

Loss Leading with Salient Thinkers*

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Abstract

In various countries, competition laws restrict retailers' freedom to sell their products below cost. A common rationale, shared by policymakers, consumer interest groups and brand manufacturers alike, is that such "loss leading" of products would ultimately lead to a race-to-the-bottom in product quality. Building on Varian's (1980) model of sales, we provide a foundation for this critique, though only when consumers are salient thinkers, putting too much weight on certain product attributes. But we also show how a prohibition of loss leading can backfire, as it may make it even less attractive for retailers to stock high-quality products, decreasing both aggregate welfare and consumer surplus.

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1 Introduction

In 2002, Germany’s highest court passed a landmark decision against Wal-Mart’s attempt to gain market share through heavy discounts and in particular below-cost selling.¹ Apart from Wal-Mart’s exit from the German market in 2006, this sparked a subsequent change of the national competition law: it now bans loss leading explicitly in the food retailing industry.² A proclaimed main objective of this regulation was to prevent a race-to-the-bottom in product quality and a crowding-out of high-quality products, promoting consumer well-being and food safety.³ Next to Germany, prohibitions of loss leading exist in various other, notably European, countries.⁴ Overall, policymakers seem to be particularly worried when loss leading affects staple goods of the national cuisine, such as dairy products in the UK or olives and wine in southern European countries,⁵ reflecting concerns for quality, consumer health, and safety.

From an economic point of view, the alleged positive effects of an imposition of price floors may seem dubious, and such prohibitions have frequently been criticized by economists. In this article, we show however that a ban of below-cost selling can indeed prevent retailers from stocking products of inefficiently low quality, though only when consumers are salient thinkers who put too much attention on certain product attributes. Then, heavy discounting of loss leaders makes the provision of high-quality products unattractive as consumers focus too much on low prices rather than on high quality. We expose the precise mechanism in detail below.

¹This goes back to a decision of the German competition authority in 2000 to prohibit below-cost selling by Wal-Mart, as well as Germany’s two leading discounters Aldi and Lidl. Wal-Mart challenged this decision and lost on appeal.

²See §20 Abs. 3 Satz 1 GWB (most recent amendment June 1, 2017). For an English translation, see https://www.gesetze-im-internet.de/englisch_gwb/englisch_gwb.html#p0066.

³See e.g. page 9 of the legislative proposal 16/5847 from June 27, 2007 by the then German government where, concerning the below-cost selling prohibition, it is stated that (own translation from German): “In the long run, the competition between retailers, which is characterized by low-price strategies, also poses a threat to the quality of food. With the general ban on the sale of food below cost, the Federal Government therefore wants to send out a signal for a high standard of food safety and counteract low-price strategies.” The full document (in German) can be accessed at <http://dip21.bundestag.de/dip21/btd/16/058/1605847.pdf>.

⁴These include Belgium, France, and Ireland. While U.S. federal law does not forbid below-cost selling or loss leading *per se*, several states have enacted below-cost selling laws. California goes even further and rules in its Business and Professions Code Section 17044 that “[i]t is unlawful for any person engaged in business within this State to sell or use any article or product as a ‘loss leader’ [...]”.

⁵For instance, in Spain the government repeatedly undertook efforts to curb discounts on olive oil (e.g., “Deal Could Stop Use of Olive Oil as Loss Leader”, *OliveOilTimes*, 4 March, 2013). These efforts are helped by Spain’s legal prohibition of loss leading, of which recently the German discounter Lidl fell foul when selling wine at a discount (“Lidl rapped for selling wines at a loss”, *TheDrinksBusiness*, October 16, 2014).

In our model, retailers compete for consumers by selecting which product to stock and which price to set in a prominent product category. Retailers may choose between either a high-quality (branded) variant or a low-quality (possibly private-label) alternative. We are primarily interested in the case where the provision of the high-quality product is more efficient. Consumers are one-stop shoppers who make their shopping decision based solely on the (observable) offers in the prominent product category. Competition on the (potentially loss-leading) prominent items occurs through promotional discounts, as in Varian (1980). Varian’s model seems to capture well the concerned retail markets, with frequent and largely unpredictable price promotions.⁶ Moreover, even when populated with salient thinkers, it allows for surprisingly tractable results.⁷

We follow the seminal analysis in Bordalo et al. (2013) in stipulating that salient-thinking consumers put too much weight on the attribute of a product, that is price or quality, along which the product differs more, in relative terms, compared to the market average of the respective attribute. When consumers are salient thinkers in this sense, we establish that increased competition and the thereby decreasing prices in the prominent category induce a gradual replacement of high-quality products, as then even small absolute price differences appear relatively large in the eyes of consumers, putting low-cost low-quality firms at an advantage.

In terms of positive implications, our model provides new insights into the rapid growth of private labels⁸ as a consequence of increased retail competition and the spread of one-stop shopping. We also derive precise conditions on when the high-quality outcome is restored by a prohibition of below-cost pricing and when such a prohibition instead accelerates a race-to-the-bottom in product quality. Which outcome prevails depends on whether the regulation constrains retailers’ pricing more for high- or low-cost products. The perception that such a prohibition would inadvertently promote high quality is thus

⁶Promotions (sales) are a defining feature of modern retailing competition and account for a large share of the observed price variation in retailing. Recent empirical studies documenting the ubiquity of retail promotions include Volpe (2013), Nakamura and Steinsson (2008), Berck et al. (2008), and Hosken and Reiffen (2004).

⁷This holds even though the subsequently introduced model of salient thinking entails a discontinuity in firms’ profits.

⁸The market share of private labels in European food retailing has risen significantly, with now more than 40% in some countries, such as the UK. Frequently, but not exclusively, private labels are positioned at the lower end of the quality and price range. See European Commission (2011) for an overview across Europe. Bergès-Sennou et al. (2004) provide an earlier guide to the academic literature, stressing different roles for the introduction and positioning of private labels. Especially in Europe, concerns about a potentially inefficient proliferation of private labels, combined with an exercise of buyer power, have recently resurfaced in sector inquiries and have led to various policy reports (e.g., European Commission (2014)).

wrong. And even when this is the case, we show that there is potentially a trade-off between higher social efficiency and lower consumer rent in case such a ban dampens retail competition too much.⁹

While economists have typically been skeptical about prohibitions of loss leasing,¹⁰ recent contributions have somewhat changed this picture. Chen and Rey (2012) show how with asymmetric retailers and a lack of competition loss leading may be used as exploitative device to screen consumers according to their shopping cost. They establish that a ban of loss leading would unambiguously increase consumer surplus and social welfare.¹¹ In our model, potentially deep discounts on one product arise as (multi-product) retailers make a positive margin on other products that consumers purchase on their shopping trip and for which consumers do not (ex-ante) compare prices,¹² and welfare losses are observed particularly when there is intense rather than low competition between retailers.¹³

Several related papers have incorporated different behavioral biases to explain the occurrence of loss leading and low-price strategies, such as loss aversion in Rosato (2016). There, the loss leader works as a “bait” that lures consumers into the store and shapes their reference point. In Johnson (2017), it pays firms to attract consumers with below-cost offers as they underestimate their propensity to make further, unplanned purchases. In Apffelstaedt and Mechtenberg (2018), consumers who are attracted by a competitively priced “bait” product are induced to purchase a different, higher-margin product by exploiting a “local-thinking” bias once they are in the shop. Their analysis can be seen as complementary to ours, as in our model consumers’ choice in the prominent, loss-leading category distorts their selection between retailers, rather than the selection at the chosen point of sale, as in Apffelstaedt and Mechtenberg (2018). Our mechanism may be particularly applicable to the aforementioned staple goods that are frequently used as loss

⁹Allain and Chambolle (2005) and Rey and Vergé (2010) show how also intrabrand, next to interbrand competition, can thereby be dampened, as below-cost pricing regulations may allow manufacturers to impose price floors.

¹⁰Cf. OECD (2007).

¹¹Chen and Rey (2016) show that with competitive loss leading, where multi-product firms differ in their products’ comparative advantages, welfare implications are more nuanced.

¹²This broadly follows Lal and Matutes (1994), albeit we do not endogenize advertising in our model. There is also a small literature that analyzes the possible incidence of below-cost pricing when consumers are equally informed about all prices, e.g., due to differences in demand elasticities (see Bliss (1988), Beard and Stern (2008), Ambrus and Weinstein (2008)).

¹³We abstract from theories of harm that focus on other, potentially smaller and less competitive retailers. Absent an outright prohibition by law, a prosecution of loss leading along such lines would typically require to find predatory behavior. For a detailed treatment of predatory behavior see Bolton et al. (2000). Chen and Rey (2012) discuss in some detail relevant case material and findings from recent sector inquiries.

leaders, where consumers indeed compare offers at a distance.¹⁴

Limited attention and context-specific preferences have also been applied more broadly in the field of Industrial Organization. Indeed, based on their concept of salience proposed in Bordalo et al. (2013), Bordalo et al. (2016) analyze a model of undifferentiated competition. Dertwinkel-Kalt and Köster (2017) and Helfrich and Herweg (2018) extend the idea of (salience-induced) context-dependent preferences to a manufacturer’s own offer across different sales channels, such as online and brick-and-mortar, thereby analyzing high-quality manufacturers’ motives to impose vertical restraints on their retailers. In Herweg et al. (2017), a high-quality manufacturer may introduce a decoy product to distort competition with a low-quality competitive fringe.¹⁵

In the fields of psychology, marketing and behavioral economics, context-dependent preferences, including consumers’ selective attention to particular product features, have long been recognized (see e.g. Huber et al. (1982), Thaler (1985), Simonson (1989), and Tversky and Simonson (1993)).¹⁶ Next to the recent formalization of salient thinking in Bordalo et al. (2013), Koszegi and Szeidl (2013) have proposed a related modeling framework according to which consumers’ utility weighting of product attributes increases in the range of the respective attributes across consumers’ consideration set.¹⁷ Consumers’ inattention to certain product characteristics may also be driven by rational consumers’ efficient allocation of attention. Such models of “rational inattention” include Matějka and McKay (2012), Gabaix (2014), and Persson (2017).

While we heavily rely on Bordalo et al. (2013) for the aforementioned formalization of consumers’ comparison in relative terms, motivated by our specific focus we digress from their concept in two ways. First, we limit firms’ opportunities to influence consumers

¹⁴Further research may seek to combine this with a potentially more impulse-driven choice for certain products, notably those where consumption is more immediate (e.g., directly at the point of sale).

¹⁵In Armstrong and Chen (2009), who also use Varian’s model, some consumers are always inattentive to product quality.

¹⁶For example, Huber et al. (1982) show that the choice among two alternatives can crucially be affected if a third, dominated alternative is added (the so-called “attraction effect”). Similarly, Simonson (1989) demonstrates that adding an alternative that is particularly good on one dimension, but bad on another (e.g., a product with very high quality, but also a very high price) may tilt consumers’ choice among the initially available alternatives (“compromise effect”). Overall, the literature stresses the importance of the choice context for the weights consumers put on different product attributes. Wedell (1991) provides a psychological analysis of the associated “weight-shift” of attributes. We do however not claim that the chosen framework is fully consistent with these earlier theories and their implications.

¹⁷Other papers in the field of behavioral economics have focused on the potentially biased formation of a decision-maker’s consideration set, rather than the weighting of attributes when choosing within this set (see e.g. Ellison and Ellison (2009), Eliaz and Spiegel (2011), de Clippel et al. (2014), and Hefti and Liu (2016)).

by unattractive offers, as we assume that consumers’ attention is not affected by clearly dominated choices. Second, in our framework consumers’ attention is only (potentially) distorted by salient thinking *within* a given product category, where choices can be compared along the same dimensions, such as price and quality, whereas consumers’ choice across entirely different alternatives, such as that of shopping and that of not shopping, is not distorted by such salient thinking. We discuss these modeling choices and their implications in more detail below.¹⁸

The remainder of this article is organized as follows. We set out the model in Section 2, where we also discuss its key assumptions. In Section 3, we solve the game and provide a full equilibrium characterization, including results on equilibrium product choice, pricing, welfare and consumer surplus. Section 4 investigates the consequences of a ban on below-cost pricing, showing its potentially ambiguous effects on welfare and consumer surplus. Several concluding remarks are provided in Section 5. Technical proofs are relegated to Appendix A. We also collect additional material in an online Appendix B.

