Excessive Competition on Headline Prices

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Abstract

In a variety of purchasing situations, consumers may focus primarily on headline prices, ignoring the full costs associated with acquiring and maintaining a product or service contract. Even when this is the case, it is widely believed that intense competition would adequately protect consumers (the so-called "waterbed effect"). However, in a tractable model of imperfect competition and vertical differentiation, we show that when consumers exhibit context-dependent preferences, competition may rather exacerbate their and society's harm. Then, consumer protection policy must sufficiently constrain hidden costs and fees so that competition, along with high-quality firms' incentives to educate consumers, can restore efficiency.

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1 Introduction

For various reasons consumers may focus primarily or even exclusively on so-called headline prices, ignoring the full costs involved in the acquisition and maintenance of a product or the signing of a service contract. Even when consumer protection policy seeks to ensure full and immediate transparency of all costs,[1] firms may find ways around it, for example by devoting more advertising space on the headline price or by disclosing additional charges only in small print. And even when consumers are generally aware of the presence of such additional costs, they may be inattentive to them during the act of purchase.

Still, when competition is sufficiently fierce, firms will compete away most of the profits earned from hidden charges. This "waterbed effect" is generally believed to protect consumers,[2] and negative welfare implications should then arise only when other choices by consumers are inefficient, such as when they take costly evasive actions.[3] We show that this logic no longer holds when consumers exhibit context-dependent preferences: By leading to excessively

[1]For instance, so-called "drip pricing" is de facto prohibited in the European Union as of 2011 (Directive 2011/83, Article 22), though in the US the approach seems to be more fragmented (Friedman, 2020), with the exception of airline tickets (cf. U.S. Department of Transportation (2011), "Enhancing Airline Passenger Protections", Docket DOT-OST-2010-0140 (April 25), `https://www.regulations.gov/document?D=DOT-OST-2010-0140-2051`, accessed March 11, 2021). Another example is that of retail investment products, where under current European regulation all costs must be displayed transparently (cf. the respective documentation by the European regulator, ESMA, `https://www.esma.europa.eu/sites/default/files/library/2015/11/2014-726_enhanced_protection_for_retail_investors_-_mifid_ii_and_mifir.pdf`, accessed March 11, 2021).

[2]A similar mechanism also arises with switching costs (cf. Farrell and Klemperer (2007)). Heidhues and Kőszegi (2018) refer to this as "safety-in-markets". Clearly, this does not work when consumers underestimate all price components (Johnson (2017); Chetty et al. (2009)) or when there is a "price floor" for the transparent component. Ellison (2005) shows how add-on prices, which are not ex-ante observed by all consumers, reduce competition when price cuts attract "low types".

[3]Cf. notably Gabaix and Laibson (2006), where the welfare loss arises from attentive consumers' inefficient actions to circumvent the consumption of the overpriced component. This is also the inefficiency on which Armstrong and Vickers (2012) focus, while pointing out that otherwise welfare implications hinge on distributional priorities (e.g., between more or less sophisticated consumers).

low headline prices, the waterbed effect may distort the provision of product quality and thereby indirectly harm consumers via their own distorted choice.

In our model, without effective consumer policy, intense competition alone is thus unable to protect consumers. When hidden charges are however sufficiently constrained, competition plays an important role as it motivates otherwise disadvantaged high-quality firms to unshroud hidden charges.[4] This contrasts with earlier contributions, which tend to be more pessimistic about the potential for unshrouding in the market (cf. Gabaix and Laibson (2006) or, more recently, Heidhues et al. (2017)). We note that when quality is endogenous in our model, unshrouding does not arise on equilibrium, but it becomes an effective off-equilibrium threat when a rival would choose a lower quality. Hence, the observed prevalence of unshrouding in the market may not be informative about its actual role.

Analyzing imperfect competition between firms that are potentially vertically differentiated, we need to rely on a simple model of consumer choice. For this, we follow our approach in Inderst and Obradovits (2020), which blends Varian's seminal model of sales (Varian (1980)) with context-dependent (relative) preferences. Since in Varian (1980) only a share of consumers actively shops among offers, consumers have different consideration sets (and thus, also a different choice context).[5] In our main analysis we stipulate for simplicity that when comparing offers in the respective choice set, consumers focus only on the most salient attribute, which proves equivalent to choosing the offer that delivers the highest "quality-per-dollar". We term this choice rule "relative thinking". With this simplification, the characterization of the full equilibrium, including the different stages of product choice, possible unshrouding, and pricing, remains highly tractable.[6]

[4]Inderst and Ottaviani (2013) also stress differences between consumer protection and competition policy, but there more competition is unambiguously positive as it constrains firms' ability to extract (inflated) consumer rent.

[5]Using a model of sales links our contribution to Heidhues et al. (2021), who analyze the trade-off when consumers either analyze fewer products in detail, thereby detecting all charges, or "browse" more products. Such an allocation of attention could be an interesting research avenue also in our model of salient or relative thinking.

[6]Still, in an extension, we follow Bordalo et al. (2013, 2016) in that the non-salient attribute is only partially discounted. This gives rise to various additional implications,

When hidden charges together with fierce competition and the resulting waterbed effect lower headline prices sufficiently, for such relative thinkers a given nominal price reduction looms relatively larger compared to when headline prices were higher. Consequently, quality differences become relatively less relevant in the eyes of consumers. We analyze how this shifts consumer choice towards lower-quality offers and ultimately distorts firms' quality choice. We show that the costs of the resulting inefficiencies are fully borne by consumers.

