

Price Pressure Indices, Innovation and Mergers Between
Commonly Owned Firms

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ONLINE APPENDIX

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A. Derivation of the PPI and merger delta under common ownership

This appendix contains some additional formal derivations that add to the main text, allowing to apply the analysis more generally. For this we first extend the equilibrium pricing (or mark-up) equation from the benchmark case without common ownership to the case with common ownership. Again, we confine ourselves to the statement and discussion of the respective formula, from the perspective of the pricing incentives of firm 1:²

$$P_1 = M_1 \left[c_1 + \sum_{k=2,3,\dots} \left(\left(\frac{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{ik}}{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i1}} \right) \delta_{k1} m_k \right) \right],$$

where we have made use of the “sum operator” Σ (so that, for instance, $\Sigma_{k=2,3,\dots}$ simply indicates that all the following terms are calculated for $k = 2, 3, \dots$ and then summed up). Though the notation looks unwieldy in its generality, it is, provided that the underlying assumptions such as the effectiveness of common ownership are true, relatively easy to calculate and apply. The key term in square brackets, which would be zero with a stand-alone firm without any ownership linkages to other firms, captures the degree to which the management of firm 1 internalizes the profits of all other firms $k = 1, 2, \dots$. While the respective product $\delta_{k1} m_k$ of the diversion ratio and the margin has already been introduced in the main text, for each considered rival firm k the ratio of the sums

$$w_{1k} = \frac{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{ik}}{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i1}}$$

is novel. The variable w_{1k} , which is used in the main text, essentially represents the weight that the management of firm 1 places on profits of firm k relative to that of its own firm 1. Here, the numerator sums up over all potential owners i of firm 1 the respective ownership shares in the considered rival k , β_{ik} , multiplied by the weight γ_{i1} that management of firm 1 places on this owner. The denominator, which refers to ownership in firm 1, essentially normalizes this, allowing us to say that the respective weight placed on firm k is relative to the weight of one placed on the firm’s own profits. Using the short-hand notation w_{1k} , we can simplify the mark-up formula as in the main text, again using the “sum operators” to obtain generality:

² See Daniel P. O’Brien & Steven C. Salop, *supra* note 13.

$$P_1 = M_1 \left[c_1 + \sum_{k=2,3,\dots} w_{1k} \delta_{k1} m_k \right].$$

We now extend this to the post-merger case. For simplicity, we suppose that the considered set of owners has not been widened in the course of the merger and now denote for the merged firm (12) the respective shares in the firm's profits by $\hat{\beta}_{i(12)}$ (with the "hat" indicating the post-merger scenario). The respective weights that the management of the merged firm puts on the respective owner are denoted likewise by $\hat{\gamma}_{i(12)}$. Possibly in course of the merger also the respective shares of profits change for owners of other firms, and with it possibly the respective weights that the management of other firms put on these owners. To capture this generally, we simply extend the post-merger notation (with the "hat") to all other firms j , thus denoting all ownership weights by $\hat{\gamma}_{ij}$ and all profit shares by $\hat{\beta}_{ij}$.

With this at hands, we can simply extend the derived (pre-merger) mark-up formula to the post-merger scenario, where, of course, P_1 now denotes the equilibrium price of the respective "business unit" in the merged firm. And with this we can finally extend the "merger delta" formula, which captures the upwards pricing pressure, under common ownership. Writing out all the intermediate step, before inserting the weight variables, we have for the delta:

$$\begin{aligned} \text{delta} &= \left(1 - \frac{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i2}}{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i1}} \right) \delta_{21} m_2 \\ &+ \sum_{k=3,4,\dots} \left(\left(\frac{\sum_{i=A,B,\dots} \hat{\gamma}_{i(12)} \hat{\beta}_{ik}}{\sum_{i=A,B,\dots} \hat{\gamma}_{i(12)} \hat{\beta}_{i(12)}} - \frac{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{ik}}{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i1}} \right) \delta_{k1} m_k \right) \\ &= (1 - w_{12}) \delta_{21} m_2 + \sum_{k=3,4,\dots} (\hat{w}_{(12)k} - w_{1k}) \delta_{k1} m_k. \end{aligned}$$