2 Model

We consider a market where consumers, albeit they potentially purchase a basket of products, focus their attention both on a particular (prominent) product category as well as on particular (“salient”) attributes of products in that category. We first describe the general market environment and then proceed to outline how consumers’ salient thinking affects their choice.

There are $N \geq 2$ symmetric retailers indexed by $n = 1, \dots, N$. Retailers compete for consumers by choosing which product to stock in the prominent product category and which price to set for it. Consumers are one-stop shoppers and thus purchase additional products at the chosen retailer. Prices for these products are not observable by consumers before visiting the respective retailer. In line with our subsequent discrete-choice specification also for the prominent product category, retailers set the monopoly price for those

¹⁸Although we motivate our assumptions for the general case, they seem particularly suitable for our key application of loss leading of staple goods in grocery retailing. There, loss leading is feared to rather affect choices among products in a particular category, rather than consumers’ choice between purchasing, say, milk instead of any other product or instead of not going to the supermarket at all. The modeling framework in the present paper is shared by our companion papers Inderst and Obradovits (2015a, 2016), which both focus on different normative and positive implications. Inderst and Obradovits (2015a) extends Narasimhan’s (1988) influential work with asymmetric firms to analyze how salience affects promotion intensity together with product choice. Inderst and Obradovits (2016) combines the analysis of salience with that of shrouding. Both papers also differ in other aspects, such as the timing of firms’ choices.

additional products and fully extract consumers’ respective surplus. This allows us to capture the one-stop shopping feature of our model by a single variable, $v \geq 0$, which is the additional profit that a retailer obtains from each attracted consumer. The parameter v thus captures also the extent of consumers’ one-stop shopping.¹⁹

In the prominent product category, each retailer n may fill its shelf space with either a high-quality or a low-quality product (but not both), for which it sets a retail price $p_n \geq 0$. The different products’ respective quality levels (measured in rational consumers’ homogeneous maximal willingnesses to pay) are denoted by $q_H > q_L > 0$, with associated (constant) marginal costs of $c_H \in (0, q_H)$ and $c_L \in (0, q_L)$. As we want to investigate whether claims regarding the negative implications of loss leading on the provision of high-quality products are justified (cf. the Introduction), we focus on the interesting case where providing high quality is more efficient: Denoting $\Delta_q = q_H - q_L$ and $\Delta_c = c_H - c_L$, we thus suppose that $\Delta_q > \Delta_c$.²⁰

We now turn to consumers. There is a unit mass of consumers with unit demand and an outside-option value that is normalized to zero. We follow Varian’s (1980) seminal paper in stipulating that a fraction $(1 - \lambda)/N$ of consumers, called “non-shoppers”, can only shop at their (local) retailer n (for each $n \in N$), such that a total fraction $1 - \lambda$ of consumers does not compare offers (in the prominent category). In contrast, the remaining fraction λ of consumers, called “shoppers”, is free to choose any retailer, so that λ also affects the intensity of competition. While shoppers thus observe all offers (q_n, p_n) in the prominent category, non-shoppers observe the respective offer only at one retailer and are also constrained to purchase from this retailer.

A rational consumer’s utility when purchasing from retailer n is given by $u_n = q_n - p_n$. Hence, taking into account consumers’ outside option of value 0, a rational non-shopper would only buy from its local retailer n if $u_n \geq 0$, while a rational shopper would only buy from retailer n if $u_n \geq 0$ and $u_n \geq \max_{n' \neq n} u_{n'}$. We now describe in detail how this differs for salient thinkers.

Salient Thinking. We suppose that salient-thinking consumers follow a hierarchical decision process. First, they compare all offers *within* the single prominent product category, selecting a favorite alternative according to their salience bias as specified below. Having

¹⁹In an earlier working-paper version (Inderst and Obradovits (2015b)), we explicitly modeled the additional supply of $I - 1 \geq 0$ products, so that there I captured the extent of one-stop shopping.

²⁰However, we have also fully solved the case where $\Delta_q < \Delta_c$, for which we find that there is no scope for beneficial policy intervention. Results can be obtained from the authors upon request.

determined the (perceived) best alternative in this category, this is then compared to the outside option of not buying, and a final purchase decision is made. Hence, salient thinking may only sway preferences within a choice set in which options are indeed comparable across the same attributes, here price and quality. In what follows, we fully specify and justify the chosen decision-making process.

For the decision within the prominent category, where shoppers have the choice between offers that are comparable in price and quality, we further assume that only genuine trade-offs matter for shaping consumers' (potentially distorted) attention. That is, all options that are strictly dominated are immediately "edited out" (discarded). Let thus N_+ denote the set of all remaining non-dominated offers in the prominent category, which consists of all offers $n \in N$ for which there does not exist a rival offer $n' \in N$ with both $q_{n'} \geq q_n$ and $p_{n'} \leq p_n$ (with one inequality strict).²¹

Define now the averages $P = \frac{1}{|N_+|} \sum_{n \in N_+} p_n$ and $Q = \frac{1}{|N_+|} \sum_{n \in N_+} q_n$. These averages describe, within a consumer's consideration set, the respective "reference good". Suppose that some retailer's price is below the average price in the consideration set, $p_n < P$, and also its quality is below the average quality in the consideration set, $q_n < Q$. Then, we say that price is salient for the retailer's prominent product when $\frac{p_n}{P} < \frac{q_n}{Q}$ and that quality is salient when the converse holds strictly. That is, a product's lower price, but not its lower quality, is salient in the eyes of consumers when its price is relatively lower (that is, in percentage terms), compared to the average price of all considered offers, than its quality, compared to the average quality of all considered offers. Suppose next instead that a retailer's price and quality are both higher than the respective averages in the consideration set, as $p_n > P$ and $q_n > Q$. Then price is salient for the retailer's prominent product when now $\frac{p_n}{P} > \frac{q_n}{Q}$, while when the converse holds strictly, quality is salient.

The preceding specification of "relative thinking" borrows from Bordalo et al. (2013), as discussed in the Introduction. They motivate this on the basis of results from cognitive psychology, whereby stimuli are perceived with diminishing sensitivity.²² As we point out below, this feature is key for our results to hold. The following is now an immediate consequence of our salience conditions and consumers' editing of dominated offers.

²¹This, of course, shuts down the (decoy) channel through which clearly dominated offers could affect consumers' decision making. Furthermore, in our static model, also past observed prices can not play a role as reference points for the determination of which attributes appear salient. While subsequently, our equilibrium will typically be in mixed prices, we also do not include expectations into the formation of such a reference point.

²²The respective simple ratio property is then obtained from a set of additional axioms. For these as well as further motivation, we refer to their work (and the Introduction).

Result. *The same attribute will be salient for all offers in the prominent category and the respective condition simplifies to a pairwise comparison: When p_L is the lowest price for a low-quality product and $p_H > p_L$ that for a high-quality product in the market, price is salient for all $n \in N_+$ offers when*

$$\frac{p_L}{p_H} < \frac{q_L}{q_H} \quad (1)$$

and quality is salient when the converse of (1) holds strictly.

Proof. Suppose that $L \geq 1$ retailers charge the lowest price p_L for a low-quality product, whereas $H \geq 1$ retailers charge the lowest price p_H for a high-quality product, so that the average price in N_+ is thus $P = \frac{Lp_L + Hp_H}{L+H}$ and the average quality $Q = \frac{Lq_L + Hq_H}{L+H}$. Then, for a low-quality offer price is salient if $\frac{p_L}{P} < \frac{q_L}{Q}$, which indeed transforms to (1), and likewise the condition $\frac{p_H}{P} < \frac{q_H}{Q}$ for a high-quality offer also transforms to (1). The same transformations apply when quality is salient. ■

Implications for Decision-Making. In any pairwise comparison between different offers in the prominent category, for each offer the respective non-salient attribute is “discounted” by some factor $\delta \in [0, 1]$, compared to what a rational consumer would do. Then, when quality is salient, the best high-quality offer (q_H, p_H) will be strictly preferred to the best low-quality offer (q_L, p_L) when $q_H - \delta p_H > q_L - \delta p_L$ (i.e., $\delta(p_H - p_L) < q_H - q_L$), while when price is salient, it is only preferred when $\delta q_H - p_H > \delta q_L - p_L$ (i.e., $p_H - p_L < \delta(q_H - q_L)$). Clearly, the two conditions coincide when $\delta = 1$, while as δ becomes smaller, the wedge between the two conditions becomes larger. To complete the specification, we suppose that when a consumer is indifferent, he randomizes with equal probability over all respective offers.²³

Note that the preceding description was guided by the application to shoppers’ choice. It however also applies to non-shoppers, albeit then the description is trivial as N_+ contains a single element (the local retailer’s prominent product). To summarize, while non-shoppers always consider only a single product, for shoppers the respective consideration set is larger, which is why salient thinking may matter (for $\delta < 1$) and distort their choice.

The final step is that consumers compare the best perceived offer $n^* \in N_+$ to the outside option of not buying. Hence, a consumer with best perceived offer n^* in the prominent category buys from retailer n^* if and only if $q_{n^*} - p_{n^*} \geq 0$.

²³This specific “tie-breaking condition” is however not needed to derive our results.

Market Game. The following description of the market game fully summarizes our model.

- $t = 1$: *Retailer choice.* Retailers simultaneously choose which product $q_n \in \{q_H, q_L\}$ to stock in the prominent product category and its price p_n .
- $t = 2$: *Consumer choice.* Of the mass $1 - \lambda$ of non-shoppers, each purchases from his local retailer n , provided that $q_n \geq p_n$, and otherwise takes up his outside option. The mass λ of shoppers first consider all non-dominated alternatives $n \in N_+$ in the prominent category, which determines whether price or quality (or neither) will be the salient attribute in that category. Given this, they compare alternatives, discounting each product's non-salient attribute by the factor δ . Finally, having picked a best alternative n^* according to their potential salience bias, they compare this alternative with the outside option of not purchasing, and choose n^* if and only if $q_{n^*} - p_{n^*} \geq 0$.
- $t = 3$: *Payoffs realized.* Having bought from firm n , a consumer obtains the true payoff of $q_n - p_n$.²⁴ A firm that only sells to its loyal consumers obtains a profit of $\frac{1-\lambda}{N}(p_n - c_n + v)$. A firm that sells to both its loyal consumers and all shoppers obtains a profit of $(\frac{1-\lambda}{N} + \lambda)(p_n - c_n + v)$.

Discussion of Consumer Choice. We borrow from the extant literature on salience two features: First, the (ratio) condition for when price or quality becomes salient; second, the specification that a salient attribute weighs more in decision making.

Salient thinking affects consumers' ordering of choices in a given category, in which choices can be readily compared along the described attributes. Alternatives outside such a category, that is in our case the alternative of not-buying, can however not be compared along the same attributes, which is why we posit that salient thinking does not affect the respective comparison. More generally, our hierarchical model of decision-making has consumers to first select their preferred choice within a given category, subject possibly to salient thinking, before making rational comparisons across categories, which in our case boils down to the decision whether to shop at all. This reflects our view that the non-shopping decision, which is the source of consumers' reservation value, can not be meaningfully compared along the attributes of price and quality, which is why it is not obvious to us why the choice between the option of non-shopping and that of purchasing

²⁴Cf. the discussion of consumer welfare in Section 3.3.

a given product should be distorted. In a model of imperfect competition, the comparison with the outside option of not buying represents however a key ingredient, which is why we now comment in more detail on this modeling choice.

Essentially, next to tractability, this modeling choice allows us to focus on the impact of salient thinking on consumer choice between competing products within a category, rather than on consumers’ decision whether to shop at all. Salient thinking would however also affect the latter decision when we followed more closely Bordalo et al. (2013) by essentially modeling the outside option of not buying as $(q_0, p_0) = (0, 0)$, i.e., when we presumed that the outside option could be evaluated and ordered along the very same dimensions, namely price and quality.