Our model predicts that in markets where, due to lack of consumer protection and its enforcement or the mere complexity of prices and costs, headline prices are particularly low, we should observe lower product quality. Other positive implications of our model relate to the role of consumers' consideration sets, which may be observed by empiricists (or constructed by experimentalists), and the market outcome. In particular, an increase of shoppers with a wider consideration set makes it more likely that low quality prevails. We also dedicate a separate discussion to the waterbed effect as, interestingly, it is also dampened by consumers' salient or relative thinking, broadening the positive implications of our model.

Our normative implications relate to the role of competition and consumer protection policies. We delineate circumstances when, if not accompanied by sufficiently stark consumer protection policy, competition alone may exacerbate welfare losses. We also show how consumer education (unshrouding) in the market can play an important role specifically in the presence of vertically differentiated firms, and that to be effective such unshrouding need not be observed on equilibrium. While we do not claim that our analysis applies to every market, perception biases of the presently assumed form may be of particular relevance in markets where, due to frequently changing promotions, consumers must constantly reassess the relative attractiveness of offers. Such biases should also be of greater relevance when the experience of quality does not immediately derive from (physical) interaction with the product.

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such as how the degree of salient thinking affects efficiency, although this comes at the cost of substantial added complexity.

Our interest in the implications of hidden charges is shared with a growing literature. In fact, the recent survey on Behavioral Industrial Organization by Heidhues and Kőszegi (2018) dedicates its first part to the "economics of hidden prices". Next to providing various examples, they also point out that such hiding or shrouding may occur in practice through an increase in the complexity of offers (cf. Carlin (2009)). Our focus on longer-term inefficiencies arising from firms' inefficient product choice is shared with Heidhues et al. (2016, 2017), where firms rather invest in their potential to increase hidden prices, which they term "exploitative" innovation. In contrast to the present model, there inefficiencies arise in particular when the waterbed effect is relatively ineffective, as then firms can earn higher profits from hidden charges.[7] Our contribution also relates to various other applications of context-dependent or reference-point-dependent preferences to Industrial Organization, such as Bordalo et al. (2016), Dertwinkel-Kalt and Köster (2017), Helfrich and Herweg (2020) and Apffelstaedt and Mechtenberg (2021).[8] More generally, the notion that consumers assess different offers relative to a reference point has, in different forms, gained wide acceptance in Behavioral Economics and Marketing (there, dating back at least to Monroe (1973)).[9]

A recent addition to the literature are experimental and field studies that directly test implications, such as from salience theory (e.g., Dertwinkel-Kalt et al. (2017) or Hastings and Shapiro (2013)). Together with available psychological evidence, this may justify drawing out the (policy) implications from

[7]Michel (2018) points yet to another inefficiency that may arise from firms screening between more or less wary consumers.

[8]Like us, these build on conceptual groundwork provided, amongst others, by Bordalo et al. (2013), Kőszegi and Szeidl (2013) and Bushong et al. (2021). We acknowledge that these models differ considerably in their assumptions and implications. For instance, the assumption of range normalization made in Bushong et al. (2021) implies that with only two products in the market, choices would not be distorted.

[9]Much of the literature in Marketing has however focused on how a single firm's offers can shape consumer perceptions. For example, Huber et al. (1982) show that the choice among two alternatives can crucially be affected if a third, dominated alternative is added (the so-called "attraction effect"). Similarly, Simonson (1989) demonstrates that adding an alternative that is particularly good on one dimension, but bad on another (e.g., a product with very high quality, but also a very high price), may tilt consumers' choice among the initially available alternatives ("compromise effect").

such behavioural assumptions. In addition, our further positive implications may provide a screen for whether our model captures the mechanisms that are at work in a particular market. For instance, when such data is available, one could test whether, ceteris paribus, there is indeed a negative relationship between hidden charges and headline prices, or crucially between an increase in the market share of lower-quality products and measures of competition, such as increased shopping intensity and greater transparency over different offers (though clearly such correlations, when considered in isolation, may have different reasons).

The rest of this paper is organized as follows. Section 2 introduces the model. Firms' price competition with exogenous product qualities is analyzed in Section 3. In Section 4, we add, as initial step, firms' endogenous quality choice. Section 5 introduces the possibility of unshrouding. In Section 6, we finally derive policy implications from our results and provide an additional analysis of (consumer) welfare. Section 7 concludes. All proofs are contained in the Appendix. In an extensive Online Appendix, we derive results for a generalized version of salience, for an arbitrary number of firms, and for a modified model where only a share of consumers is subject to the salience bias.

2 The Model

The market. In our baseline model, we consider $I = 2$ firms that compete for a mass one of consumers.[10] We stipulate that a fixed fraction $\lambda \in (0, 1)$ of consumers is aware of all offers, while the remaining fraction $1 - \lambda$ only considers (randomly) the offer of a single firm. The former consumers are thus akin to "shoppers" in Varian's model of sales (Varian (1980)). The key difference between consumers lies thus in the larger consideration or choice set

[10] In the Online Appendix, we extend our results to more than two firms ($I > 2$).

of the former group of consumers. In what follows, we refer to these consumers as "market savvy".[11]

Each consumer demands at most one product. Firms' offers may differ in qualities and prices, where we index quality by some positive real number q. A firm's price involves a headline price p as well as a hidden or shrouded charge h. The price component h is hidden to all consumers, irrespective of whether they are market savvy or not. The (maximum) size of h depends on consumer protection policy, as well as possible unshrouding by firms, as we discuss below. The total true price paid by a consumer is thus $p + h$. Firms' offers are indexed by i and we suppose that they have constant marginal costs $c_i \in (0, q_i)$. We simplify the exposition by supposing that there is a single low-quality and a single high-quality variant, $q_i \in \{q_L, q_H\}$, with $q_L < q_H$ and associated marginal costs $c_L < c_H$. For each consumer we normalize the (reservation) value from any alternative outside the considered market to zero.

