Payment System Self-Regulation through Fee Caps

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Abstract

This paper considers the organization of a single (domestic) payment system. When card issuers that are members of a payment system set their fees individually, this gives rise to a free-riding problem, as in providing access to different customers, card issuers are complements from the perspective of each merchant. When payment systems can threaten to exclude, in particular, card issuers with a smaller customer base that do not adhere to a common cap on fees, this allows to restore the full internalization outcome, leading to lower fees but higher profits and higher welfare. When payment systems cannot threaten to exclude card issuers, the full internalization outcome arises only when card issuers are sufficiently symmetric.

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1 Introduction

Payment systems have been the focus of much regulatory scrutiny over the last decades, culminating in various antitrust cases against the large international (four-party) systems Mastercard and Visa, as well as outright price regulations in various countries.[1] For instance, the European Commission has capped so-called Multilateral Interchange Fees (MIFs) in 2015,[2] and is currently reviewing this regulation.[3]

With a focus on these international (four-party) systems and the two-sided nature of these markets, the academic literature and much of the policy debate has ignored, however, at least for Europe, the importance of national payment systems that are prevalent, though to a much lesser degree as previously, in various countries.[4] For instance, as discussed below in greater detail, the German girocard (debit) system covers the vast majority of all debit card payments and all banks, national as well as foreign, have access to this payment system. Other previous or still existing national payment systems that covered almost all (debit) card payments include the Dutch "PIN", the Finnish "Pankkikortti" or the Luxembourgish "Bancomat" network.

As I discuss below also in greater detail, for the operation of such domestic payment systems the two-sidedness is of little concern. Put differently, national payment systems are best studied from the perspective of keeping each card issuer's customer base as fixed. When customer bases are considered fixed, the focus lies on card issuers' charges vis-á-vis merchants. This is due to that a merchant typically cannot steer a customer towards another card than the one that the customer presents (e.g., the customer has only the

[1]In 2007, the European Commission found that Mastercard's interchange fees for international payment card transactions in the European Economic Area harm competition (European Union, 2009). Thereafter, in 2009, to meet the European Commission's competition concerns, Mastercard reduced its interchange fees applied by its member banks to a maximum weighted average of 0.2% for debit cards and 0.3% for credit cards (European Union, 2019b). Similarly, in 2008, the European Commission also opened formal antitrust proceedings against Visa for its debit cards, for which interchange fees were then reduced to 0.2% (European Union, 2019a).

[2]A report of the European Commission evaluating the functioning of the MIF regulation is expected by mid-2020.

[3]The potential market failure that competition between card schemes could lead to *higher* rather than *lower* fees to attract issuing banks - in contrast to the usual price depressing effect of competition - has led the European Parliament to adopt an interchange fee regulation (European Union, 2015), which from December 2015 onward capped interchange fees for cards issued and used in Europe to a maximum of 0.2% for debit cards and 0.3% for credit cards.

[4]With respect to payment systems, the literature focuses almost exclusively on the international credit card industry (see, e.g., Belleflamme and Peitz, 2015; Evans et al., 2011; Rochet and Tirole, 2003).

card issued by the bank at which he also has his current account). With respect to such "single-homing" customers, the card issuer thus becomes a monopolistic supplier vis-á-vis merchants with respect to his customer base. It is this situation on which the analysis in this paper focuses and for which I obtain the following set of results.

I show first that card issuers offer complementary services to a particular merchant, implying that a higher merchant fee charged by one card issuer reduces the demand also for all other card issuers. I derive this result for the case where a merchant must decide in general whether to invest in card acceptance, but I also delineate other decisions of the merchant, such as a maximum payment for card acceptance. From this follows my second result, namely that in equilibrium fees are too high within a single payment system with card issuers having the smallest customer base charging the highest fees. Thus, smaller card issuers, in particular, free-ride on the lower fees charged by larger card issuers.

The next set of results concerns the key topic of this paper: the possible introduction of a self-regulatory cap by card issuers. It is well known that with complements, firm profits and welfare would both be higher when all prices were set by a single monopolistic firm. In this case, free-riding is absent and all benefits from reducing fees are internalized. I refer to this as the "full internalization outcome". Then, I ask whether card issuers can jointly achieve a welfare improvement, possibly up to the full internalization outcome, with the instrument of setting a voluntary fee cap. To answer this question, I distinguish between two cases. In the first scenario, card issuers cannot threaten to exclude other card issuers from the joint payment system in case of non-compliance with the cap, notably as competition authorities may consider this to be unlawful. In the second scenario, such a threat is feasible and credible by any group of card issuers. Absent such, potentially unwarranted, intervention by competition authorities, I find that the full internalization outcome can indeed be realized. Under the threat of being excluded from the (large) payment system, meaning that the excluded member must operate a separate payment system, all card issuers agree to the full internalization fee. However, when notably large card issuers cannot threaten to exclude small card issuers and when customer base asymmetries are too large, the full internalization outcome can no longer be achieved. While there is still a single payment system in equilibrium, only sufficiently large card issuers adhere to a cap, while smaller card issuers set a strictly higher fee, which further increases as a card issuer becomes smaller.

Related Literature. The results in this paper relate to the empirical findings of Valverde et al. (2016). They examine the effects of the Spanish interchange fee regulations between 1997 and 2007 on the usage of debit and credit cards.[5] Their key finding is that, even though usage fees of credit and debit decreased due to these regulations, card issuers' revenues increased due to an even larger increase in the number of transactions. As in my baseline scenario when self-regulation is not feasible, this points to an initial free-riding problem among Spanish card issuers: while jointly they would have benefitted from a reduction in fees, individually they lacked the incentives to lower fees. While there regulation lead to lower fees in case of the analyzed domestic payment scheme, card issuers may solve or at least mitigate the free-riding problem by jointly determining a capped fee that is lower than what would arise when fees were set individually. Faced with such uniformly or at least widely applied capped fees, merchants have much higher incentives to adopt card payments and foster their usage by customers.

The payment network literature has largely neglected this issue, with a focus on potential competition between the large international (four-party) payment systems Mastercard and Visa. This obviously limits the applicability to environments where card issuers cooperate within a domestic payment system. The identified free-riding problem among card issuers operating within the same payment system and its analysis are clearly closely related to the problem faced by "competition" with complements. The present analysis bears also some resemblance to that of "standard-essential patents" in Lerner and Tirole (2015). There, complementarities arise in the case of patent pools. Once such a patent pool has been created, when a user wants to adopt the specific technology (e.g., 4G with mobile phones), he essentially has to adopt the patent of each member of the pool and has to pay the corresponding royalties. The adoption thus hinges on the total royalties that a user must pay, leading to the same internalization problem as in the considered case of the adoption of a payment system by a merchant.[6]

[5]For instance, interchange fees for credits cards in Spain were reduced step-by-step from 3.50% in 1997 to 1.50% in 2007.

