profits are

$$PS^s = \frac{2a^2 - b^2\left[(c_A^o)^2 + (c_B^o)^2\right]}{4b}.$$

Recall further that then $w_n^s = c_n^o$ and with this $q_n^s = (a - bc_n^o)/2$ and $p_n^s = (a + bc_n^o)/2b$. Consequently, consumer surplus is

$$CS^s = \int_0^{q_A^s} \frac{a - q_A}{b} dq_A - p_A^s q_A^s + \int_0^{q_B^s} \frac{a - q_B}{b} dq_B - p_B^s q_B^s = \frac{(a - bc_A^o)^2 + (a - bc_B^o)^2}{8b}.$$

Aggregating both industry profits and consumer surplus yields total welfare:

$$W^s = \frac{6a^2 - 2ab\left(c_A^o + c_B^o\right) - b^2\left[(c_A^o)^2 + (c_B^o)^2\right]}{8b}.$$

We turn next to the case where the manufacturer is not constrained on his strong product A, so that $w_A^s = w_A^m$. This case applies when $c_A^o \geq \hat{c}_A^o = w_A^m$, i.e., when $c_A^o \geq a/2b$. Aggregating the previously derived profits yields

$$\Pi^s = \frac{a^2 + 4abc_B^o - 4b^2(c_B^o)^2}{8b} \text{ and } \pi^s = \frac{5a^2 - 8abc_B^o + 4b^2(c_B^o)^2}{16b},$$

so that total industry profits are now

$$PS^s = \frac{7a^2 - 4b^2(c_B^o)^2}{16b}.$$

Substituting the derived wholesale prices yields $p_A^s = 3a/4b$ and $p_B^s = (a + bc_B^o)/2b$, so that $q_A^s = a/4$ and $q_B^s = (a - bc_B^o)/2$. This yields for consumer surplus and total welfare

$$\begin{aligned} CS^s &= \frac{5a^2 - 8abc_B^o + 4b^2(c_B^o)^2}{32b}, \\ W^s &= \frac{19a^2 - 8abc_B^o - 4b^2(c_B^o)^2}{32b}. \end{aligned}$$

Industry Profits and Welfare under Tied Contracting. We suppose first that the joint participation constraint binds. In that case $\pi^t = [(a - bc_A^o)^2 + (a - bc_B^o)^2]/4b$, and after the substitution of w_n^t for the manufacturer, we obtain

$$\Pi^t = \frac{a\sqrt{2\left[(a - bc_A^o)^2 + (a - bc_B^o)^2\right]} - (a - bc_A^o)^2 - (a - bc_B^o)^2}{2b},$$

so that industry profits are

$$PS^t = \frac{2a\sqrt{2\left[(a - bc_A^o)^2 + (a - bc_B^o)^2\right]} - (a - bc_A^o)^2 - (a - bc_B^o)^2}{4b}.$$

Substituting the wholesale prices, we have for the retail prices and quantities

$$\begin{aligned} q_A^t = q_B^t &= \frac{\sqrt{2\left[(a - bc_A^o)^2 + (a - bc_B^o)^2\right]}}{4}, \\ p_A^t = p_B^t &= \frac{a}{b} - \frac{\sqrt{2\left[(a - bc_A^o)^2 + (a - bc_B^o)^2\right]}}{4b}, \end{aligned}$$

so that

$$CS^t = 2\left[\int_0^{q_n^t} \frac{a - q}{b} dq - p_n^t q_n^t\right] = \frac{(a - bc_A^o)^2 + (a - bc_B^o)^2}{8b},$$

and total welfare is thus

$$W^t = \frac{4a\sqrt{2\left[(a - bc_A^o)^2 + (a - bc_B^o)^2\right]} - (a - bc_A^o)^2 - (a - bc_B^o)^2}{8b}.$$

We now suppose that the joint participation constraint is slack. As then $w_n^t = w_m^t$, it is immediate that $\Pi^t = a^2/4b$, $\pi^t = a^2/8b$, and thus $PS^t = 2a^2/8b$. With $p_n^t = 3a/4b$ and $q_n^t = a/4$, consumer surplus equals $CS^t = a^2/16b$ and total welfare is thus $W^t = 7a^2/16b$.

Comparison of Industry Profits and Welfare (Proposition 3) Based on the derived expressions we can now turn to the proof of Proposition 3. We consider first the range of costs up to $c_A^o \leq \hat{c}_A^o$. We note here that by definition $\pi^s = \pi^t$, so that together with $\Pi^t > \Pi^s$, we thus have that $PS^t > PS^s$. With linear demand, we have from inspection of the respective derivations that $CS^s = \pi^s/2$ and $CS^t = \pi^t/2$, so that also $CS^s = CS^t$. Together with $PS^t > PS^s$, we thus have that $W^t > W^s$. Next, we turn to the other extreme, where $c_A^o \geq \bar{c}_A^o$. In this case, $W^t < W^s$ follows immediately from $w_A^t = w_A^s$ and $w_B^t > w_B^s$. In what follows, we thus deal with the intermediate case $\hat{c}_A^o < c_A^o < \bar{c}_A^o$.

In this intermediate case, the joint participation constraint binds under tied contracting, and with separate contracting, the participation constraint binds only for product B. Importantly, in this case, wholesale prices under separate contracting do not depend on c_A^o, so that profits Π^s and welfare W^s are unaffected by a change in c_A^o. This is not the case under tied contracting, where wholesale prices do depend on c_A^o. There, for example, as the joint participation constraint binds, π^t is strictly decreasing in c_A^o. In terms of how total welfare W^t changes in c_A^o, we note that the derivative with respect to the term $(a - bc_A^o)^2$ in W^t can change sign at most once. It follows that $W^t > W^s$ at $c_A^o = \hat{c}_A^o$ and $W^t < W^s$ at $c_A^o = \bar{c}_A^o$, together with continuity, establishes the existence of the unique threshold c_W.