To see the difference that this alternative formulation would make, suppose that all firms had the same quality q and charged the same price p , so that neither price nor quality was salient. When now one firm deviated even only slightly to some price $p' < p$, so that price became salient, consumers who previously bought as $q - p > 0$ would now abstain from buying if $\delta q - p' < 0$. Hence, as in consumers’ perception this would essentially “degrade” the offer relative to the outside option of not buying, undercutting may not be profitable, at least not for sufficiently low initial price levels p , thus potentially supporting a continuum of equilibria with symmetric prices.²⁵ The hierarchical decision-making process also allows to shut down this channel through which salient thinking can affect the overall price level.

3 Equilibrium Analysis

In this section, we solve for and analyze the product-choice and pricing equilibrium of the considered game. We do so in three steps. First, we postulate existence of an (efficient) high-quality equilibrium in which each retailer stocks the high-quality product, such that consumers’ salient thinking plays no role on equilibrium. We ask under which circumstances this type of equilibrium exists, and interpret the relevant condition. In the second part of our equilibrium analysis, we proceed to characterize the symmetric mixed-strategy equilibrium for the case where the high-quality equilibrium does not exist. We also consider how firms’ equilibrium stocking decision is influenced by changes in the extent of

²⁵Incidentally, in a potentially enriched model with a more continuous differentiation between retailers, this would also have the following stark, possibly testable implications: A retailer’s own price cut diverts consumers away from other retailers, but it equally diverts consumers who would have shopped at the retailer at, say, the regular price into the option of “not buying at all”.

one-stop shopping or the fraction of shoppers in the market. In the third part of the equilibrium analysis, we discuss the outcome in terms of (consumer) welfare. After this, we analyze in Section 4 whether a ban of loss leading could improve market outcomes.

Before starting with the analysis, we streamline the exposition by restricting our attention to the interesting case where consumers' salience bias is sufficiently strong: Consumers discount non-salient product attributes sufficiently much (δ is sufficiently small) such that

$$\delta\Delta_q < \Delta_c. \tag{2}$$

If instead $\delta\Delta_q \geq \Delta_c$, then *even* if price is salient, consumers' *perceived* difference between high and low quality exceeds the respective cost difference. By optimality, firms must then always offer high quality in equilibrium. This is immediate as irrespective of other firms' choices (of prices and qualities), a firm that considered offering low quality at a price p would always be better off by instead offering high quality at a price $p' = p + \delta\Delta_q$: even when price was salient, consumers would be indifferent between the old and the new offer, while the firm's margin would be higher.²⁶ In what follows, we focus on the interesting case where from inequality (2) the salience bias is sufficiently strong such as to potentially give rise to inefficient quality choices. We finally stipulate throughout that v is not too high so that in what follows we can restrict case distinctions to those with positive equilibrium prices in the prominent category. A sufficient condition is that $v \leq c_L$ (cf. however below for a relaxed condition).

3.1 High-Quality Equilibrium

We note first that a retailer can guarantee a profit of

$$\pi^* = \frac{1-\lambda}{N} (q_H - c_H + v) \tag{3}$$

by stocking the more efficient high quality q_H , setting the highest possible price $p_n = q_H$, and only selling to its share $\frac{1-\lambda}{N}$ of loyal consumers. When all retailers stock high quality, we know from Baye et al. (1992) that this is also the equilibrium profit, even though for $N > 2$ there is a continuum of asymmetric pricing equilibria. It is also well-known since Varian (1980) that in the unique symmetric pricing equilibrium, given $q_n = q_H$ and

²⁶Strictly speaking, the firm's margin would only be strictly higher if $\delta\Delta_q > \Delta_c$, such that in the knife-edged case where $\delta\Delta_q = \Delta_c$, in principle also equilibria in which some firms choose low quality may exist. Here, we appeal to the standard refinement of ignoring equilibria in weakly dominated strategies.

effective marginal costs of $c_H - v$, each firm draws prices randomly from the atomless CDF

$$F(p) = 1 - \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)} \quad (4)$$

with convex support $[\underline{p}, q_H]$, where the lower bound is given by

$$\underline{p} = c_H - v + \frac{1-\lambda}{1-\lambda + \lambda N} (q_H - c_H + v). \quad (5)$$

While with rational consumers $q_n = q_H$ must always hold, so that this would complete the equilibrium characterization, this is no longer the case with salient thinkers.

Proposition 1 *If*

$$\underline{p} \geq \frac{\Delta_c}{\Delta_q} q_H, \quad (6)$$

all firms choose the efficient high quality and make an identical expected profit of π^ as given in (3). In contrast, if condition (6) does not hold, there exists no equilibrium in which all firms choose high quality.*

Proof. We prove in Appendix A that when (6) holds, $q_n = q_H$ is indeed the unique equilibrium outcome. We thus confine ourselves to showing that if the converse holds, this is no longer an equilibrium outcome. We argue to a contradiction. Knowing that π^* must be the equilibrium profit in any pricing equilibrium where $q_n = q_H$, from this we derive, across all equilibria, the lowest price that any retailer may charge: The respective price $p = \underline{p}$, as given by (5), is obtained from the requirement

$$(p - c_H + v) \left(\frac{1-\lambda}{N} + \lambda \right) = \pi^*.$$

Consider now a deviation of retailer n to stocking $q_n = q_L$. This deviating retailer can attract all shoppers with probability one by pricing at the minimum of $\underline{p} \frac{q_L}{q_H}$ (which guarantees that price is salient) and $\underline{p} - \delta \Delta_q$ (which guarantees that the retailer attracts the shoppers, provided that price is salient). If the minimum is given by the former ($\underline{p} > \delta q_H$), the deviating retailer's profit with this price satisfies

$$\left(\underline{p} \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1-\lambda}{N} + \lambda \right) > (\underline{p} - c_H + v) \left(\frac{1-\lambda}{N} + \lambda \right) = \pi^* \quad \text{if } \underline{p} < \frac{\Delta_c}{\Delta_q} q_H.$$

If instead the minimum is given by the latter ($\underline{p} \leq \delta q_H$), the deviating retailer's profit with this price satisfies

$$(\underline{p} - \delta \Delta_q - c_L + v) \left(\frac{1-\lambda}{N} + \lambda \right) > \pi^*,$$

where the inequality follows from condition (2). Hence, no matter whether $\underline{p} > \delta q_H$ or $\underline{p} \leq \delta q_H$, if it holds that

$$\underline{p} < \frac{\Delta_c}{\Delta_q} q_H, \quad (7)$$

a deviation to choosing $q_n = q_L$ is strictly profitable. This proves that condition (6) (the converse of condition (7)) is indeed necessary. ■

Non-existence of an equilibrium with $q_n = q_H$ whenever (6) does not hold is the main result in this section. To establish this we have simply shown that a firm deviating to stocking q_L and pricing at the minimum of $\frac{q_L}{q_H} \underline{p}$ (which guarantees that price will be salient) and $\underline{p} - \delta \Delta_q$ (which guarantees that the shoppers will be attracted, given price salience) makes a strictly higher profit than π^* . The crucial observation is now that condition (6) is less likely to hold, so that the high-quality equilibrium is less likely to exist, when \underline{p} is low. This is the main implication of the stipulated salience bias: Which feature is salient is decided not by an absolute but by a relative comparison. When the lowest rival price is lower, a firm offering low quality needs to undercut this price by less in *absolute* terms to make price salient, so that a deviation of stocking the (inefficient) low quality may become worthwhile.

The following is now an immediate implication of inserting for \underline{p} and rewriting condition (6) in terms of consumers' one-stop shopping parameter v .

Corollary 1 *All retailers choose the efficient high quality in any equilibrium if and only if the extent of one-stop shopping is sufficiently small, with*

$$v \leq \tilde{v} := \frac{q_H c_L - q_L c_H}{\Delta_q} + \frac{1 - \lambda}{\lambda N} \left[\frac{q_H (\Delta_q - \Delta_c)}{\Delta_q} \right]. \quad (8)$$

Inequality (8) becomes less likely to hold (the high-quality equilibrium becomes less likely to exist) as the fraction of shoppers λ or the number of retailers N increases. This is because both the presence of more shoppers or a larger number of firms in the market tend to depress the minimum price that will be charged for high quality, which, in light of consumers' distorted attention towards low prices if price becomes salient, makes it more profitable for retailers to deviate and stock low quality.

3.2 Provision of High and Low Quality

When condition (6) does not hold, there can be no equilibrium in which all firms choose high quality. But it is also immediate that it can not be an equilibrium either that all

retailers choose low quality in the prominent category. Each retailer would then earn strictly less than when offering instead high quality and setting $p_n = q_H$.

Denote now the likelihood that, in a symmetric equilibrium, a retailer chooses high quality by α^* . Depending on quality, a retailer then uses different pricing strategies, which we denote by F_H when stocking q_H and F_L when stocking q_L . Proposition 2 provides a full characterization, on which we comment subsequently.²⁷

Proposition 2 *If condition (6) does not hold, the high-quality equilibrium does not exist. In the unique symmetric equilibrium, each retailer offers the high quality product only with probability*

$$\alpha^* = \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left[\frac{(\Delta_q - \Delta_c)q_H}{v\Delta_q - (q_H c_L - q_L c_H)} \right]} \in (0, 1) \quad (9)$$

and the low quality product with probability $1 - \alpha^$. Conditional on stocking q_H , retailers draw prices from the CDF*

$$F_H(p) = 1 - \frac{1}{\alpha^*} \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)} \quad (10)$$

with support

$$[\underline{p}_H, \bar{p}_H] = \left[\frac{\Delta_c}{\Delta_q} q_H, q_H \right].$$

Conditional on stocking q_L , retailers draw prices from the CDF

$$F_L(p) = 1 - \frac{\sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_L + v} - 1 \right)} - \alpha^*}{1 - \alpha^*} \quad (11)$$

with support

$$[\underline{p}_L, \bar{p}_L] = \left[\underline{p} - \Delta_c, \frac{\Delta_c}{\Delta_q} q_L \right].$$

Under the corresponding pricing strategies, F_H and F_L , price is always salient if both qualities coexist, and all shoppers surely choose the low-quality product unless all retailers stock the high-quality product. Each firm makes an expected profit of $\pi^ = \frac{1-\lambda}{N} (q_H - c_H + v)$.*

Proof. See Appendix A. ■

Despite the potential discontinuity in demand and profits at price realizations at which the salience condition (1) is just satisfied, we are able to obtain a closed-form characterization of the (mixed-strategy) equilibrium prices and quality choices when no high-quality

²⁷Note now that with the condition $\underline{p}_L \geq 0$ we ensure that on equilibrium prices are always non-negative.

equilibrium exists. What helps this derivation, as well as our subsequent tractable analysis of (consumer) welfare and policy, is the feature that on equilibrium, albeit clearly not off equilibrium, price becomes salient with probability one whenever not all products are of the same quality. Consequently, the salience condition (1) shows up prominently in the two supports of F_H and F_L : When high quality is offered at the lowest price \underline{p}_H , the highest price at which low quality is offered, \bar{p}_L , is just sufficient to tilt shoppers' perception towards prices.

As lower (market) prices allow firms to ensure with a smaller absolute price reduction that price becomes salient, it is again intuitive that low-quality products are offered and bought with higher probability when prices are overall lower in equilibrium. The following is an immediate consequence of the preceding discussion and a comparative analysis of (9).

Corollary 2 *When condition (6) does not hold, the likelihood that any retailer provides high quality, α^* , as well as the likelihood with which any given consumer purchases high quality, which is α^* for non-shoppers and $(\alpha^*)^N$ for shoppers, are strictly lower when either the extent of one-stop shopping (v) or the fraction of shoppers (λ) increases.*

In summary, we observe that salient thinking may indeed lead to the inefficient provision of low quality, provided that the prevailing (market) prices in the prominent category are sufficiently low, which in our model depends crucially on the extent of one-stop shopping. To provide the background for our ensuing policy analysis, we now turn to a more detailed analysis of (consumer) welfare.