[6]Typically, the literature, such as the seminal paper by Lerner and Tirole (2015), considers here various other stages, such as the definition of a "standard" through users, that complement the formation of patent pools, in which owners of patents can then jointly set royalties for individual licensing agreements. The literature, however, does not focus on caps and in that respect does not analyze the difference between exclusion and non-exclusion.

Organization. This paper proceeds as follows. After setting up the model in Section 2, I analyze the case without self-regulation in Section 3. Subsequently, self-regulation is approached from two perspectives: Section 4 examines the scenario when card issuers cannot be excluded from a joint payment system, while Section 5 assumes that exclusion is feasible and points out its implications. Finally, Section 6 closes the paper with some concluding remarks. Additional material is provided in Appendix A.

2 The Model

Institutional Background. The model is motivated by the examples of national payment systems in Europe. To be specific, consider the case of the German girocard system which is jointly operated by and open to all banks. A girocard is almost always part of the current account service package that banks offer.[7] While by now all larger retailers accept the girocard, this was not the case till the mid 2000s when the card was first adopted, in particular, by larger food retailers. The subsequent model captures such a first-adoption decision by merchants. Even in 2020, however, many smaller retailers have not adopted card payments.[8] Moreover, the subsequently modeled decision whether to accept card payments may relate to the setting of a particular threshold from which on card payments are profitable for the particular merchant. In addition, recall from the Introduction the other examples of domestic card payment schemes in Europe.

With these particular cases in mind, I consider in what follows a situation where each bank (card issuer) is in a monopolistic (gatekeeper) situation with respect to its customers (card holders). I thus explicitly exclude any interaction between banks on the card issuing side of the market, focusing exclusively on the interaction with merchants.

Players. I analyze a setting with I card issuers (henceforth abbreviated as issuers) indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$. For simplicity, I consider on the other side of the market a

[7]In Germany, for instance, 98% of all persons have at least one debit card, while only around 11% have more than one debit card (Deutsche Bundesbank, 2018). For that reason, when a card is not accepted by a merchant, it is very unlikely that the payment can still be made by means of an alternative debit card. Moreover, other payment cards are not common. More precisely, only 1.5% of all transactions were carried out by credit card in 2017 (Deutsche Bundesbank, 2018).

[8]These smaller retailers include bakeries, kiosks or pharmacies. For instance, it was estimated that still 71% of all revenue of (in Germany typically owner-lead) pharmacies was made with cash in Germany in 2017 (Deutsche Bundesbank, 2018).

single merchant and suppose that in the examined time period each customer wishes to undertake one interaction with the merchant. Letting v_i denote the customer base of issuer i, the number of possible transactions is thus given by $v = \sum_{i \in \mathcal{I}} v_i$. When a customer pays by card instead of cash, I stipulate that this saves the merchant the costs $s > 0$. Alternatively, when the merchant does not accept the customer's card, then $s > 0$ may also represent the expected lost margin, as the customer may then, with some probability, refuse to undertake the transaction.

In general, any subgroup of issuers can operate a card payment system. The adoption of any card payment system comes at costs k to the merchant. While in equilibrium, as I show in Section 4, there will be a single payment system to which all issuers belong, in principle, multiple payment systems could form and the merchant may then decide whether to invest in the acceptance of only one or of more than one card. The cost of such investment is the merchant's private information with $k \in [0, K]$ distributed according to the the cumulative distribution function $H(k)$ with density $h(k) > 0$ in the interior of the support. A particular example that I consider throughout the analysis is that of an uniform distribution with $h(k) = 1/K$ and $H(k) = k/K$.

When the merchant decides to adopt a particular payment system to which issuer i belongs, issuer i sets the transaction fee p_i. Note that my pricing strategy ignores the possibility of charging membership fees to customers. Importantly, in my model, no further payments or transfers are allowed, neither between issuers nor between issuers and the merchant. A transaction comes at cost c_i to the issuer, where I presently assume that $c_i = c < s$ for all issuers. Moreover, I stipulate that $p_i \leq s$, as otherwise the merchant would simply refuse acceptance of cards from issuer i.

Game. Throughout the analysis I consider the following sequence of moves. In $t = 1$, issuers decide whether to join a particular payment system and, as described in more detail below, whether to thereby accept certain common rules, notably a cap on fees. In $t = 2$, issuers set their fees and, subsequently, in $t = 3$, the merchant decides which payment systems to adopt, where each adoption comes at cost k. Finally, in $t = 4$, customers arrive at the merchant and pay either with cash or card.

While steps $t = 2$ to $t = 4$ are immediate, I flesh out the details of step $t = 1$. The subsequent analysis proceeds as follows. I first simply suppose that there is a single payment system and derive the monopoly outcome (of full internalization), which I compare to the equilibrium fee structure with independent issuers. The main part of my analysis then examines a potential agreement to cap on fees and the implications of allowing exclusion.

3 Equilibrium Fees within a Single Payment System

3.1 Optimal Monopoly Fee

Though I am interested in deriving the equilibrium when issuers jointly operate a single payment system and do not impose further rules, an analysis of the monopoly outcome is helpful as a benchmark.

Obviously, a monopolist will only set up a single payment system. Given cost savings $s > 0$ and provided that the merchant has made the respective investment at cost k, the merchant will accept any card payment when $p_i = p \leq s$. The merchant's investment is optimal if

$$k \leq v(s-p),$$

so that $H(v(s-p))$ denotes the probability that the merchant adopts the payment system. The optimal fee of the monopolistic card issuer, p^{FI}, thus maximizes expected profits

$$\pi = v(p-c)H\left(v(s-p)\right).$$

Note that the superscript *FI* refers to the fact that the monopolist obviously fully internalizes profits from all accepted card transactions. Given the support $[0, K]$ and $h(k) > 0$ in the interior of the support, the optimal fee must be interior and solve the first-order condition $d\pi/dp = 0$:

$$v(p^{FI} - c) = \frac{H\left(v(s-p^{FI})\right)}{h\left(v(s-p^{FI})\right)}. \tag{1}$$

Throughout, I restrict welfare comparison to the joint surplus of issuers and the merchant. Hence, I exclude possible benefits from card payments that accrue to final customers. With this restriction, total welfare in the full internalization (monopoly) equilibrium is thus

$$W^{FI} = \left[v(s-c) - k\right] H\left(v(s-p^{FI})\right).$$ [9]

[9] This is net of other surplus, notably that accruing to the merchant from the considered transactions (irrespective of the respective means of payment). Moreover, it should be noted that the socially optimal fee corresponds to c, simply covering the issuers' marginal costs.

