Persuasion through Selective Disclosure: Implications for Marketing, Campaigning, and Privacy Regulation*

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January 2019

Abstract

This paper characterizes equilibrium persuasion through selective disclosure based on the personal information that senders acquire about the preferences and orientations of receivers, with applications to strategic marketing and campaigning. We derive positive and normative implications depending on: the extent of competition among senders, whether receivers are wary of senders collecting personalized data, and whether firms are able to personalize prices. Privacy laws requiring senders to obtain consent to acquire information are beneficial when there is little or asymmetric competition among senders, when receivers are unwary, and when firms can price discriminate. Otherwise, policy intervention has unintended negative welfare consequences.

Keywords: Selective disclosure, hypertargeting, limited attention, privacy regulation.

JEL Classification: D83 (Search; Learning; Information and Knowledge; Communication; Belief), M31 (Marketing).

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*This paper supersedes and generalizes the analysis contained in an earlier draft circulated under the title: "Hypertargeting, Limited Attention, and Privacy: Implications for Marketing and Campaigning". We are grateful to Matteo Camboni and Marco Petterson for outstanding research assistance. Hoffmann and Inderst gratefully acknowledge financial support from the ERC Advanced Grant 22992, Ottaviani from the ERC Advanced Grant 295835, and Inderst also from the DFG through the Research Unit "Design & Behavior" (FOR 1371).

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1 Introduction

Firms traditionally had two distinct ways to persuade consumers. First, they could broadcast their messages through old media (leaflets, billboards, newspapers, and television), thereby achieving only a coarse segmentation of the audience, mostly along channel types and regional boundaries. Alternatively, they could customize their communication strategies through direct marketing aimed at persuading single individuals or small groups. To implement this second strategy, firms could hire experienced salespeople to gather critical knowledge about their audiences, making it possible to tailor their messages via face-to-face contacts.

Nowadays, the greater availability of personally identifiable data on the internet blurs the distinction between these two traditional communication strategies. Developments in computer technology increasingly allow sellers to systematically collect personal and detailed data about an individual's past purchasing behavior, browsing activity, and credit history, as well as the personal likes and dislikes the individual shares on social networking sites.[1] When conducting what might appear to be an impersonal transaction through the internet, a great deal of personal information may be used to finely target consumers. On Facebook, for example, ski resorts advertise family activities to married users with kids, but stress snowboarding and party options to younger users interested in winter sports. Behavioral targeting or hypertargeting along these lines combines features of remote broadcasting with features of personal selling.[2]

Concerns are often raised that some consumers might suffer if they remain blithely unaware of the ability of firms to collect information and communicate selectively.[3] An active debate is under way among policymakers about reforming the regulatory framework for consumer privacy with an emphasis on the collection and use of personal data on the internet. While in this area the U.S. currently relies mostly on industry self regulation, policymakers and Congress are considering stricter regulation of consumer privacy.[4] In

[1] Information can be either collected directly or acquired from search engines and specialized data vendors. In its privacy policy, Facebook writes: "We allow advertisers to choose the characteristics of users who will see their advertisements and we may use any of the non-personally identifiable attributes we have collected (including information you may have decided not to show to other users, such as your birth year or other sensitive personal information or preferences) to select the appropriate audience for those advertisements." *https://www.facebook.com/note.php?note_id=+322194465300*

[2] "Tailor your ads and bids to specific interests: Suppose you sell cars and want to reach people on auto websites. You believe that the brand of cars you sell appeals to a wide variety of people, but some of them may react more positively than others to certain types of ads. For example, . . . you could show an image ad that associates a family-oriented lifestyle with your car brand to auto website visitors who're also interested in parenting." *Google AdWords, http://support.google.com/adwords/answer/2497941?hl=en*

[3] Tucker (2012) discusses the extent of informational asymmetry between consumers and firms in online advertising about how much personal data is being collected.

[4] See American Association of Advertising Agencies (2009) for a widely adopted set of self-regulatory principles for online behavioral advertising. On the U.S. policy debate, see White House (2012), Federal

recent years, European legislators have intervened more directly by raising barriers to the collection and use of personally identifiable data about past purchases or recent browsing behavior, including a requirement that firms seek explicit consent to collect information.[5] The prevailing presumption—see Shapiro and Varian (1997)—is that efficiency is achieved by granting property rights over information to consumers, for example by requiring consumer consent.

Motivated by these policy questions, this paper analyzes personalization of marketing communication from a positive and normative standpoint. Our baseline framework features firms acting as senders who attempt to persuade receivers with limited attention. Each receiver is a buyer who individually chooses among the offerings of the different senders. In the absence of regulation, senders privately choose whether to acquire better information about receiver preferences and then attempt to persuade receivers by selectively disclosing information about their horizontally differentiated offerings.

Specifically, a receiver's valuation for a sender's offering is the sum of two i.i.d. components associated with two attributes of the sender's offering. Each sender privately decides whether to observe the receiver's attribute valuations before disclosing "hard" information about the respective offering. The scope of communication is naturally restricted by factors such as airtime and screen space, or simply by the receiver's limited attention.[6] Given these restrictions, senders can disclose only one of the two attributes, making the selection of the information to disclose a strategic decision. In particular, a sender who has learned the receiver's attribute valuations selects the attribute to disclose so as to increase the chance of a favorable decision, as in the example about ski resorts reported in the second paragraph.

Consider initially a single sender attempting to persuade a single receiver to accept its offering rather than an outside option with given reservation utility. In this baseline

Trade Commission (2012), and the discussion of the Do Not Track legislation proposals on wikipedia.

[5] See the Data Protection Directive (1995/46/EC) and the Privacy and Electronic Communications Directive (2002/58/EC), also known as the E-Privacy Directive, which regulates cookies and other similar devices through its amendments, such as Directive 2009/136/EC, the so-called EU Cookie Directive, and the Privacy and Electronic Communications (EC Directive) (Amendment) Regulations 2011. The current prescription is that "cookies or similar devices must not be used unless the subscriber or user of the relevant terminal equipment: (a) is provided with clear and comprehensive information about the purposes of the storage of, or access to, that information; and (b) has given his or her consent." More recently, European authorities have been pressuring internet giants such as Facebook and Google to limit the collection of personal data without user consent.

[6] Senders might be unable to communicate all the attributes they know because of space or time constraints or simply because (too much) information "consumes the attention of its recipients" (Simon 1971). The limited capacity of individuals to process information is currently being investigated in a number of other areas, ranging from macroeconomics (e.g., Sims 2003) to organization economics (Dessein, Galeotti, and Santos 2016). In our model, it is the sender who must choose a particular attribute to disclose given the limitation of the communication channel, rather than the receivers having to choose how to optimally direct their limited attention and information processing capacity.

setting, we compare the payoff of the receiver in the following two disclosure regimes:

- Under **non-selective disclosure** the sender reports to the receiver the realization of one out of two i.i.d. attribute valuations at random.
- Under **selective disclosure** the sender reports to the receiver the highest realization of the two attribute valuations.

In our main application to marketing, non-selective disclosure corresponds to broadcasting of ads via billboards, newspapers or television, while selective disclosure captures direct marketing or online advertising targeted at the individual. Beyond marketing, our comparison of non-selective vs. selective disclosure is also relevant for political campaigning. Non-selective disclosure corresponds to broadcasting of campaign messages through traditional public communication channels. Selective disclosure, instead, results when political candidates hire skilled campaigners to gather critical knowledge about individual voters' preferences and orientations. Candidates could tailor their messages through ground-game campaigning, canvassing, face-to-face conversation or via social media. More generally, non-selective disclosure consists of a randomized experiment, whereas selective disclosure can be seen as a manipulated experiment in which the receiver is fed with the more favorable of the two pieces of evidence.[7] To account for these additional applications, our first set of results abstracts from optimal pricing focusing entirely on the welfare implications of equilibrium disclosure.

When the receiver expects the sender to disclose non-selectively, selective disclosure shifts upward the distribution of the receiver's perceived valuation, in the sense of first-order stochastic dominance. Thus, if the receiver does not observe whether the sender collects information about receiver preferences (as a prerequisite for selective disclosure), in equilibrium the sender acquires this information and discloses selectively. We show that a wary receiver—who correctly anticipates selective disclosure—benefits from selective disclosure for a broad set of distributions satisfying logconcavity. For these distributions of attribute valuations, selective disclosure rotates the ex-ante distribution of the receiver's expected valuation, thereby making both low and high expected valuations more likely. Such a rotation improves the receiver's information, so that selective disclosure necessarily increases the ex ante welfare of the receiver. In contrast, selective disclosure benefits or harms the sender, depending on whether the receiver's reservation utility lies above or below the rotation point.

Thus, the sender faces a commitment problem—selective disclosure arises in equilibrium but damages the sender when the receiver's reservation utility is low, or equivalently

[7] Similarly, in the context of project evaluation, an independent reviewer may report on a randomly selected attribute, while an in-house reviewer selectively reports the most favorable one.

when competition from other senders is relatively weak. The receiver, however, benefits from selective disclosure by any sender, independent of the reservation utility and, thus, under competition, other senders' disclosure strategies. Privacy regulation that either fully prohibits the collection of personal data—or that just requires receivers' consent—can provide senders with commitment power not to disclose selectively, in which case even relatively light-handed regulation can backfire.

Given that regulation is often motivated by the protection of unsophisticated consumers, we also analyze selective disclosure when receivers remain unwary of selective disclosure.[8] Even though selective disclosure biases upward a receiver's perceived valuation, we identify two channels through which even unwary receivers can benefit from selective disclosure under a broad range of circumstances. First, we show this happens when the increase in false positives from erroneously accepting a sender's offer are more than outweighed by the reduction in false negatives from erroneously rejecting the offer. Second, we show that biases can completely cancel out when senders compete, so that selective disclosure unambiguously leads to more informed decisions by unwary receivers if it does so in case of wary receivers. Once we take into account senders' preferences to acquire personal information and disclose selectively, we show how ignorance can be bliss, as unwary receivers end up making better decisions than wary receivers.

Within the main application of our selective disclosure model to marketing, the extent to which the efficiency gains associated with more informative communication are shared between firms and consumers depends on whether firms can price discriminate according to the perceived expected valuation of a particular consumer. When a firm is in a monopolistic position, price discrimination can result in exploitative behavior, making regulatory intervention desirable. With the introduction of competition, perceived product differentiation matters. Selective disclosure dampens competition by increasing perceived differentiation, from an ex-ante perspective. However, when competition is sufficiently strong, this negative effect for consumers is reduced, so that consumer again tend to benefit from selective disclosure. Further, consumer ignorance about selective disclosure reduces differentiation, spurs rivalry among firms for a particular consumer, and lowers prices. With competition and price discrimination, consumer unwariness again becomes a blessing.

[8]In a disclosure setting in which the fraction of receivers who fail to update their beliefs following the lack of disclosure (analytical failure) is higher than the fraction of receivers who do not attend to the disclosure (cue neglect), Hirshleifer, Lim, and Teoh (2004) obtain an equilibrium in which the sender only discloses high realizations. Unwary consumers, instead, attend to the disclosed attribute but fail to make the appropriate inference about the undisclosed attribute, which is chosen selectively by the sender. Thus, relative comparisons across different dimensions of information play a key role in our model. Relative comparisons across dimensions also play a role in the construction of cheap-talk equilibria by Chakraborty and Harbaugh (2007, 2010) and Che, Dessein, and Kartik (2013).

The paper contributes to the literature on information control, disclosure, and persuasion. In our model, senders are able to control receivers' information indirectly by selectively disclosing information based on their knowledge of receiver preferences, rather than directly and truthfully as in the literature on information control in markets à la Lewis and Sappington (1994), Johnson and Myatt (2006), and Ganuza and Penalva (2010).[9] As we stipulate that senders cannot disclose all attributes, in the spirit of Fishman and Hagerty's (1990) notion of limited attention (see also Glazer and Rubinstein (2004)), lack of disclosure does not trigger complete unraveling, thus departing from the baseline models of Grossman (1981), Milgrom (1981), and Milgrom and Roberts (1986). Our model also adds hidden information acquisition prior to the stage of selective disclosure. In addition, senders in our model cannot commit to the information structure, akin to signal-jamming à la Holmström (1999), and so can fall victim to their own incentives to secretly acquire information and disclose selectively. In these cases, regulation helps senders achieve commitment, which in turn damages wary receivers. The non-commitment assumption also distinguishes our paper from models of *optimal* persuasion with commitment à la Rayo and Segal (2010) and Kamenica and Gentzkow (2011), where a sender commits to an information structure in an unconstrained fashion.[10]

In our model, privacy affects consumer welfare through the restrictions it imposes on the selection of disclosed information. Instead, the law and economics literature on transparency focusses mainly on the direct costs of information acquisition. Here, incentives to collect information may be too high when the prime purpose of information is to affect the distribution of surplus (Hirshleifer 1971), as is possibly the case when information allows firms to better price discriminate.[11] To better trade-off the social costs and benefits of collecting and using personally identifiable data, instead of prohibiting these practices, it has been proposed to essentially grant agents property rights over such information (e.g., Shapiro and Varian 1997). Our analysis reveals a particular twist to this policy. We show that a policy that requires consumer consent may allow firms to commit to abstain from selective communication even when this would benefit consumers.

[9] See also Kamenica, Mullainathan, and Thaler (2011) for a discussion of situations in which firms might know more about consumer preferences than consumers know themselves.

[10] Our analysis of equilibrium persuasion with multiple senders is particularly tractable given our focus on horizontal differentiation with independently distributed values. See Gentzkow and Kamenica (2017) for an analysis of optimal persuasion with multiple senders releasing information simultaneously, and Board and Lu (2018) for a related setting with consumer search and sequential information provision. Bhattacharya and Mukerjee (2013) analyze strategic disclosure by multiple senders who share the same information; in our horizontal-differentiation model, instead, senders are endogenously informed about the values of their offerings, which are independently distributed. See also DellaVigna and Gentzkow (2010) for a survey of the literature on persuasion, including applications to marketing and political settings.

[11] The literature on law and economics has also discussed more broadly the benefits of greater transparency for expanding efficiency-enhancing trade (Stigler 1980, Posner 1981). Hermalin and Katz (2006) show, however, that trade efficiency may not monotonically increase with information.

A different twist on the costs of transparency has been recently offered in the marketing literature on targeted advertising, which allows firms to better restrict the scope of their marketing to those consumers who are likely to purchase in the first place (cf., Athey and Gans 2010 for its impact on media competition). Several recent papers (e.g., Goldfarb and Tucker 2011; Campbell, Goldfarb, and Tucker 2015, and Shen and Villas-Boas 2018) analyze, both theoretically and empirically, how more restrictive privacy rights affect competition and welfare by potentially making advertising campaigns less cost-effective. Combined with the insights from our analysis, the protection of privacy rights should thus always be considered while taking into account competition and its benefits to consumers.[12]

The paper proceeds as follows: Section 2 sets the stage by analyzing how individual receivers update their valuations when faced with selective or non-selective disclosure by a single sender and how the choice of disclosure strategy accordingly affects receivers' expected utility. Building on this baseline analysis of the effect of selective vs. non-selective disclosure on receiver demand, Sections 3 and 4 consider equilibrium disclosure in a model that allows for multiple senders who can privately choose whether to acquire information about receiver preferences before disclosing information. While Section 3 derives equilibrium for fixed prices, Section 4 allows for individualized price discrimination. For both cases, we analyze the impact of different regulatory regimes aimed at restricting senders' ability to secretly acquire information and how this impact depends on receivers' potential naiveté, the extent of competition among senders as well as senders' potential asymmetry. Section 5 concludes and suggests avenues for future research. All proofs are in Appendix A and supplementary material in Appendix B.

2 Selective Disclosure

2.1 Baseline Setting with One Sender and One Receiver

Consider a single sender who aims at persuading a single receiver to accept an offering. The receiver has reservation utility R (which will be endogenized in the equilibrium model in Section 3) and utility from acceptance

$$u^1 + u^2 \tag{1}$$

in the spirit of Lancaster (1966). The valuations u^1 and u^2 are identically and independently distributed with atomless distribution $F(u)$,[13] expectation $E[u]$, and (possibly

[12] Another strand of the literature assumes that information disclosure is directly inconvenient for consumers, as in Casadesus-Masanell and Hervas-Drane (2015).

[13] The case where receivers place different weights on the two components in (1) is analyzed in Appendix B.2; when F is uniform we obtain a clean characterization and can show that our main comparison of selective and non-selective disclosure still applies.

unbounded) support $(\underline{u}, \overline{u})$,[14] with $\underline{u} + E[u] < R < \overline{u} + E[u]$.

The receiver is interested in learning about the realizations of u^i, $i = 1, 2$, based on the ("hard") information disclosed by the sender. The key restriction we impose is that the sender can disclose only a single attribute $d \in \{1, 2\}$, thus, allowing the receiver to observe only a single realization, u^1 or u^2.[15] Our main motivation for this is that the receiver has limited attention; in practice such a restriction also arises naturally from limitations of (air) time and (advertising) space. From (1) and the fact that under any feasible disclosure regime the receiver observes only a single realization u^1 or u^2, the receiver must make an inference on the non-disclosed variable which clearly depends on the conjectured disclosure strategy. In light of our subsequent equilibrium analysis in which the sender privately chooses whether to learn the values of u^i, $i = 1, 2$, prior to disclosure, we compare the following disclosure regimes:

(N) Under **non-selective disclosure**, the sender discloses a single variable, u^i, independently of the realization.[16] The receiver thus infers that the expected valuation conditional on the observed u^i is

$$E_N \left[u^1 + u^2 | u = u^i \right] = u^i + E\left[u\right]. \tag{2}$$

(S) Under **selective disclosure**, the sender discloses the highest realization of the random variables, $\max \langle u^1, u^2 \rangle$. If the receiver is aware that disclosure is selective, the expected valuation conditional on observing $u_d = \max \langle u^1, u^2 \rangle$ is

$$\mathcal{U}\left(u_d\right) := E_S \left[u^1 + u^2 | \max \left\langle u^1, u^2 \right\rangle = u_d \right] = u_d + E\left[u | u \leq u_d\right]. \tag{3}$$

($\hat{S}$) Under **unwary selective disclosure**, the sender discloses $u_d = \max \langle u^1, u^2 \rangle$, but the receiver now wrongly believes that disclosure is non-selective. The receiver's perceived expected sum is

$$E_{\hat{S}} \left[u^1 + u^2 | \max \left\langle u^1, u^2 \right\rangle = u_d \right] = u_d + E\left[u\right]. \tag{4}$$

We consider two cases regarding the receiver's anticipation that disclosure is selective: a wary receiver is aware of (and properly adjusts for) the selectivity of disclosure while an

[14] Without loss of generality we take the support as open, so as to allow both for the possibility that it is bounded and unbounded (with either $\underline{u} = -\infty$ or/and $\overline{u} = \infty$).