Our baseline game consists of the following sequence of moves. In $t = 1$, firms with potentially different qualities q_i choose their headline price p_i and their hidden charge h_i. In $t = 2$, non-savvy consumers only have the choice whether to take up the observed single offer or whether not to purchase at all; market-savvy consumers consider both firms' offers instead. Their respective choice criterion is formalized below. In $t = 3$, all payoffs are realized.

After solving this simple game, we introduce two extensions, both of which are crucial for our positive and normative implications. First, we let firms choose their quality q_i endogenously. Second, we allow firms to potentially unshroud the hidden price component. More precisely, for the first modification we introduce an initial stage $t = 0$ where firms simultaneously choose which product variety to offer: $q_i \in \{q_L, q_H\}$. For the second modification, after product choice we allow firms to educate consumers. Such possible unshrouding takes place in $t = 0.5$. For instance, this may be achieved by a different advertising of the full price, which also induces consumers to rethink all other offers in the market.

[11] While a consumer's type is exogenous in our model, we briefly discuss below the possibility that consumers' consideration sets are determined endogenously.

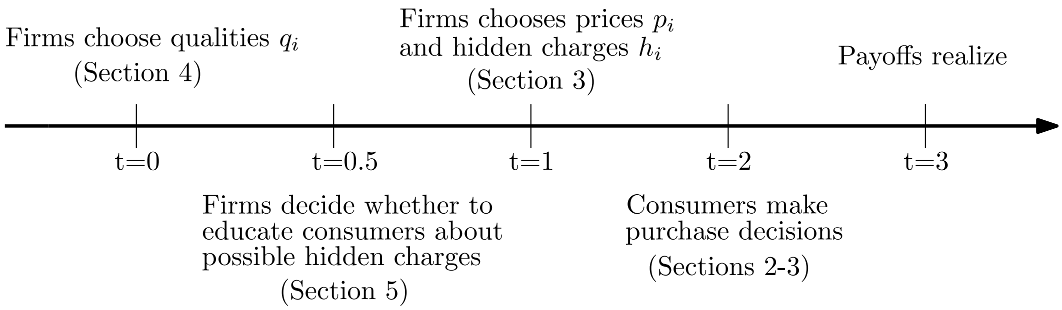

Figure 1: Timeline of the (full) game.

For easier reference, the timeline of the full game is summarized in Figure 1.

Hidden or shrouded charges. The extent to which firms are able to shroud part of their charges should depend crucially on consumer protection and its enforcement. We employ a reduced-form parametrization of the respective policy and the effectiveness of its enforcement. This is captured by a boundary up to which firms can shroud their charges: $h \geq 0$, such that $h_i \leq h$. We further suppose that there is always some minimum level of consumer protection, $h \leq c_H$.

Preferences of market-savvy consumers. The key feature of the subsequently introduced choice criterion is that consumers compare offers not in absolute terms, but relative to some reference point, which in turn depends on their choice set. Thereby, a given price or quality difference between offers will weigh more or less, depending on comparable offers in a consumer's consideration set. The subsequent specification follows that in Inderst and Obradovits (2020). We outline below how this borrows from the literature.

With a slight abuse of notation, we stipulate that the reference point is given by the average price $P = \frac{p_L+p_H}{2}$ and the average quality $Q = \frac{q_L+q_H}{2}$ of the two offers.[12] Take the low-quality product: Whether its low price or its

[12]We thus do not include the outside option into the reference point, albeit stipulating an outside option of $(0, 0)$ would presently not change results. This follows the hierarchical approach as described in Inderst and Obradovits (2020); cf. however the additional analysis

low quality is salient depends on a comparison of the specific values p_L and q_L with those of the reference point, P and Q. Its (low) price is salient when

$$\frac{p_L}{P} < \frac{q_L}{Q}, \tag{1}$$

while its (low) quality is salient when the converse of (1) holds strictly. Likewise, for the high-quality product, its (high) quality is salient when $\frac{p_H}{P} < \frac{q_H}{Q}$, while its (high) price is salient when the converse of this holds strictly. Note that the same attribute is salient for both offers: When (1) holds such that the low-quality offer's low price (low quality) is salient, this implies that $\frac{p_H}{P} > \frac{q_H}{Q}$, i.e., that also the high-quality offer's high price (high quality) is salient.[13] Bordalo et al. (2013) motivate this specific criterion of salience with evidence from psychology, which supports an underlying notion of a diminishing sensitivity. In our main analysis, we now make the stark assumption that consumers compare offers only on the salient attribute, so that when (1) holds (and price is salient), consumers strictly prefer the low-quality product, while otherwise, they prefer the high-quality product. In the Online Appendix, we show, however, that the main features of the equilibrium characterization fully survive when the non-salient attribute is only partially discounted by some factor $\delta \in (0, 1)$.[14] While the presently analyzed case is thus particularly tractable, it is not knife-edge. Note also that consumers do not differ with respect to some inherent propensity for such a bias, but they differ only in the size of their consideration set.[15]

under a generalized version of salience in the Online Appendix. We also acknowledge that there exist other notions of reference-point formation, e.g., when offers are evaluated (also) relative to expected prices and qualities (with expectations formed over firms' mixed pricing strategies; cf. below).