Uniformly Distributed Costs. When the merchant's adoption costs $k \in [0, K]$ are uniformly distributed, I obtain the following closed-form solutions: $h(k) = 1/K$ and $H(k) = k/K$. Applying these properties to the first-order condition (1) implies that the optimal markup in the full internalization (monopoly) case is

$$p^{FI} - c = \frac{s-c}{2},$$

so that in equilibrium the monopolist earns

$$\pi^{FI} = \frac{1}{4K} v^2 (s-c)^2,$$

and total welfare is

$$W^{FI} = \frac{1}{2K} v(s-c) \left[v(s-c) - k \right].$$

3.2 Equilibrium Fees with Independent Card Issuers

Now, I suppose that all I issuers are party to the same payment system, which the merchant adopts when the respective costs k are sufficiently low. With potentially different fees, p_i, the respective decision criterion becomes

$$k \leq \sum_{i \in \mathcal{I}} v_i (s - p_i),$$

so that there is adoption with likelihood $H\left(\sum_{i \in \mathcal{I}} v_i (s - p_i)\right)$. Each issuer i chooses thus the individually optimal fee, p_i^*, that maximizes expected profits

$$\pi_i = v_i (p_i - c) H \left(\sum_{j \in \mathcal{I}} v_j (s - p_j) \right),$$

where the asterisk labels the situation when fees are set individually optimal.

For notational convenience, I occasionally denote the merchant's expected gross benefits by $B = \sum_{j \in \mathcal{I}} v_j (s - p_j)$ and the total fees that the merchant has to pay by $P = \sum_{j \in \mathcal{I}} v_j p_j$. In the case that the individually optimal fee is interior, it follows from $d\pi_i / dp_i = 0$ that

$$v_i (p_i^* - c) = \frac{H(B)}{h(B)} =: r(B). \tag{2}$$

Accordingly, all issuers for which the optimal fee is determined by the first-order condition (2) will make in equilibrium the same profits, irrespective of the transaction volume. The respective first-order condition obviously applies only when this leads to $p_i^* \leq s$, as otherwise, that is for higher values of the fee, the merchant would simply exclude cards from the respective issuers. For the further formal analysis, I assume additionally that $r(\cdot)$ is strictly increasing. Then, I obtain the following result:

Lemma 1 *Consider the equilibrium outcome when all issuers participate in the same payment system and fees are set so as to maximize individual profits. Holding all other equilibrium fees $p_j^* \leq s$ fixed, the optimal choice for some card issuer i is unique and characterized as follows. When*

$$r\left(\sum_{j\neq i} v_j(s-p_j^*)\right) - v_i(s-c) \geq 0,$$

it holds that $p_i^* = s$, while otherwise $p_i^* < s$ is uniquely determined by the first-order condition (2). This implies that two issuers i, j for which the respective first-order condition holds realize the same profits and set fees inversely proportional to their customer base: $(p_i^* - c)/(p_j^* - c) = v_j/v_i$.

Proof. Holding all other fees constant, I consider π_i as a function of p_i. Then, I rewrite the derivative of π_i with respect to p_i as follows:

$$\frac{d\pi_i}{dp_i} = v_i h(B)\left[r(B) - v_i(p_i - c)\right].$$

As $r(B)$ is strictly increasing in its argument and B strictly decreasing in p_i, the term $r(B) - v_i(p_i - c)$ is strictly decreasing. From this, it follows that π_i is strictly quasi-concave so that there are two cases to be considered. In the first case, I obtain a unique interior solution where p_i^* is determined by the first-order condition (2) (i.e., $d\pi_i/dp_i|_{p_i=s} < 0$). In the second case, there is a corner solution with $p_i^* = s$ (i.e., $d\pi_i/dp_i|_{p_i=s} \geq 0$), for which I recall that for $p_i > s$ the merchant would refuse acceptance of cards from issuer i. From strict quasi-concavity, I can also conclude that exactly one of the two cases applies and that the corner solution with $p_i^* = s$ applies if and only if $r(B) - v_i(s-c) \geq 0$. ■

To conclude this section, I additionally point out the following:

Lemma 2 *When all issuers participate in the same payment system, individually set fees are strategic substitutes.*

Proof. To see this, note first that p_i^* clearly only changes following the (anticipated) adjustment of another issuer's fee p_j^* when p_i^* is interior with $p_i^* < s$. Applying the implicit function theorem to the first-order condition (2) and using that the optimal fee is always uniquely determined, I obtain:

$$\frac{dp_i^*}{dp_j^*} = -\frac{1}{SOC}\left[r'(B)\frac{dB}{dp_j^*}\right] < 0,$$

where SOC stands for second-order condition. The negative sign follows the fact that $SOC < 0$, $r'(B) > 0$ and $dB/dp_j^* = -v_j < 0$. ■

For that reason, card transactions of different issuers are complementary services, implying that the corresponding (Cournot) equilibrium is inefficient. To be more specific, each issuer acts as a monopolist with respect to his market share, however, ignores that lowering his own fee increases the likelihood of acceptance of the joint payment system and thus positively affects all issuers' profits. This "positive externality" causes the equilibrium to entail inefficiently high fees and lower expected joint profits as compared to the full internalization equilibrium. As higher total fees obviously also harm the merchant, a non-cooperative setting of fees deteriorates welfare (i.e., $W^{FI} > W^*$).

3.2.1 The Case with Sufficiently Symmetric Card Issuers

In the case of an interior solution for all $i \in \mathcal{I}$, all issuers' first-order conditions from (2) hold and thus must sum up as follows:

$$\sum_{i \in I} v_i(p_i^* - c) = Ir\left(\sum_{j \in I} v_j(s - p_j^*)\right), \tag{3}$$

which can be can be rewritten with the alternative notation as

$$P = vc + Ir(vs - P),$$

and implicitly defines the equilibrium fee level, P. Since $r(\cdot)$ is strictly increasing in its argument and thus strictly decreasing in P, an unique equilibrium fee level exists. This leads me to the following intuitive results:

Lemma 3 *Consider the equilibrium outcome when all issuers participate in the same payment system and fees are set so as to maximize individual profits. Suppose that for each* *issuer p_i^* is determined by the respective first-order condition (2). This case applies when it holds for all $i \in \mathcal{I}$ that*

$$v_i > \frac{r\,(vs - P)}{s - c}. \tag{4}$$

Then, total fees paid by the merchant if he invests in adoption of the payment system uniquely solve

$$P = vc + Ir(vs - P) \tag{5}$$

and are characterized as follows: total fees are strictly increasing in the number of issuers I and are always strictly higher than the full internalization fee level vp^{FI}.