[15] When a sender is informed about the values u^i a given receiver attaches to the two components of his offering, this restriction on the communication channel prevents full unravelling, thus, creating scope for disclosure strategies that depend on the receiver's preferences. Note further, that the key restriction, thus, is that the sender can disclose at most one component, while disclosure of at least one component will be optimal in equilibrium due to a standard unraveling argument.

[16] For example, the sender could always disclose u^1 or u^2 or randomly disclose either u^1 or u^2 with any (fixed) probability. All these situations are clearly equivalent because u^1 and u^2 are i.i.d..

unwary receiver believes that disclosure is non-selective even when disclosure is selective. In particular, given selective disclosure by the sender, regime S applies when the receiver is aware of the relevant disclosure regime when forming expectations, which clearly will be the case in equilibrium of our game with strategic sender, when the receiver forms rational expectations. As a building block of the construction of the equilibrium, we also analyze the (off-equilibrium) regime $\hat{S}$ in which the receiver does not anticipate the selectivity of disclosure. The case with unwary selective disclosure is also relevant more broadly when the receiver remains unaware of the sender's incentives for selective disclosure.

In the remainder of this section we characterize the impact of selective disclosure on sender and receiver welfare, depending on whether the receiver is wary or unwary; proofs of results are contained in Appendix A.1, while Appendix B.1 collects supplementary material and illustrations for parametric distributions.

2.2 Impact of Selective Disclosure on Receiver: Decomposition

To compare the receiver's expected payoff under selective and non-selective disclosure, it is useful to decompose the effect of the disclosure strategy on the expected sum into two channels:

- **Factual Channel**. First, the sender's disclosure strategy affects the receiver through the distribution of the disclosed variable. When the sender selectively reports $u_d = \max\langle u^1, u^2\rangle$, high realizations become more likely compared to when the sender reports a random realization u^i. This factual channel is the only one active for an unwary receiver who perceives disclosure to be non-selective such that the expected value of the undisclosed variable remains $E[u]$ even under selective disclosure.

- **Inference Channel.** When the sender's disclosure strategy is selective, the disclosed realization $u_d = \max\langle u^1, u^2\rangle$ contains indirect information about the undisclosed variable; it must be that the undisclosed realization is smaller than u_d. Only a wary receiver, who knows the sender's disclosure strategy, is able to exploit this additional information to correctly estimate the undisclosed variable as $E[u|u < u_d]$ rather than $E[u]$, adjusting for selection bias.

The impact of selective disclosure on the receiver depends on a comparison of these two channels across the disclosure regimes. To see this denote the receiver's expected payoff in disclosure regime $j = N, S, \hat{S}$ by V_j,[17] and note that the value obtained by the wary

[17] As the receiver accepts the sender's offering if and only if his expected valuation is greater than the outside option R, we obtain $V_N = \int \max\left\langle R, u^1 + E[u]\right\rangle dF(u^1)$, $V_S = \int \max\left\langle R, u_d + E[u|u \leq u_d]\right\rangle dF(u_d)^2$ and $V_{\hat{S}} = RF(R - E[u])^2 + \int_{R-E[u]}^{\infty} [u_d + E[u|u \leq u_d]]\, dF(u_d)^2$.

receiver under selective disclosure can be decomposed as

$$V_S = \underbrace{V_{\hat{S}}}_{\text{Factual Channel}} + \underbrace{(V_S - V_{\hat{S}})}_{\text{Inference Channel}},$$

where $V_S - V_{\hat{S}} > 0$ is the gain from the inference channel.

Under non-selective disclosure, given that the sender discloses u^i independently of its actual realization, the receiver cannot make any strategic inference. Thus, because the receiver is equally likely to observe $\max\langle u^1, u^2\rangle$ and $\min\langle u^1, u^2\rangle$, the receiver's value under non-selective disclosure is

$$V_N = \underbrace{\frac{V_{\hat{S}} + V_{\hat{T}}}{2}}_{\text{Factual Channel}} + \underbrace{0}_{\text{Inference Channel}},$$

where $V_{\hat{T}}$ is the unwary receiver's value from observing $\min\langle u^1, u^2\rangle$. Overall,

$$V_S - V_N = (V_{\hat{S}} - V_N) + (V_S - V_{\hat{S}}) = \underbrace{\frac{V_{\hat{S}} - V_{\hat{T}}}{2}}_{\vartriangle \text{ Factual Channel}} + \underbrace{(V_S - V_{\hat{S}})}_{\vartriangle \text{ Inference Channel}}. \tag{5}$$

The impact of selective disclosure on the wary receiver's value through the factual channel (first term) crucially depends on whether the unwary receiver prefers observing $\max\langle u^1, u^2\rangle$ or $\min\langle u^1, u^2\rangle$ at a given reservation utility R, while the impact through the inference channel (second term) is unambiguously positive. Overall, when also the factual channel is more valuable under selective disclosure, the wary receiver unambiguously benefits:

Observation 1 *A sufficient condition for the wary receiver to always (for all R) benefit from selective disclosure is that the unwary receiver's expected value is always (for all R) higher when observing* $\max\langle u^1, u^2\rangle$ *rather than* $\min\langle u^1, u^2\rangle$*: $V_{\hat{S}} \geq V_{\hat{T}} \Leftrightarrow V_{\hat{S}} \geq V_N$ is sufficient for $V_S \geq V_N$.*

For instance, as verified in Example B.1 in Appendix B.1, if u is exponentially distributed, the unwary receiver prefers observing $\max\langle u^1, u^2\rangle$ rather than $\min\langle u^1, u^2\rangle$; the wary receiver then benefits from selective disclosure. On the flip side, for the wary receiver to lose from selective disclosure it becomes *necessary*—rather than sufficient—that $V_{\hat{S}} < V_{\hat{T}}$:

Observation 1* *A necessary condition for the wary receiver's expected payoff to be always (for all R) higher under non-selective than under selective disclosure is that the unwary receiver's expected payoff is always (for all R) higher when observing* $\min\langle u^1, u^2\rangle$ *rather than* $\max\langle u^1, u^2\rangle$*: $V_{\hat{S}} < V_{\hat{T}} \Leftrightarrow V_{\hat{S}} < V_N$ is necessary for $V_S < V_N$.*

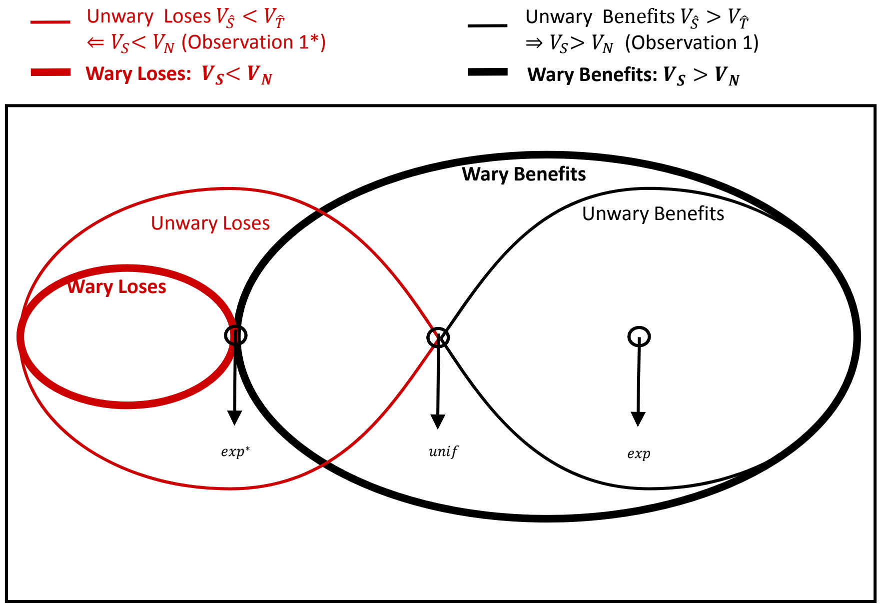

Figure 1: Illustration of Decomposition of Welfare Impact of Selective Disclosure

In general, by taking the mirror image (reflected around $u = 0$) of any distribution satisfying $V_{\hat{S}} > V_{\hat{T}}$ we obtain a distribution satisfying $V_{\hat{S}} < V_{\hat{T}}$. Thus, the two sets representing when the unwary receiver benefits or loses for all R in Figure 1 have the same size.[18] However, note the stark asymmetry between the conditions in Observation 1 and 1*, sufficient the former and necessary the latter. Thus, in Figure 1, the wary receiver benefits from selective disclosure in a superset (bold, on the right-hand side) of the set of distributions for which the unwary benefits, but loses in a subset (bold and red, on the left-hand side) of the set for which the unwary receiver loses. The underlying force that creates this asymmetry and handicaps non-selective disclosure is the positive impact of the strategic inference channel, present only under selective disclosure for the wary receiver.[19]

To see this in more detail, recall that when disclosure is selective the receiver observes $\max\langle u^1, u^2\rangle$, while under non-selective disclosure the receiver is equally likely to observe $\max\langle u^1, u^2\rangle$ or $\min\langle u^1, u^2\rangle$ without knowing which one is observed. Thus, when the non-selectively disclosed value u^i is equal to $\max\langle u^1, u^2\rangle$, the receiver benefits from selective

[18] Observation 1 and 1* of course hold equally also for fixed R. Thus distributions for which the value comparison depends on R are outside of the receiver benefits/loses regions in Figure 1, cf. Figure 5 in Appendix B.1.

[19] By (5) we have $V_{\hat{S}} > V_{\hat{T}} \Rightarrow V_S > V_{\hat{T}} \Rightarrow V_S > V_N$, Observation 1's sufficient condition can be replaced by the less stringent $V_S > V_{\hat{T}}$. Similarly, from $V_{\hat{S}} < V_{\hat{T}} \Leftarrow V_S < V_{\hat{T}} \Leftarrow V_S < V_N$, Observation 1*'s necessary condition can be replaced by the more stringent $V_S < V_{\hat{T}}$. This additional asymmetry further highlights that there are many more distributions for which selective disclosure benefits rather than harms the wary receiver.

disclosure because of the value of knowing to have observed $\max\langle u^1, u^2\rangle$. When u^i is equal to $\min\langle u^1, u^2\rangle$, the disclosed value under selective and non-selective disclosure differ. Again, selective disclosure entails the benefit that the receiver knows that $\max\langle u^1, u^2\rangle$ was observed, but now there is also a potential cost, if observing $\min\langle u^1, u^2\rangle$ is more beneficial via the factual channel. Writing

$$V_S - V_N = \frac{1}{2}\left(V_S - V_{\hat{S}}\right) + \frac{1}{2}\left(V_S - V_T\right) + \frac{1}{2}\left(V_T - V_{\hat{T}}\right),$$

we see that the first and third term are positive, while the second term has an ambiguous sign in general—unless seeing the $\min\langle u^1, u^2\rangle$ is more valuable than seeing the $\max\langle u^1, u^2\rangle$ selective disclosure dominates non-selective disclosure. In the following we provide precise conditions for selective disclosure to dominate non-selective disclosure.

Impact of Selective Disclosure on Unwary Receiver. Putting Observation 1 to work, we now characterize when an unwary receiver benefits from selective disclosure: $V_{\hat{S}} \geq V_{\hat{T}}$. By biasing upward an unwary receiver's perceived utility, the mistake of erroneously accepting the offering, even though $u^1 + u^2 < R$, evidently becomes larger under selective disclosure. But this is only one side of the equation. At the same time, with selective disclosure, it becomes less likely that the receiver erroneously decides against the offering, namely when actually $u^1 + u^2 > R$ holds. For symmetric distributions we obtain a clear-cut result for how the two errors trade off:

Proposition 1 *If F has a symmetric and unimodal density the unwary receiver benefits from selective disclosure, $V_{\hat{S}}(R) \geq V_N(R)$, if and only if $R \geq 2E[u]$.*

We state Proposition 1 for the natural case of unimodal densities. For distributions with U-shaped density, as an immediate corollary to this proposition, we obtain the reverse result: the unwary receiver benefits from selective disclosure $V_{\hat{S}}(R) \geq V_N(R)$ if and only if $R \leq 2E[u]$.

To get some intuition for the result in Proposition 1 consider the Cartesian product of the supports of u^1 and u^2 as depicted in the two panels of Figure 2 which correspond to two different values of the outside option R. With non-selective disclosure the receiver who observes u^1 is indifferent between accepting and rejecting the offering at $u^1 = R - E[u]$. This threshold can also be derived as the crossing between the iso-payoff of level R and the horizontal line at $u^2 = E[u]$. Under selective disclosure, the unwary receiver still accepts when observing a realization above $R - E[u]$ as under non-selective disclosure, but now accepts not only when $u^1 \geq R - E[u]$ but also when $u^2 \geq R - E[u]$. Thus, all points (u^1, u^2) in the shaded region $[\underline{u}, R - E[u]] \times [R - E[u], \overline{u}]$ in the figure correspond to

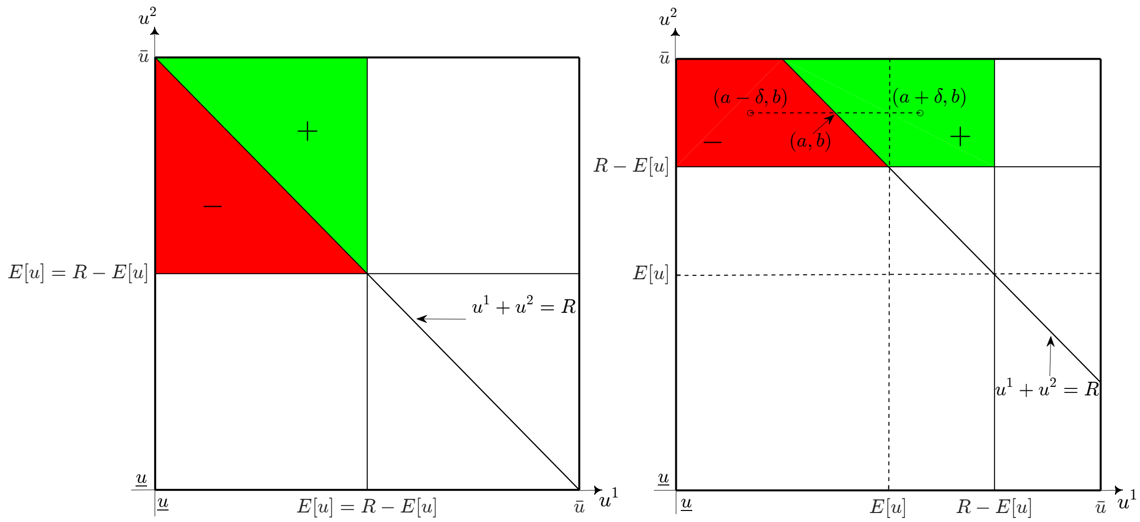

Figure 2: Welfare impact on unwary receiver.

additional acceptances by the unwary receiver induced by the selectivity of disclosure.[20]

Within the shaded region, the receiver loses when $u^1 + u^2 < R$, in (red) subregions to the left of the iso-payoff corresponding to R (increase in false positives, marked with a $-$ sign), and gains in (green) subregions to the right (increase in correct positives, marked with a $+$ sign). As is easily seen from both panels of the figure, for symmetric distributions, the total area of gains (+) and losses (–) is the same. The net effect then depends on the relative probability of points corresponding to comparable gains and losses.

Consider now, first, the left-hand panel of Figure 2 depicting the case where $R = 2E[u]$ such that the receiver is indifferent at the prior. As is immediate from the graph, given symmetry of F, in this case, points corresponding to comparable gains and losses have the same probability. Hence, the expected payoff of the unwary receiver is unaffected by selective disclosure, $V_{\hat{S}}(R) = V_N(R)$; this indifference between the offering and the outside option at the prior can be seen as a condition of equipoise.

Consider next, the right-hand panel of Figure 2 depicting a case where $R \geq 2E[u]$. Due to symmetry of F, the total area of gains (+) and losses ($-$) is again the same. Further, as illustrated in the figure, for each point in the $+$ region with a given gain δ we can again find a corresponding point in the $-$ region with a loss of same size. However, the relative probability of these two points now no longer needs to be the same. In particular,

[20] This region can alternatively be obtained by noting that $V_{\hat{S}} - V_N = \frac{1}{2}\left(V_{\hat{S}} - V_{\hat{T}}\right)$. Hence, given that u^1 and u^2 are identically distributed, the impact of selective disclosure for the unwary receiver is half of the difference of the value from observing $\max\left\langle u^1, u^2\right\rangle$ rather than $\min\left\langle u^1, u^2\right\rangle$ while accepting when the realization is above $R - E\left[u\right]$.

the points corresponding to gains in the + region to the right of the iso-payoff of level R are closer to the expectation $E[u]$ than the points corresponding to similar losses in the $-$ region to the left of the iso-payoff. Thus, if the distribution F is unimodal, gains (generated by the increase in correct positive decisions) weigh more than losses (generated by the increase in false positive decisions) and the unwary receiver benefits from selective disclosure.

When the distribution F is uniform, all points in Figure 2 are equally likely, so that for all reservation utilities R the increase in expected payoff associated to the additional correct positives is exactly offset by the increase in false positives, so that the unwary receiver always obtains the same expected payoff under selective and non-selective disclosure. The uniform is at the center of Figure 1, on the boundary of the set of distributions for which the unwary receiver benefits and loses.[21] By Observation 1, we thus conclude that for uniform F, the wary receiver always benefits from selective disclosure as $V_S - V_N = V_S - V_{\hat{S}} > 0$. As we will see in the next section, the wary receiver benefits from selective disclosure much more generally.