3.3 (Consumer) Welfare

While consumers' perceived utility guides their choice, we assume that their truly experienced (consumption) utility is given by the undistorted utility a rational consumer would obtain.²⁸ As a consequence, the purchase of a high-quality product generates the total (true) surplus of $q_H - c_H + v$, while that of a low-quality product the surplus $q_L - c_L + v$. We know from Proposition 2 that in the mixed-product equilibrium shoppers will only purchase high quality if all retailers stock high quality (probability $(\alpha^*)^N$), while non-shoppers

²⁸As pointed out by a referee, however, notably when the product is consumed "immediately", that is under the still biased perception, also a different welfare criterion may be justified. Still, even then policymakers may have a (paternalistic) concern for consumers' supposedly "true" welfare, e.g., when quality relates to safety and health concerns.

simply buy at a random firm (that stocks high quality with probability α^*). Letting $\alpha^* = 1$ if condition (6) holds, social welfare can be written as

$$W := [q_H - c_H + v] - [(1 - \lambda)(1 - \alpha^*) + \lambda(1 - (\alpha^*)^N)] (\Delta_q - \Delta_c), \quad (12)$$

where the term in the second bracket captures the expected loss of welfare from the provision of the less efficient low-quality product, given $\Delta_q > \Delta_c$. As firms' profits are fully pinned down by the fraction of non-shoppers, $N\pi^* = (1 - \lambda)(q_H - c_H + v)$, this welfare loss is fully borne by consumers. Precisely, subtracting firm profits from welfare, we obtain for total consumer surplus

$$CS := W - N\pi^* = [\lambda(q_H - c_H + v)] - [(1 - \lambda)(1 - \alpha^*) + \lambda(1 - (\alpha^*)^N)] (\Delta_q - \Delta_c). \quad (13)$$

Corollary 3 *When salient thinking combined with low pricing in the prominent category induce firms to inefficiently offer low quality (i.e., when condition (6) does not hold), the resulting loss in welfare is fully borne by consumers.*

In summary, our results for (consumer) welfare speak right to the concerns of policy-makers and consumer interest groups, as discussed in the Introduction. Policymakers and consumer interest groups are rightly concerned about the possibility that loss leading may lead to inefficiently low product quality. We will subsequently analyze whether a ban of such loss leading could mitigate these problems.

To provide more background for such a policy analysis, we end this section with some observations on how different groups of consumers fare. In the benchmark case with only rational consumers, ignoring possible costs of information gathering and shopping, clearly shoppers can not fare worse, neither ex-ante nor ex-post, and in fact are ex-post strictly better off compared to any non-shopper with probability $(N - 1)/N$ (as with this probability another retailer provides a better offer, given symmetry). Now, with salient thinking, having a larger consideration set is a blessing and a curse at the same time.²⁹ We show this with the following example, for which we simplify the exposition by setting $N = 2$. Suppose then that (6) does not hold, so that $\alpha^* < 1$, and also suppose that different qualities are offered, with $q_1 = q_L$ and $q_2 = q_H$.

The first interesting observation is that as shoppers *always* purchase at $n = 1$ in this case (cf. Proposition 2), for all incidences where different qualities are offered they are *not*

²⁹As noted by a referee, biases may realistically be “correlated” in the sense that consumers who make “wrong” decisions for one reason may also behave suboptimally for a different reason (or bias). If, in future work, our model was enriched so as to incorporate salience-biased decision making also across comparable products within a shop, one could possibly disentangle erroneous choices both across and within shops.

better off than customers loyal to $n = 1$. The second observation is that shoppers may be (at least ex-post) strictly worse off than customers loyal to, in this example, $n = 2$. To see this, take the lowest possible price for the high quality and the highest possible price for the low quality, in which case shoppers still choose low quality, as they perceive price to be salient. Without such a salience-induced distortion, however, they would strictly prefer the high-quality offer in this case. If they later realized their “true” preferences, they would thus suffer from ex-post regret, and having had a larger consideration set than loyal customers would then have worked to their disadvantage.

4 Prohibiting Below-Cost Pricing

The preceding results open up a potential role for policymakers. Precisely, the reason for why salient thinking increases a retailer’s incentives to deviate from the provision of high quality in the prominent product category seems to provide a rationale in particular for a ban on below-cost pricing (or loss leading), i.e. for the requirement that $p_n \geq c_i$ when stocking quality q_i (for $i \in \{H, L\}$). Given its practical relevance, in what follows we focus on an assessment of this specific policy. We start by establishing a condition which guarantees restoration of full efficiency.

Proposition 3 *Suppose that*

$$\frac{q_H}{q_L} > \frac{c_H}{c_L}. \quad (14)$$

Then a prohibition of below-cost pricing ensures that only high quality is offered in equilibrium. This leads to a strict increase of social welfare if and only if $v > \tilde{v}$.

Proof. See Appendix A. ■

Intuitively, under condition (14) the regulation works as intended because it constrains the pricing of low-quality offers more than that of high-quality offers. Given the relative advantage (in terms of the quality-cost ratio) of high quality, the price of low-quality offers can never be so low to give low-quality firms a competitive advantage, even if price salience is achieved. We relegate a characterization of the symmetric pricing equilibrium to Appendix B.

The opposite case, where for simplicity we assume that the converse of condition (14) holds strictly, is considerably more complex to analyze. For this it is instructive to focus first on a particular situation, namely that where the extent of one-stop shopping is sufficiently large so that when loss leading is banned and all retailers stock the same quality,

prices lie deterministically at the permissible lower bound. To see when this arises, note that firms then split the market evenly and realize profits of v/N (given that the prominent product is sold just at cost). As only an upward deviation is possible, this can only target loyal customers, yielding maximum deviation profits of $\frac{1-\lambda}{N}(q_H - c_H + v)$ when choosing the efficient high quality and setting $p_{dev} = q_H$. This is lower than v/N if

$$v \geq \bar{v} := \frac{1-\lambda}{\lambda}(q_H - c_H). \quad (15)$$

Consider now the situation in which all retailers offer *low* quality and price at $p_n = c_L$. From the converse of (14), a high-quality product offered at the lowest permissible price $p_{dev} = c_H$ would not be attractive to shoppers as then price, but not quality, would be salient (with also a lower perceived surplus due to condition (2)). Moreover, offering a high-quality product at the highest permissible price $p_{dev} = q_H$ would not be profitable either as $\frac{1-\lambda}{N}(q_H - c_H + v) \leq \frac{v}{N}$ for $v \geq \bar{v}$. Hence, when the converse of (14) holds strictly and the extent of one-stop shopping is sufficiently large, so that (15) holds, only low quality is offered in equilibrium, and welfare is then surely lower than without regulation. In this case, a prohibition of loss leading backfires.

In the proof of Proposition 4, we show that the latter finding extends also to $v < \bar{v}$, albeit the likelihood with which high quality is stocked strictly decreases only when v is not too small, as otherwise such regulation does not constrain pricing at all. The proof of Proposition 4 also contains a full characterization of the mixed-strategy pricing equilibrium for intermediate values of v .

Proposition 4 *Suppose that the converse of (14) holds strictly. Then a prohibition of below-cost pricing strictly reduces the probability that each retailer stocks high quality whenever the extent of one-stop shopping is sufficiently large, with $v > \hat{v} \in (\tilde{v}, \bar{v})$. For $v \geq \bar{v}$, all retailers deterministically stock low quality under the regulation.*

Proof. See Appendix B. ■

(Consumer) Welfare. The ban on below-cost pricing never reduces retailer profits, as each retailer can still earn π^* by choosing $q_n = q_H$ and $p_n = q_H$. As the ban can not increase overall welfare when the converse of (14) holds strictly, but strictly decreases it when the extent of one-stop shopping is sufficiently large, in this case it can only negatively affect consumer surplus. We now focus on the case where (14) holds.

Then, in terms of consumer surplus, there are now potentially two conflicting forces at work: First, total welfare is increased when $v > \tilde{v}$; second, as the policy dampens competition, firm profits are higher when $v > \bar{v}$. To analyze this trade-off, note first that for $v \in (\tilde{v}, \bar{v}]$, consumers pocket the full welfare gain resulting from the ban of below-cost pricing. When we now slightly increase the extent of one-stop shopping, by continuity consumers are still better off with such a policy – and now also retailers’ profits are higher. Only for sufficiently high values of v all of the welfare gain goes to retailers, and when we further increase v , retailers benefit from the regulation even at the detriment of consumers.

Corollary 4 *Suppose condition (14) holds. While a ban on below-cost pricing always increases social welfare (and strictly so when $v > \tilde{v}$), consumers are only strictly better off when the extent of one-stop shopping v is not too large, but they are worse off when v is sufficiently high. Retailers strictly gain from such a policy when $v > \bar{v}$.*

Figure 1 offers an illustration of this comparative analysis. For any given extent of one-stop shopping v , it contrasts the unregulated outcome with that under a ban of below-cost pricing. We see that $\alpha^* < \alpha_{reg}^* = 1$ when $v > \tilde{v}$ and we see that with respect to consumer surplus, there are three different cases: When $v \leq \tilde{v}$ the ban has no impact on consumer surplus. Then there is an intermediate range where the ban increases consumer surplus, $CS_{reg} > CS$, while for sufficiently high v the ban leads to strictly lower consumer surplus with $CS_{reg} < CS$. This is notably the case as for $v > \bar{v}$, all additional benefit from one-stop shopping under regulation is pocketed by retailers (as then $CS_{reg} = q_H - c_H$).

5 Conclusion

Our analysis in this paper is motivated by the following two observations. First, one-stop shopping leads consumers to base their choice of retailers only on a comparison of a selected number of products (“known-value items”). These are consequently the products on which price competition occurs, leading potentially to loss leading. Second, consumers’ attention to different attributes of a product, notably price and quality, may not be “fixed”, but may depend instead on market circumstances, precisely on whether a particular offer is “saliently different” along the respective attribute, compared to other offers in the market. Our analysis captures these two features by combining a model of one-stop stopping and limited information about prices, set into a model of sales (Varian (1980)), with recent developments in behavioral economics (precisely, the formalization of salience in Bordalo et al. (2013)).

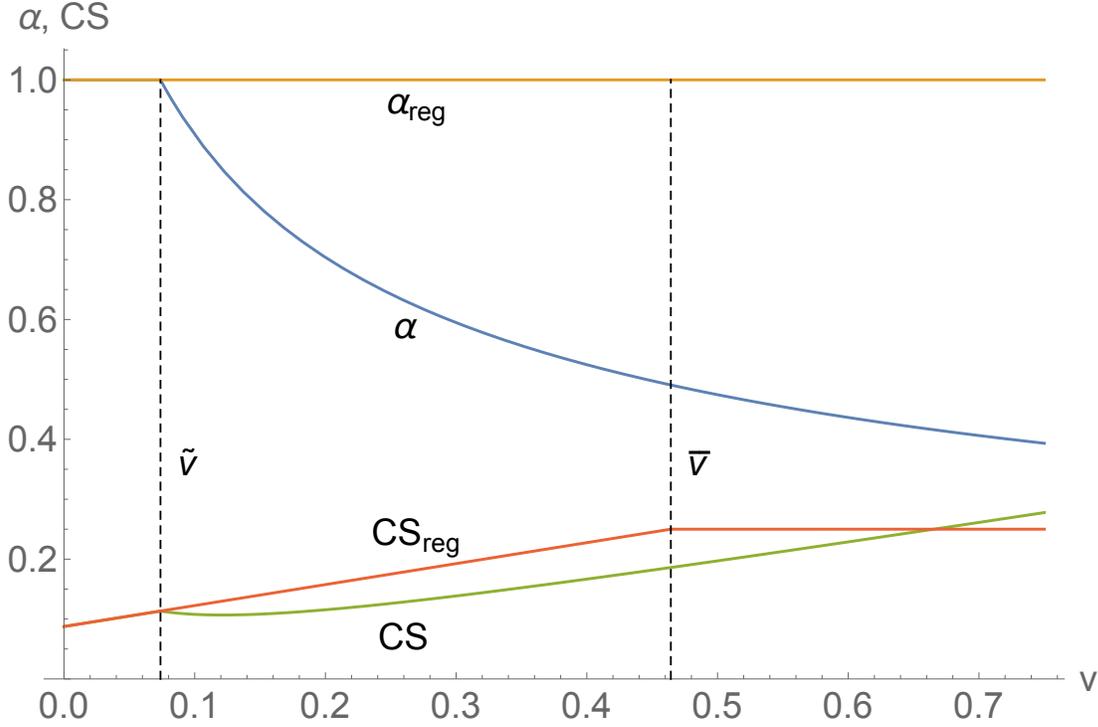


Figure 1: Depiction of the equilibrium product-choice-probability and consumers' expected surplus, both under a ban of below-cost pricing and without, for the case where $\frac{q_H}{c_H} > \frac{q_L}{c_L}$. The parameters used are $q_H = 1$, $c_H = 0.75$, $q_L = 0.5$, $c_L = 0.35$, $N = 3$, $\lambda = 0.35$, $\delta < \Delta_c/\Delta_q$.