Proof. First, note that I obtain condition (4) by evaluating the derivative of π_i with respect to p_i at $p_i = s$ and the fact that π_i is strictly quasi-concave.

Next, let me prove that $dP/dI > 0$. Applying the implicit function theorem to equation (5) yields

$$\frac{dP}{dI} = \frac{r(vs - P)}{1 + Ir'(vs - P)} > 0.$$

As $r(\cdot)$ is strictly increasing in its argument and non-negative as $vs \geq P$, it follows that total fees are strictly increasing in the number of issuers. ■

The intuition behind this result is straightforward: as the number of issuers increases, each issuer sees its influence on the total fee level diminish and is therefore more willing to increase its fee. As a result, total fees increase and inefficiencies intensify with the number of issuers.

Uniformly Distributed Costs. When the merchant's adoption costs are uniformly distributed, I can explicitly determine the equilibrium fee level as $r(B) = B$. With this specification, equilibrium fees sum up to

$$P = \frac{v(Is + c)}{I + 1},$$

and are on average (i.e., per transaction) equal to

$$p_{\varnothing} = \frac{Is + c}{I + 1},$$

which implies that $p_{\varnothing} > p^{FI}$ when $I \geq 2$. Then, the first-order condition (2) simplifies to

$$v_i(p_i^* - c) = vs - P,$$

which leads me to the following optimal markup

$$p_i^* - c = \left(\frac{v_i}{v}\right)^{-1} \frac{s-c}{I+1}. \tag{6}$$

This pricing formula indicates that issuers set equilibrium fees inversely proportional to their market shares: smaller issuers with less customers choose higher fees as compared to larger issuers. Note that all issuers' fees are interior and determined by the price-cost margin (6) when issuers are sufficiently symmetric. That is, when it holds for all $i \in \mathcal{I}$ that

$$\frac{v_i}{v} > \frac{1}{I+1}.$$

Then, the merchant is left with an average benefit (gross of investment) of

$$s - p_{\varnothing} = \frac{s-c}{I+1}, \tag{7}$$

whereas issuers receive on average

$$p_{\varnothing} - c = \frac{I(s-c)}{I+1}. \tag{8}$$

Obviously, equation (7) is strictly decreasing while equation (8) is strictly increasing in the number of issuers I. Importantly, however, as the number of issuers increases, the likelihood of acceptance decreases due to a higher total fee level. From the full internalization equilibrium, I know that issuers are then jointly strictly worse off. Hence, as I increases, the merchant's profits and issuers' joint profits decrease, and so does welfare (i.e., $W^{FI} > W^*$). Thus, I obtain the following result in case of a uniform distribution:

Lemma 4 *Consider the equilibrium outcome when all issuers participate in the same payment system and fees are set so as to maximize individual profits. When for each p_i^* the respective first-order condition (2) applies, I obtain in the case that $H(\cdot)$ is uniformly distributed the following characterization:*

$$p_i^* - c = \left(\frac{v_i}{v}\right)^{-1} \frac{s-c}{I+1}, \tag{9}$$

with

$$p_{\varnothing} = \frac{Is+c}{I+1}.$$

The case of an interior solution for all p_i^* applies when issuers are sufficiently symmetric in customer bases, that is for all $i \in \mathcal{I}$

$$\frac{v_i}{v} > \frac{1}{I+1}.$$

3.2.2 Full Characterization

In the preceding analysis, I focused on the case when there is an interior solution for all p_i^* so that all issuers set $p_i^* < s$. This is the case when issuers are sufficiently symmetric in customer bases. Otherwise, it may also be the case that at least one issuer finds it optimal to set $p_i^* = s$. This leads me to the following more general characterization:

Proposition 1 *Suppose that all issuers participate in the same payment system and that fees are set so as to maximize individual profits. There is a unique equilibrium which is characterized as follows. Suppose that issuers are ordered such that for all $i \geq j$ and it holds that $v_i \geq v_j$. Then there exists a threshold issuer i_T so that issuers $i \leq i_T$ set $p_i^* = s$, while all issuers $i > i_T$ set $p_i^* < s$ satisfying the first-order condition (2), which implies that for any two issuers $i > i_T$, $j > i_T$ it holds that $(p_i^* - c)/(p_j^* - c) = v_j/v_i$. Consequently, fees are decreasing in the size of an issuer's customer base, and strictly so for all $i > i_T$. Total fees paid by the merchant in case of investment in the payment system are always strictly higher than in the full internalization (monopoly) case.*

Thus, I cannot claim anymore that total fees are strictly increasing in I. This is due to the corner solutions, as splitting the customer base of small issuers with already $p_i^* = s$ even further has no impact on the total fee level. Lastly, note that now the sum of all paid fees does not only depend on I but also on the distribution of customers between issuers.

Uniformly Distributed Costs and Two Issuers. When there are only two issuers $\mathcal{I} = \{1, 2\}$ and $H(\cdot)$ is uniformly distributed, there is an interior solution for p_1 and p_2 if and only if market shares are sufficiently symmetric. More precisely, this situation applies when market shares of both issuers are strictly larger than 1/3, for which it follows from Lemma 4 that

$$p_i^* - c = \left(\frac{v_i}{v}\right)^{-1} \frac{s - c}{3}.^{10}$$

If this is the case, equilibrium profits are for both issuers identical as

$$\pi_i^* = \frac{1}{9K} v^2 (s - c)^2.$$

[10] Interestingly, both issuers set inefficiently high fees (i.e., $p_i^* > p^{FI}$), suggesting that not only smaller issuers "free-ride".