[13]This can be seen immediately after substituting for P and Q.

[14]There, we also conduct a comparative analysis of the equilibrium characterization in δ.

[15]From an empirical perspective, this could be proxied by observed (purchasing) behavior, e.g., from homescan panel data. Still, in the Online Appendix, we provide a discussion of a model variant where, instead of having more or less savvy consumers who all share the same proclivity to salient thinking, all consumers sample both offers, but only a fraction θ are salient thinkers.

We now offer some alternative interpretations of the invoked consumer choice criterion. Substituting for P and Q, (1) transforms to

$$\frac{q_L}{p_L} > \frac{q_H}{p_H}. \tag{2}$$

The criterion that a market-savvy consumer compares offers only according to the salient attribute is thus equivalent to a comparison in terms of the "quality-per-dollar" ratio, choosing low quality when condition (2) holds.[16] The same criterion is also retrieved when we let consumers compare relative differences in qualities and prices. For instance, the price increment is relatively larger than the quality increment for the high-quality product when

$$\frac{p_H - p_L}{p_L} > \frac{q_H - q_L}{q_L}, \tag{3}$$

which immediately transforms to condition (2) and thus (1). In what follows, it will be helpful to use the different expressions (1), (2) and (3) interchangeably. While this should give our assumptions a broader foundation, we note that ours is not a conceptual contribution. Our objective is to explore the implications of this specification, and thereby offering as well a highly tractable model.

3 Equilibrium of the Baseline Model: "Shrouding Meets Salience"

In this section, we solve the pricing subgame in $t = 1$. Since (without unshrouding) any hidden charges are unobservable to consumers, it is immediate that both firms set h_i as high as possible, $h_i = h$, so that it remains to characterize their choice of the headline price p_i. The unique pricing equilibrium is in mixed strategies, such that firm i's price choice will be a random variable $\tilde{p}_i$. The technical steps of our characterization follow from the seminal work of Varian (1980).

[16] Indeed, some contributions in the literature, such as Azar (2014), start right with similar choice rules.

We denote firm i's price strategy by the cumulative distribution function (CDF) $F_i(p_i) = \Pr(\tilde{p}_i \leq p_i)$ with lower and upper support bounds $\underline{p}_i$ and $\overline{p}_i$, respectively. Over the pricing support of firm i, the rival's CDF $F_j(p_j)$ must be such that firm i is indifferent. Denoting firm i's equilibrium profit by π_i, we thus have the following identity that implicitly defines $F_j(p_j)$:

$$\begin{aligned} \pi_i &= (p_i - c_i + h)\left[\frac{1-\lambda}{2} + \lambda \Pr\left(\frac{p_i}{q_i} < \frac{\tilde{p}_j}{q_j}\right)\right] \\ &= (p_i - c_i + h)\left[\frac{1-\lambda}{2} + \lambda\Big(1 - F_j\Big(\frac{q_j}{q_i}p_i\Big)\Big)\right]. \end{aligned} \tag{4}$$

The right-hand side of equation (4) contains the following terms: the respective margin, $p_i - c_i + h$, the mass of non-savvy consumers who are always attracted, $\frac{1-\lambda}{2}$, and the expected mass of attracted savvy consumers who compare both offers, $\lambda\Big(1 - F_j\big(\frac{q_j}{q_i}p_i\big)\Big)$. When firms have symmetric qualities q_i, we are back to the standard case, where it is well known that supports are convex with upper boundary $\overline{p}_i = q_i$, there are no mass points, and firms realize profits

$$\pi_i = \frac{1-\lambda}{2}\left(q_i - c_i + h\right), \tag{5}$$

i.e., exactly the profits that they would make when charging the highest price q_i and only selling to non-savvy consumers. To characterize the outcome with heterogeneous qualities, denote the threshold

$$\widetilde{h} = \frac{q_H c_L - q_L c_H}{q_H - q_L}, \tag{6}$$

which is smaller than c_L and strictly positive if and only if $\frac{q_H}{c_H} > \frac{q_L}{c_L}$.[17]

Lemma 1 *There is a unique pricing equilibrium in mixed strategies, where supports are convex and the CDFs have at most a mass point at* $\overline{p}_i = q_i$. *When firms have the same quality, the equilibrium is symmetric and without a mass point, while firms realize profits (5). When firms have different qualities, then we have the following case distinction: i) when the maximum feasible*

[17] In fact, $\widetilde{h}$ is derived from the requirement that $\frac{q_H}{c_H - \widetilde{h}} = \frac{q_L}{c_L - \widetilde{h}}$.

shrouded charges are sufficiently small, $h < \widetilde{h}$*, only* F_L *has a mass point,*

$$\pi_H = \frac{1-\lambda}{2}(q_H - c_H + h) + \lambda\left[\frac{q_H}{q_L}(c_L - h) - (c_H - h)\right], \tag{7}$$

and π_L *is given by (5); ii) when* $h > \widetilde{h}$*, only* F_H *has a mass point,*

$$\pi_L = \frac{1-\lambda}{2}(q_L - c_L + h) + \lambda\left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right], \tag{8}$$

and π_H *is given by (5); when* $h = \widetilde{h}$*, there are no mass points and* π_i *is given by (5) for both firms.*

We relegate a full explicit characterization of the CDFs (and the mass points) to the proof of the lemma. The fact that the equilibrium is always in mixed strategies should lend our model additional support in the following sense. When prices are in mixed strategies, this essentially implies that, compared to the expected price $E[\tilde{p}_i]$, on the equilibrium path consumers are always "surprised" by the respective deviation $p_i - E[\tilde{p}_i]$.[18] Compared to a model with deterministic equilibrium prices, in our model consumers are therefore "forced" to (re-)assess their optimal choice between different products after the actual realization of prices. In such situations, salient or relative thinking may be of particular relevance.