We derive novel normative and positive implications from this model, showing in particular the difference that the presence of salient thinkers makes, compared to a situation where consumers have rational attention. As our main finding, we document that high-quality products may be crowded out inefficiently when these are used for promotions and when competition is fierce or the extent of one-stop shopping is large.

As discussed in the Introduction, these findings directly relate to the ongoing policy debate about possible detrimental effects of retailers' deep discounting in loss-leading product categories. We thus explicitly consider a policy of prohibiting below-cost pricing. This only affects product choice when consumers are salient thinkers, but not so otherwise. And we identify the precise circumstances when this intervention increases or decreases overall efficiency. In particular, we show that such a regulation might be detrimental to welfare and consumer surplus when it is overlooked that also high-quality products' pricing becomes constrained, facilitating a substitution to less efficient low-quality products.

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Appendix A Technical Proofs

Proof of Proposition 1. The proof proceeds in a series of lemmas. We start by noting that in any equilibrium, a firm stocking q_H or q_L can never price below the level where, even if it attracted the mass λ of shoppers with certainty, its profits would fall short of π^* . The subsequent lemma is then immediate.

Lemma 1 *There exists no equilibrium in which any firm stocking q_H prices below \underline{p} and no equilibrium in which any firm stocking q_L prices below*

$$\underline{p}_L = \underline{p} - \Delta_c. \quad (16)$$

We next show that price-salience is necessary for a low-quality firm to attract shoppers.

Lemma 2 *Suppose both qualities are offered with positive probability, and denote the lowest price of any high-quality (low-quality) firm by $p_H \leq q_H$ ($p_L \leq q_L$). Then the firm offering (q_L, p_L) cannot attract shoppers unless price is salient.*

Proof. If price is not salient, either no attribute is salient, (a) $\frac{q_H}{q_L} = \frac{p_H}{p_L}$, or quality is salient, (b) $\frac{q_H}{q_L} > \frac{p_H}{p_L}$. But (a) can be transformed to $q_H - p_H = \frac{p_H}{p_L}(q_L - p_L) > q_L - p_L$, while (b) can be transformed to $q_H - \delta p_H > \frac{p_H}{p_L}(q_L - \delta p_L) > q_L - \delta p_L$. Hence, in both cases, shoppers strictly prefer the high-quality offer. ■

We continue with an important characteristic of firms' expected demand.

Lemma 3 *Consider a firm stocking q_L and setting $p \leq q_L$. Then the firm can achieve a weakly higher expected demand by stocking q_H and setting $p' = \frac{q_H}{q_L}p \leq q_H$ instead.*

Proof. In light of Lemma 2, when offering (q_L, p) , a firm's expected demand is bounded above by

$$D(q_L, p) \leq \bar{D}(q_L, p) = \frac{1 - \lambda}{N} + \lambda \mathbb{P}\left\{p_L \geq p \wedge \frac{p}{p_H} < \frac{q_L}{q_H}\right\},$$

where p_L (p_H) denotes the minimum price of a rival low-quality firm (high-quality firm), respectively.

When offering (q_H, p') instead, it once again follows from Lemma 2 that a firm's expected demand is bounded below by

$$D(q_H, \frac{q_H}{q_L}p) \geq \frac{1 - \lambda}{N} + \lambda \mathbb{P}\left\{p_H > \frac{q_H}{q_L}p \wedge \frac{\frac{q_H}{q_L}p}{p_L} \leq \frac{q_H}{q_L}\right\} = \bar{D}(q_L, p).$$

Hence, the offer (q_H, p') indeed guarantees a weakly higher expected demand. ■

We are now in a position to rule out certain price choices by low-quality firms. In particular, a low-quality firm setting some price $p \leq q_L$ makes an expected profit of

$$\pi(q_L, p) = (p - c_L + v)D(q_L, p),$$

where $D(q_L, p)$ denotes its expected demand when offering (q_L, p) . Using Lemma 3, we know that if the same firm offered $(q_H, \frac{q_H}{q_L}p)$ instead, its expected profit would be bounded below by

$$\pi\left(q_H, \frac{q_H}{q_L}p\right) \geq \left(\frac{q_H}{q_L}p - c_H + v\right)D(q_L, p).$$

Hence, independent of rivals' choices, offering $(q_H, \frac{q_H}{q_L}p)$ is strictly preferred to (q_L, p) if $\frac{q_H}{q_L}p - c_H + v > p - c_L + v$, which transforms to $p > \frac{\Delta_c}{\Delta_q}q_L$.

Lemma 4 *In any equilibrium, no low-quality firm may ever price strictly above $\frac{\Delta_c}{\Delta_q}q_L$.*

Putting together Lemmas 1 and 4, it now becomes evident under which condition all firms must unanimously choose high quality in any equilibrium. From Lemma 1, we know that no low-quality firm will ever price strictly below $\underline{p} - \Delta_c$. From Lemma 4, we know that no low-quality firm will ever price strictly above $\frac{\Delta_c}{\Delta_q}q_L$. But then all prices are ruled out for low-quality firms, such that choosing low quality is strictly dominated, if $\underline{p} - \Delta_c \geq \frac{\Delta_c}{\Delta_q}q_L$, which transform to $\underline{p} \geq \frac{\Delta_c}{\Delta_q}q_H$. This completes the proof of Proposition 1. **Q.E.D.**

Proof of Proposition 2. We show first that in the characterized candidate equilibrium, choosing high quality and setting any price in the support of F_H , or choosing low quality and setting any price in the support of F_L , always yields the same expected profit of $\pi^* = \frac{1-\lambda}{N}(q_H - c_H + v)$. We then show that retailers have no incentive to deviate to any price outside the respective supports.

For the former, we start by proving that the characterized pricing supports in the candidate equilibrium are such that whenever high and low quality coexist, price must necessarily be salient, and the shoppers will always be attracted by the lowest-priced low-quality firm. Because $p_H > \underline{p}_H = \frac{\Delta_c}{\Delta_q}q_H$ with probability 1, while also $p_L < \bar{p}_L = \frac{\Delta_c}{\Delta_q}q_L$ with probability 1 (since there are no mass points in the respective pricing supports), it holds that $\frac{p_H}{p_L} > \frac{\underline{p}_H}{\bar{p}_L} = \frac{q_H}{q_L}$ with probability 1, hence price must always be salient in the candidate equilibrium. Given this, the lowest-priced high-quality firm could only attract the shoppers if $\delta q_H - p_H \geq \delta q_L - p_L$, that is, if $p_H - p_L \leq \delta \Delta_q$. But $p_H - p_L > \underline{p}_H - \bar{p}_L = \Delta_c$ with probability 1. Hence, the shoppers could only be attracted by the considered high-quality firm if $\delta \Delta_q > \Delta_c$, which however is ruled out by condition (2). Hence, we have established that in the candidate equilibrium, a high-quality firm can only attract the shoppers if there are no low-quality firms (and obviously, if no other high-quality firm has a lower price).

From these preliminary observations, it follows that for any price p in the support of $F_H(\cdot)$, a high-quality firm's probability of attracting the shoppers is

$$(\alpha^* [1 - F_H(p)])^{N-1}, \quad (17)$$

while for any price p in the support of $F_L(\cdot)$, a low-quality firm's probability of attracting the shoppers is

$$(\alpha^* + (1 - \alpha^*) [1 - F_L(p)])^{N-1}. \quad (18)$$

With this we can immediately confirm that the retailers are indeed indifferent between choosing low or high quality and setting a price in the respective supports $[\underline{p}_H, \bar{p}_H]$ and $[\underline{p}_L, \bar{p}_L]$. Precisely, using (17) we have that

$$\pi_H(p) = (p - c_H + v) \left\{ \frac{1 - \lambda}{N} + \lambda [\alpha^* (1 - F_H(p))]^{N-1} \right\} = \frac{1 - \lambda}{N} (q_H - c_H + v) = \pi^*,$$

while using (18) we have that

$$\pi_L(p) = (p - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda [\alpha^* + (1 - \alpha^*) (1 - F_L(p))]^{N-1} \right\} = \pi^*.$$

Below we show that there is no profitable deviation to prices $p \notin [\underline{p}_H, \bar{p}_H]$ and $p \notin [\underline{p}_L, \bar{p}_L]$. Before doing so, we show that the distribution functions are well-behaved. For this, note first that $\alpha^* \in (0, 1)$ holds if and only if

$$\frac{1 - \lambda}{\lambda N} \left[\frac{(\Delta_q - \Delta_c) q_H}{v \Delta_q - (q_H c_L - q_L c_H)} \right] \in (0, 1).$$

To see this, observe that a violation of condition (6), as required by the proposition, is equivalent to

$$v > \tilde{v} = \frac{q_H c_L - q_L c_H}{\Delta_q} + \frac{1 - \lambda}{\lambda N} \left[\frac{q_H (\Delta_q - \Delta_c)}{\Delta_q} \right]$$

(compare with Corollary 1), from which straightforward algebra yields the claim. Note next that both $F_H(\cdot)$ and $F_L(\cdot)$ are clearly (strictly) increasing over their respective supports. Finally, substitution of α^* reveals that they are also well-behaved at the boundaries.

Consider now deviations to prices outside the respective supports. If some retailer n chooses $q_n = q_H$ but deviates to a price $p_{dev} < \underline{p}_H$, its offer is clearly preferred to any other offer of a high-quality product and it is also preferred to the lowest-price offer of a low-quality product if, for the respective minimum \tilde{p}_{min} , it holds that $\tilde{p}_{min} > p_{dev} \frac{q_L}{q_H}$, so

that quality becomes salient, or $\tilde{p}_{min} > p_{dev} - \delta\Delta_q$, so that the deviating offer is preferred even if price is salient. It thus follows for the expected deviation profits that

$$\pi_H(p_{dev}) = (p_{dev} - c_H + v) \cdot \left\{ \frac{1 - \lambda}{N} + \lambda \left[\alpha^* + (1 - \alpha^*) \left(1 - F_L \left(\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta\Delta_q \right\} \right) \right) \right]^{N-1} \right\},$$

which from inserting $F_L(\cdot)$ transforms to

$$\pi_H(p_{dev}) = \frac{1 - \lambda}{N} (q_H - c_H + v) \left(\frac{p_{dev} - c_H + v}{\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta\Delta_q \right\} - c_L + v} \right).$$

The sign of the function's derivative is equal to the sign of $v\Delta_q - (q_H c_L - c_H q_L)$ if $p_{dev} \frac{q_L}{q_H} \leq p_{dev} - \delta\Delta_q$ or to the sign of $\Delta_c - \delta\Delta_q$ if $p_{dev} \frac{q_L}{q_H} > p_{dev} - \delta\Delta_q$, respectively. The former is strictly positive since by assumption condition (6) is violated (such that $v > \tilde{v}$; compare once again with Corollary 1), while the latter is strictly positive due to condition (2). Thus, we have shown that $\pi_H(p_{dev}) < \pi^*$ if $p_{dev} < \underline{p}_H$ for a high-quality firm.