Suppose now that instead $v_1/v \leq 1/3$ so that a corner solution arises. Then, the small issuer will set $p_1^* = s$, while the profit-maximizing fee of the large issuer solves

$$\arg\max_{p_2} \quad v_2(p_2 - c)H(v_2(s - p_2)),$$

being equal to the monopolist's problem with "market size" v_2. This implies that

$$p_2^* - c = \frac{s - c}{2},$$

which constitutes an equilibrium where the small issuer realizes higher expected profits (i.e., $\pi_1^* > \pi_2^*$) as

$$\frac{1}{2K} v_1 v_2 (s - c)^2 > \frac{1}{4K} v_2^2 (s - c)^2.$$

Note that the small issuer earns then twice as much profits per customer as the large issuer since $\pi_1^*/v_1 = 2(\pi_2^*/v_2)$. Overall, however, expected joint profits are strictly higher in the full internalization outcome (i.e., $\pi_1^* + \pi_2^* < \pi^{FI}$). This holds independent of whether the interior solution applies as

$$\frac{1}{2K} v(s - c) \left[v(s - c) - k \right] > \frac{1}{3K} v(s - c) \left[v(s - c) - k \right],$$

or the corner solution applies as

$$\frac{1}{2K} v(s - c) \left[v(s - c) - k \right] > \frac{1}{2K} v_2(s - c) \left[v(s - c) - k \right].$$

Recall that the total fee level is strictly higher with independent card issuers, and this has a negative effect on the expected profits of the merchant. Thus, both sides of the market, issuers and the merchant, are strictly worse off so that total welfare decreases (i.e., $W^{FI} > W^*$) when fees are set independently. From the case just analyzed, I derive the following results:

Proposition 2 *Consider the equilibrium outcome when two issuers $i \in \mathcal{I} = \{1, 2\}$ participate in the same payment system and fees are set so as to maximize individual profits. When $v_2 \geq v_1$ and $H(\cdot)$ is uniformly distributed, I obtain the following full characterization:*

$$p_i^* - c = \left(\frac{v_i}{v} \right)^{-1} \frac{s - c}{3},$$

when issuers are sufficiently symmetric with

$$\frac{v_1}{v} > \frac{1}{3},$$

while otherwise

$$p_1^* = s,$$
$$p_2^* = p^{FI}.$$

Expected joint profits and welfare are always strictly higher in the full internalization equilibrium as compared to the case when fees are set independently.

4 Self-Regulation without Exclusion

4.1 Preliminary Results

As shown in the preceding sections, the monopoly outcome (of full internalization) leads to higher welfare than the outcome when fees are set independently. This suggests that in my setting any increase in market concentration raises aggregate surplus. Importantly, however, I assume that mergers between issuers are not possible. This is motivated by the fact that issuers also compete in other markets (e.g., in the credit market), where mergers tend to raise prices. For that reason, it seems reasonable that antitrust authorities consider horizontal mergers between issuers to be harmful. Nonetheless, in what follows, I propose an alternative mechanism that resolves the free-riding problem, that is a (binding) self-regulatory fee cap.

On a more general level, I consider the possibility that issuers can try to increase profits (and potentially also welfare) by self-regulating their fees. To be more precise, I allow for the following strategy: issuers belonging to the same payment scheme can sign a contract that commits them not to set fees above a certain common level, p^{SR}, where the superscript stands for "self-regulation".[11]

Generally, the respective game that determines which payment systems issuers join, whether they potentially agree to such a cap and how this cap is determined may become quite complex. Moreover, there may be various ways how to determine the "protocol" of the determination of such a cap within a given payment system. As it turns out, however, one can determine the outcome of any such game without having to specify the details. This holds for the following reasons.

[11] Crucially, I do not allow for "side payments". This has two implications: (i) issuers and the merchant cannot attain the Coasian equilibrium in which fees are equal to marginal costs, and (ii) issuers cannot collude and maximize joint profits.

The first key reason is that presently I do not allow issuers to exclude any other issuer from joining a payment system. Ex-post issuers will indeed want to integrate any issuer who wants to join (and strictly so when $p_i^* < s$) because it increases the likelihood that the merchant adopts the payment system. What is then also immediate is that again only a single payment system forms, as well as that ultimately all issuers join this payment system, which is then adopted or not by the particular merchant.

With a single payment system, there is a single cap, p^{SR}, and a subset of issuers $\mathcal{I}_{SR} \subseteq \mathcal{I}$ jointly sign a contract to adhere to this cap, while issuers $i \notin \mathcal{I}_{SR}$ do not. The reason why I can further dispose of an elaborate "protocol" to determine p^{SR} is that all $i \in \mathcal{I}_{SR}$ have exactly the same preference over which (binding) cap to set, irrespective of their size v_i. To see this, note that for all $i \in \mathcal{I}_{SR}$ expected profits are

$$\pi_i = v_i(p^{SR} - c)H\left(vs - P\right), \tag{10}$$

whereas all $i \notin \mathcal{I}_{SR}$ choose their individually optimal fees, p_i^*, maximizing

$$\pi_i = v_i(p_i - c)H\left(vs - P\right).$$

Then, with a voluntary fee cap, total fees paid by the merchant can be rewritten as

$$P = p^{SR} \sum_{i \in \mathcal{I}_{SR}} v_i + \sum_{i \notin \mathcal{I}_{SR}} v_i p_i^*. \tag{11}$$

It should be stressed that for all $i \in \mathcal{I}_{SR}$ expected profits are given by equation (10), and as this only differs with respect to the multiplicative factor v_i, I have indeed:

Lemma 5 *Consider the case of self-regulation without exclusion. All issuers that adhere to the maximum fee p^{SR}, $i \in \mathcal{I}_{SR}$, have the same preference over which level of p^{SR} to set.*

So far, I have not specified how $\mathcal{I}_{SR}$ is actually formed. For the sake of simplicity, I assume that each issuer decides unilaterally whether to join or not, and once joined, all prefer the same cap. To ensure that I however solve a properly specified game, I stipulate that any member $i \in \mathcal{I}_{SR}$ is randomly chosen to dictate p^{SR}.

While all $i \in \mathcal{I}_{SR}$ thus set p^{SR}, all $i \notin \mathcal{I}_{SR}$ charge individually optimal fees. These issuers outside the self-regulation coalition set p_i^* in the knowledge of which other issuers have joined the coalition. It still needs to be specified, however, whether their choice p_i^* is made simultaneously to that of p^{SR} or whether the latter precedes (and self-regulation acts as a “Stackelberg leader”). While the subsequent insights do not depend on this, to stay close to the game without self-regulation, I assume that all fees, including the voluntary cap, are set simultaneously.

4.2 Equilibrium

For those who apply the cap, the chosen fee level p^{SR} maximizes equation (10). This is clearly the same choice as that made by an issuer with volume $v_{SR} = \sum_{i \in \mathcal{I}_{SR}} v_i$. Consequently, once $\mathcal{I}_{SR}$ is determined, the outcome is then the same as that without self-regulation but where $\mathcal{I}_{SR}$ is represented by a single issuer.