2.3 Impact of Disclosure on Distributions of Expected Valuation

To characterize the impact of selective disclosure on a wary receiver we now turn to a more detailed analysis of the distributions of expected valuations induced by different disclosure regimes. Given the information disclosed by the sender and the conjectured disclosure strategy, the receiver updates the valuation for the sender's offering before deciding whether to accept or reject, according to (2), (3), and (4). Next, we derive the ex ante distribution of the receiver's expected valuation resulting in the different disclosure regimes:

Non-Selective Disclosure. Given that the sender discloses a single variable, u^i, independently of its realization, the distribution N of the expected valuation $u^i + E[u]$ for the receiver conditional on observing a single variable u^i is

$$N(U) = F(U - E[u]) \text{ with } U \in (\underline{u} + E[u], \overline{u} + E[u])$$

for all possible realizations of the expected sum U. For example, for a uniform $F(u^i) = u^i$ on $(0,1)$, we have $N(U) = U - 1/2$ with support $(1/2, 3/2)$, as depicted in Figure 3.

Selective Disclosure. Rewriting expression (3), the receiver who is aware that the sender discloses $\max\langle u^1, u^2\rangle$ infers that the expected sum conditional on the observed realization

[21] Among symmetric distributions, the uniform is at the boundary between distributions with unimodal and U-shaped densities. We stated Proposition 1 for the natural case of unimodal densities. For distributions with U-shaped density, as an immediate Corollary to Proposition 1 we obtain the reverse result: the unwary receiver benefits from selective disclosure $V_{\hat{S}}(R) \geq V_N(R)$ if and only if $R \leq 2E[u]$.

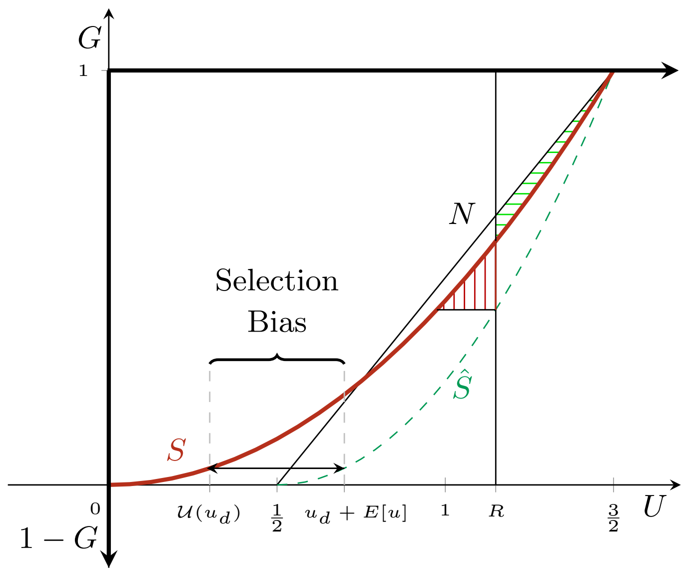

Figure 3: Comparison of distributions of posterior evaluations $G = N, S, \hat{S}$ for the uniform distribution, satisfying Property 2.

$u_d = \max \langle u^1, u^2 \rangle$ is

$$\mathcal{U}(u_d) = u_d + \left(\int_{\underline{u}}^{u_d} u \frac{f(u)}{F(u_d)} du \right) = u_d + \left(u_d - \int_{\underline{u}}^{u_d} \frac{F(u)}{F(u_d)} du \right) = 2u_d - \frac{L(u_d)}{F(u_d)}, \tag{6}$$

where we used integration by parts and the definition $L(u) := \int_{\underline{u}}^{u} F(y)\, dy$ of the left-hand integral of the distribution function. The term $L(u_d)/F(u_d)$ is equal to the *mean-advantage-over-inferiors* function, as defined by Bagnoli and Bergstrom (2005, page 249). The distribution S of the expected sum $\max \langle u^1, u^2 \rangle + E[u | u \leq \max \langle u^1, u^2 \rangle]$ conditional on observing $\max \langle u^1, u^2 \rangle$ then is

$$S(U) = F\left(\mathcal{U}^{-1}(U)\right)^2 \text{ with } U \in (2\underline{u}, \overline{u} + E[u]),$$

where the inverse function $\mathcal{U}^{-1}$ is well defined given that $\mathcal{U}$ is monotone increasing. In the uniform example, we have $E[u | u \leq u_d] = u_d/2$ so that $\mathcal{U}(u) = 3u_d/2$. Thus, the distribution of the expected valuation is $S(U) = (2U/3)^2$ with support $(0, 3/2)$, bold in Figure 3.

Unwary Selective Disclosure. In this case, the receiver is overoptimistic about the undisclosed variable and fails to adjust downward the expectation to account for selection bias

$$\delta(u_d) = \underbrace{E[u] - E[u | u \leq u_d]}_{\text{Selection Bias Adjustment}} > 0, \tag{7}$$

the difference between (3) and (4). The distribution $\hat{S}$ of the perceived valuation U, given the receiver's wrong beliefs about the disclosure regime, then is

$$\hat{S}(U) = F(U - E[u])^2 \text{ with } U \in [\underline{u} + E[u], \overline{u} + E[u]].$$

In the uniform example, the distribution of the expected perceived valuation by the unwary receiver is $\hat{S}(U) = (U - 1/2)^2$ with support $(1/2, 3/2)$, dashed in Figure 3.

Impact on Unwary Receiver. Figure 3 illustrates the following relation between the distribution N of expected valuation under non-selective disclosure and the distribution $\hat{S}$ of the expected valuation under selective disclosure as perceived by the unwary receiver:

Property 1 (FOSD) *Distribution $\hat{S}$ first-order stochastically dominates distribution N,*

$$\hat{S}(U) < N(U) \text{ for } U \in (\underline{u} + E[u], \overline{u} + E[u]).$$

This FOSD Property is completely general and is based on the stochastic dominance property of order statistics: $F(u)^2 < F(u)$. The FOSD Property will be a key determinant of the sender's incentives for choosing the disclosure regime in the equilibrium analysis of the full model presented in Section 3. Holding fixed the beliefs of the receiver, a move to selective disclosure results in an increase of the probability that the offering is accepted for any given reservation utility R.

Impact on Wary Receiver. For the uniform distribution in Figure 3 the following rotation property holds:

Property 2 (Clockwise Rotation) *Distribution S is a mean-preserving clockwise rotation of distribution N, meaning that S crosses N once and from above at one $\tilde{U}$ in the interior of the support*

$$S(U) \gtreqless N(U) \text{ for } U \lesseqgtr \tilde{U}.$$

The (strict) Clockwise Rotation Property has two important welfare implications. First, combined with the preservation of the mean that follows from the law of iterated expectations, Clockwise Rotation directly implies that S is a mean-preserving spread of N. Thus, we must have

$$V_S = \int \max\langle U, R\rangle \, dS(U) > \int \max\langle U, R\rangle \, dN(U) = V_N$$

for all choices of R because $\max\langle U, R\rangle$ is a convex function. Hence, the wary receiver unambiguously benefits from selective disclosure.[22] In Figure 3 the shaded area between

[22] Clockwise Rotation is a strengthening of the convex order—see e.g. Shaked and Shantikumar (2007, Theorem 3.A.44 on page 133) and, thus, sufficient for the wary receiver to benefit from selective disclosure.

the two distributions and to the right of the reservation utility (drawn as a vertical line at R) represents the difference in the receiver's value.[23] Second, clockwise rotation also determines whether the sender prefers selective disclosure when facing a wary receiver, which is the case if and only if $R > \tilde{U}$.[24]

In Appendix B.1, we provide a characterization of the set of distributions for which S is a clockwise rotation of N (Proposition B.1) as well as sufficient conditions (Lemma B.1). There we also verify that all the random variables with logconcave F listed in Bagnoli and Bergstrom (2005), such as power distributions (including uniform), gamma(α, β) as well as Weibull with shape $\alpha \leq 1$ (including exponential), extreme value Gumbel, Pareto with $\alpha > 1$ (for which the expectation exists), normal, lognormal, Fisher-Snedecor F, and beta(α, β) with $\beta \geq 1$ satisfy the clockwise rotation property. In fact, whenever the distributions S and N cross only once, the rotation is clockwise if and only if the left-hand integral $L(u) = \int_{\underline{u}}^{u} F(y)\,dy$ is logconcave at the upper bound, a property that is implied by logconcavity of the distribution F, in turn implied by logconcavity of the density f.[25] In the boundary case with loglinear L (corresponding to F positive exponential, the mirror image of the negative exponential), selective and non-selective disclosure induce identical distributions $S = N$ (see Example B.6 in Appendix B.1). As represented in Figure 1, the positive exponential distribution thus is the boundary case between the sets of distributions for which the wary receiver always benefits and always loses from selective disclosure. It is then easy to see that logconvexity of L at the upper bound of the support implies that S crosses N from below at the last crossing (see Lemma B.2 in Appendix B.1) and thus that the receiver is harmed by selective disclosure when R is sufficiently high.[26]

The preceding discussion illustrates the subtleties of the impact of selective disclosure on the wary receiver's welfare. Still, as shown in Section 2.2, the direct benefit of the strategic inference channel tilts the welfare comparison in favor of selective disclosure (cf. Figure 1). In particular, the wary receiver always benefits from selective disclosure for the large class of commonly used log-concave distributions, for which the Clockwise Rotation property holds. Hence, for the remainder of this paper we assume that Property 2

[23] For an economic interpretation, turn around axes and place the origin at the top-left corner. The reliability function $1 - G(U)$ can then be seen as the quantity demanded when the marginal consumer is offered utility U. Thus, the expected consumer surplus at R is equal to the area below the demand (i.e., to left of the distribution) and above the reservation utility R (to the right of R with the initial origin).

[24] This is easily seen by noting that the probability of acceptance is higher under selective disclosure, $1 - S(R) > 1 - N(R)$, when the outside option is high, $R > \tilde{U}$ (*niche market*), whereas it is higher under non-selective disclosure, $1 - S(R) < 1 - N(R)$, when the outside option is low, $R < \tilde{U}$ (*mass market*).

[25] These results follow from Prékopa's (1973) Theorem, which guarantees that logconcavity is preserved by integration; see, for example, An (1998) and Bagnoli and Bergstrom (2005).

[26] Appendix B.1 displays distributions with logconvex L (i.e., with bottom tail thicker than the positive exponential distribution) for which the wary receiver always loses from selective disclosure. In particular, there we also provide formal statements characterizing when the wary receiver prefers non-selective rather than selective disclosure.

(Clockwise Rotation) is satisfied, which in turn is sufficient (but not necessary, cf. Example B.11 in Appendix B.1) for the wary receiver to benefit from selective disclosure.[27]

Building on the baseline results obtained in this section, the paper now proceeds to derive the equilibrium disclosure strategy when multiple senders compete for receivers who make individual (purchasing) decisions. In the context of our main application to marketing, we also analyze the impact of privacy regulation limiting firms' scope for selective disclosure, depending on the wariness of consumers, the extent of competition among possibly asymmetric firms, and their capability to practice individualized price discrimination.

3 Equilibrium Selective Disclosure

We now turn to a setting where the choice of disclosure rule is made strategically by (multiple) senders. Let thus $M \geq 2$ be the set, as well as the number, of the alternatives from which the receiver can choose one. For each offering, we denote the component valuations by u_m^i for $i = 1, 2$, whose sum gives the receiver's utility from acceptance as in (1). These two components of the receiver's utility may correspond to two attributes $i = 1, 2$ of the sender's offering, so that the realization of u^i represents the match (or fit) of the attribute's characteristics with the receiver's idiosyncratic preferences; see Appendix B.3 for a fully worked-out foundation along these lines.[28] Receiver preferences are independent across senders and we allow for heterogeneous distributions $F_m(u_m^i)$. To ensure that the different offerings are indeed in competition we stipulate that the valuations are not too different, so that for any pair (m, m') it holds that

$$\overline{u}_m + E[u_m] > \underline{u}_{m'} + E[u_{m'}], \tag{8}$$

where we dropped the superscripts i denoting the attributes.[29]

Unless we explicitly state otherwise, we focus on the case in which all alternatives are offered by strategic senders, so that M also corresponds to the set of senders. Still, the framework allows for the possibility that a subset of alternatives comprises outside

[27] Our analysis below also extends to situations in which the receiver's utility depends on additional, possibly only privately known determinants. To see this, suppose that the receiver's *total* perceived utility under option m is equal to U *plus* some additional, independently distributed component V, i.e., equal to $\widehat{U} = U + V$, now with respective distribution $\widehat{G}(\widehat{U})$. As long as V has a logconcave density, when G undergoes a rotation, the same holds for $\widehat{G}$ (see e.g. Theorem 4 in Jewitt 1987).

[28] Appendix B.3 uses a Salop circle formulation to disentangle the actual location of a sender's attributes from the location that corresponds to a particular receiver's bliss point. According to this interpretation, disclosure of a dimension by a sender allows a receiver to learn the distance between his ideal location and the offering's location along that dimension.

[29] Recall that distribution S has support $(2\underline{u}, \overline{u} + E[u])$, while the one of N is $(\underline{u} + E[u], \overline{u} + E[u])$. Condition (8) thus is necessary and sufficient for the supports of any combination of $G_m = S_m, N_m$ and $G_{m'} = S_{m'}, N_{m'}$ to strictly overlap.

options, for which the perceived value is not affected by senders' strategies. In particular, we will repeatedly consider the case of a single (monopolistic) sender, where the receiver's alternative is an outside option of fixed value R as in the preceding foundational analysis.[30]

With respect to senders' preferences, we only need to specify that each sender m is strictly better off when the receiver chooses his option m.[31] Senders thus choose their disclosure strategy—selective or non-selective disclosure—in order to persuade the receiver to choose their offering. For a sender to use selective disclosure, he must have learned the receiver's preferences. In this section we first suppose that all senders can do so without (regulatory) restrictions and denote the strategies by $s_m \in \{y, n\}$, so that y (yes) means that sender m learns the receiver's preferences and n (no) corresponds to not acquiring the information.

We thus consider now the following game:

- At $t = 1$, each sender chooses whether or not to acquire information, $s_m \in \{y, n\}$. The baseline assumption in Section 3.1 is that information acquisition is an unobservable hidden action, while in Sections 3.2 and 3.3 we discuss how regulation may imply observability (and thereby commitment).

- At $t = 2$, each sender discloses a particular attribute, $d_m \in \{1, 2\}$, of his offering thereby revealing to the receiver the value u_m^i with $i = d_m$ and inducing an updated perceived valuation U_m, which clearly depends both on what the receiver observed as well as on his belief about the sender's disclosure strategy. When senders are firms selling a product or service they may, at this stage, also engage in personalized pricing as analyzed in Section 4.

- Finally, at $t = 3$, the receiver chooses offering $m \in M$ that has the highest perceived utility, U_m, and randomizes with equal probability in case of a tie.

Unless we explicitly state otherwise, we focus on the case in which all alternatives are offered by strategic senders, so that M also corresponds to the set of senders. Still, the framework allows for the possibility that a subset of alternatives comprises outside options, for which the perceived value is not affected by senders' strategies. In particular, we will repeatedly consider the case of a single (monopolistic) sender, where the receiver's alternative is an outside option of fixed value R as in the preceding foundational analysis.[32]

[30] Such a fixed outside option can be easily accomodated by specifying that $u_m^i = R/2$ with probability one.

[31] Given that the distributions of preferences across alternatives are independent, we in particular need not specify whether sender m receives a different (or the same) payoff when alternatives $m' \neq m$ or $m'' \neq m$ are chosen.

[32] Such a fixed outside option can be easily accomodated by specifying that $u_m^i = R/2$ with probability one.

3.1 Non-Commitment Regime

Recall that, for our baseline model without regulation, we assume that the choice of s_m is an unobservable hidden action and that there is also no other way for the senders to commit to a certain information acquisition or disclosure strategy. Denote then the receiver's belief about sender m's information acquisition strategy for the case of pure strategies by $\widehat{s}_m \in \{y, n\}$. We solve the game backwards. At $t = 2$, a sender who chose $s_m = n$ can only disclose non-selectively. When this was anticipated by the receiver, $\widehat{s}_m = n$, recall that the distribution of U_m is given by $G_m(U_m) = N_m(U_m)$. Suppose now that $s_m = y$. Given that receivers place the same weight on both attributes and given that the fit for each attribute i is distributed according to the same distribution function $F_m(u_m^i)$ it is then easy to show that such an informed sender will disclose selectively, independently of receivers' beliefs $\widehat{s}_m$.[33] Hence, a receiver with belief $\widehat{s}_m = y$ also anticipates selective disclosure, such that, when a receiver rightly anticipates the sender's acquisition of information, $\widehat{s}_m = s_m = y$, then $G_m(U_m) = S_m(U_m)$, while when the receiver holds the wrong beliefs $\widehat{s}_m = n$, though it holds that $s_m = y$, we have $G_m(U_m) = \hat{S}_m(U_m)$.

We can then formalize the senders' incentives to become better informed in order to disclose selectively as follows. For a given realization of U_m, $w_m(U_m) = \prod_{m' \in M \setminus m} G_{m'}(U_m)$ is the "winning" likelihood with which the receiver chooses alternative m.[34] Hence, from an ex-ante perspective, alternative m is chosen with probability

$$q_m = \int w_m(U_m) dG_m(U_m).$$

The following is now an immediate implication of the fact that $G_m(U_m) = \hat{S}_m(U_m)$ dominates $G_m(U_m) = N_m(U_m)$ in the strict FOSD order and that $w_m(U_m)$ is non-decreasing.

Lemma 1 *Suppose senders are unable to commit to their information acquisition strategy. Then, in the unique equilibrium all senders choose to acquire information about the receiver's preferences and then disclose selectively ($s_m = y$ for all $m \in M$).*

We showed already that when the receiver compares this offering to an exogenous reservation value, the receiver benefits from such selective disclosure. This property now extends to sender competition. To formalize this insight, note first that the receiver realizes $U^{(1)} = \max_{m \in M} U_m$. Note also that in equilibrium the receiver's expectations about senders' strategies hold true ($\widehat{s}_m = s_m$). Pick now some $m \in M$ and denote by $U^{(1:M \setminus m)}$ the maximum over all remaining senders, with distribution $G^{(1:M \setminus m)}(U^{(1:M \setminus m)})$. For given U_m, the receiver accordingly realizes $E\left[\max\left\{U^{(1:M \setminus m)}, U_m\right\}\right]$. Given that the expression

[33] See however the Appendix B.2 for the case with asymmetric weights.