Consider finally deviations by low-quality firms, where we only need to consider deviations $p_{dev} > \bar{p}_L$. Clearly, the deviation profit can only exceed π^* if the deviating firm still attracts the shoppers, for which it is necessary that all other retailers choose high quality and that price remains salient. Just respecting the salience constraint, the deviation profit thus satisfies

$$\pi_L(p_{dev}) \leq (p_{dev} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[\alpha^* \left(1 - F_H \left(\frac{q_H}{q_L} p_{dev} \right) \right) \right]^{N-1} \right\},$$

which from inserting $F_H(\cdot)$ transforms to

$$\pi_L(p_{dev}) \leq \frac{1 - \lambda}{N} (q_H - c_H + v) \left(\frac{p_{dev} - c_L + v}{p_{dev} \frac{q_H}{q_L} - c_H + v} \right).$$

The sign of the derivative of $\frac{p_{dev} - c_L + v}{p_{dev} \frac{q_H}{q_L} - c_H + v}$ with respect to p_{dev} has the same sign as $-[v\Delta_q - (q_H c_L - c_H q_L)]$, which is strictly negative since by assumption condition (6) is violated (see above). Hence, the RHS of the above inequality is maximized for $p_{dev} = \bar{p}_L$, such that $\pi_L(p_{dev}) \leq \pi^*$. **Q.E.D.**

Proof of Proposition 3. We know from Lemma 4 in the proof of Proposition 1 that no firm may ever stock q_L and set a price $p > \frac{\Delta_c}{\Delta_q} q_L$ in any unregulated equilibrium, as this is strictly dominated by choosing q_H and setting $p' = p \frac{q}{q_L}$ instead. But if low-quality

firms are prohibited to price below c_L , they are forced to set such prices, as $p \geq c_L$ implies $p > \frac{\Delta_c}{\Delta_q} q_L$ if condition (14) holds. At the same time, a deviation from $p \geq c_L$ with $q_n = q_L$ to $p' = \frac{q_H}{q_L} p$ with $q_n = q_H$ is still feasible, since $\frac{q_H}{q_L} p \geq \frac{q_H}{q_L} c_L > c_H$ due to the regulation and condition (14). **Q.E.D.**

Appendix B Omitted Material and Proofs

Appendix B.1 Symmetric Pricing Equilibrium under the Regulation and $\frac{q_H}{q_L} > \frac{c_H}{c_L}$

Lemma 5 *Suppose that below-cost pricing is prohibited and $\frac{q_H}{q_L} > \frac{c_H}{c_L}$, such that all retailers must stock q_H in any equilibrium. Then for $v \leq \underline{v} := \frac{1-\lambda}{\lambda N}(q_H - c_H)$, the prohibition does not bind and thus does not affect the unique symmetric pricing equilibrium. For $v \in (\underline{v}, \bar{v})$, we have the following characterization:*

There exists a unique symmetric pricing equilibrium in which retailers set $p_n = c_H$ with probability $\beta^ \in (0, 1)$, whereas with the remaining probability, they sample prices continuously from the CDF*

$$F_r(p) := 1 - \frac{N^{-1} \sqrt{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)}}{1 - \beta^*} \quad (19)$$

with support $[\underline{p}_r, q_H]$, where β^ is defined implicitly by the unique solution to*

$$\frac{1 - (1 - \beta)^N}{\beta} = \frac{1 - \lambda}{\lambda v} (q_H - c_H), \quad (20)$$

and

$$\underline{p}_r := c_H - v + \frac{q_H - c_H + v}{1 + \frac{\lambda N}{1-\lambda}(1 - \beta^*)^{N-1}} \in (c_H, q_H). \quad (21)$$

In equilibrium, each retailer makes an expected profit of $\pi^ = \frac{1-\lambda}{N}(q_H - c_H + v)$.*

Proof. Recall that $\underline{p} < c_H$ holds if and only if $v > \underline{v}$. Since in any pricing equilibrium with high-quality products, no matter whether symmetric or asymmetric, no firm samples prices below \underline{p} , the set of pricing equilibria is clearly not affected if $v \leq \underline{v}$.

Suppose now that $v \in (\underline{v}, \bar{v})$. It is straightforward to verify that $p_n = c_H$ and $p_n \in [\underline{p}_r, q_H]$ all yield the same profit of $\frac{1-\lambda}{N}(q_H - c_H + v)$. Precisely, for \underline{p}_r note that the expected profit is $(\underline{p}_r - c_H + v) \left[\frac{1-\lambda}{N} + \lambda(1 - \beta)^{N-1} \right]$, as shoppers can only be attracted if none of the $N - 1$ rivals samples c_H . And with $p \in (\underline{p}_r, q_H)$, the expected profit is $(p - c_H + v) \left\{ \frac{1-\lambda}{N} + \lambda [(1 - \beta)(1 - F_r(p))]^{N-1} \right\}$, as shoppers can only be attracted if all rivals sample a price above p . Provided that $\beta \in (0, 1)$, which will be verified below, $F_r(p)$

in (19) is strictly increasing in p , with $F_r(\underline{p}_r) = 0$ and $F_r(q_H) = 1$. Finally, if a retailer samples c_H , its expected profit can be written as

$$\pi_i(c_H) = (c_H - c_H + v) \left[\frac{1 - \lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} \beta^j (1 - \beta)^{N-1-j} \frac{\lambda}{j+1} \right],$$

as it has to share the shoppers with $j \in \{0, \dots, N-1\}$ rivals, which happens with probability $\binom{N-1}{j} \beta^j (1 - \beta)^{N-1-j}$, respectively. Using that

$$\sum_{j=0}^{N-1} \binom{N-1}{j} \beta^j (1 - \beta)^{N-1-j} \frac{1}{j+1} = \frac{1}{\beta N} [1 - (1 - \beta)^N],$$

which follows from the binomial theorem, this uniquely pins down $\beta = \beta^*$. (Note that the left-hand side of (20) is strictly decreasing in β for $\beta \in (0, 1)$, as it can be rewritten as $f(\beta) = \sum_{k=0}^{N-1} (1 - \beta)^k$.) Note finally that no retailer can profitably deviate to $p_n \in (c_H, \underline{p}_r)$ as, by construction, a strictly higher profit is realized with \underline{p}_r instead. ■

Appendix B.2 Omitted Proof for the Case where the Regulation Backfires (Proposition 4)

We first prove the following claim, which describes the different regimes for equilibrium product choice under the pricing regulation and $\frac{q_H}{c_H} < \frac{q_L}{c_L}$.

Claim 1 *If consumers are salient thinkers and $\frac{q_H}{c_H} < \frac{q_L}{c_L}$, a prohibition of below-cost pricing has the following consequences for retailers' product choice:*

(I) *If $v \leq \tilde{v}$, retailers always stock the high-quality product.*

(II) *If $\tilde{v} < v \leq \hat{v}$, where $\hat{v} \in (\underline{v}, \bar{v})$ is defined implicitly by the unique solution to*

$$v \left(\frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H),$$

retailers stock the high-quality product with probability $\alpha^(v) \in (0, 1)$ (as defined in Proposition 2), while they stock the low-quality product with complementary probability. Moreover, $\alpha^*(v)$ is strictly decreasing in v .*

(III) *If $\hat{v} < v < \bar{v}$, retailers stock the high-quality product with probability $\tilde{\alpha}(v) \in (0, \alpha^*(v))$, where $\tilde{\alpha}(v)$ is defined implicitly by the unique solution to*

$$\frac{1 - \alpha^N}{1 - \alpha} = \frac{1 - \lambda}{\lambda v} (q_H - c_H),$$

while they stock the low-quality product with complementary probability. With $\tilde{\alpha}(\hat{v}) = \alpha^(\hat{v})$ and $\tilde{\alpha}(\bar{v}) = 0$, it holds that $\tilde{\alpha}(v) < \alpha^*(v)$ for all $v \in (\hat{v}, \bar{v}]$.*

Proof of Claim 1. The claim is proven by a series of lemmas.

Lemma 6 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and below-cost pricing is prohibited, a high-quality equilibrium as characterized by Lemma 5 exists if and only if $v \leq \tilde{v}$.*

Proof of Lemma 6. Note that since $\tilde{v} < \underline{v}$ for $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, on-equilibrium pricing is not affected under the regulation given $v \leq \tilde{v}$. Hence, at worst the regulation also does not restrict the set of optimal deviations. From Proposition 1 it thus follows that a high-quality equilibrium exists if $\underline{p} \geq \frac{\Delta_c}{\Delta_q} q_H$, which is equivalent to $v \leq \tilde{v}$.

We proceed to show that no high-quality equilibrium can exist in the complementary case where $v > \tilde{v}$. If $v \in (\tilde{v}, \underline{v}]$, the equilibrium pricing of a high-quality equilibrium would still be unaffected, as high-quality firms' lowest price \underline{p} would satisfy $\underline{p} \geq c_H$. Then there are two cases. If $\delta \leq \frac{p}{q_H}$, a deviating retailer can attract the shoppers deterministically by pricing at $\underline{p} \frac{q_L}{q_H}$ (as the salience constraint is binding for low δ). This price is feasible because $\underline{p} \frac{q_L}{q_H} > c_L$ due to $\underline{p} \geq c_H$ and $\frac{q_H}{q_L} < \frac{c_H}{c_L}$. It is then easy to show that the resulting deviation profit $\pi_{dev} = \left(\underline{p} \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1-\lambda}{N} + \lambda \right)$ exceeds the profit $(q_H - c_H + v) \frac{1-\lambda}{N} = (\underline{p} - c_H + v) \left(\frac{1-\lambda}{N} + \lambda \right)$ in a hypothetical high-quality equilibrium if $\underline{p} < \frac{\Delta_c}{\Delta_q} q_H$, i.e., if $v > \tilde{v}$. Second, if it instead holds that $\delta > \frac{p}{q_H}$, a deviating retailer can attract the shoppers deterministically by pricing at $\underline{p} - \delta \Delta_q$ (as the competition constraint is binding for high δ). This price is feasible because $\underline{p} - \delta \Delta_q > c_L$ due to $\underline{p} \geq c_H$ and $\delta \Delta_q < \Delta_c$. The corresponding deviation profit of $\pi_{dev} = (\underline{p} - \delta \Delta_q - c_L + v) \left(\frac{1-\lambda}{N} + \lambda \right)$ again exceeds the hypothetical equilibrium profit in a high-quality equilibrium due to $\delta \Delta_q < \Delta_c$.

We now show that no high-quality equilibrium can exist for $v \in (\underline{v}, \bar{v})$. This is because, although the regulation becomes binding and retailers' pricing in a hypothetical high-quality equilibrium becomes restricted to prices at or above c_H , retailers' expected profit stays at $(q_H - c_H + v) \frac{1-\lambda}{N}$ (compare with Lemma 5). Hence, similar to the case where $v \in (\tilde{v}, \underline{v}]$ discussed before, if $\delta \leq \frac{c_H}{q_H}$, a deviating retailer can attract all shoppers by pricing at $c_H \frac{q_L}{q_H} - \epsilon$, which is permissible as $c_H \frac{q_L}{q_H} > c_L$. It is then easy to show that the resulting deviation profit of $\left(c_H \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1-\lambda}{N} + \lambda \right)$ strictly exceeds $(q_H - c_H + v) \frac{1-\lambda}{N}$ for $v > \underline{v}$. If it holds in contrast that $\delta > \frac{c_H}{q_H}$, a deviating retailer can attract all shoppers by pricing at $c_H - \delta \Delta_q - \epsilon$, which is feasible because $c_H - \delta \Delta_q > c_L$. The corresponding deviation profit of $(c_H - \delta \Delta_q - c_L + v) \left(\frac{1-\lambda}{N} + \lambda \right)$ also strictly exceeds $(q_H - c_H + v) \frac{1-\lambda}{N}$, as follows from $\delta \Delta_q < \Delta_c$ and $v > \underline{v}$. ■

The following sequence of lemmas characterizes the symmetric equilibrium in product choice and pricing under the regulation if $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $v > \tilde{v}$ such that no high-quality

equilibrium exists.