Lemma 6 *Consider the case of self-regulation without exclusion and suppose that a subset of issuers $\mathcal{I}_{SR} \subseteq \mathcal{I}$ agrees to a cap. Then, the unique fee levels, that is of $p_i = p^{SR}$ for all $i \in \mathcal{I}_{SR}$ and of all independently chosen p_j for all $j \notin \mathcal{I}_{SR}$, are equal to those without regulation but where $\mathcal{I}_{SR}$ (and thus v_{SR}) is represented by a single issuer.*

A first observation is that the equilibrium may not be unique. In fact, though this may seem at first intuitive given that $v_{i+1} \geq v_i$, the set of issuers that apply a cap may not be convex, i.e., $\mathcal{I}_{SR} = \{i_{SR}, ..., I\}$ may not always hold. To see this, take some i and $j = i + 1$. Suppose first that $v_j = v_i$. Now, suppose that $\mathcal{I}_{SR} = \{i_{SR}, ..., I\}$ and that $i_{SR} = j$. Given that all issuers expect that j adheres to a cap, it can indeed be strictly for i not to do so, while at the same time it is strictly optimal for j to declare that he will accept a cap. Intuitively, the key difference between the two decision problems is that from i's perspective already one issuer more will stick to the cap, namely $j = i + 1$. From this observation it then follows that when $v_j < v_i$, while the difference stays small, it can indeed be that j accepts a cap while i does not.

The stage game where issuers have to decide whether to agree to a cap bears a close resemblance to takeover games with strategic substitutes (as typically the case with Cournot competition). As shown above, the formation of an agreement to a cap is equivalent to a merger of the respective issuers. The (takeover) literature has pointed both to the free-riding problem by outsiders and to the potential multiplicity of equilibria.[12]

[12]See, e.g., Salant et al. (1983) or Kamien and Zang (1990).

Uniformly Distributed Costs and Two Issuers. As before, I assume that there are two issuers $i \in \mathcal{I} = \{1, 2\}$, which now have to decide whether to participate in self-regulation and thereby accept a fee cap, p^{SR}, or to set p_i individually. In the special case with two issuers, I have to distinguish thus only between two scenarios: (i) no self-regulation; (ii) complete self-regulation (i.e, $\mathcal{I}_{SR} = \mathcal{I}$).

In the first case, when there is no self-regulation at all, the outcome is simply characterized as outlined in Proposition 2. In the second case, when all issuers participate in self-regulation, expected profits of both issuers are

$$\pi_i^{SR} = v_i(p^{SR} - c)H(v(s - p^{SR})).$$

Recall that p^{SR} is independent of which issuer is randomly chosen to dictate the payment cap. Therefore, the optimal cap is unique and characterized by the first-order condition $d\pi_i^{SR}/dp^{SR} = 0$:

$$v(p^{SR} - c) = r(v(s - p^{SR})).$$

As the merchant's adoption costs are assumed to be uniformly distributed, I can explicitly derive the equilibrium fee cap:

$$p^{SR} - c = \frac{s - c}{2},$$

so that indeed $p^{SR} = p^{FI}$. Then, equilibrium profits are

$$\pi_i^{SR} = \frac{1}{4K} v_i v(s - c)^2.$$

implying that expected aggregate profits are equal to those in the full internalization equilibrium (i.e, $\pi_1^{SR} + \pi_2^{SR} = \pi^{FI}$).

Nevertheless, I still need to check whether both issuers are better off in the case of self-regulation as compared to the unregulated case. Importantly, when an issuer deviates and does not agree to accept the cap, then, in the subsequent stage, both issuers, not only the deviating issuer, will freely choose the individually optimal fee. Keeping the previous assumption that $v_2 \geq v_1$, I have to consider again two possible equilibria: an interior solution (i.e., $p_i^* < s$) and a corner solution (i.e., $p_1^* = s$ and $p_2^* < s$). If the interior solution applies, issuer i strictly prefers a cap (i.e., $\pi_i^{SR} > \pi_i^*$) if

$$\frac{v_i}{v} > \frac{4}{9}.$$

In contrast, if the corner solution applies, the larger issuer always benefits from the cap (i.e., $\pi_2^{SR} > \pi_2^*$) since

$$\frac{1}{4K} v_2 v (s-c)^2 > \frac{1}{4}\frac{1}{K} v_2^2 (s-c)^2,$$

whereas the smaller issuer has no incentive to participate (i.e., $\pi_1^{SR} < \pi_1^*$) as $v_1/v \leq 1/3$ and thus $v_2/v \geq 2/3$ imply that

$$\frac{1}{2}\frac{1}{K} v_1 v_2 (s-c)^2 > \frac{1}{4}\frac{1}{K} v_1 v (s-c)^2.$$

In general, I focus on Pareto dominant equilibria. That is, when one equilibrium is better for all issuers than another, and strictly so for one, then the Pareto dominant equilibrium is selected. Otherwise, one would obviously always have the equilibrium where no issuer chooses to adhere to a cap, simply as he expects nobody else to do so as well. Thus, I obtain:

Proposition 3 *Consider the case of self-regulation without exclusion and suppose that there are only two issuers and that the merchant's adoption costs are uniformly distributed. Then, there is a unique (Pareto dominant) equilibrium, characterized as follows:*

i) When issuers are sufficiently symmetric with $v_i/v > 4/9$, both issuers participate in self-regulation and the corresponding maximum fee is equal to the full internalization level p^{FI}.

ii) Otherwise, self-reguation is not feasible and the outcome is as characterized in Proposition 2 so that total fees strictly exceed those under full internalization.

5 Self-Regulation with the Threat of Exclusion

5.1 Preliminary Discussion

I already noted that even when a set of issuers had threatened to exclude another issuer in case he would set an excessively high fee, such exclusion may not be credible: ex-post all issuers are better off when another issuer joins the payment system (and strictly so when $p_i^* < s$). This follows immediately from the fact that their offerings are complementary from the merchant's perspective. Still, in a realistic, long-run relationship issuers may be able to uphold such a threat of exclusion, as long as they are permitted to do so by competition authorities. Despite the fact that, under the model's assumptions, issuers are not competitors but instead complementary in their services, competition authorities may not be open to such distinctions and may thus oppose, in particular, the exclusion of a smaller issuer, i.e., of an issuer with a smaller customer base. A key result of this section is that such an antitrust policy will typically increase average fees and reduce welfare.

While I keep the timing of the overall game as introduced in the preceding section, the various decisions that issuers can make, i.e., which payment system to join, whether to accept a cap, which cap to choose and whether to exclude a subset of issuers from the payment system, can easily blur the economic analysis. Recall however that, as already discussed in the case without exclusion, a larger payment system is always beneficial to all participants and when a fee cap is chosen, all participants agree on the same level. Also, when exclusion is only an out-of-equilibrium threat, it does not decrease efficiency but may enhance stability of a mutually beneficial agreement. These observations would suggest that once again the particular "protocol" of the game has little impact on the outcome. However, with exclusion the following conflict of interests arises.