[34] We use here that, for all possibilities, $G_m(U_m)$ does not have mass points, though the subsequent analysis can be readily extended to the case where distributions have atoms.

in brackets is a convex function of U_m, reflecting the fact that taking m is an *option* for the receiver, the receiver obtains a higher expected utility after a mean-preserving spread in $G_m(U_m)$ (which is implied by the rotation property).

Lemma 2 *A switch from non-selective to selective disclosure by any sender m for which Property 2 holds strictly benefits a receiver who is aware of this, regardless of other senders' strategies.*

From now on, we assume that Property 2 holds for every sender m, so that S_m is a clockwise rotation of N_m. Combining Lemmas 1 and 2, we have the following equilibrium characterization:

Proposition 2 *When senders are unable to commit to an information acquisition strategy, the unique equilibrium outcome maximizes receiver's utility through selective disclosure by all senders.*

3.2 Equilibrium with Commitment

We now turn to circumstances in which senders have the ability to credibly commit to a certain information acquisition strategy.[35] This is so, in particular, when information acquisition requires either the direct cooperation or at least the consent of receivers, in which case receivers in fact directly observe a sender's attempt to become better informed about receiver preferences in order to disclose more selectively. We relate this to the question of (optimal) privacy regulation after the analysis of the equilibrium outcome.

In contrast to the preceding analysis without commitment, now a given sender m's preferred choice depends crucially on the distribution of a receiver's next best alternative, which we denoted by $G^{(1:M\backslash m)}(U^{(1:M\backslash m)})$. To illustrate, we return to the simple case analyzed in the preceding section, where the receiver's best alternative has a deterministic value (that is, $U^{(1:M\backslash m)} = R$). The receiver would then choose sender m's preferred alternative with probability $1 - S(R)$ when the sender discloses more selectively, rather than with probability $1 - N(R)$. The probability that the receiver accepts the sender's offering is strictly higher under more selective disclosure if and only if R lies to the right of the intersection of $S(U_m)$ and $N(U_m)$, while otherwise it is strictly lower. That is, in this example the sender prefers information acquisition and subsequent selective disclosure if the receiver's preferred alternative is sufficiently attractive (high R), while he otherwise prefers not to acquire information, which then essentially commits the sender to disclose

[35] While we do not assume that senders can commit to a particular disclosure strategy, since information acquisition is a prerequisite for any disclosure strategy that conditions on the realization of u^i, $i \in \{1, 2\}$, commitment not to acquire information in fact also implies commitment not to disclose selectively.

non-selectively. We now generalize this insight to the case where $U^{(1:M\setminus m)}$ is stochastic. For this we use the following auxiliary result:

Lemma 3 *Consider a single sender m and suppose the distribution of the receiver's next best alternative, $U^{(1:M\setminus m)}$, undergoes a shift resulting in a distribution that dominates in the likelihood ratio order. Then, if the sender weakly prefers selective disclosure before the shift, where the choice is observed by the receiver, he strictly does so after the shift. Likewise, if the sender weakly prefers non-selective disclosure after the shift, he strictly did so before the shift.*

To use this result, we need to map the comparative analysis into the model's primitives. Here, we focus on a comparative analysis in terms of competition, as expressed by the number of senders M.[36] To do so in a compact way, so that M is the only relevant variable to consider, we suppose that all senders $m \in M$ are symmetric, i.e., that the distributions $N_m(U_m)$ and $S_m(U_m)$ are identical across senders. Then pick any sender m and suppose that all other senders $m' \neq m$ disclose selectively such that $G_{m'}(U_{m'}) = S(U_{m'})$ (where we dropped the subscript due to symmetry). The receiver's best alternative (to m) is then distributed according to $G^{(1:M\setminus m)}(U_m) = S^{M-1}(U_m)$. Thus, as the number of senders increases from M to $M+1$ we obtain for the likelihood ratio of the receiver's outside option that

$$\frac{g^{(1:(M+1)\setminus m)}(U_m)}{g^{(1:M\setminus m)}(U_m)} = \frac{M}{(M-1)} S(U_m), \tag{9}$$

which is increasing in U_m. By property (9) we can now invoke Lemma 3 to obtain the following result:

Proposition 3 *Suppose that senders are symmetric and can commit to a disclosure strategy (e.g., as their information acquisition strategy s_m is observed by the receiver). Then there is a finite threshold M' such that for all $M \geq M'$ there exists a unique equilibrium where all senders disclose selectively ($s_m = y$ for all $m \in M$), while for $M < M'$ this outcome is not an equilibrium.*

Proposition 3, thus, provides an equilibrium characterization for (sufficiently) large M. Intuitively, as the number of senders increases, it becomes more and more likely that the receiver obtains a highly valuable offer elsewhere, so that from the perspective of sender

[36] An alternative way would be to hold M fixed and to consider a switch in the respective distribution $G_{m'}$ of any other sender. To illustrate, suppose $M = 2$ and that u_1^i is distributed uniformly over $[0, 1]$. For sender $m = 2$ take $G_2(U_2) = N_2(U_2)$ and suppose that u_2^i is also distributed uniformly but with support $[0, \overline{u}_2]$, for $0 < \overline{u}_2 < 3$. Then $m = 1$ prefers to become informed and disclose selectively, resulting in $G_1(U_1) = S_1(U_1)$, if $\overline{u}_2 > 3/4$ (i.e., if the receiver's alternative is relatively attractive), but prefers not to become informed if $\overline{u}_2 < 3/4$ (i.e., if the receiver's alternative is relatively unattractive).

m, more and more probability mass shifts to the upper tail in the distribution of the receiver's best "outside option", $U^{(1:M\setminus m)}$. As a consequence, each sender m prefers to disclose selectively in order to increase the probability of particularly high realizations of U_m.[37] Proposition 3 also covers the converse case: When M falls below the threshold M', there no longer is an equilibrium where all senders choose to disclose selectively.[38] There, the same intuition applies as in case of a deterministic outside option $U^{(1:M\setminus m)} = R$ with a low value of R: A sender does not want to disclosure selectively when, given a low M, the receiver's alternative is likely to be relatively unattractive. Taken together, receiver welfare is, thus, maximized if and only if competition among senders is sufficiently strong.

3.3 Implications for Privacy Regulation

Proposition 3 assumes that receivers are aware of a sender's attempt to become informed about their preferences in order to disclose selectively. Absent regulation, receivers may not be able to control the extent to which firms collect personal data, such as past purchases that reflect also on consumer preferences for the current offering. By Proposition 2, all senders would then indeed want to disclose selectively. Instead, from Proposition 3 this may no longer be the case when the receiver can observe senders' actions to collect such information, in which case we know from Lemma 2 that the receiver would be harmed. Consumer regulation that entails an outright prohibition of practices through which firms collect personalized data would then hurt consumers. Interestingly, even a less restrictive regulation requiring firms to seek consumer *consent* can backfire and lead to a reduction in consumer surplus, as we explain next.

This is the case when firms would like to commit not to become better informed about consumers' preferences before disclosing selectively, but cannot do so because consumers do not observe whether firms collect and use personalized data. Regulation that prescribes consumer consent then provides such commitment, which is in the interest of firms, at least when M is low, but not in the interest of consumers. Then, even regulation that, rather than prohibiting the collection and use of personal data, just requires consumer permission can backfire and decrease consumer welfare. We should however observe already now that when, for a particular application, firms can price discriminate based on their knowledge of consumers preferences, there is indeed scope for regulation, albeit only when competition is sufficiently weak. We return to this discussion in Section 4 below.

[37] This result can easily be generalized to the case of asymmetric senders, as long as distributions N_m and S_m have the same support across senders and sender types, as characterized by these distributions, are drawn from a finite set.

[38] We do not provide a complete characterization of equilibrium for the case with low M, in which existence can only be guaranteed by allowing for mixed strategies.

Unwary Receivers. Given the novelty of hypertargeting technology, receivers may well be unaware of senders' capability to collect and use data for selective disclosure. Suppose, thus, that receivers are unwary in that they believe that $\widehat{s}_m = n$ irrespective of the senders' incentives to acquire information and disclose selectively.[39] Then, when a sender chooses $s_m = y$ instead of $s_m = n$, the distribution of perceived valuation of an unwary receiver shifts in the FOSD order from $N_m(U_m)$ to $\hat{S}_m(U_m)$.[40] When there are no (regulatory) restrictions this immediately implies that senders will always choose selective disclosure if receivers are unwary of this (cf. Lemma 1). Our subsequent analysis thus focuses exclusively on receiver welfare.

In equilibrium, when an unwary receiver observes u_m^i, his perceived value U_m is inflated and exceeds the *true* conditional expected value by $E[u_m^i] - E[u_m^i \mid u_m^i \leq u_{d_m}]$. In Section 2.2 we already remarked that this shuts down one channel through which wary receivers can benefit from selective disclosure (strategic inference). Interestingly, with a monopolistic sender (and thus an outside option of fixed value for the receiver) we also have shown there that, despite his inflated expectations, an unwary receiver may still benefit from selective disclosure (cf. Proposition 1), although not to the same extend as wary receivers do. However, when there are multiple competing senders, matters are substantially different, as we show next.

This becomes most transparent when senders are symmetric, that is when $F_m(u_m^i) = F(u_m^i)$. Then, as all senders will follow the same selective disclosure strategy in equilibrium and the respective utilities are drawn from identical distributions, the receiver's perception of any sender's option will be equally inflated. This is the key difference to the previously considered case with a monopolistic sender, where the receiver compared an inflated perception to the fixed value of an outside option (R).[41] In fact, when there is symmetry across senders, the respective distortions exactly cancel out and selective disclosure affects an unwary receiver exactly in the same way as a wary receiver. We next derive this insight more formally.

[39] I.e., such unwary receivers do not adjust for the the fact that the informed sender discloses the most favorable attribute. Thus, they are effectively cursed, as in Eyster and Rabin (2005).

[40] Our analysis with unwary receivers could also be adapted to study settings where receivers have rational expectations but there is uncertainty about the selectivity of disclosure, e.g., because receivers do not perfectly know the state of a sender's targeting technology as in Grubb (2011). For instance, there might be uncertainty about a sender's costs of acquiring information, $c_m \in \{c_m^L, c_m^H\}$ with $p_L := \Pr(c_m = c_m^L)$. Then, faced with a low-cost sender, rational receivers indeed underestimate the selectivity of disclosure when only low-cost senders acquire information, and our analysis applies with $N_m(U_m)$ now denoting the distribution of the receiver's perceived utility in the (hypothetical) case where the sender is low-cost with probability p_L and high-cost with probability $1-p_L$, and $\hat{S}_m(U_m)$ the respective distribution when costs are indeed low $c_m = c_m^L$. The case of a high-cost sender, where receivers now overestimate the selectivity of disclosure, would then require a separate analysis. Still, in what follows, we stick to the interpretation of unwary receivers.

[41] This case, in fact, corresponds to an extreme form of asymmetry across senders.

For this recall first that in equilibrium each sender discloses $u_{d_m} = \max\{u_m^1, u_m^2\}$. As both the true expected valuation, $u_{d_m} + E[u_m^i | u_m^i \leq u_{d_m}]$, as well as the one perceived by the unwary receiver, $u_{d_m} + E[u_m^i]$, are strictly increasing in u_{d_m}, and, by symmetry, for a given u_{d_m} constant over m, an unwary receiver's decision rule is the same as that of a wary receiver, namely to choose the option for which the disclosed value u_{d_m} is maximal. Note that this observation clearly makes use of symmetry across senders, as we further discuss below. We thus have the following striking implication:

Proposition 4 *Consider competition between at least two symmetric senders with unwary receivers. Then, all senders acquire information and disclose selectively, and the resulting outcome is the same, in terms of receiver decisions and receiver welfare, as when receivers are wary.*

Proposition 4 is, at the same time, both stronger and weaker than the corresponding results we obtained with wary receivers. It is weaker as we only argue that receivers surely benefit from a given sender's selective disclosure when also all other senders disclose selectively, thereby preserving symmetry.[42] But it is stronger because, when receivers are unwary, senders always disclose selectively, while this was not necessarily the case when wary receivers observed senders' strategies. In this sense, once we take into account senders' equilibrium strategies, unwary receivers can be strictly better off than wary receivers. Hence, receivers' ignorance is bliss—receivers reap the benefits of more informed decision making even though they make the wrong inferences. This immediately leads on to the following observations regarding policy.

When senders compete on a level playing field, Proposition 4 shows that such competition already sufficiently protects unwary receivers, providing them with the same advantage of selective disclosure as wary receivers. Policy intervention that restricts selective disclosure would then be unwarranted, as in the case of wary receivers. What is now different however is that the previously discussed light-touch regulation of requiring consumer consent (to collect and use personalize data) would now no longer risk backfiring, as senders no longer benefit from committing not to selectively disclose information.

We conclude this section with some observations that apply when the economy contains a mix of wary and naive receivers. When senders can choose their information acquisition strategies individually for each receiver, then the preceding results apply (on a case-by-case logic) irrespective of whether all receivers are wary or unwary or whether there is a mixed composition of them. Suppose now instead that, while firms can disclose selectively to each receiver, their (now observed) investment in the collection of personalized data

[42] When only some but not all senders disclose selectively, the insights from the analysis with a monopolistic sender still apply qualitatively, given that the value of some options is then inflated (more) than those of others, so that we have to trade off two types of errors (i.e., false positives and negatives).

applies across all receivers. In this situation wary receivers can benefit from the presence of unwary receivers in case the (sufficiently large) presence of the latter induces senders to (observably) collect such data, while (notably for low M) they would not do so when the fraction of unwary receivers is smaller. Interestingly, this positive externality of unwary on wary consumers arises without hurting the former.[43]

Political Campaigning. While we so far focused on situations in which receivers take individual decisions, our equilibrium analysis and regulatory implications extend in a straightforward way also to cases of collective decision-making. To see this suppose, within an application to political campaigning, that $M = 2$ ex-ante symmetric candidates compete for voters. Denote by $U_m(z)$ voter z's expected utility when candidate m wins. In terms of motivation, candidates' platforms could comprise issues on which a candidate's stance can more or less coincide with the preference and political orientation of a particular voter. This generates scope for tailoring campaign messages to the political preferences of individual voters via selective disclosure.[44]

Clearly, this setting is identical to our previous analysis when the number of voters Z is equal to one. However, also for $Z > 1$, our previous analysis can be applied by noting that whenever a given voter is pivotal, his conditional expected utility equals $E[U^{(1)}]$, while it equals $E[U]$ else, as his vote then does not influence the collective decision. Hence, a voter's ex-ante expected utility becomes

$$E[U] + y\left\{E[U^{(1)}] - E[U]\right\},$$

where $y = \binom{Z-1}{\frac{Z-1}{2}}\frac{1}{2}^{Z-1}$ denotes the probability with which any given voter will become pivotal.[45] Now, we already know that $E[U^{(1)}]$, and, thus, the term in braces is strictly higher when one of the candidates discloses selectively, and even more so when both candidates do. Through selective disclosure, individual votes better reflect the preferences of individual voters and it is in this sense that our previous analysis can be applied.[46]

[43] See Armstrong (2015) for a survey of how the presence of both savvy (well informed and strategically sophisticated) as well as non-savvy consumers in a market can give rise to search externalities, according to which the non-savvy are protected by the savvy consumers, or ripoff externalities, which are present whenever the savvy consumers benefit from the presence of non-savvy types. Our externality is clearly different, as the presence of unwary consumers induces firms to choose (disclosure) strategies that also benefit wary consumers.

[44] This setting with voting relates to the problem of persuading a group to take a collective decision considered by Caillaud and Tirole (2007); in our model voters cast their ballot simultaneously rather than sequentially. See Alonso and Câmara (2016) for a model of optimal (rather than equilibrium) persuasion of voters in a setting with vertical (rather than horizontal) differentiation.

[45] Note that, in equilibrium, the ex-ante likelihood of a vote for any of the two candidates must be equal to $\frac{1}{2}$.

[46] While we cast our analysis into a framework where disclosure can be personalized for each individual receiver, the results also apply when communication is more coarse as it can be targeted (only) to groups

Corollary 1 *Suppose Z voters decide by majority rule over $M = 2$ ex-ante symmetric candidates. A switch from non-selective to selective disclosure by either (both) candidate(s) benefits each wary (unwary) voter, strictly so conditional on being pivotal. When candidates are unable to commit to their information acquisition strategy, in the unique equilibrium both choose to acquire information and then disclose selectively, maximizing voter utility. Privacy regulation restricting information acquisition by campaigners strictly reduces welfare.*

4 Equilibrium with Personalized Pricing

Within our main application to marketing, a distinctive feature of our baseline analysis, where now senders represent firms and each receiver represents an individual consumer deciding which product to purchase, is that firms do not adjust prices individually, based on their knowledge of consumer preferences. Given that each firm offers all consumers the same product, even when it selectively gives them different information, such personalized pricing may be difficult with physical goods that can be easily resold. Price discrimination would then create scope for arbitrage, either through a grey (or parallel) market between consumers or through the activity of intermediaries.[47] These arguments motivate why there are circumstances under which our baseline analysis seems suitable. In other markets, however, because of transaction costs arbitrage may be less of a concern. Accordingly, this section turns to situations in which firms are not only able to learn about the preferences of consumers and target their communication accordingly, but are also able to use this detailed information to charge personalized prices to customers.[48]

4.1 Firm Preferences with Personalized Pricing

With competition, we stipulate that firms learn the utility that the consumer perceives for each product, for example, on the basis of some commonly collected information. When no firm chooses weakly dominated prices, this ensures that, first, the consumer still purchases

of voters through the particular media that they consume. In this sense, our analysis also apply when candidates strategically adjust their messages to each individual channel in an increasingly fragmented media landscape.

[47] Also, price discrimination may be limited when consumers are concerned about fairness. Price (or rate) parity has become a major objective for firms, e.g., hotels, given the increasing transparency via online channels. Furthermore, when the considered channel may only represent one among several (online or offline) distribution channels, the firm's pricing flexibility for this channel may be seriously compromised, so that we may indeed abstract away from pricing differences depending on the firm's disclosure policy.