Lemma 7 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $v \in (\tilde{v}, \underline{v}]$, the equilibrium is still characterized by Proposition 2.*

Proof of Lemma 7. From the proof of Proposition 2, we know that in this candidate equilibrium, the lowest price a low-quality firm samples is $\underline{p}_L = c_L - v + (q_H - c_H + v) \frac{1-\lambda}{1-\lambda+\lambda N}$, while the lowest price a high-quality firm samples is $\underline{p}_H = \frac{\Delta c}{\Delta q} q_H$. Observe first that \underline{p}_H exceeds c_H for every v , so the regulation clearly does not bind for high-quality firms. And it also does not bind for low-quality firms, provided that $v \leq \underline{v}$. Hence, as the equilibrium pricing is not affected for $v \in (\tilde{v}, \underline{v}]$, which was part of an equilibrium without the regulation (see the proof of Proposition 2), the corresponding strategy-combination still constitutes an equilibrium with the regulation. ■

The following technical lemma is needed for a characterization of the remaining case where $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $v \in (\underline{v}, \bar{v})$.

Lemma 8 *There exists a unique $\hat{v} \in (\underline{v}, \bar{v})$ such that*

$$v \left(\frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H), \quad (22)$$

where

$$\alpha^*(v) = \sqrt[N-1]{\frac{1 - \lambda}{\lambda N} \left[\frac{(\Delta_q - \Delta_c) q_H}{v \Delta_q - (q_H c_L - q_L c_H)} \right]},$$

as defined in Proposition 2.

Proof of Lemma 8. Note first that the RHS of equation (22) is independent of v . Hence, it is sufficient to prove that (1) $\underline{v} \left(\frac{1 - [\alpha^*(\underline{v})]^N}{1 - \alpha^*(\underline{v})} \right) < \frac{1 - \lambda}{\lambda} (q_H - c_H)$, (2) $\bar{v} \left(\frac{1 - [\alpha^*(\bar{v})]^N}{1 - \alpha^*(\bar{v})} \right) > \frac{1 - \lambda}{\lambda} (q_H - c_H)$, and (3) $v \left(\frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)} \right)$ is strictly increasing in v over the relevant range. For (1), note that since $\tilde{v} < \underline{v}$, $\alpha^*(\tilde{v}) = 1$, $\alpha^*(v)$ is strictly decreasing in v , and $\frac{1 - \alpha^N}{1 - \alpha}$ is strictly increasing in α , the LHS for $v = \underline{v}$ must fall short of $\underline{v} \left(\lim_{\alpha \rightarrow 1} \frac{1 - \alpha^N}{1 - \alpha} \right) = \underline{v} N$, which is the RHS of equation (22). For (2), note that $\frac{1 - \alpha^N}{1 - \alpha}$ strictly exceeds 1 for all $\alpha \in (0, 1)$. Hence, the LHS of equation (22) for $v = \bar{v}$ must exceed \bar{v} , which is the RHS of the equation.

For (3), we first make use of the implicit definition of $\alpha^*(v)$, which is given by

$$\pi_H(\underline{p}_H) = \left(\frac{\Delta_c}{\Delta_q} q_H - c_H + v \right) \left(\frac{1 - \lambda}{N} + \lambda \alpha^{N-1} \right) \stackrel{!}{=} \frac{1 - \lambda}{N} (q_H - c_H + v).$$

Implicit differentiation establishes that

$$\frac{d\alpha^*(v)}{dv} = -\frac{\alpha^*(v)}{\left(\frac{\Delta_c}{\Delta_q}q_H - c_H + v\right)(N-1)}.$$

As $v \left(\frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)}\right)$ can be rewritten as $v \sum_{j=0}^{N-1} \alpha^*(v)^j$, the derivative of the latter with respect to v is then given by

$$\begin{aligned} & \sum_{j=0}^{N-1} \alpha^*(v)^j + v \sum_{j=0}^{N-1} j \alpha^*(v)^{j-1} \frac{d\alpha^*(v)}{dv} \\ &= \sum_{j=0}^{N-1} \alpha^*(v)^j - v \sum_{j=0}^{N-1} j \alpha^*(v)^{j-1} \left(\frac{\alpha^*(v)}{\left(\frac{\Delta_c}{\Delta_q}q_H - c_H + v\right)(N-1)} \right) \\ &> \sum_{j=0}^{N-1} \alpha^*(v)^j - v \sum_{j=0}^{N-1} j \alpha^*(v)^j \left(\frac{1}{v(N-1)} \right) \\ &> \sum_{j=0}^{N-1} \alpha^*(v)^j - \sum_{j=0}^{N-1} (N-1) \alpha^*(v)^j \left(\frac{1}{N-1} \right) = 0, \end{aligned}$$

where the third line follows from $\frac{q_H}{q_L} < \frac{c_H}{c_L}$. ■

Lemma 9 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $v \in (\underline{v}, \hat{v})$, the following constitutes an equilibrium. With probability $\alpha^* \in (0, 1)$ as defined in Proposition 2, retailers choose the high-quality product and sample prices according to the CDF $F_H(p)$, again as defined in Proposition 2. With probability $1 - \alpha^*$, retailers choose the low-quality product. If they do so, they choose $p_n = c_L$ with probability $\tilde{\beta}(v) \in (0, 1)$, where $\tilde{\beta}(v)$ is defined implicitly by*

$$v \left(\frac{1 - \left[1 - (1 - \alpha^*(v))\tilde{\beta}\right]^N}{(1 - \alpha^*(v))\tilde{\beta}} \right) = \frac{1 - \lambda}{\lambda}(q_H - c_H), \quad (23)$$

with $\tilde{\beta}(\underline{v}) = 0$, $\tilde{\beta}(\hat{v}) = 1$, and $\tilde{\beta}'(v) > 0$. With the remaining probability $1 - \tilde{\beta}(v)$, low-quality retailers sample prices continuously from a CDF $F_{L,r}(p)$ with support $[\underline{p}_{L,r}, \frac{\Delta_c}{\Delta_q}q_L]$, where

$$\underline{p}_{L,r} := c_L - v + \frac{(q_H - c_H + v)^{\frac{1-\lambda}{N}}}{\frac{1-\lambda}{N} + \lambda \left[1 - (1 - \alpha^*(v))\tilde{\beta}(v)\right]^{N-1}} \in (c_L, \frac{\Delta_c}{\Delta_q}q_L),$$

and

$$F_{L,r}(p) := 1 - \frac{N-1 \sqrt{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_L + v} - 1 \right)} - \alpha^*(v)}{(1 - \alpha^*(v)) \left(1 - \tilde{\beta}(v)\right)}.$$

Proof of Lemma 9. Because the lower support bound of high-quality firms (which is given by $\frac{\Delta_c}{\Delta_q} q_H$ and thus strictly exceeds c_H , as follows from $\frac{q_H}{q_L} < \frac{c_H}{c_L}$) and the upper support bound of low-quality firms are the same as in the mixed-product equilibrium without the regulation (see the proof of Proposition 2), price will always be salient and high-quality firms cannot serve shoppers if both low-quality and high-quality products are introduced to the market. Hence, from the proof of Proposition 2, high-quality firms are still indifferent between sampling any price in their specified support, and make an expected profit of $(q_H - c_H + v) \frac{1-\lambda}{N}$. A low-quality firm's expected profit when sampling $\bar{p}_L = \frac{\Delta_c}{\Delta_q} q_L$ is $(\frac{\Delta_c}{\Delta_q} q_L - c_L + v) (\frac{1-\lambda}{N} + \lambda(\alpha^*)^{N-1})$, as it can only attract the shoppers if all of its rivals stock q_H . By construction of α^* , this also yields an expected profit of $(q_H - c_H + v) \frac{1-\lambda}{N}$. If a low-quality firm samples $\underline{p}_{L,r}$, its expected profit is given by

$$\pi_L(\underline{p}_{L,r}) = (\underline{p}_{L,r} - c_L + v) \left\{ \frac{1-\lambda}{N} + \lambda \left[\alpha^* + (1-\alpha^*) (1-\tilde{\beta}) \right]^{N-1} \right\},$$

as it can only attract the shoppers if all rivals either stock q_H , or stock q_L , but do not sample c_L . Setting this equal to $(q_H - c_H + v) \frac{1-\lambda}{N}$, we find $\underline{p}_{L,r}$. If a low-quality firm samples an arbitrary price p in its support, its expected profit is given by

$$(p - c_L + v) \left\{ \frac{1-\lambda}{N} + \lambda \left[\alpha^* + (1-\alpha^*) (1-\tilde{\beta}) (1 - F_{L,r}(p)) \right]^{N-1} \right\},$$

as it can only attract shoppers if all of its rivals either choose q_H , or choose q_L but do not charge a lower price than p . Setting this equal to $(q_H - c_H + v) \frac{1-\lambda}{N}$, we find the CDF $F_{L,r}(p)$ reported in the lemma.

If a low-quality firm samples c_L , its expected profit is

$$\begin{aligned} \pi_L(c_L) &= (c_L - c_L + v) \left\{ \frac{1-\lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} \left[(1-\alpha^*)\tilde{\beta} \right]^j \left[1 - (1-\alpha^*)\tilde{\beta} \right]^{N-1-j} \frac{\lambda}{j+1} \right\} \\ &= v \left[\frac{1-\lambda}{N} + \lambda \left(\frac{1 - \left[1 - (1-\alpha^*)\tilde{\beta} \right]^N}{N(1-\alpha^*)\tilde{\beta}} \right) \right], \end{aligned}$$

as it has to share the shoppers with $j \in \{0, \dots, N-1\}$ rivals which also sample c_L with probability $\binom{N-1}{j} \left[(1-\alpha^*)\tilde{\beta} \right]^j \left[1 - (1-\alpha^*)\tilde{\beta} \right]^{N-1-j}$, and where the second line is obtained by proper manipulation and making use of the binomial theorem. Setting this equal to $(q_H - c_H + v) \frac{1-\lambda}{N}$ and simplifying, we find the implicit definition of $\tilde{\beta}(v)$ as provided in the lemma.

Taken together, each price in retailers' support thus indeed yields the same expected profit. Further, low-quality retailers have no profitable deviation price as pricing below c_L is prohibited, pricing between c_L and $\underline{p}_{L,r}$ is strictly dominated by pricing at $\underline{p}_{L,r}$, and pricing above their upper support bound $\frac{\Delta_c}{\Delta_q}q_L$ was already shown to be inferior in the proof of Proposition 2, where the high-quality product is stocked with the same probability and high-quality firms use the same strategy as in the present candidate equilibrium. Deviating high-quality firms can never guarantee to attract all shoppers, as they would have to price below $c_L \frac{q_H}{q_L}$, which falls short of c_H . If a high-quality firm deviates to $p_{dev} \in [\underline{p}_{L,r} \frac{q_H}{q_L}, \frac{\Delta_c}{\Delta_q}q_H)$, its expected profit is given by

$$\begin{aligned} \pi_H(p_{dev}) &= (p_{dev} - c_H + v) \cdot \\ &\quad \left\{ \frac{1 - \lambda}{N} + \lambda \left[\alpha^* + (1 - \alpha^*) (1 - \tilde{\beta}) \left(1 - F_{L,r} \left(\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta_q \right\} \right) \right) \right]^{N-1} \right\} \\ &= (p_{dev} - c_H + v) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta_q \right\} - c_L + v} \right), \end{aligned}$$

where the second line follows from inserting $F_{L,r}(\cdot)$, as found in the lemma, and simplifying. (Note that the term $\min \{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta_q \}$ appears because a deviating high-quality firm can win the shoppers if either quality is salient, or price is salient but its offer still provides a higher perceived utility). The derivative of $\pi_H(p_{dev})$ with respect to p_{dev} is strictly positive for all p_{dev} , which implies that also firms with q_H have no profitable deviation.