B. Illustration of the mitigating price effect of common ownership

Substituting back the expression for the pre-merger weight w_{12} , we have again, starting from the general expression,

$$\begin{aligned} \text{delta} &= \left(1 - \frac{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i2}}{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i1}} \right) \delta_{21} m_2 \\ &= \left(1 - \frac{\gamma_{A1} \beta_{A2} + \gamma_{B1} \beta_{B2}}{\gamma_{A1} \beta_{A1} + \gamma_{B1} \beta_{B1}} \right) \delta_{21} m_2 \\ &= \left(\frac{\gamma_{B1} (\beta_{B1} - \beta_{B2}) - \gamma_{A1} (\beta_{A2} - \beta_{A1})}{\gamma_{A1} \beta_{A1} + \gamma_{B1} \beta_{B1}} \right) \delta_{21} m_2, \end{aligned}$$

so that the mitigating effect is now expressed in terms of the pre-merger profit shares and ownership weights. In the literature, a common, though not necessarily always realistic, specification of the ownership weights is that of so-called “proportional control”³. Here, the weight that management places on a given owners equals the owner’s cash-flow rights: $\gamma_{ij} = \beta_{ij}$. With two owners only, we can further replace $\beta_{B2} = 1 - \beta_{A2}$ and $\beta_{B1} = 1 - \beta_{A1}$ and so we obtain that the sign of delta depends on the sign of $(\beta_{B1} - \beta_{A1}) \cdot (\beta_{A2} - \beta_{A1})$.

With this simplification we can now even ask when the delta of the merger is *negative*. For this we stipulate that A has a larger stake in firm 1 than owner B. Then we can see immediately that the delta is *negative* when owner A has a still larger share in the former rival 2, $\beta_{A2} > \beta_{A1}$. While this effect, whereby the merger exerts a (gross) downward pricing pressure, seems somewhat unrealistic, it is a direct effect of the combination of the PPI-methodology and the assumptions on how common ownership affects firm incentives (under, in addition, proportional control). In the considered extreme case, the pre-merger management of firm 1 essentially favored profits of firm 2 above profits of firm 1 as its main owners had a greater interest in the rival firm. The merger sets this “imbalance” straight, so that profits from both firms (respectively, business segments) enter the objective with same weights.

3 Note that the expression “control” in this sense is not tantamount to control in terms of Article 3 EUMR, *see supra* note **Fehler! Textmarke nicht definiert.**

C. Illustrating the reinforcing effect of common ownership

Taking up the illustrative case from the main text, we again express explicitly the respective weight put on other rivals, which is now $\widehat{w}_{(12)3}$ (i.e., the post-merger weight put in firm 3 by the management of the integrated firm (12)). As by assumption firm 2 was owned by B, who had and still keeps a stake in firm 3, with respective shares in the profits of the integrated firm and with the respective weights, the merger delta is then

$$\delta_{21}m_2 + \frac{\gamma_{B(12)}\beta_{B3}}{\gamma_{A(12)}\beta_{A(12)} + \gamma_{B(12)}\beta_{B(12)}}\delta_{31}m_3.$$

Obviously, when B keeps a larger stake in firm 3, as expressed by β_{B3} , and when the merged firm's management puts a larger weight $\gamma_{B(12)}$ on B, that is, relatively to its other owner $\gamma_{A(12)}$, then the additional upward pricing pressure arising from a widening of common ownership is higher.

D. Merger effect on innovations

As discussed in the main text, we return to the original derivation of management's objective under common ownership. Making now use of the general notation (via the sum operator), we have the following objective function relating to the pre-merger management of firm 1. The management maximizes a weighted sum of its owner's profits, that is, with the previously introduced notation, $\Pi_1 = \sum_{i=A,B,\dots} \gamma_{i1} \pi^i$. These profits in turn derive, under the given assumptions, from the owners' stakes β_{ik} in firms k in the considered industry. For instance, we have for owner A profits of $\pi^A = \sum_{k=1,2,\dots} \beta_{Ak} \pi_k$, where the subscript in π_k denotes the respective firm k . Substitution for owners' profits thus yields the objective for firm 1's management

$$\Pi_1 = \sum_{i=A,B,\dots} \gamma_{i1} \pi^i = \sum_{i=A,B,\dots} \gamma_{i1} \left(\sum_{k=1,2,\dots} \beta_{ik} \pi_k \right).$$

We can rearrange this expression, collecting terms referring to each firm $k = 1, 2, \dots$ in the industry, as follows:

$$\Pi_1 = \sum_{k=1,2,\dots} \pi_k \left(\sum_{i=A,B,\dots} \gamma_{i1} \beta_{ik} \right).$$

We now finally "normalize" this expression as follows:

$$\begin{aligned} \Pi_1 &= \left(\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i1} \right) \left[\pi_1 + \sum_{k=2,3,\dots} \pi_k \left(\frac{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{ik}}{\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i1}} \right) \right] \\ &= \left(\sum_{i=A,B,\dots} \gamma_{i1} \beta_{i1} \right) \left[\pi_1 + \sum_{k=2,3,\dots} \pi_k w_{1k} \right]. \end{aligned}$$

As the first term is just a multiplicative factor, the objective of firm 1's management is indeed to maximize $\pi_1 + \sum_{k=2,\dots} \pi_k w_{1k}$, where the weights w_{1k} have been constructed so as to compare with the weight of 1 that is placed on the firm's own profits.