[48] The industrial organization literature on behavior-based price discrimination has focused on personalized pricing where, in particular, the past purchasing history of consumers is used; see, for example, Villas-Boas (1999) and Acquisti and Varian (2005). As we abstract from this dynamic feature, our analysis will be quite different.

the product with the highest perceived utility $U^{(1)} = \max_{m \in M} U_m$, and that, second, the price that the consumer pays is equal to the incremental utility relative to the second-highest such value, denoted by $U^{(2)}$. This is, thus, in the spirit of so-called mill pricing in a Hotelling model, where firms know the location of each consumer, which perfectly reflects the valuation for each individual product, and can make prices specific for each consumer location. Consequently, a consumer realizes the second order-statistic $U^{(2)}$.[49] We first establish that with personalized pricing all firms prefer to disclose more selectively, now regardless of whether this is observed by consumers or not and irrespective of the intensity of competition (and thus in contrast to our previous findings without personalized pricing; see Proposition 3).

Recall our notation $U^{(1:M \backslash m)}$ for the highest expected utility over all other $M \backslash m$ firms. Then, the expected profit of firm m is given by

$$\int \left[\int \max \left\{ U_m - U^{(1:M \backslash m)}, 0 \right\} dG^{(1:M \backslash m)}(U^{(1:M \backslash m)}) \right] dG_m(U_m).$$

As the term in rectangular brackets is a convex function of U_m,[50] it is higher after a mean-preserving spread in $G_m(U_m)$. With personalized pricing, a firm that offers a consumer's preferred choice—and can thus make a profit—wants to maximize the distance between the consumer's expected utility for the firm's own product and the utility for the product of its closest rival, because the firm extracts exactly this difference. From an ex-ante perspective, the firm thus prefers a greater dispersion of U_m. As, trivially, the firm also benefits from a FOSD shift in $G_m(U_m)$, we have the following result:

Proposition 5 *With personalized pricing, any given firm m prefers to disclose selectively, irrespective of whether this is anticipated by the consumer or not. Thus, with personalized pricing it is the unique equilibrium outcome that all firms disclose selectively ($s_m = y$ for all $m \in M$).*

4.2 Consumer Preferences with Personalized Pricing

From the perspective of consumers, the effect of selective disclosure depends now crucially on the degree of competition. Before we derive these results, note the stark contrast to the baseline case without personalized pricing, where the degree of competition affects firms' but not consumers' preferred choice of disclosure policy. The opposite holds with personalized pricing: the degree of competition affects consumers' but not firms' preferences with regards to selective disclosure.

[49] With a monopolistic seller, consumers are, thus, always pushed down to their reservation utility R.

[50] Its derivative with respect to U_m is $G^{(1:M \backslash m)}(U_m)$ which is increasing in U_m.

As a starting point, consider a duopoly with $M = 2$, where the differences between the two cases are particularly stark. While without personalized pricing a consumer realized the maximum of the two expected utilities $U^{(1)} = \max\{U_1, U_2\}$, with personalized pricing the consumer now realizes the second-highest value, which for $M = 2$ is the minimum $U^{(2)} = \min\{U_1, U_2\}$. The consumer is now strictly worse off when any of the presently considered two firms discloses selectively. Formally, this can be seen in complete analogy to the argument for why without personalized pricing the consumer was strictly better off.

More concretely, with personalized prices the consumer's expected utility is

$$E[U^{(2)}] = \int \left[\int \min \{U_m, U_{m'}\} \, dG_{m'}(U_{m'}) \right] dG_m(U_m).$$

As the expression in rectangular brackets is now a strictly concave function of U_m,[51] while it was a strictly convex function when without personalized pricing we applied the maximum, it is lower after a mean-preserving spread in $G_m(U_m)$. The intuition for this is that when firm m discloses selectively, a consumer's updating makes firms, from an ex-ante perspective, more differentiated. This ensures that in expectation the (winning) firm with the highest perceived value can extract a higher price. As we show next, however, this detriment to consumers from increased differentiation is reduced as M increases, when it becomes increasingly likely that each firm has a close competitor, in which case the efficiency benefits resulting from selective disclosure again accrue to the consumer, as stated in Lemma 2.

Proposition 6 establishes that regardless of what all other firms do, when there are sufficiently many firms a consumer strictly benefits when a particular firm discloses selectively, so that $S_m(U_m)$ instead of $N_m(U_m)$ applies. More precisely, consumers benefit from selective disclosure by firm m also under personalized pricing if and only if

$$\int [N_m(U_m) - S_m(U_m)] \left[G^{(2:M \backslash m)}(U_m) - G^{(1:M \backslash m)}(U_m) \right] dU_m > 0. \tag{10}$$

Expression (10) intuitively captures the fact that, from a consumer's perspective, with personalized pricing the *precise* realization of U_m only matters when it falls between the first and second highest realizations of all other $M - 1$ utilities, which happens with probability $G^{(2:M \backslash m)}(U_m) - G^{(1:M \backslash m)}(U_m)$. Furthermore, given single-crossing of $N_m(U_m)$ and $S_m(U_m)$, we can sign expression (10) unambiguously to be positive whenever there are sufficiently many firms (high M), as then, somewhat loosely speaking, both the first and the second highest value of all other $M - 1$ utilities take on high realizations (that is,

[51] It can be written as $U_m - \int_{\underline{U}}^{U_m} [U_m - U_{m'}] dG_{m'}(U_{m'})$, with first derivative $1 - G_{m'}(U_m)$, which is decreasing.

to the right-hand side of the rotation point). To express our results succinctly, we again impose symmetry across senders.[52]

Proposition 6 *Suppose that senders are symmetric. Then, with personalized pricing, wary consumers are indifferent between selective and non-selective disclosure by a monopolist ($M = 1$). With a duopoly ($M = 2$), consumers are always strictly worse off when a firm m switches to selective disclosure, regardless of the disclosure strategy of the rival firm. However, irrespective of the other firms' choices, consumers strictly benefit when any firm m chooses selective disclosure provided that there is sufficient competition (M large).*

Proposition 6 relates to results by Board (2009) and Ganuza and Penalva (2010) on the effect of providing bidders with private information in a private-values second-price auction. With personalized pricing, a comparison of consumer surplus in our model effectively amounts to comparing the expectation of the second-order statistic $E[U^{(2)}]$, as in a second-price auction. In these papers, however, the question that is asked is whether providing more information to *all* bidders increases the auctioneer's expected payoff, while for Proposition 6 we ask whether more information held by a *single* firm benefits the consumer.

4.3 Implications for Regulation

As previously, we summarize our findings in terms of potential regulation of consumer privacy. Consumers can now be worse off under selective disclosure if this is combined with personalized pricing. Without personalized pricing, this could not be the case. A second difference arises from senders' (firms') preferences. As with personalized pricing firms will always want to collect personal data and disclose selectively, regulation that requires consumers' consent can no longer backfire by granting firms commitment power, as in the baseline case. Regulation that strictly prohibits the collection and use of personal data will, however, reduce efficiency and consumer welfare when competition is sufficiently intense.

Suppose now again that consumers remain unwary of firms' capability to collect and use personally identifiable data for selective disclosure such that consumers' perceived value of the respective products U_m risks being inflated. While for the baseline analysis we showed that consumers may still be better off even with a monopolistic firm, with personalized pricing this is clearly no longer the case. Personalized pricing allows the firm to extract a consumer's *perceived* incremental utility relative to her next best choice. If the

[52] The subsequent results easily generalize to the case of asymmetric senders as long as distributions N_m and S_m have the same support across senders and sender types, as characterized by these distributions, are drawn from a finite set.

perceived utility is inflated, this generates the potential for consumer exploitation. Facing a monopolistic firm, an unwary consumer would be better off staying out of the market.

Competition between senders, however, protects naive consumers, also with personalized pricing. As in the baseline case, this is most immediate when, under symmetry, the perception of the different offers is equally inflated, so that the decision of an unwary consumer fully matches that of a wary consumer. Interestingly, in this case an unwary consumer is now strictly better off than a wary one when personalized pricing is feasible, as he pays a lower price. This striking result follows because unwary consumers do not adjust expectations for firms' selection bias, which works towards reducing the perceived difference between the first-best and second-best alternative. Precisely, if $u^{(1)}$ is the highest disclosed fit (attribute) and $u^{(2)}$ is the second highest, an unwary consumer pays the price $\widehat{p} = u^{(1)} - u^{(2)}$, given that the expectations about the non-disclosed attribute of either firm wrongly remain unchanged at $E[u^i]$. A wary consumer pays, instead, the strictly higher price

$$p = u^{(1)} - u^{(2)} + \left(E[u^i \mid u^i \leq u^{(1)}] - E[u^i \mid u^i \leq u^{(2)}] \right),$$

given that the term in brackets, equal to the difference in the updated conditional expectations for the attributes not disclosed by firms 1 and 2, is strictly positive. In other words, in the eyes of an unwary consumer, the firms offering the first-best and the second-best fit appear to be less differentiated, compared to the perceptions of wary consumers. We have thus the peculiar situation where any regulation that would *not* affect firms' strategies to gather and disclosure information but would *only* increase receivers' (consumers') awareness would not be in the interest of consumers, but in the interest of firms. Abstracting from these finer results, we summarize our implications for regulation as follows, based notably on Proposition 6:

Proposition 7 *When symmetric firms can not only selectively disclose information but also price discriminate based on collected personal data, regulation that prohibits the collection of such data benefits consumers if and only if there is insufficient competition.*

5 Conclusion

The greater availability of personally identifiable data opens up new opportunities for tailoring (advertising) messages to the perceived preferences of particular consumers. This is the main application for the preceding analysis, and much of the analyzed regulation is also discussed in light of the greater availability of data on the internet. Before we summarize our main results, we would like to point out that our analysis applies however more broadly. For this consider a face-to-face interaction between a salesperson and an individual consumer. Even when meeting a consumer for the first time, an experienced salesperson should be able to draw inferences about the consumer's needs and preferences and use the limited time available (or the consumer's limited attention) to communicate only those product attributes that dovetail nicely with those preferences. We leave it for further research to develop a theory of the skills of a good salesperson based on his ability to learn about consumers' preferences and to build up his sales talk accordingly. Furthermore, also old media may allow at least for a segmentation of receivers into coarse groups, so that different messages can be sent to groups with different preferences. Data collection on the internet naturally increases firms' ability to target their communication to individual receivers.

To summarize, in our baseline setting with individual decisions at fixed prices, wary receivers benefit from selective disclosure for a broad set of distributions satisfying log-concavity. However, senders' incentives to become better informed—as a basis of (more) selective disclosure—are more subtle. In the absence of policy intervention, we naturally assume that receivers *do not observe* the senders' choice of information acquisition. Thus, for given expectations by the receivers, senders have an incentive to become better informed because, off-equilibrium, they would increase the chances that their offering is chosen. Even though senders' own incentives force them to become better informed, (more) selective disclosure ends up either benefitting or hurting them in equilibrium. Notably, senders benefit from increased information only when competition is intense. Policy intervention that makes information acquisition *observable*, for example by requiring consumer consent, may end up hurting consumers because senders are then able to commit not to acquire information by not requiring consent.

The introduction of personalized pricing changes the outcome of our baseline analysis in one important way.[53] The extent to which the efficiency gains associated with more informative communication are shared between firms and consumers depends now on the degree of competition. As selective disclosure based on better information is also found to dampen competition by increasing perceived differentiation, there is thus a trade-off from consumers' perspective. Policy intervention is notably not warranted when there is sufficient competition. For policy purposes we also consider the case where (some) receivers remain naive about senders' capabilities, thereby not properly discounting their valuation for the adverse selection that is implicit in the fact that the disclosed attribute is the most favorable. Again, a particularly clear-cut case is that with personalized pricing and insufficient competition, as naive consumers then risk being exploited, so that they would be better off staying out of the market. On the other hand, (sufficient) competition allows naive receivers to benefit as much, and sometimes even more, from selective disclosure as wary consumers. The key to the latter observation is that when senders are equally

[53] Such price discrimination may only be feasible for services or low-value products, when customers or intermediaries have little scope for arbitrage.

positioned to selectively disclose, the effect of inflated values effectively cancels out.

To conclude, we obtained our results in a stylized model where senders can freely acquire and divulge information only about their own offerings in a private-value environment.[54] An extension could allow senders to disclose information about their competitors. Information could be costly, as in the law and economics literature on transparency. Information could be also sold by an information broker as in Taylor (2004), Bergemann and Bonatti (2015), and Montes, Sand-Zantman, and Valletti (2018).

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[54] Our analysis abstracts away from externalities across the communication strategies of firms due to congestion effects and information overload; see, for example, Van Zandt (2004) and Anderson and de Palma (2012) for analyses in this direction using models à la Butters (1977). See also Johnson (2013) for a welfare analysis of the impact of targeted advertising in the presence of advertising avoidance by consumers.

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Appendix A Proofs

A.1 Impact of Selective Disclosure

In this Appendix we collect the results and proofs omitted from Section 2; for additional material and verification of the examples see Appendix B.1.

Proof of Proposition 1. Suppose that $R \in [2E[u], \overline{u} + E[u]]$. Slice the shaded regions A, B, Y, and Z into iso-payoffs symmetrically with respect to R, as shown in Figure 4. For every offset level $\delta > 0$ draw two iso-payoffs on either side of R, with equations $u^2 = R - \delta - u^1$ and $u^2 = R + \delta - u^1$.

First, consider the iso-payoffs for which $\delta \in [0, R - (\overline{u} + \underline{u})]$ in areas B and Y. To show that the expected gain in Y is higher than the expected loss in B, on the iso-payoff of level $R - \delta$ take a generic point $b \in B$ with coordinates $(u^1 = -\delta - \varepsilon + E[u], u^2 = R - E[u] + \varepsilon)$ where $\varepsilon \in [0, \overline{u} - R + E[u]]$ is the vertical distance of u^2 from $R - E[u]$. The expected loss relative to R at this point is

$$-\delta f(-\delta - \varepsilon + E[u]) f(R - E[u] + \varepsilon). \tag{11}$$

At the corresponding point $y \in Y$, with coordinates $(u^1 = \delta - \varepsilon + E[u], u^2 = R - E[u] + \varepsilon)$ on the iso-payoff of level $R + \delta$, the expected gain relative to R is

$$\delta f(\delta - \varepsilon + E[u]) f(R - E[u] + \varepsilon). \tag{12}$$

Note that the horizontal distance of y to $E[u]$ is lower than the horizontal distance of b to $E[u]$, i.e., $|(\delta - \varepsilon + E[u]) - E[u]| \leq |E[u] - (-\delta - \varepsilon + E[u])|$ given that $\varepsilon \geq 0$. If F is unimodal, the density decreases the farther away the realization is from $E[u]$, thus, using symmetry we have $f(-\delta - \varepsilon + E[u]) \leq f(\delta - \varepsilon + E[u])$. Summing the gains (12) and losses (11) over δ and ε in regions Y and B, we conclude that the expected gain is greater than the expected loss

$$\int_0^{R-(\overline{u}+\underline{u})} \int_0^{\overline{u}-R+E[u]} \delta f(R - E[u] + \varepsilon) [f(E[u] - \varepsilon + \delta) - f(E[u] - \varepsilon - \delta)] \, d\varepsilon d\delta \geq 0. \tag{13}$$

Second, construct paired iso-payoffs with offset $\delta \in [R - (\overline{u} + \underline{u}), E[u] - \underline{u}]$ in regions A and Z. The following one-to-one function maps each point in A to each point in Z

$$(E[u] - \delta - \varepsilon, R - E[u] + \varepsilon) \to (R - E[u] - \varepsilon, E[u] + \delta + \varepsilon),$$

where $\varepsilon \in [0, E[u] - \underline{u} - \delta]$, illustrated by the stars in Figure 4. To compare the density of points in A and Z, we map all points (such as the original star) of A to points in X (such as the dot within X marked in Figure 4) through the function $(E[u] - \delta - \varepsilon, R - E[u] + \varepsilon) \to (E[u] + \delta + \varepsilon, R - E[u] + \varepsilon)$. Given that region X is symmetric to region A with respect to $E[u]$ and that F is symmetric, this first mapping preserves the density of the

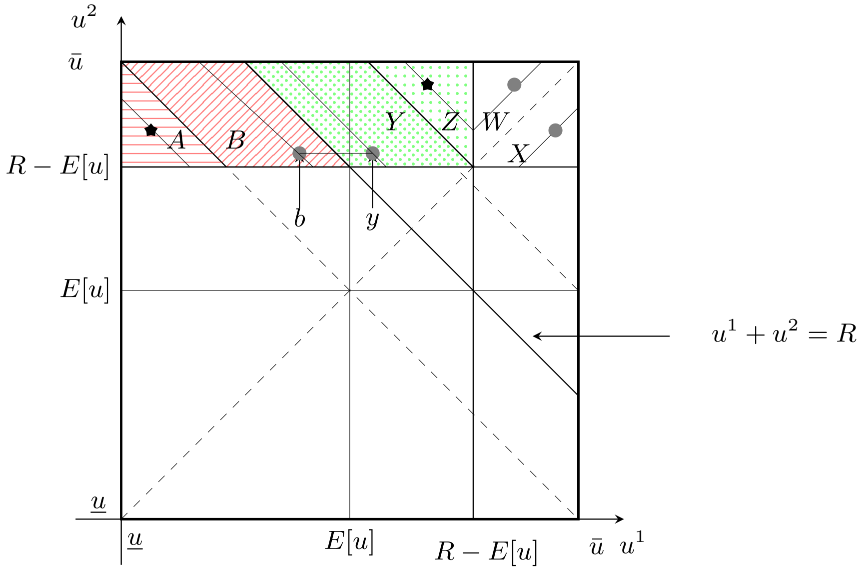

Figure 4: Welfare impact on unwary receiver.

points. Second, map each point in X to a point in W through the inverse function $(E[u]+\delta+\varepsilon, R-E[u]+\varepsilon) \to (R-E[u]+\varepsilon, E[u]+\delta+\varepsilon)$. Given that u^1 and u^2 are i.i.d., the density of the points is preserved. Thus, the initial star in A has the same density as the final dot in W. Comparing the density of points in A and Z is therefore equivalent to comparing the density of points in W and Z. The density of the star in Z is higher than the density of the dot in W because it is closer to $E[u]$: $f(R-E[u]-\varepsilon) \geq f(R-E[u]+\varepsilon)$. We conclude that the expected gain is greater than the expected loss

$$\int_{R-(\overline{u}+\underline{u})}^{E[u]-\underline{u}} \int_0^{E[u]-\underline{u}-\delta} \delta f(\delta+E[u]+\varepsilon)\left[f(R-E[u]-\varepsilon)-f(R-E[u]+\varepsilon)\right] d\varepsilon d\delta \geq 0. \tag{14}$$

We have established that if $R \in [2E[u], \overline{u}+E[u]]$ and the distribution F is unimodal, the unwary receiver is better off under selective than non-selective disclosure.[55]

To complete the proof, note that if $R \geq \overline{u}+E[u]$ the receiver never accepts the

[55] Alternatively, to compare the density of points in regions A and Z we calculate the radiuses of the circles, with center $(E[u], E[u])$, that pass through two corresponding points. The i.i.d. and symmetry assumptions imply that points on the same circle have the same density. Unimodality then implies that points with a higher distance from the center of Figure 4 have a lower density. The radius of a point in A is

$$(E[u]-\delta-\varepsilon-E[u])^2+(R-E[u]+\varepsilon-E[u])^2,$$

while the radius of a point in Z is

$$(R-E[u]-\varepsilon-E[u])^2+(E[u]+\delta+\varepsilon-E[u])^2.$$

Thus, the density of a point in Z is higher than the corresponding point in A if and only if $4E[u]\varepsilon-2\varepsilon R \leq 2\varepsilon R-4E[u]\varepsilon$, i.e., $R \geq 2E[u]$.

offering and thus always obtains utility R regardless of the disclosure regime. If, instead, $R \leq 2E[u]$, the points corresponding to losses (to the left of the iso-payoff of level R) are now closer to $E[u]$ and thus become more likely than those corresponding to gains (with the same offset level to the right of the iso-payoff). Because of this reversal in the distance of the points from $E[u]$, if F is unimodal and $R \leq 2E[u]$, the unwary receiver is better off under non-selective disclosure than under selective disclosure. **Q.E.D.**

A.2 Equilibrium Analysis

In this Appendix we collect all omitted proofs from Sections 3 and 4.