Recall that in the preceding game without exclusion, I specified that a subset of issuers within a payment system could first commit to adhere to a common fee level, which was then set by an arbitrary member of this subset, given that preferences within this subset are always identical. All issuers belonging to the payment system benefited from such a cap, including those who did not agree to adhere to it. With exclusion this is clearly different if non-adherence to such a cap means that an issuer is credibly excluded from the payment system. Consequently, there is now a new conflict of interests as, in particular, with rather asymmetric customer bases, some issuers may wish to enforce a cap with the threat of exclusion, while other issuers are better off without any self-regulation. This is why the specified sequence of moves may now matter.

To first abstract from this, I consider a situation where a single payment system has formed with the given rule that issuers who do not adhere to the commonly specified cap must exit the payment system. Again, as previously, a member of the payment system is chosen randomly to set the cap.

5.2 Stability of the Full Internalization Outcome with the Threat of Exclusion

Suppose, as described above, that a single payment system forms. If the subset $\mathcal{I}_{SR}$ adheres to the rule, given that any other issuers $i \notin \mathcal{I}_{SR}$ are now excluded, the optimal fee cap for any of the issuers in $\mathcal{I}_{SR}$ solves the full internalization problem, once the set of participating issuers is constrained to $\mathcal{I}_{SR}$. Suppose now that, starting from the "grand coalition" with $\mathcal{I}_{SR} = \mathcal{I}$, only one issuer i decides to be excluded so that now $I-1$ issuers apply a common cap.

Obviously, for issuer i expected profits from participating in the payment system are

$$\pi_i^{SR} = v_i \arg\max_{p^{SR}} (p^{SR} - c) H\left(v(s - p^{SR})\right).$$

which strictly exceed expected profits from non-participation

$$\pi_i^{O} = v_i \arg\max_{p_i^{O}} (p_i^{O} - c) H\left(v_i(s - p_i^{O})\right),$$

where the superscript O labels the "outside option" of setting up an individual payment system.[13] From this follows:

Proposition 4 *If issuers who do not adhere to a fee cap (self-regulation) can be excluded from a payment system, then there is an equilibrium in which a single payment systems forms to which all issuers belong and where the common fee level (and thus also total fees paid by the merchant) correspond to the full internalization outcome.*

Thus, the full internalization outcome is stable as, given the threat of exclusion, no issuer indeed wants to deviate.

Uniformly Distributed Costs and Two Issuers. Suppose that there are two issuers so that $\mathcal{I} = \{1, 2\}$ and $v_2 > v_1$. Then, the small issuer strictly prefers the unregulated to the self-regulated outcome if customer bases are sufficiently asymmetric. More precisely, this holds when $v_1/v \leq 4/9$.

Suppose next that the large issuer can exclude the small issuer from the joint payment system. By consequence, both issuers must set up their own payment system in which

[13] Formally, this can easily be seen by taking the maximizer of π_i^O, p_i^O, and noting that obviously $(p_i^O - c)H\left(v(s - p_i^O)\right) > (p_i^O - c)H\left(v_i(s - p_i^O)\right)$ as $v > v_i$.

they act as a monopolist, indicating that expected profits are then

$$\pi_i^O = \frac{1}{4K} v_i^2 (s-c)^2.$$

It should be noted that joint profits with two payment systems are strictly lower than those in the fully integrated single payment system (i.e., $\pi^{FI} > \pi_1^O + \pi_2^O$) because

$$\frac{1}{4K} v^2 (s-c)^2 > \frac{1}{4K} (v_1^2 + v_2^2)(s-c)^2,$$

as $(v_1 + v_2)^2 > v_1^2 + v_2^2$.

Now, given that the outside option payoff is no longer the unregulated outcome but the, surely for the small issuer, lower disagreement payoff under two payment systems (i.e., $\pi_i^{SR} > \pi_i^O$), there is always an agreement to a cap since

$$\frac{1}{4K} v_i v (s-c)^2 > \frac{1}{4K} v_i^2 (s-c)^2,$$

as $v > v_i$.

6 Concluding Remarks

Arguably, the described outcomes, where either a cap is agreed on (potentially under the threat of exclusion) or where fees are not capped, are obtained under the restriction on issuers' feasible contractual space. As already indicated, if issuers could agree at the same time on individual fee levels and on "side payments" made to or received from other issuers, then one would anticipate from the Coase Theorem that the fully integrated outcome, where all externalities from the setting of fees are internalized, is always achieved.

Alternatively, a richer contractual space may allow issuers to agree on individual (capped) fees for each participant in the payment system. Without side payments, this would amount to a non-standard (bargaining) game between potentially I players in a non-transferable utility setting, which I consider to be beyond the scope of this paper. Still, in the Appendix A, I provide the following interesting insight, albeit restricted to a particular tractable situation. Precisely, I consider the case with $I = 2$, as for most of the preceding discussion. This allows me to apply a standard Nash bargaining solution (as defined for bargainig games with two players only). In the Appendix A, I characterize the respective bargaining problem when the contractual space is (p_1, p_2) and there is mutual exclusion upon disagreement. I derive the bargaining solution and show for the explicitly solvable case with uniformly distributed costs that, despite this additional freedom, still $p_1 = p_2 = p^{FI}$ is realized as the unique outcome. Intuitively, that the full internalization outcome arises, despite a possible extreme asymmetry in market sizes, is due to the fact that the outside options are equally asymmetric. This generates the following additional insight: a single, uniform cap would thus also arise as the outcome of bilateral negotiations between issuers, again with the threat of (mutual) exclusion, but despite potentially large asymmetries in size and thus even a fundamental conflict of interests with respect to whether a cap should be applied at all (in the absence of which the smaller issuer could free-ride to the detriment of the merchant on the lower fee charged by the larger issuer).

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Appendix A Nash Bargaining over Individually Set Fees

I now consider an axiomatic Nash bargaining game with $i \in \mathcal{I} = \{1, 2\}$. The two issuers negotiate within a single payment scheme over their fees (p_1, p_2). In what follows we show that both agree under the threat of "mutual exclusion". Recall that

$$\pi_1 = v_1(p_1 - c)H(vs - v_1p_1 - v_2p_2),$$

while π_2 is defined analogously. These define the mapping $(p_1, p_2) \to (\pi_1, \pi_2)$, where I can restrict consideration to the Pareto non-dominated outcome. The resulting bargaining frontier can now be constructed as follows. Suppose that $i = 2$ must leave $i = 1$ with a certain value of $\pi_1 \geq \widetilde{\pi}_1$. The "contract" on the frontier then realizes

$$\pi_2 = \arg\max_{p_1,p_2} v_2(p_2 - c_2)H(vs - v_1p_1 - v_2p_2) \text{ s.t. } \pi_1 = v_1(p_1 - c_1)H(vs - v_1p_1 - v_2p_2) \geq \widetilde{\pi}_1.$$

It is obvious that the constraint always binds, otherwise p_1 could be decreased which would increase π_2 due to a higher acceptance probability. Then, I can write the bargaining frontier as $\pi_2 = \pi_2^*(\pi_1)$.