Turning to Lemma 1, below we will use the characterization of profits for the endogenization of product qualities. Here, we first focus on the the pricing distributions. When $h < \widetilde{h}$, the low-quality firm's distribution has a mass point at the upper boundary, while otherwise this holds for the high-quality firm's distribution. When pricing at the upper boundary, the respective firm only attracts its share of non-savvy consumers, but not consumers who compare offers. This already suggests that that when firms can more easily shroud charges, $h > \widetilde{h}$, the low-quality firm will become more competitive in the marketplace and will attract a larger expected number of consumers. To make

[18]Strictly speaking, this holds when $F_i(p_i)$ has no mass point at $E[\tilde{p}_i]$, which is always the case.

this precise, denote for given price distributions the likelihood that a savvy consumer buys product i by

$$\sigma_i = \Pr\left(\frac{q_i}{\tilde{p}_i} > \frac{q_j}{\tilde{p}_j}\right),$$

where we have used the re-formulation of the choice criterion in (2). An explicit characterization of σ_L for the low-quality product and of $\sigma_H = 1 - \sigma_L$ for the high-quality product is provided in the subsequent proof. Proposition 1 is one of our main results.

Proposition 1 *Suppose that firms have different qualities in $t = 1$. Then, if the maximum feasible shrouded charges are sufficiently large with $h > \widetilde{h}$, the low-quality firm has a larger expected market share ($\sigma_L > \sigma_H$). When instead $h < \widetilde{h}$ holds, the picture is reversed as then $\sigma_H > \sigma_L$. Generally, across both cases, when the maximum feasible shrouded charges h increase, σ_H decreases strictly and σ_L increases strictly, so that consumers who observe both firms' offers become more likely to buy low instead of high quality.*

In the proof of this proposition, we are able to derive the comparative analysis of σ_i in h after a transformation of the random variable (prices). There, we also observe that indeed for both firms the expected price $E[\tilde{p}_i]$ strictly decreases in h: When firms' can hide more charges, this intensifies competition on the headline price. Below we will explore in more detail the extent of this waterbed effect.

The observation that headline prices decrease when hidden charges are higher is key to understand the resulting shift in market share to the low-quality firm.[19] With the chosen preferences, the low-quality firm wins the savvy consumers if it provides a *relatively* better deal. Formally, making now use of the formulation in condition (3),[20] to win the savvy consumers, for a

[19]This is shared with Inderst and Obradovits (2020), though the characterization of the pricing equilibrium is different, as there firms choose qualities and prices simultaneously, which, amongst others, implies that in case of different qualities, the low-quality product is always bought by the contested share of the market. The subsequent analysis, including that of unshrouding, is different as well.

[20]Recall that conditions (1), (2), and (3) can be used interchangeably.

given price p_H of its rival the low-quality firm's discount $\Delta_p = p_H - p_L$ must satisfy $\Delta_p > \Delta_q \frac{p_H}{q_H}$. Hence, a lower *absolute* price difference is required when p_H is lower. When hidden charges h are higher and, by the waterbed effect, headline prices are lower, it thus becomes less expensive for the low-quality firm to win the savvy customers, which increases σ_L and decreases σ_H.

4 Endogenous Product Choice

Continuing with our backward induction, we now consider firms' choice of products in $t = 0$. We denote the respective likelihood with which either firm chooses high quality by γ_i.

We first deal with a particularly clear-cut case. When $q_H - c_H < q_L - c_L$, only low quality will be provided in equilibrium. This is intuitive as in this case low quality both affords firms a strictly larger margin with non-savvy consumers (for any given utility $u_i = q_i - p_i$ offered) and it generates a higher quality-per-dollar when priced at costs, $\frac{q_L}{c_L} > \frac{q_H}{c_H}$.[21] Formally, the result is obtained from a comparison of the respective profits in Lemma 1 and after noting that $q_H - c_H < q_L - c_L$ implies $h > \widetilde{h}$ so that case ii) applies. This observation is the reason for why in what follows we restrict consideration to the case where

$$q_H - c_H > q_L - c_L. \tag{9}$$

It is only in this case that both qualities may arise endogenously.[22] In the remainder, we will thus always invoke this restriction. For ease of exposition, denote the profit of a firm that chooses high quality H, while its rival chooses low quality L, by $\pi_{H,L}$. Profits for all other permutations are denoted

[21] This follows as $q_H - c_H < q_L - c_L$ can be rewritten as $q_H \left(1 - \frac{c_H}{q_H}\right) < q_L \left(1 - \frac{c_L}{q_L}\right)$, which implies that $1 - \frac{c_H}{q_H} < 1 - \frac{c_L}{q_L}$ and therefore $\frac{q_L}{c_L} > \frac{q_H}{c_H}$.