Proof of Lemma 2. The receiver's expected utility can be written as

$$E[U^{(1)}] = \int \left[\int \max \left\{ U^{(1:M\setminus m)}, U_m \right\} dG^{(1:M\setminus m)}(U^{(1:M\setminus m)}) \right] dG_m(U_m). \tag{15}$$

Given that the expression in brackets is a convex function of U_m,[56] it is higher after a mean-preserving spread in $G_m(U_m)$. **Q.E.D.**

Proof of Lemma 3. For any given distribution of the receiver's next best alternative $G^{(1:M\setminus m)}(\cdot)$, the difference in the likelihood that option m is chosen when sender m switches from $N_m(\cdot)$ to $S_m(\cdot)$ is given by[57]

$$\Delta q_m = \int_{\underline{U}}^{\overline{U}} G^{(1:M\setminus m)}(U_m) d\left[S_m(U_m) - N_m(U_m)\right] = \int_{\underline{U}}^{\overline{U}} Z_m(U_m) dG^{(1:M\setminus m)}(U_m), \tag{16}$$

where, with $Z_m(U_m) = N_m(U_m) - I_m(U_m)$, the second line follows from integration by parts. Now consider two choices for the distribution $G^{(1:M\setminus m)}(\cdot)$: $H'(\cdot)$ and $H''(\cdot)$, where the latter dominates in the likelihood ratio order. Further, denote the support of H' by $\left[\underline{U}', \overline{U}'\right]$ and by $\left[\underline{U}'', \overline{U}''\right]$ the respective support of H'', where from the likelihood ratio property we must have that $\underline{U}' \leq \underline{U}''$ as well as $\overline{U}' \leq \overline{U}''$. We now apply the following

[56] It can be written as $U_m + \int_{U_m}^{\infty} [U^{(1:M\setminus m)} - U_m] dG^{(1:M\setminus m)}(U^{(1:M\setminus m)})$ with first derivative $G^{(1:M\setminus m)}(U_m)$, which is increasing.

[57] The upper and lower bounds of the integral are chosen to contain the supports of $N_m(\cdot)$ and $S_m(\cdot)$.

transformations, focusing on the non-trivial case where $\widetilde{U}_m$ satisfies $\underline{U}'' \leq \widetilde{U}_m \leq \overline{U}'$:[58]

$$\begin{aligned}
\Delta q''_m &= \int_{\underline{U}'}^{\overline{U}''} Z_m(U_m)h''(U_m)dU_m \\
&= \int_{\underline{U}'}^{\widetilde{U}_m} Z_m(U_m)\frac{h''(U_m)}{h'(U_m)}h'(U_m)dU_m + \int_{\widetilde{U}_m}^{\overline{U}''} Z_m(U_m)\frac{h''(U_m)}{h'(U_m)}h'(U_m)dU_m \\
&> \frac{h''(\widetilde{U}_m)}{h'(\widetilde{U}_m)}\int_{\underline{U}'}^{\overline{U}'} Z_m(U_m)h''(U_m)dU_m = \frac{h''(\widetilde{U}_m)}{h'(\widetilde{U}_m)}\Delta q'_m,
\end{aligned}$$

where, for the second line, we have used that $Z_m(U_m) < 0$ for $U_m < \widetilde{U}_m$ and $Z_m(U_m)$ for $U_m > \widetilde{U}_m$ and, for the third line, that $\frac{h''(U_m)}{h'(U_m)}$ is increasing in U_m. **Q.E.D.**

Proof of Proposition 3. Given Lemma 3 it only remains to prove that such a (finite) cutoff M' indeed exists. Take some $U \in (\widetilde{U}, \overline{U})$ (where we have dropped the subscript due to sender symmetry). Then for arbitrary small, but strictly positive, $\varepsilon > 0$ there exists a finite boundary $\widehat{M}$ such that for all $M > \widehat{M}$ we have, by construction, that $G^{(1:M\backslash m)}(U) = S^{M_S}(U)N^{M-M_S-1}(U) < \varepsilon$, where M_S denotes the number of firms other than m that disclose selectively. I.e., almost all mass for the second-best alternative lies above the considered value $U > \widetilde{U}$. The assertion follows then immediately from expression (16) in the preceding proof and the single-crossing mean-preserving spread between $N(U)$ and $S(U)$. **Q.E.D.**

Proof of Proposition 6. It remains to show the result for large M. To simplify the exposition, without loss of generality we consider the choice of firm $m = 1$ and thus a switch from $G_1(U_1) = N_1(U_1)$ to $G_1(U_1) = S_1(U_1)$. Note also that the case with $M = 2$ was already fully solved in the main text and that, presently, we are interested in the case for high M, which is why without loss of generality we can assume that $M \geq 3$. Denote by $v_1(U_1)$ a consumer's expected utility for given U_1, so that the respective difference in ex-ante utility is given by

$$\int_{\underline{U}}^{\overline{U}} v_1(U_1)d\left[S_1(U_1) - N_1(U_1)\right]. \tag{17}$$

Next, note that $v_1(U_1)$ has derivative

$$\eta(U_1) = G^{(2:M\backslash 1)}(U_1) - G^{(1:M\backslash 1)}(U_1) = \sum_{m\in M\backslash 1}\left([1 - G_m(U_1)]\prod_{m'\notin\{1,m\}} G_{m'}(U_1)\right).$$

[58] Note that from the restrictions that condition (8) imposes on $H'(\cdot)$ and $H''(\cdot)$ it holds that $N_m(\cdot)$ and $S_m(\cdot)$ have strictly positive mass for $U_m \in \left[\underline{U}'', \overline{U}'\right]$. Then, if $\underline{U}'' > \widetilde{U}_m$, it is immediate that $S_m(\cdot)$ first-order stochastically dominates $N_m(\cdot)$ on the relevant support, such that we must have $\Delta q''_m > 0$; similarly, if $\overline{U}' < \widetilde{U}_m$, we must have that $\Delta q'_m < 0$.

So, using integration by parts, we can transform (17) to obtain

$$\int_{\underline{U}}^{\overline{U}} \eta(U_1)[N_1(U_1) - S_1(U_1)]dU_1. \tag{18}$$

We now drop the respective subscripts due to sender symmetry and define $\phi(U) = \eta(U)/\eta(\widetilde{U})$. (Recall that $\widetilde{U}$ denotes the rotation point.) Then, extending the expression in (18), for each realization of U, by multiplying and dividing with the term $\eta(\widetilde{U})$, and noting that $\eta(\widetilde{U}) > 0$, a sufficient condition for (18) to be greater than zero is that

$$\int_{\underline{U}}^{\widetilde{U}} [N(U) - S(U)]\, \phi(U)dU + \int_{\widetilde{U}}^{\overline{U}} [N(U) - S(U)]\, \phi(U)dU > 0. \tag{19}$$

Then, noting that for $U < \widetilde{U}$ it holds that $\phi(U) \leq \frac{1-N(U)}{1-N(\widetilde{U})}\left(S(U)/S(\widetilde{U})\right)^{M-2}$, the first (negative) integral in (19) converges to zero as $M \to \infty$, regardless of whether $G_m = N$ or $G_m = S$ for any other sender $m \neq 1$. It remains to show that the second (positive) integral remains bounded away from zero, which follows immediately as, for any $\widetilde{U} < U < \overline{U}$, we can write $\phi(U) \geq \frac{1-N(U)}{1-N(\widetilde{U})}\left(N(U)/N(\widetilde{U})\right)^{M-2} > 0$. **Q.E.D.**

Appendix B Supplementary Material

B.1 Impact of Selective Disclosure

Verification of Material Reported in Text. Here we collect additional material relevant for the analysis and examples reported in Sections 2.2 and 2.3.

Verification of Observation 1. For all R the difference between the receiver's values under selective and non-selective disclosure is

$$V_S - V_N = \frac{(V_S - V_{\hat{S}}) + (V_S - V_{\hat{T}})}{2} > \frac{(V_S - V_{\hat{T}})}{2} > 0, \tag{20}$$

where the first inequality follows from $V_S - V_{\hat{S}} > 0$ and the second inequality uses the assumption $V_S > V_{\hat{T}}$.

Verification of Observation 1*. Suppose by contradiction that at some R we have $V_{\hat{S}} > V_{\hat{T}}$. Then at that R, (20) holds, i.e., $V_S > V_N$. This contradiction proves the claim.

Example B.1 (Exponential) *The unwary receiver's expected payoff is always (for all R) higher under selective rather than non-selective disclosure, $V_{\hat{S}} > V_N$, if u is exponentially distributed, $f(u) = \lambda e^{-\lambda u}$ with $\lambda > 0$ and support $[0, \infty)$.*

Verification of Example B.1. Evaluating $V_N = \int_0^\infty \max \langle R, u + E[u] \rangle \, dF(u)$ and $V_{\hat{S}} = RF(R - E[u])^2 + \int_{R-E[u]}^\infty [u_d + E[u|u \leq u_d]] \, dF(u_d)^2$ using $F(u) = 1 - e^{-\lambda u}$, $E[u] = 1/\lambda$ and $E[u|u \leq u_d] = 1/\lambda - u_d e^{-\lambda u_d} / \left(1 - e^{-\lambda u_d}\right)$, we obtain after some transformations

$$V_N - V_{\hat{S}} = e^{-\lambda\left(R - \frac{1}{\lambda}\right)} \left[R e^{-\lambda\left(R - \frac{1}{\lambda}\right)} - \frac{1}{\lambda} \right].$$

The result then follows from noting that $Re^{-\lambda\left(R-\frac{1}{\lambda}\right)}$, with $R \geq 1/\lambda$, is maximized at $R = 1/\lambda$ where it takes on value $1/\lambda$.

We next establish the result that S is a clockwise rotation of N for a large set of commonly used distributions F that satisfy logconcavity. To reframe the problem, note that observation of $x = \max \langle u^1, u^2 \rangle$ under selective disclosure corresponds to a crossing point between S and N if and only if $F(x)^2 = F(\mathcal{U}(x) - E[u])$.[59] Because F is strictly increasing and continuous, we can conveniently define the function

$$\gamma(x) := F^{-1}\left(\sqrt{F(\mathcal{U}(x) - E[u])}\right) \text{ for } x \in (\mathcal{U}^{-1}(\underline{u} + E[u]), \overline{u}),$$

whose fixed points exactly correspond to crossings between S and N. In the following Proposition we exploit this property of function γ to characterize the set of distributions F satisfying clockwise rotation.

[59] To any crossing point $\tilde{U}$ between S and N, where by definition $N(\tilde{U}) = S(\tilde{U})$, there correspond two realizations $x = \max \langle u^1, u^2 \rangle$ and u^i of the disclosed variable that induce receiver's expected utility $\tilde{U} = \mathcal{U}(x)$ and $\tilde{U} = u^i + E[u]$, respectively under selective and non-selective disclosure. Thus, x corresponds to a crossing point between N and S if and only if $S(\mathcal{U}(x)) = F(x)^2 = F(\mathcal{U}(x) - E[u]) = N(\mathcal{U}(x))$.

Proposition B.1 *The distribution S of $E\left[u^1+u^2\right|\max\left\langle u^1,u^2\right\rangle]$, the expected sum given the maximum, crosses only once and from above the distribution N of the expected sum given a single variable, $E\left[u_1+u_2|u_1\right]$, in the interior of the support if and only if:*
(A) the left-hand integral $L(u)=\int_{\underline{u}}^{u}F(y)\,dy$ is logconcave at the upper bound and
(B) the function γ has a unique fixed point in the interior of the domain $\left(\mathcal{U}^{-1}\left(\underline{u}+E[u]\right),\overline{u}\right)$.

Proof of Proposition B.1. We first establish that the left-hand integral $L(u)=\int_{\underline{u}}^{u}F(y)\,dy$ is (strictly) logconcave at the upper bound $\Leftrightarrow x<\gamma(x)$, or equivalently $S(U)<N(U)$, in a left neighborhood of the upper bound of the support. To prove this, note that there exists a value x_l such that $x<\gamma(x)$ for all $x>x_l$ if and only if at the upper bound $\left.\frac{d\gamma(x)}{dx}\right|_{x=\overline{u}}<1$. From $\gamma(x)=F^{-1}\left(\sqrt{F\left(\mathcal{U}(x)-E[u]\right)}\right)$ and $\mathcal{U}(x)=2x-\frac{L(x)}{F(x)}$ we have

$$\begin{aligned}\frac{d\gamma(x)}{dx} &= \frac{\mathcal{U}'(x)}{2}\cdot f\left(\mathcal{U}(x)-E[u]\right)\cdot\frac{1}{2}\left(F\left(\mathcal{U}(x)-E[u]\right)\right)^{-\frac{1}{2}}\cdot\frac{1}{f(\gamma(x))}\\ &= \frac{\mathcal{U}'(x)}{2}\cdot\frac{\frac{f(\mathcal{U}(x)-E[u])}{F(\mathcal{U}(x)-E[u])}}{\frac{f(\gamma(x))}{F(\gamma(x))}} \end{aligned}\tag{21}$$

where for the second equality we have multiplied and divided by $F\left(\mathcal{U}(x)-E[u]\right)$ and used the fact that $\left(F\left(\mathcal{U}(x)-E[u]\right)\right)^{\frac{1}{2}}=F\left(F^{-1}\left(\sqrt{F\left(\mathcal{U}(x)-E[u]\right)}\right)\right)=F(\gamma(x))$. Since at the upper bound $\gamma(\overline{u})=\overline{u}=\mathcal{U}(\overline{u})-E[u]$, we have

$$\frac{d\gamma(\overline{u})}{dx}=\frac{\mathcal{U}'(\overline{u})}{2}\cdot 1=\frac{1+\frac{L(\overline{u})f(\overline{u})}{F(\overline{u})^2}}{2}.$$

Thus $\frac{d\gamma(\overline{u})}{dx}\leq 1\Leftrightarrow L$ is logconcave at the upper bound.

Second, by construction γ has a unique fixed point in the interior of the domain $\left(\mathcal{U}^{-1}\left(\underline{u}+E[u]\right),\overline{u}\right)$ if and only if there is a unique crossing between N and S in the interior of their common support, i.e. if and only if S is a rotation of N. By the observation in the first paragraph, the left-hand integral $L(u)=\int_{\underline{u}}^{u}F(y)\,dy$ is (strictly) logconcave at the upper bound if and only if $x<\gamma(x)$ for all x sufficiently high (i.e. at the last interior fixed point γ must cross the 45-degree line from below so that $S(U)<N(U)$ in a left neighborhood of the upper bound of the support). Thus, conditions (A) and (B) hold if and only if S is clockwise rotation of N. **Q.E.D.**

Logconcavity of the left-hand integral L is a relatively weak condition, which is implied by logconcavity of the distribution F, in turn implied by logconcavity of the density f.[60] As verified in below, a large class of distributions satisfying Proposition B.1's condition (A), logconcavity of L, also satisfy condition (B). In fact, we show next that under Proposition B.1's condition (A), logconcavity of L at the upper bound, concavity of γ implies condition (B), uniqueness of the interior fixed point of γ. Indeed, if γ is concave, it can cross the

[60] These results follow from Prékopa's (1973) Theorem, which guarantees that logconcavity is preserved by integration; see, for example, An (1998) and Bagnoli and Bergstrom (2005).