The standard procedure to derive the Nash bargaining solution is now as follows. To obtain a unique outcome, it is sufficient that the bargaining frontier is concave. The Nash bargaining solution (π_1, π_2) maximizes the Nash product $(\pi_1 - \pi_1^O)(\pi_2 - \pi_2^O)$, where the factors are the differences with respect to the bargaining frontiers and the outside options.

I now apply immediately the uniform distribution to $H(k)$ and assume that $c_i = c$. From the binding constraint, I obtain

$$\pi_2^*(\pi_1) = \pi_1 \frac{v_2}{v_1} \arg\max_{p_1,p_2} \frac{p_2 - c}{p_1 - c}.$$

This suggests that on the bargaining frontier $(p_2 - c)/(p_1 - c)$ must be highest, however, note that I clearly cannot freely choose both p_i. Turning again to the uniform distribution so that

$$\pi_1 = v_1(p_1 - c)\frac{1}{K}(sv - v_1p_1 - v_2p_2),$$

which I can rewrite as

$$p_2 = \frac{1}{v_2}\left(sv - v_1p_1 - \frac{K\pi_1}{v_1(p_1 - c)}\right).$$

Now, simplifying by setting $c = 0$, I thus want to maximize

$$\frac{p_2}{p_1} = \frac{svv_1p_1 - (v_1p_1)^2 - K\pi_1}{v_1v_2p_1^2}.$$

The sign of the derivative with respect to p_1 is then given by

$$\begin{aligned} & v_1v_2p_1^2\left(svv_1 - 2v_1^2p_1\right) - 2v_1v_2p_1(svv_1p_1 - v_1^2p_1^2 - K\pi_1) \\ = \quad & 2K\pi_1 - svv_1p_1, \end{aligned}$$

from which I obtain

$$p_1 = \frac{2K\pi_1}{sv_1(v_1+v_2)}.$$

Inserting yields

$$\begin{aligned} \frac{p_2}{p_1} &= \frac{svv_1p_1 - (v_1p_1)^2 - K\pi_1}{v_1v_2p_1^2} \\ &= \frac{s(v_1+v_2)v_1\frac{2K\pi_1}{sv_1(v_1+v_2)} - \left(v_1\frac{2K\pi_1}{sv_1(v_1+v_2)}\right)^2 - K\pi_1}{v_1v_2\left(\frac{2K\pi_1}{sv_1(v_1+v_2)}\right)^2} \\ &= \frac{1}{4K\pi_1v_2}\left(s^2v_1^3 + 2s^2v_1^2v_2 + s^2v_1v_2^2 - 4K\pi_1v_1\right), \end{aligned}$$

so that

$$\begin{aligned} \pi_2^*(\pi_1) &= \pi_1\frac{v_2}{v_1}\arg\max_{p_1,p_2}\frac{p_2}{p_1} \\ &= \pi_1\frac{v_2}{v_1}\frac{1}{4K\pi_1v_2}\left(s^2v_1^3 + 2s^2v_1^2v_2 + s^2v_1v_2^2 - 4K\pi_1v_1\right) \\ &= \frac{1}{4Kv_1}(s^2v_1^3 + 2s^2v_1^2v_2 + s^2v_1v_2^2) - \pi_1. \end{aligned}$$

I have thus shown that at the optimally chosen fees (p_1, p_2) the bargaining frontier is in fact linear (i.e., $\pi_1 + \pi_2^*(\pi_1)$ is the same and thus independent of how profits are distributed). I can thus indeed apply the Nash bargaining solution, which now simplifies to the requirement that

$$(\pi_1 - \pi_1^O) = (\pi_2 - \pi_2^O).$$

It remains to specify the outside options, where each issuer operates a separate payment system. Hence, with $\pi_1 = v_1(p_1 - c)\frac{1}{K}(sv_1 - v_1p_1)$ and now $c_i = 0$, I obtain $p_1 = s/2$ and thus $\pi_1^O = \frac{1}{4K}s^2v_1^2$. This holds analogously for $i = 2$. Now, I substitute for the outside options and for $\pi_2 = \pi_2^*(\pi_1)$ to obtain

$$\begin{aligned}
(\pi_1 - \pi_1^O) &= (\pi_2 - \pi_2^O) \\
\pi_1 - \frac{1}{4K}s^2 v_1^2 &= \frac{1}{4Kv_1}(s^2 v_1^3 + 2s^2 v_1^2 v_2 + s^2 v_1 v_2^2) - \pi_1 - \frac{1}{4K}s^2 v_2^2.
\end{aligned}$$

This yields the equilibrium profits

$$\pi_1 = \frac{1}{4K}s^2 v_1 \left(v_1 + v_2\right)$$

and

$$\begin{aligned}
\pi_2 &= \frac{1}{4Kv_1}(s^2 v_1^3 + 2s^2 v_1^2 v_2 + s^2 v_1 v_2^2) - \pi_1 \\
&= \frac{1}{4Kv_1}(s^2 v_1^3 + 2s^2 v_1^2 v_2 + s^2 v_1 v_2^2) - \frac{1}{4K}s^2 v_1 \left(v_1 + v_2\right),
\end{aligned}$$

which simplifies to

$$\pi_2 = \frac{1}{4K}s^2 v_2 \left(v_1 + v_2\right).$$

The corresponding values of p_1, p_2 on which they thus agree must jointly solve

$$\begin{aligned}
v_1 p_1 \frac{1}{K}\left[s(v_1 + v_2) - v_1 p_1 - v_2 p_2\right] &= \frac{1}{4K}s^2 v_1 \left(v_1 + v_2\right), \\
v_2 p_2 \frac{1}{K}\left[s(v_1 + v_2) - v_1 p_1 - v_2 p_2\right] &= \frac{1}{4K}s^2 v_2 \left(v_1 + v_2\right),
\end{aligned}$$

leading to the symmetric full internalization outcome because $p_1 = p_2 = s/2$, independent of any asymmetries in size.

I thus have shown that with two parties Nash bargaining over the fees (p_1, p_2) and the threat of exclusion, the full internalization outcome is obtained.