[22] Strictly speaking, both qualities may also arise as part of an asymmetric pure-strategy equilibrium when (9) holds with equality, though in what follows we ignore this knife-edge case. Moreover, in this case, it would be irrelevant from a social point of view which product is offered and bought.

accordingly. In the following part, we derive step-by-step the product-choice equilibrium.

As obviously $\gamma_i = 0$ for both firms is not an equilibrium since $\pi_{L,L} < \pi_{H,L}$ given (9), we turn first to the candidate equilibrium with $\gamma_i = 1$ for both firms, which can be supported when $\pi_{H,H} \geq \pi_{L,H}$. Intuitively, from Lemma 1 this holds for sure when $h \leq \widetilde{h}$, as when hidden charges are sufficiently small, the (deviating) low-quality firm is still at a disadvantage in the market.[23] There exists however a strictly higher cutoff $h^* > \widetilde{h}$ so that for all $h > h^*$, such a deviation becomes profitable and $\gamma_i = 1$ can no longer be an equilibrium. Making use of the analytical tractability of our model, we can directly compare the candidate equilibrium payoff with high quality to that from deviating to low quality in case of $h > \widetilde{h}$, so that, using expression (8), $\pi_{H,H} \geq \pi_{L,H}$ becomes

$$\frac{1-\lambda}{2}\left[(q_H - q_L) - (c_H - c_L)\right] \geq \lambda \left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right]. \qquad (10)$$

Condition (10) captures a firm's trade-off for the case where $h > \widetilde{h}$: The left-hand side captures the additional margin (for any given utility offered) when selling only to non-savvy consumers, while the right-hand side captures the advantage vis-à-vis savvy consumers when a firm offers low quality while the rival offers high quality. At $h = \widetilde{h}$, price is equally likely to be salient as quality and the right-hand side of (10) is zero, such that the condition is always satisfied. But as h increases, the right-hand side increases, and we denote the level at which (10) is satisfied just with equality by h^*. The monotonicity in h reflects the comparative analysis in Proposition 1, from which the low-quality firm's market share increases with h.

When there is no longer an equilibrium where firms choose high quality for sure, there exist multiple equilibria: one where both firms choose a symmet-

[23] Formally, the (deviating) low-quality firm's profit would then be equal to that obtained just with its share of non-savvy consumers, $\pi_{L,H} = (q_L - c_L + h)\frac{1-\lambda}{2}$, which from (9) is clearly strictly smaller than $\pi_{H,H} = (q_H - c_H + h)\frac{1-\lambda}{2}$.

ric mixed strategy and one where firms choose asymmetric but deterministic strategies.

Proposition 2 *Suppose still that unshrouding (in $t = 0.5$) is not a possibility. Then the size of firms' maximally feasible shrouded charges h determines the provision of qualities in equilibrium as follows: When $h \leq h^*$, with*

$$h^* = \widetilde{h} + \frac{1-\lambda}{2\lambda}\frac{q_H}{q_H - q_L}[(q_H - c_H) - (q_L - c_L)], \tag{11}$$

only high quality is provided ($\gamma_i = 1$ for $i = 1, 2$). When $h > h^*$, there exist multiple equilibria as follows: In the unique equilibrium in deterministic strategies, one firm chooses high and the other firm low quality ($\gamma_i = 1$ and $\gamma_j = 0$ for $i \in \{1, 2\}$, $j \neq i$). The unique equilibrium in mixed strategies is symmetric, $\gamma_1 = \gamma_2 = \gamma \in (0, 1)$, so that both qualities are offered with strictly positive probability, where

$$\gamma = \frac{1-\lambda}{2\lambda}\left[\frac{(q_H - c_H) - (q_L - c_L)}{\frac{q_L}{q_H}(c_H - h) - (c_L - h)}\right], \tag{12}$$

which is strictly decreasing in h.

While we already know that the level of h determines whether low quality will be provided, we also learn from Proposition 2 that in the mixed-strategy equilibrium, where both firms randomize which quality to offer, the provision of low quality becomes more likely as (the maximum) hidden charges are higher and thus headline prices lower. The following comparative analysis summarizes Propositions 1 and 2.

Proposition 3 *Suppose still that unshrouding (in $t = 0.5$) is not a possibility. As the maximum hidden charges h increase, it becomes more likely that low quality is provided and purchased: When $h > h^*$, high quality is no longer offered with probability one, and in the symmetric mixed-strategy equilibrium, an increase in h further strictly increases the likelihood with which either firm chooses to offer low quality, $1 - \gamma$. In case of different qualities, an increase* *in h also increases the likelihood with which savvy consumers purchase low instead of high quality, σ_L.*

Note that we are so far silent regarding an interpretation in terms of efficiency and welfare, to which we turn only after fully solving the model, including firms' potential unshrouding. There, we also comment on the interaction of competition and consumer protection policy.

5 The Potential for Unshrouding

We have used so far that both firms fully exploit any leeway that results from a slack in consumer protection legislation or its enforcement and thus choose $h_i = h$. We analyze now how the outcome changes when firms can educate consumers in $t = 0.5$. When this is the case, consumers become wary of any supposedly hidden charges, which effectively eliminates firms' scope for shrouding, setting $h = 0$. For simplicity and following the literature (cf., e.g., Gabaix and Laibson (2006)), we abstract from any costs that would be associated with such unshrouding (arising either directly for the respective firm or for consumers, who may have to devote more time to understand the respective offers). As discussed above, such unshrouding may be achieved by the design of (pricing) labels, which induce consumers to look for such information also when contemplating other offers. Educating consumers could also occur through respective information as part of an advertising campaign.