45-degree line at most twice and, if there are exactly two crossings, the first crossing is from below and the second from above. Combined with the law of iterated expectations (implying existence of an interior crossing) and logconcavity of L at the upper bound (implying that at the last crossing γ crosses the 45-degree line from below), concavity of γ is sufficient for uniqueness of the interior fixed point of γ. Differentiating (21), γ is concave if

$$\frac{d^2\gamma(x)}{dx^2} = \underbrace{\frac{\mathcal{U}''(x)}{2}}_{\leq 0 \Leftrightarrow \frac{L(x)}{F(x)} \text{ is convex}} \frac{\frac{f(\mathcal{U}(x)-E[u])}{F(\mathcal{U}(x)-E[u])}}{\frac{f(\gamma(x))}{F(\gamma(x))}} + \underbrace{\frac{\mathcal{U}'(x)}{2}}_{\geq 0} \underbrace{\frac{d}{dx}\left(\frac{\frac{f(\mathcal{U}(x)-E[u])}{F(\mathcal{U}(x)-E[u])}}{\frac{f(\gamma(x))}{F(\gamma(x))}}\right)}_{\leq 0 \text{ if } F \text{ is logconcave } \& \ \frac{f'F}{f^2} \text{ is increasing.}} \leq 0. \quad (22)$$

A first force leading toward concavity of γ is concavity of $\mathcal{U}$ defined in (6), equivalent to convexity of the mean-advantage-over-inferiors L/F. Given that $\mathcal{U}'(x) > 0$, the second addend is also negative whenever

$$\frac{f'(\mathcal{U}(x)-E[u])F(\mathcal{U}(x)-E[u])-f(\mathcal{U}(x)-E[u])^2}{F(\mathcal{U}(x)-E[u])^2}\mathcal{U}'(x)\frac{f(\gamma(x))}{F(\gamma(x))} \leq \frac{f(\mathcal{U}(x)-E[u])}{F(\mathcal{U}(x)-E[u])}\frac{f'(\gamma(x))F(\gamma(x))-f(\gamma(x))^2}{F(\gamma(x))^2}\gamma'(x). \quad (23)$$

Using (21) and logconcavity of F, (23) is equivalent to

$$\left(1-\frac{f'(\mathcal{U}(x)-E[u])F(\mathcal{U}(x)-E[u])}{f(\mathcal{U}(x)-E[u])^2}\right)+\left(\frac{f'(\gamma(x))F(\gamma(x))}{f(\gamma(x))^2}-\frac{f'(\mathcal{U}(x)-E[u])F(\mathcal{U}(x)-E[u])}{f(\mathcal{U}(x)-E[u])^2}\right) \geq 0.$$

Given that $\mathcal{U}(x) - E[u] \leq \gamma(x)$ we conclude that the second addend in (22) is negative if F is logconcave and $f'F/f^2$ is non-decreasing. For example, if F is a power distribution (with uniform as a special case), L/F is convex (being linear), F is logconcave, and $f'F/f^2$ is constant, so that γ is concave. Lemma B.1 extends this logic:

Lemma B.1 *Distribution S crosses only once and from above distribution N in the interior of the support if:*
(i) the distribution F is logconcave,
(ii) the mean-advantage-over-inferiors L/F is convex, and
(iii) the ratio of the distribution to the density F/f is logconcave.

Proof of Lemma B.1. Note the following facts:

(a) By the law of iterated expectations S and N have the same expectation so that they must cross at least once in the interior of the common support, i.e., γ has at least one interior fixed point.

(b) Assumption (i), logconcavity of F, implies logconcavity of L by Prékopa's (1973) Theorem. Then, by the first result shown in the proof of Proposition B.1, at the last interior fixed point γ must cross the 45-degree line from below.

To claim that there is a single interior crossing between S and N it is sufficient to show that the domain of γ can be partitioned into two connected regions: a lower region $\underline{\mathcal{S}}$ in which γ can cross the 45-degree line only from below and a higher region $\overline{\mathcal{S}}$ in which γ can cross the 45-degree line only from above. At a fixed point x^*, γ crosses the 45-degree line from below if and only if

$$\left.\frac{d\gamma(x)}{dx}\right|_{x=x^*} = \underbrace{\frac{U'(x)}{2}}_{\leq 1 \text{ if } F \text{ is logconcave}} \cdot \frac{\frac{f(\mathcal{U}(x^*)-E[u])}{F(\mathcal{U}(x^*)-E[u])}}{\frac{f(\gamma(x^*))}{F(\gamma(x^*))}} > 1,$$

If the inequality is reversed, at x^*, γ crosses the 45-degree line from above.

To prove that $\underline{\mathcal{S}}$ and $\overline{\mathcal{S}}$ are connected and that $\overline{\mathcal{S}} \leq \underline{\mathcal{S}}$ in the set order (i.e., all elements of $\overline{\mathcal{S}}$ are lower than all elements of $\underline{\mathcal{S}}$) we show that under (i), (ii), (iii) there exists $\bar{y} \in [\mathcal{U}(\underline{u}+E[u]), \overline{u}]$ such that if y is fixed point of γ

$$\begin{cases} \frac{\frac{f(\mathcal{U}(x^*)-E[u])}{F(\mathcal{U}(x^*)-E[u])}}{\frac{f(x^*)}{F(x^*)}} > \frac{2}{\mathcal{U}'(x^*)} & \text{if } y \in [\mathcal{U}(\underline{u}+E[u]), \bar{y}] \\ \frac{\frac{f(\mathcal{U}(x^*)-E[u])}{F(\mathcal{U}(x^*)-E[u])}}{\frac{f(x^*)}{F(x^*)}} < \frac{2}{\mathcal{U}'(x^*)} & \text{if } y \in [\bar{y}, \overline{u}]. \end{cases} \tag{24}$$

Note that, from (6) the right-hand side $\frac{2}{\mathcal{U}'(x^*)}$ is increasing if (ii) $\frac{L}{F}$ convex. So it is sufficient to show that the left-hand side is decreasing. Indeed the ratio of reverse hazard rates in (24) is decreasing in x^* if

$$\frac{d}{dx}\frac{\frac{f(\mathcal{U}(x)-E[u])}{F(\mathcal{U}(x)-E[u])}}{\frac{f(x)}{F(x)}} = \frac{\frac{f'(\mathcal{U}(x)-E[u])F(\mathcal{U}(x)-E[u])-f(\mathcal{U}(x)-E[u])^2}{F(\mathcal{U}(x)-E[u])^2}\mathcal{U}'(x)\frac{f(x)}{F(x)} - \frac{f(\mathcal{U}(x)-E[u])}{F(\mathcal{U}(x)-E[u])}\frac{f'(x)F(x)-f(x)^2}{F(x)^2}}{\left(\frac{f(x)}{F(x)}\right)^2} < 0.$$

Rewriting this using logconcavity of F, (i), which implies $f'F - f^2 < 0$, we have

$$\frac{\frac{F(\mathcal{U}(x)-E[u])}{f(\mathcal{U}(x)-E[u])}}{\frac{F(x)}{f(x)}}\frac{\frac{f(\mathcal{U}(x)-E[u])^2-f'(\mathcal{U}(x)-E[u])F(\mathcal{U}(x)-E[u])}{F(\mathcal{U}(x)-E[u])^2}}{\frac{f(x)^2-f'(x)F(x)}{F(x)^2}} = \frac{\frac{f(\mathcal{U}(x)-E[u])}{F(\mathcal{U}(x)-E[u])} - \frac{f'(\mathcal{U}(x)-E[u])}{f(\mathcal{U}(x)-E[u])}}{\frac{f(x)}{F(x)} - \frac{f'(x)}{f(x)}} > \frac{1}{\mathcal{U}'(x)}. \tag{25}$$

Recalling that $x \geq \mathcal{U}(x) - E[u]$ for all x, by logconcavity of $\frac{F}{f}$, (iii), we conclude that the left-hand side of (25) is larger than 1. Then (25) holds because $\frac{1}{\mathcal{U}'(x)} \leq 1$ for all x.

Because $\underline{\mathcal{S}}$ and $\overline{\mathcal{S}}$ are connected and $\overline{\mathcal{S}} \leq \underline{\mathcal{S}}$ under the set order, claim (b) implies that there cannot be a fixed point in the interior of $\overline{\mathcal{S}}$. To see this, suppose, by way of contradiction, that there is such a fixed point in the interior of $\overline{\mathcal{S}}$, call it $\tilde{x}$; then, given that at $\tilde{x}$, γ crosses the 45-degree line from above and that there is no further crossing from above at any $x > \tilde{x}$, all points $x > \tilde{x}$ in the domain would satisfy $\gamma(x) < x$ violating claim (b), thus reaching a contradiction.

Given that by claim (a) there must be an interior crossing and that crossings must alternate in sign, the set $\underline{\mathcal{S}}$ contains a unique interior crossing, while $\overline{\mathcal{S}}$ can contain at most one crossing, located at the upper bound of the support. If $\overline{u}$ is bounded there is exactly one fixed point in set $\overline{\mathcal{S}}$ at the upper bound, where $\frac{d\gamma(x)}{dx} < 1$. If, instead, $\overline{u}$ is unbounded there is no crossing in $\overline{\mathcal{S}}$. **Q.E.D.**

Condition (i) implies a global version of Proposition B.1's condition (A); it is relatively weak given that logconcavity is preserved under integration. Condition (ii) is a mild regularity condition, which is equivalent to concavity of the expected sum $\mathcal{U}(u_d)$ as a function of the selectively disclosed value; see (6). Condition (iii) means that F is logconcave relative to f (see Whitt, 1985); logconcavity of F/f is automatically satisfied if f is logconvex and F is logconcave. Under these sufficient conditions, the domain of the function γ can be partitioned into two connected regions, a lower region in which $\gamma(x)$ can cross the 45-degree line x only from below and a higher region in which $\gamma(x)$ can cross x only from above. This property, weaker than concavity of γ, implies clockwise rotation because by definition $\gamma(x) > x \Leftrightarrow S(\mathcal{U}(x)) < N(\mathcal{U}(x))$. Below we report a number of examples of distributions satisfying Lemma B.1's three sufficient conditions: power distributions (including uniform), gamma(α, β) as well as Weibull with shape $\alpha \leq 1$ (including exponential), extreme value Gumbel, and Pareto with $\alpha > 1$ (for which the expectation exists).

Next, we show that logconcavity of L—which is implied by logconcavity of F by Prékopa's (1973) Theorem—is necessary for S to be a clockwise rotation of N.

Lemma B.2 *Distribution S is a clockwise rotation of distribution N only if the left-hand integral L is logconcave at the upper bound $\overline{u}$.*

Proof of Lemma B.2. We show by contradiction that if left-hand integral L is logconvex at the upper bound, S cannot be a clockwise rotation of N. Note that, if S is a clockwise rotation of N, then for all U above the rotation point $S(U) \leq N(U)$. However, as shown in the proof of Proposition B.1, if L is logconvex then $x > \gamma(x)$, or equivalently $S(U) > N(U)$, in a left neighborhood of the upper bound, reaching a contradiction. **Q.E.D**.

Reversing Proposition B.1's condition (A) we have:

Proposition B.1* *Distribution S crosses only once and from below distribution N in the interior of the support if and only if:*
(A) the left-hand integral $L(u) = \int_{\underline{u}}^{u} F(y)\,dy$ is logconvex at the upper bound and*
(B) the function γ has a unique fixed point in the interior of the domain $(\mathcal{U}^{-1}(\underline{u} + E[u]), \overline{u})$.

Similarly, turning Lemma B.1 on its head, the following result provides sufficient conditions for *anti-clockwise* rotation.

Lemma B.1* *Distribution S crosses only once and from below distribution N in the interior of the support if:*
(i) the distribution F is logconvex,*
(ii) the mean-advantage-over-inferiors L/F is convex, and
(iii) the ratio of the distribution to the density F/f is logconcave.

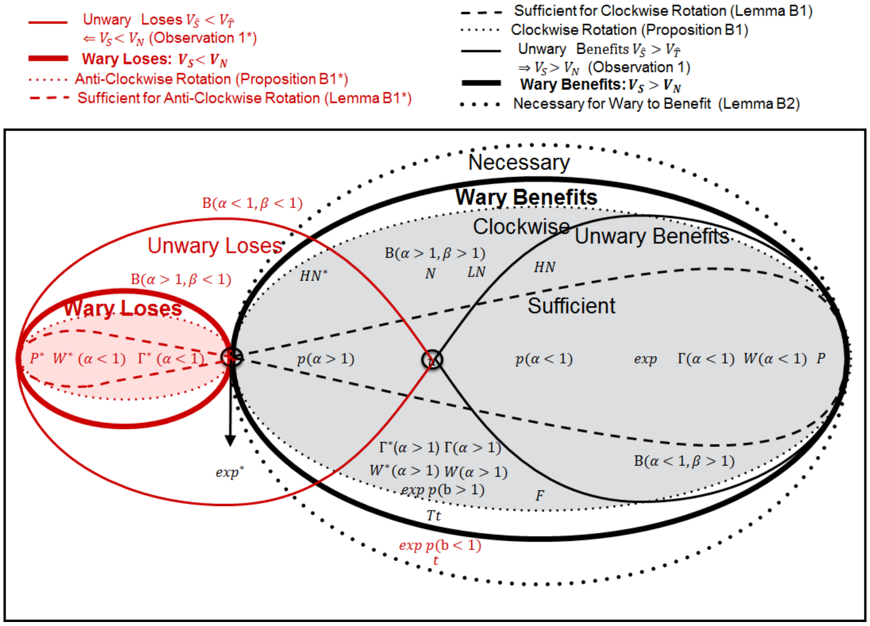

Figure 5: Summary of main results on impact of selective disclosure.

Proof of Lemma B.1*. The proof follows the same steps as for Lemma B.1 and reversing the sign of (25). **Q.E.D.**

Figure 5 summarizes the implications of selective disclosure for receiver welfare for a large set of parametric distributions (see below for verification):

N: normal (clockwise rotation)
LN: log-normal (clockwise rotation)
HN: half-normal ($V_{\hat{S}} > V_N$ and clockwise rotation)
$B(\alpha < 1, \beta > 1)$: beta ($V_{\hat{S}} > V_N$ and clockwise rotation)
$B(\alpha > 1, \beta > 1)$: beta (clockwise rotation)
$B(\alpha > 1, \beta < 1)$: beta ($V_{\hat{S}} < V_N$)
$B(\alpha < 1, \beta < 1)$: beta (first interior crossing from below, second from above)
$p\,(\alpha < 1)$: power ($V_{\hat{S}} > V_N$ and sufficient for clockwise rotation)
u: uniform ($V_{\hat{S}} \equiv V_N$ and sufficient for clockwise rotation)
G: extreme value Gumbel (sufficient for clockwise rotation)
$p\,(\alpha > 1)$: power ($V_{\hat{S}} < V_{\hat{T}}$ and sufficient for clockwise rotation)
$\Gamma\,(\alpha < 1)$: gamma ($V_{\hat{S}} > V_{\hat{T}}$ and sufficient for clockwise rotation)
$\Gamma\,(\alpha > 1)$: gamma ($V_{\hat{S}} > V_{\hat{T}}$ and clockwise rotation)
exp: negative exponential ($V_{\hat{S}} > V_{\hat{T}}$ and sufficient for clockwise rotation)
$W(a < 1)$: Weibull ($V_{\hat{S}} > V_{\hat{T}}$ and sufficient for clockwise rotation)
$W(a > 1)$: Weibull ($V_{\hat{S}} > V_{\hat{T}}$ and clockwise rotation)
P: Pareto ($V_{\hat{S}} > V_{\hat{T}}$ and sufficient for clockwise rotation)
t: Student's t (necessary for clockwise rotation)

F: Fisher-Snedecor F (clockwise rotation)
Tt: truncated Student's t ($V_S > V_T$)
HN^*: mirror half-normal ($V_{\hat{S}} < V_{\hat{T}}$ and clockwise rotation)
Γ^* ($\alpha > 1$): mirror gamma ($V_{\hat{S}} < V_{\hat{T}}$ and clockwise rotation)
Γ^* ($\alpha < 1$): mirror gamma ($V_{\hat{S}} < V_{\hat{T}}$ and anticlockwise rotation)
$\exp^*$: positive exponential ($V_S \equiv V_N$ and $S \equiv N$)
W^* ($\alpha < 1$): mirror Weibull ($V_{\hat{S}} < V_{\hat{T}}$ and clockwise rotation)
P^*: mirror Pareto (sufficient for anti-clockwise rotation)

Sufficient Conditions for Rotation: Verification in Examples. The following inheritance property provides a set of sufficient conditions that will prove convenient to verify condition (ii) of Proposition B.1 in examples:

Lemma B.3 *(a) The ratio L/F is convex if (i) the ratio F/f is convex and (ii) $F(\underline{u})^2/f(\underline{u}) = 0$ at the lower bound $\underline{u}$ of the support. (b) The ratio F/f is convex if (iii) the ratio f/f' is convex and (iv) $f(\underline{u})^2/f'(\underline{u}) = 0$.*

Proof of Lemma B.3. The argument is similar to Baricz (2010), Theorem 2(c), and is based on the monotone form of de l'Hôpital rule; see Wu and Debanth (2009). Clearly, $L(u)/F(u)$ is convex whenever $L(u)f(u)/F(u)^2$ decreases in u. At the lower bound $\underline{u}$ of the support we have that $F(\underline{u})^2/f(\underline{u}) = 0$ (by assumption) and $L(\underline{u}) = 0$ (by definition), so that

$$\frac{L(u)}{\frac{F(u)^2}{f(u)}} = \frac{L(u) + L(\underline{u})}{\frac{F(u)^2}{f(u)} + \frac{F(\underline{u})^2}{f(\underline{u})}}$$

which, in view of the monotone form of the l'Hôpital rule, is decreasing if the function

$$\frac{\frac{dL(u)}{du}}{\frac{d\left(\frac{F(u)^2}{f(u)}\right)}{du}} = \frac{F(u)}{\frac{2f(u)^2F(u) - F(u)^2 f'(u)}{f(u)^2}} = \frac{1}{2 - \frac{F(u)f'(u)}{f(u)^2}}$$

is decreasing in u, proving part (i). The proof of part (ii) follows the same steps. **Q.E.D.**

We now derive a further sufficient condition for the mean-advantage over inferiors to be convex:

Lemma B.4 *L/F is convex if f is logconcave and decreasing.*

Proof of Lemma B.4. The assumptions imply that F/f is convex,

$$\frac{d^2}{du^2}\frac{F(u)}{f(u)} = \frac{-\left(f(u)f''(u) - f'(u)^2\right)F(u) + \left(F(u)f'(u) - f(u)^2\right)f'(u)}{f(u)^3} \geq 0$$

given that logconcavity of f (i.e., $ff'' - f'^2 \leq 0$) guarantees that F is logconcave ($Ff' - f^2 \leq 0$) by Prepoka's Theorem. The result follows by Lemma B.3 given that we then have that $f(\underline{u}) > 0$ and thus $F(\underline{u})^2/f(\underline{u}) = 0$. **Q.E.D.**

Example B.2 (Power) *The power distribution with* $f(u) = \alpha u^{\alpha-1}$ *and support* $(0,1)$ *satisfies Lemma B.1. The uniform distribution corresponds to* $\alpha = 1$.