In the proof of Proposition 2, it has already been established that α^* and $F_H(p)$ are well-behaved. Furthermore, provided that $\tilde{\beta}(v) \in [0, 1)$, $F_{L,r}(p)$ is also well-behaved, as it is strictly increasing in p , with $F_{L,r}(\underline{p}_{L,r}) = 0$ and $F_{L,r} \left(\frac{\Delta_c}{\Delta_q} q_L \right) = 1$. It remains to show that $\tilde{\beta}(v)$ and $\underline{p}_{L,r}$ are well-behaved, where for convenience we restate the implicit definition of $\tilde{\beta}(v)$:

$$v \left(\frac{1 - \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^N}{(1 - \alpha^*(v)) \tilde{\beta}} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H).$$

Note that for $v = \underline{v}$, the above equation can only be satisfied if $(1 - \alpha^*) \tilde{\beta} = 0$, which, as $\alpha^* \in (0, 1)$ for $v > \tilde{v}$, implies that $\tilde{\beta}(\underline{v}) = 0$. Setting $\tilde{\beta} = 1$, this becomes

$$v \left(\frac{1 - \alpha^*(v)^N}{1 - \alpha^*(v)} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H),$$

where the unique solution is given by $\hat{v} \in (\underline{v}, \bar{v})$ (see Lemma 8 above). We will now prove

that $\tilde{\beta}(v)$ is strictly increasing in v . To see this, note first that

$$v \left(\frac{1 - \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^N}{(1 - \alpha^*(v)) \tilde{\beta}} \right) = v \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j.$$

Plugging this into the above implicit definition of $\tilde{\beta}(v)$ and differentiating implicitly, we find that $\tilde{\beta}(v)$ is strictly increasing in v if

$$\sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j + v \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^{j-1} \tilde{\beta} \left(\frac{d\alpha^*(v)}{dv} \right) > 0.$$

To see that this is indeed the case, note that

$$\begin{aligned} & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j + v \sum_{j=0}^{N-1} j \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^{j-1} \tilde{\beta} \left(\frac{d\alpha^*(v)}{dv} \right) \\ = & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j + v \sum_{j=0}^{N-1} j \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^{j-1} \tilde{\beta} \left(-\frac{\alpha^*(v)}{\left(\frac{\Delta c}{\Delta q} q_H - c_H + v \right) (N-1)} \right) \\ > & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j + v \sum_{j=0}^{N-1} j \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^{j-1} \tilde{\beta} \left(-\frac{\alpha^*(v)}{v(N-1)} \right) \\ > & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j - \sum_{j=0}^{N-1} (N-1) \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j \tilde{\beta} \left(\frac{\alpha^*(v)}{N-1} \right) \\ = & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta} \right]^j \left(1 - \alpha^*(v) \tilde{\beta} \right) \geq 0, \end{aligned}$$

where the second line follows from implicitly differentiating the definition of $\alpha^*(v)$, and the third line follows from $\frac{q_H}{q_L} < \frac{c_H}{c_L}$. Hence, $\tilde{\beta}(v)$ is strictly increasing in v , with $\tilde{\beta}(\underline{v}) = 0$ and $\tilde{\beta}(\hat{v}) = 1$. From this it follows directly that $\underline{p}_{L,r}(\underline{v}) = c_L$, whereas using the definition of $\alpha^*(v)$ yields $\underline{p}_{L,r}(\hat{v}) = \frac{\Delta c}{\Delta q} q_L$. A proof that $\underline{p}_{L,r}(v)$ is strictly increasing in v is omitted for brevity. ■

Lemma 10 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $v \in (\hat{v}, \bar{v})$, the following constitutes an equilibrium. Retailers choose the high-quality product with probability $\tilde{\alpha}(v)$, where $\tilde{\alpha}(v)$ is the unique solution to*

$$\frac{1 - \alpha^N}{1 - \alpha} = \frac{1 - \lambda}{\lambda v} (q_H - c_H),$$

with $\tilde{\alpha}(\hat{v}) = \alpha^*(\hat{v})$, $\tilde{\alpha}(\bar{v}) = 0$, and $\tilde{\alpha}'(v) < 0$. High-quality retailers sample prices continuously from a CDF $F_{H,r}(p)$ with support $[\underline{p}_{H,r}, q_H]$, where

$$\underline{p}_{H,r} := c_H - v + \frac{(q_H - c_H + v)^{\frac{1-\lambda}{N}}}{\frac{1-\lambda}{N} + \lambda \tilde{\alpha}^{N-1}} \in \left(\frac{\Delta_c}{\Delta_q} q_H, q_H \right)$$

and

$$F_{H,r}(p) := 1 - \frac{N^{-1} \sqrt{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)}}{\tilde{\alpha}(v)}.$$

With the remaining probability $1 - \tilde{\alpha}(v)$, retailers choose q_L and set $p_n = c_L$ deterministically.

Proof of Lemma 10. As $\underline{p}_{H,r} > \frac{\Delta_c}{\Delta_q} q_H > c_L \frac{q_H}{q_L}$, price is always salient if both high-quality and low-quality products are in the market. Then, all shoppers prefer a low-quality firm's offer, as $\delta q_H - \underline{p}_{H,r} < \delta q_L - c_L$ due to $\underline{p}_{H,r} > c_H$ and $\delta \Delta_q < \Delta_c$. If a high-quality firm samples an arbitrary price p in its support, its expected profit is given by

$$(p - c_H + v) \left\{ \frac{1 - \lambda}{N} + \lambda [\tilde{\alpha} (1 - F_{H,r}(p))]^{N-1} \right\},$$

as it can only attract the shopper if all of its rivals stock q_H and sample a higher price than p . Setting this equal to $(q_H - c_H + v)^{\frac{1-\lambda}{N}}$, we find the CDF $F_{H,r}(p)$ and $\underline{p}_{H,r}$. Clearly, provided that $\tilde{\alpha}(v) > 0$, $F_{H,r}(p)$ is well-behaved, as it is strictly increasing in p , with $F_{H,r}(\underline{p}_{H,r}) = 0$ and $F_{H,r}(q_H) = 1$.

If a retailer chooses q_L and $p_n = c_L$, its expected profit is given by

$$\begin{aligned} \pi_L(c_L) &= (c_L - c_L + v) \left[\frac{1 - \lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} (1 - \tilde{\alpha})^j \tilde{\alpha}^{N-1-j} \frac{\lambda}{j+1} \right] \\ &= v \left[\frac{1 - \lambda}{N} + \lambda \frac{1 - \tilde{\alpha}^N}{(1 - \tilde{\alpha}) N} \right], \end{aligned}$$

as it has to share the shoppers with $j \in \{0, \dots, N-1\}$ rivals, which happens with probability $\binom{N-1}{j} (1 - \tilde{\alpha})^j \tilde{\alpha}^{N-1-j}$, and where the second line again follows from the binomial theorem. Setting this equal to $(q_H - c_H + v)^{\frac{1-\lambda}{N}}$ yields the implicit definition of $\tilde{\alpha}(v)$ in the lemma. Note that the LHS of this is strictly increasing in α , which implies that $\tilde{\alpha}(v)$ must be strictly decreasing in v . It is also easy to check that $\tilde{\alpha}(\bar{v}) = 0$. Furthermore, comparing the above equation with the definition of \hat{v} , which is the unique v that satisfies

$$\frac{1 - \alpha^*(\hat{v})^N}{1 - \alpha^*(\hat{v})} = \frac{1 - \lambda}{\lambda \hat{v}} (q_H - c_H),$$

it is apparent that $\tilde{\alpha}(\hat{v}) = \alpha^*(\hat{v})$. Using the latter two results, from the definition of $\underline{p}_{H,r}$ it immediately follows that $\underline{p}_{H,r}(\bar{v}) = q_H$, whereas also using the definition of $\alpha^*(v)$ yields $\underline{p}_{H,r}(\hat{v}) = \frac{\Delta_c}{\Delta_q} q_H$. It can likewise be established that $\underline{p}_{H,r}(v)$ is strictly increasing in v .

It remains to show that no firm can have a profitable deviation. Note first that due to the regulation, it is impossible for deviating high-quality firms to render quality salient if any rival stocks q_L . Hence, the best deviation a high-quality retailer can make is to charge the highest price for which its offer wins although price, rather than quality, is salient. But this price, $p_{dev} = c_L + \delta\Delta_q$, is prohibited due to $\delta\Delta_q < \Delta_c$. If a low-quality firm chooses $p_{dev} > c_L$, its expected profit is at best (that is, for $\delta = 0$) given by

$$\begin{aligned}\pi_L(p_{dev}) &= (p_{dev} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[\tilde{\alpha} \left(1 - F_{H,r} \left(p_{dev} \frac{q_H}{q_L} \right) \right) \right]^{N-1} \right\} \\ &= (p_{dev} - c_L + v) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{p_{dev} \frac{q_H}{q_L} - c_H + v} \right),\end{aligned}$$

where the second line follows from inserting $F_{H,r}(\cdot)$ and simplifying. From this, it is easy to show that $\pi_L(p_{dev})$ is strictly decreasing in p_{dev} , from which it follows that low-quality firms' optimal deviation price is $\underline{p}_{H,r} \frac{q_L}{q_H} > c_L$ for a maximal deviation profit of

$$\left(\underline{p}_{H,r} \frac{q_L}{q_H} - c_L + v \right) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{\underline{p}_{H,r} - c_H + v} \right).$$

This could only exceed the candidate equilibrium's profit $(q_H - c_H + v) \frac{1 - \lambda}{N}$ if $\underline{p}_{H,r} < \frac{\Delta_c}{\Delta_q} q_H$, which is not satisfied. Hence, also low-quality firms do not have a profitable deviation. ■

For the comparison of $\tilde{\alpha}(v)$ and $\alpha^*(v)$, we finally establish the following.

Lemma 11 $\tilde{\alpha}'(v) < \alpha^{*'}(v)$ for all $v \in [\hat{v}, \bar{v}]$.

Proof of Lemma 11. We first note that the implicit definition of $\tilde{\alpha}(v)$ can be rewritten as

$$\sum_{j=0}^{N-1} \alpha^j = \frac{1 - \lambda}{\lambda v} (q_H - c_H).$$

Implicitly differentiating this, we obtain that

$$\begin{aligned}
|\tilde{\alpha}'(v)| &= \frac{1-\lambda}{\lambda v^2}(q_H - c_H) \left(\frac{1}{\sum_{j=0}^{N-1} j \tilde{\alpha}(v)^{j-1}} \right) \\
&= \frac{1}{v} \left(\frac{\sum_{j=0}^{N-1} \tilde{\alpha}(v)^j}{\sum_{j=0}^{N-1} j \tilde{\alpha}(v)^{j-1}} \right) \\
&> \frac{1}{v} \left(\frac{\sum_{j=0}^{N-1} \tilde{\alpha}(v)^j}{\sum_{j=0}^{N-1} (N-1) \tilde{\alpha}(v)^j} \right) = \frac{1}{v(N-1)},
\end{aligned}$$

where the second line follows from using $\frac{1-\lambda}{\lambda v}(q_H - c_H) = \sum_{j=0}^{N-1} \tilde{\alpha}(v)^j$ by the above definition. We have next that

$$|\alpha^{*'}(v)| = \frac{\alpha^*(v)}{\left(\frac{\Delta_c}{\Delta_q} q_H - c_H + v \right) (N-1)} < \frac{\alpha^*(v)}{v(N-1)},$$

where the expression for $\alpha^{*'}(v)$ has already been established in the proof of Lemma 8, and the inequality follows from $\frac{q_H}{q_L} < \frac{c_H}{c_L}$. Comparing the lower bound of $|\tilde{\alpha}'(v)|$ with the upper bound of $|\alpha^{*'}(v)|$, it is clear that $\tilde{\alpha}(v)$ must have a larger absolute slope than $\alpha^*(v)$ if $\alpha^*(v) \leq 1$, which is indeed the case for all $v \in [\hat{v}, \bar{v}]$. ■

Taken together, Lemmas 6 to 11 prove our claim regarding equilibrium product choice under the regulation and $\frac{q_H}{c_H} < \frac{q_L}{c_L}$. **Q.E.D.**

Having established Claim 1, it is evident that the below-cost pricing regulation strictly decreases efficiency (by reducing the equilibrium probability α^* of retailers choosing high quality in the prominent category) if and only if $v > \hat{v}$. Otherwise, efficiency remains unchanged.