Consider now first the case where firms have the same quality. It is immediate that in this case each firm would strictly lose from unshrouding, as this would reduce profits by $\frac{1-\lambda}{2}h$. This is different when firms are vertically differentiated. Now the high-quality firm faces the following trade-off. On the one hand, through unshrouding it loses its own ability to exploit consumers. This loss is obviously particularly high when firms can shroud charges to a large extent (high h). On the other hand, unshrouding dampens competition on headline prices. The resulting higher price level favors the high-quality firm, as then its higher quality becomes relatively more important. The latter advantage should matter more when there is a large fraction of savvy consumers in the market (high λ). Taken together, we obtain the following result:[24]

Proposition 4 *Consider the extended game where each firm can unshroud all hidden charges in $t = 0.5$. Then only a firm with high quality may ever unshroud, and only so when the rival offers low quality. Such unshrouding in case of different qualities occurs if and only if $\widetilde{h} > 0$, the fraction of savvy consumers λ is sufficiently high,*

$$\lambda \geq \underline{\lambda} = \frac{q_L}{2q_H - q_L} \in (0,1), \tag{13}$$

and the maximum feasible shrouded charges are not too high,

$$h \leq \overline{h} = \frac{2\lambda(q_H c_L - q_L c_H)}{q_L(1-\lambda)}. \tag{14}$$

Proposition 4 thus delineates the conditions for when unshrouding will occur in case of different qualities. If there is unshrouding, it is immediate that the pricing equilibrium will be different, but we can still completely rely on the characterization in Lemma 1, setting $h = 0$. As we know from Proposition 1, this will tilt purchases towards the high-quality product.

The preceding observations however do not yet describe the equilibrium outcome, but only whether, for given parameters and given qualities, unshrouding would occur in the respective subgame. As we show next, when product choice is endogenous, unshrouding will in fact never occur in equilibrium! This is the case because when, along the equilibrium path, different qualities are chosen, the parameter constellations are such that, according to Proposition 4, also the high-quality firm has no incentive to unshroud. Nevertheless, unshrouding is still effective, as the threat of subsequent unshrouding by the rival may prevent firms from choosing low quality.

[24] For the subsequent proposition, we assume that if a firm is indifferent between shrouding and unshrouding it unshrouds, though this only makes a difference at the parameter boundaries.

Proposition 5 *Consider the extended game where firms can unshroud all hidden charges in* $t = 0.5$*. While in the equilibrium of the full game, where qualities are chosen in* $t = 0$*, unshrouding never occurs, the possibility of unshrouding affects the choice of qualities when* $\widetilde{h} > 0$*. This is the case when the share of market-savvy consumers is sufficiently large,* $\lambda > \hat{\lambda}$ *for some* $\hat{\lambda} \in (\underline{\lambda}, 1)$*, and the maximum feasible shrouded charges are in an intermediate range,* $h \in (h^*, \overline{h}]$*. Then, when unshrouding is possible, both firms choose high quality for sure, while otherwise low quality would be chosen with strictly positive probability.*

Thus, the possibility that a high-quality firm may unshroud charges ensures that, provided that the conditions of Proposition 5 hold, only high quality is chosen. The threat of unshrouding disciplines both firms in that it makes a deviation to low quality unprofitable, while this would be profitable without such a threat. That unshrouding is not observed in equilibrium may then provide a misleading picture, as firms' ability to educate consumers is still an effective threat against rivals and renders the equilibrium outcome efficient. This requires, however, that consumer protection policy already sufficiently constrains hidden charges ($h \leq \overline{h}$). We keep this interaction of unshrouding and consumer protection in mind when we return below to a full discussion of policy implications.

Figure 2 visualizes the equilibrium outcomes when accounting for firms' option to unshroud. Region I arises if the maximum feasible hidden charges are small, $h \leq h^*$ (or alternatively, if the fraction of market-savvy consumers is not too high for given $h > \widetilde{h}$). We know that in this case both firms choose high quality even without the ability to unshroud hidden charges. Obviously, the potential to unshroud thus has no effect on the equilibrium outcome in this region. This also holds for region II, though there in equilibrium low quality is (still) chosen with positive probability as there is no effective threat of unshrouding. This is because the per-customer benefits from shrouding are high compared to the number of market-savvy consumers, $h > \overline{h}(\lambda)$, so that the gains for a high-quality firm from competing more effectively for these consumers by unshrouding are insufficient. Finally, in region III, the