Verification of Example B.2. (i) F is logconcave, see Bagnoli and Bergstrom (2005).
(ii) $d(L(u)/F(u))/du = 1/(\alpha+1)$ so that L/F is linear.
(iii) $d^2[\ln(F(u)/f(u))]/du^2 = -\alpha/u^2 + (\alpha-1)/u^2 = -1/u^2 < 0$.

Example B.3 (Gamma) *The gamma*(α,β) *distribution with* $\alpha \leq 1$ *satisfies Lemma B.1.*

Verification of Example B.3. (i) F is logconcave, see Bagnoli and Bergstrom (2005).
(ii) L/F is convex by Lemma B.3.a given that

$$\lim_{x\to 0^+} \frac{F(x)^2}{f(x)} = 0,$$

as $\lim_{x\to 0^+} F(x) = 0$ and $\lim_{x\to 0^+} f(x) = +\infty$ for $\alpha < 1$, and that F/f is convex by Lemma B.3.b because

$$\frac{\partial^2}{\partial x^2}\frac{f(x)}{f'(x)} = \frac{\partial^2}{\partial x^2}\left(\frac{x\beta}{\beta(\alpha-1)-x}\right) = \frac{2\beta^2(\alpha-1)}{[\beta(\alpha-1)-x]^3} > 0$$

and

$$\lim_{x\to 0^+}\frac{f(x)^2}{f'(x)} = \lim_{x\to 0^+}\frac{x^\alpha e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^{\alpha-1}[\beta(\alpha-1)-x]} = 0.$$

(iii) F/f is logconcave because F is logconcave and f is logconvex for $\alpha \leq 1$.

Example B.4 (Gumbel) *The extreme value Gumbel distribution satisfies Lemma B.1.*

Verification of Example B.4. (i) F is logconcave.
(ii) L/F is convex by verifying the conditions of Lemma B.3.a.
(iii) F/f is loglinear.

Example B.5 (Pareto) *The Pareto distribution* $F(u) = 1 - \underline{u}^\alpha u^{-\alpha}$ *with* $\underline{u} > 0$ *and* $\alpha > 1$ *(so that the expectation exists) satisfies Lemma B.1.*

Verification of Example B.5. (i) F is logconcave, see Bagnoli and Bergstrom (2005).
(ii) By Lemma B.3.a, L/F is convex because (i) F/f is convex, given that $d(F/f)/du = [(1+a)u^a - \underline{u}^\alpha]/(\alpha\underline{u}^\alpha)$ is clearly increasing in u, and (ii) $F(\underline{u})^2/f(\underline{u}) = (1-\underline{u}^\alpha\underline{u}^{-\alpha})^2/(\alpha\underline{u}^\alpha\underline{u}^{-1-\alpha}) = 0$.
(iii) F/f is logconcave for $\alpha > 1$ because

$$\frac{d^2}{du^2}\ln\frac{F}{f} = -\frac{\overbrace{1+\alpha-2(\underline{u}/u)^\alpha}^{>0} + \overbrace{\alpha^2(\underline{u}/u)^\alpha - \alpha(\underline{u}/u)^\alpha}^{>0} + (\underline{u}/u)^{2\alpha}}{u^2[(\underline{u}/u)^\alpha - 1]^2} < 0.$$

Failure of Clockwise Rotation. We now characterize the set of *non-logconcave* distributions for which the clockwise rotation property does not hold.

Example B.6 (Positive Exponential) *Selective and non-selective disclosure induce identical distributions $S(U) \equiv N(U)$ in the boundary case with loglinear left-hand integral L, which corresponds to the positive exponential distribution with $f(u) = \lambda e^{\lambda(u-\overline{u})}$ with $\lambda > 0$ and support $(-\infty, \overline{u}]$.*

Verification of Example B.6. From $F(u) = e^{\lambda(u-\overline{u})}$ and $E[u] = \overline{u} - 1/\lambda$ we have $N(U) = F(U - E[u]) = e^{\lambda\left(U-\left(\overline{u}-\frac{1}{\lambda}\right)-\overline{u}\right)} = e^{U\lambda - 2\overline{u}\lambda + 1}$. From $U = E[u^1 + u^2 | u_d = \max\langle u^1, u^2\rangle] = 2u_d - 1/\lambda$ we have $u_d(U) = (U + 1/\lambda)/2$, so that $S(U) = (F(u_d(U)))^2 = e^{2\lambda\left(\frac{U+\frac{1}{\lambda}}{2}-\overline{u}\right)} = e^{U\lambda - 2\overline{u}\lambda + 1}$.

The rotation property is reversed for the mirror image of the Pareto distribution, a logconvex distribution that has thicker tail than the positive exponential distribution on both sides:

Example B.7 (Mirror Pareto) *The mirror image of the Pareto distribution with $f(u) = -(\beta/\overline{u})(u/\overline{u})^{-\beta}$ and support $(-\infty, \overline{u}]$ with $\overline{u} < 0$ and $\beta > 1$ (so that the expectation exists) satisfies all three conditions of Lemma B.1*.*

Verification of Example B.7. We have $E[u] = -\beta/(\beta - 1)$, $F(u) = (u/\overline{u})^{-\beta}$, $L(u) = -u/(1-\beta)(u/\overline{u})^{-\beta}$ and $E[u|u < u_d] = \beta u_d/(\beta - 1)$, so that:
(i*) F is logconvex: $d^2 \ln F(u)/du^2 = \beta/u^2 > 0$.
(ii) L/F is concave (being linear): $d^2(L(u)/F(u))/du^2 = d(-u/(1-\beta))/du^2 = 0$.
(iii) F/f is logconcave: $d^2 \ln(F(u)/f(u))/du^2 = \beta/u^2 - (1+\beta)/u^2 = -1/u^2 < 0$.

We conclude by illustrating two classes of examples for which violation of logconcavity of L result in S and N that cross more than once in the interior. First, we give an example of a distribution with U-shaped density that is steeper than the positive exponential at the top of the support:

Example B.8 (Beta) *If F is beta(α, β) with parameter $\beta < 1$, distributions S and N cross an even number of times, so that $V_S(R) < V_N(R)$ for sufficiently high R and $V_S(R) > V_N(R)$ for sufficiently low R.*

Verification of Example B.8. Logconcavity of the left-hand integral L is violated for u sufficiently close to $\overline{u} = 1$ because then $\lim_{u\to 1} L(u) f(u) - F(u)^2 = \infty$ given that $\lim_{u\to 1} L(u) = 1 - \mu$ and $\lim_{u\to 1} f(u) = \lim_{u\to 1} u^{\alpha-1}(1-u)^{\beta-1} = \infty$ for $\beta < 1$. Thus, by Lemma B.2 at the last crossing S crosses N from below. Given that $\underline{u} > -\infty$, at the first crossing S crosses N from above. We conclude that S crosses N an even number of times.

Second, distributions F with a bottom tail that is thicker than the negative exponential (thus violating of logconcavity of L) result in S first crossing N from below:

Example B.9 (Exponential Power) *If F is exponential power with shape parameter $b \geq 1$ (including as special case Laplace for $b = 1$, Normal for $b = 2$, and uniform for $b \to \infty$), S crosses N once and from above in the interior. If $b < 1$ (so that the bottom tail is are thicker tail than negative exponential), distribution S first crosses N from below and then a second time from above.*

Example B.10 (Student's t) *If F is Student's t with at least 2 degrees of freedom (so that the expectation exists), distribution S first crosses N from below and then a second time from above.*

Example B.11 (Truncated Student's t) *If F is a left truncation of Student's t with sufficiently large variance, S crosses N three times (from above, below, and above); if an addition the variance is not too large, so that the first and second crossing points are sufficiently close, the welfare property $V_S > V_N$ is preserved in spite of violation of the clockwise rotation Property 2.*

B.2 Different Weights for Different Attributes

In this Appendix, we introduce asymmetry in the importance of the two attributes by stipulating the following specification of utility

$$u = \alpha^1 u^1 + \alpha^2 u^2,$$

where the weights satisfy without loss of generality $\alpha^1 > \alpha^2 > 0$. We focus on the tractable case where u^1 and u^2 are independent and uniformly distributed on $[\underline{u}, \overline{u}]$ and restrict attention to the characterization of a rational expectations equilibrium where the disclosure rule is linear: $d = 1$ whenever $u^1 \geq a + bu^2$.

If this rule is rationally anticipated by the receiver, then choosing $d = 1$ is indeed optimal if and only if

$$\alpha^1 u^1 + \alpha^2 E\left[u^2 | u^2 \leq \frac{u^1 - a}{b}\right] \geq \alpha^2 u^2 + \alpha^1 E\left[u^1 | u^1 \leq a + bu^2\right],$$

which can be transformed to obtain

$$u^1 \geq \frac{(\alpha^1 - \alpha^2)}{\alpha^1}\underline{u} + \frac{\alpha^2}{\alpha^1}u^2. \tag{26}$$

If (26) does not hold, $d = 2$ is disclosed. With this rule at hand, after disclosing $d = 1$ the expected utility equals

$$U = \frac{3}{2}\alpha^1 u^1 - \frac{1}{2}\left(\alpha^1 - 2\alpha^2\right)\underline{u},$$

so that $U \in \left[(\alpha^1 + \alpha^2)\underline{u}, \frac{3}{2}\alpha^1\overline{u} - \frac{1}{2}(\alpha^1 - 2\alpha^2)\underline{u}\right]$. With $d = 2$ we obtain

$$U = \frac{3}{2}\alpha^2 u^2 + \frac{1}{2}\left(2\alpha^1 - \alpha^2\right)\underline{u},$$

so that now $U \in \left[\left(\alpha^1+\alpha^2\right)\underline{u}, \frac{3}{2}\alpha^2\overline{u}+\frac{1}{2}\left(2\alpha^1-\alpha^2\right)\underline{u}\right]$. Note that from $\alpha^1 \geq \alpha^2$, which we stipulated without loss of generality, the highest value of U is attained when disclosing $u^1=\overline{u}$, $\overline{U}=\frac{3}{2}\alpha^1\overline{u}-\frac{1}{2}\left(\alpha^1-2\alpha^2\right)\underline{u}$, while $\underline{U}=\left(\alpha^1+\alpha^2\right)\underline{u}$. The following characterization can now be obtained after some calculations:

$$S(U)=\begin{cases} \frac{1}{\alpha^1\alpha^2}\left(\frac{2\left(U-\left(\alpha^1+\alpha^2\right)\underline{u}\right)}{3\left(\overline{u}-\underline{u}\right)}\right)^2 & \text{for } \underline{U}\leq U\leq U' \\ \frac{1}{\alpha^1}\frac{2\left(U-\left(\alpha^1+\alpha^2\right)\underline{u}\right)}{3\left(\overline{u}-\underline{u}\right)} & \text{for } U'<U\leq\overline{U} \end{cases}$$

where

$$U'=\frac{3}{2}\alpha^2\overline{u}+\frac{1}{2}\left(2\alpha^1-\alpha^2\right)\underline{u},$$

with $U'\in\left(\underline{U},\overline{U}\right)$ for $\alpha^1>\alpha^2$.

We next derive $\hat{S}(U)$ (where the receiver is not wary of the fact that the sender observes her preferences before disclosure). In this case, for the sender it is optimal to choose $d=1$, so as to maximize the perceived valuation, when

$$\alpha^1u^1+\alpha^2E[u^2]\geq\alpha^1E[u^1]+\alpha^2u^2,$$

which transforms to

$$u^1\geq\frac{\left(\alpha^1-\alpha^2\right)}{\alpha^1}\frac{\left(\overline{u}+\underline{u}\right)}{2}+\frac{\alpha^2}{\alpha^1}u^2,$$

and otherwise to disclose $d=2$. Again after some tedious calculations we obtain:

$$\hat{S}(U)=\begin{cases} \frac{\left[2U-\left(\alpha^1+\alpha^2\right)\left(\frac{\left(\overline{u}+\underline{u}\right)}{2}+\underline{u}\right)\right]^2-\left[\left(\alpha^1-\alpha^2\right)\left(\frac{\left(\overline{u}+\underline{u}\right)}{2}-\underline{u}\right)\right]^2}{4\alpha^1\alpha^2\left(\overline{u}-\underline{u}\right)^2} & \text{for } \underline{\hat{U}}\leq U\leq\hat{U}' \\ \frac{2U-\alpha^2\left(\overline{u}+\underline{u}\right)-2\alpha^1\underline{u}}{2\alpha^1\left(\overline{u}-\underline{u}\right)} & \text{for } \hat{U}'<U\leq\overline{\hat{U}} \end{cases}$$

where

$$\begin{aligned} \underline{\hat{U}} &= \alpha^2\underline{u}+\alpha^1\frac{\left(\overline{u}+\underline{u}\right)}{2}, \\ \overline{\hat{U}} &= \alpha^1\overline{u}+\alpha^2\frac{\left(\overline{u}+\underline{u}\right)}{2}, \\ \hat{U}' &= \alpha^2\overline{u}+\alpha^1\frac{\left(\overline{u}+\underline{u}\right)}{2}, \end{aligned}$$

with $\hat{U}'\in\left(\underline{\hat{U}},\overline{\hat{U}}\right)$ for $\alpha^1>\alpha^2$.

What now complicates the analysis is that with unequal weights $\alpha^1\neq\alpha^2$ the sender's strategy is no longer immediate even when he does not observe the receiver's preferences. This is despite the fact that the sender arguably still applies the same disclosure rule to each receiver. Without loss of generality, we can limit the sender's strategies to always disclosing the first attribute *or* to always disclosing the second attribute. For our subsequent derivations we need not determine which one is optimal. When the sender discloses $d=1$, then

$$N(U)=\frac{2U-\alpha^2\left(\overline{u}+\underline{u}\right)-2\alpha^1\underline{u}}{2\alpha^1\left(\overline{u}-\underline{u}\right)}$$

for $U \in \left[\alpha^1 \underline{u} + \alpha^2 \frac{(\overline{u}+\underline{u})}{2}, \alpha^1 \overline{u} + \alpha^2 \frac{(\overline{u}+\underline{u})}{2}\right]$. When he discloses $d = 2$, then

$$N(U) = \frac{2U - \alpha^1 (\overline{u} + \underline{u}) - 2\alpha^2 \underline{u}}{2\alpha^2 (\overline{u} - \underline{u})}$$

for $U \in \left[\alpha^2 \underline{u} + \alpha^1 \frac{(\overline{u}+\underline{u})}{2}, \alpha^2 \overline{u} + \alpha^1 \frac{(\overline{u}+\underline{u})}{2}\right]$.

Based on the derived expressions, one can show the following result. (The proof is available from the authors upon request.)

Proposition B.2 *When u^i is uniformly distributed but the receiver applies different weights, $\alpha^1 \neq \alpha^2$, the distributions for the receiver's perceived utility are still related by FOSD (naive) or Clockwise Rotation (wary).*

B.3 Disentangled Locations

This Appendix extends the primitives of the model by introducing separately the location of the attributes of a given product and the location of the attributes a given consumer would prefer.

To this end, consider one firm and suppose that two attributes (characteristics) are given by x_1, x_2 and are, from an ex-ante perspective, distributed uniformly on a Salop circle of circumference two. Each consumer has a preferred location for each attribute. The preferred location of a mass one of consumers is distributed uniformly around each circle and denoted by y_1, y_2. A particular consumer's true utility is then

$$2\alpha - \sum_{i=1}^{2} |y_i - x_i| = 2\alpha - \sum_{i=1}^{2} d_i,$$

where $\alpha > 1$ and $d_i := |y_i - x_i|$ denotes the discrepancy between characteristic x_i and the consumer's preferences y_i. With d_i, thus, distributed uniformly on $[0, 1]$, we obtain for the respective distribution of $u_i = \alpha - d_i$ the following:

$$\begin{aligned} F(u_i) &= \Pr(\alpha - d_i \leq u_i) = \Pr(d_i \geq \alpha - u_i) \\ &= u_i - (\alpha - 1), \end{aligned}$$

which is the distribution function of a uniform distribution on $[\underline{u}, \overline{u}]$ with $\underline{u} = \alpha - 1$ and $\overline{u} = \alpha$. Accordingly, this model with distances can be analyzed using the same methods as in the main text. Alternatively, for completeness, we can derive the distribution of U directly. So, assume without loss of generality that $d_1 \leq d_2$, so that, under selective disclosure, the firm reveals d_1. Then, given d_1, a wary consumer's expected valuation is

$$\begin{aligned} U &= 2\alpha - (d_1 + E[d_2 \mid d_2 \geq d_1]) \\ &= 2\alpha - \frac{1}{2}(3d_1 + 1), \end{aligned}$$

with ex-ante distribution

$$\begin{aligned} S(U) &= \Pr\left(2\alpha - \frac{1}{2}(3d_1+1) \leq U\right) \\ &= \left(\frac{2U + 4(1-\alpha)}{3}\right)^2, \end{aligned}$$

for $U \in \left[2(\alpha-1), 2\alpha - \frac{1}{2}\right]$. Next, an unwary customer's perceived expected valuation when facing selective disclosure is given by

$$U = 2\alpha - d_1 - E[d_2] = 2\alpha - d_1 - \frac{1}{2},$$

with ex-ante distribution

$$\begin{aligned} \hat{S}(U) &= \Pr\left(2\alpha - d_1 - \frac{1}{2} \leq U\right) \\ &= \left(U + \frac{3}{2} - 2\alpha\right)^2, \end{aligned}$$

for $U \in \left[2\alpha - \frac{3}{2}, 2\alpha - \frac{1}{2}\right]$. With non-selective disclosure, we have

$$U = 2\alpha - d_1 - E[d_2] = 2\alpha - d_1 - \frac{1}{2},$$

with ex-ante distribution

$$\begin{aligned} N(U) &= \Pr\left(2\alpha - d_1 - \frac{1}{2} \leq U\right) \\ &= U + \frac{3}{2} - 2\alpha \end{aligned}$$

for $U \in \left[2\alpha - \frac{3}{2}, 2\alpha - \frac{1}{2}\right]$. Thus, comparing the respective expressions for the distributions of U, we can confirm that $N(U)$ and $S(U)$ satisfy the condition of a Clockwise Rotation, while $\hat{S}(U)$ dominates $N(U)$ in the sense of FOSD.