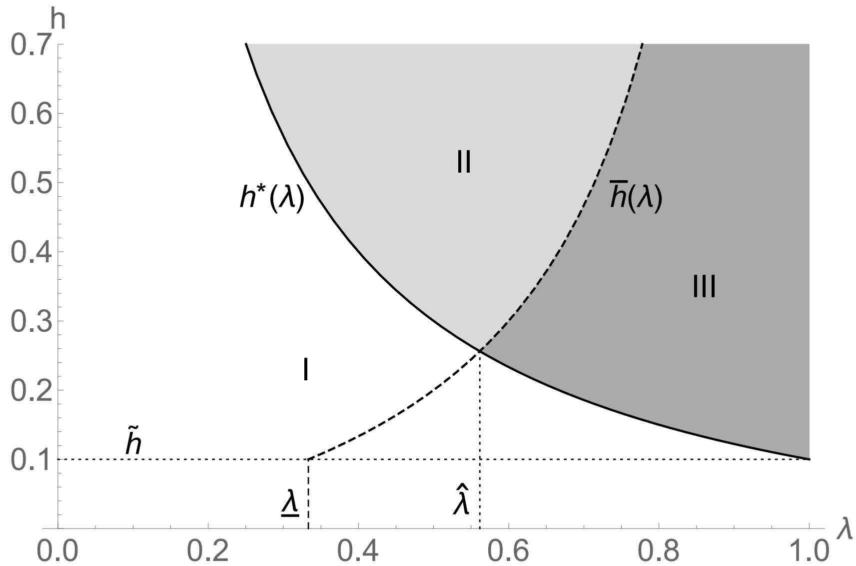

Figure 2: Equilibrium product-choice regions in (λ, h)-space when unshrouding is possible. The parameters used are $q_H = 1$, $c_H = 0.7$, $q_L = 0.5$, $c_L = 0.4$.

threat of unshrouding changes the equilibrium outcome. There, the maximum feasible hidden charges are both not too low, $h > h^*(\lambda)$, as otherwise only high quality would be provided even without the threat of unshrouding, and not too high, $h \leq \overline{h}(\lambda)$, as otherwise unshrouding would not be profitable even for a disadvantaged high-quality firm. Importantly, the threat of unshrouding becomes only effective when the share of market-savvy consumers is sufficiently high, $\lambda > \hat{\lambda}$.

6 Welfare Analysis and Policy Implications

Turning now to potential policy implications, we have to take a stance on how we define (consumer) welfare. Our measure of consumer welfare is the difference $q_i - (p_i + h_i)$, irrespective of which decision rule a consumer followed when choosing between competing offers.[25]

[25]For instance, when we interpret consumer choice in terms of salience, the same measure of consumer welfare applies irrespective of whether at the time of purchase price or quality was salient.

We first discuss total welfare, the sum of firm profits and consumer welfare. When the same quality is offered by both firms, given that the (true) price paid is just a transfer, total welfare is either $w_{LL} = q_L - c_L$ or $w_{HH} = q_H - c_H$. When firms offer different qualities, total welfare is given by

$$w_{HL} = \frac{1-\lambda}{2}(q_L - c_L) + \frac{1-\lambda}{2}(q_H - c_H) + \lambda\left[\sigma_L(q_L - c_L) + \sigma_H(q_H - c_H)\right],$$

where the first two terms capture the welfare created by the sale to non-savvy consumers and the last term the expected welfare created by the sale to market-savvy consumers. From an ex-ante perspective, the expected total welfare equals

$$W = \gamma_1\gamma_2 w_{HH} + (1-\gamma_1)(1-\gamma_2)w_{LL} + \left[\gamma_1(1-\gamma_2) + \gamma_2(1-\gamma_1)\right] w_{HL}.$$

Recall that we focus on the case where (9) holds. Otherwise, only low quality would be offered in equilibrium, regardless of the choice of all other parameters. When (9) holds, total welfare would be highest when $\gamma_1 = \gamma_2 = 1$ or when, provided that $\gamma_i < 1$ for at least one firm i, $\sigma_L = 0$ and $\sigma_H = 1$. When there is no unshrouding, we can infer from Proposition 3 that welfare gradually increases when stricter consumer protection policy and its enforcement reduce h. There is however an additional, potentially larger effect of consumer protection policy, given firms' potential to unshroud (cf. Proposition 5).

Corollary 1 *When consumer protection policy becomes sufficiently strict, so that h falls below $\overline{h}$, and when competition is sufficiently intense ($\lambda > \hat{\lambda}$), this leads to a discrete increase in welfare. This is because, due to high-quality firms' threat of unshrouding, only high quality will be offered in equilibrium. When $h > \max\{h^*, \overline{h}\}$, a marginal reduction of h through consumer protection policy has a marginal positive effect on welfare through two channels: It decreases the likelihood with which either firm inefficiently choose low quality in a symmetric mixed-strategy equilibrium, $1-\gamma$, and, in case different qualities are offered (as always applies in the pure-strategy equilibrium), it reduces the expected market share of low quality, σ_L.*

When we consider total welfare, its distribution between consumers and firms remains irrelevant. Naturally, consumer protection focuses instead on consumer welfare. As we noted in the Introduction, it is widely believed that the waterbed effect protects consumers when there is sufficient competition. This is however different in our model. Fiercer competition on headline prices may lead to inefficiencies that are, as we show, fully borne by consumers. Interestingly, we also find that with salient or relative thinking, the waterbed effect remains incomplete.

Consumer Welfare. We start with a benchmark and consider the case where (exogenously) both firms choose the same quality. Then, we know from (5) that their joint expected profits are $\Pi = (1-\lambda)(q - c + h)$; total welfare is $W = q - c$; and consequently consumer surplus is $S = \lambda(q-c) - (1-\lambda)h$. The derivative $\frac{dS}{dh} = -(1-\lambda)$ exposes the incompleteness of the waterbed effect, as long as not all consumers are market-savvy ($\lambda < 1$). Only when $\lambda \to 1$, with symmetric qualities, consumers are fully protected.

In what follows we focus on parameters for which heterogeneous qualities will arise in the market with positive probability (which, depending on whether unshrouding is possible, requires at least that $h > h^*$, where $h^* > \widetilde{h}$). Recall that in this case, there exists a symmetric mixed-strategy equilibrium in product choice and two asymmetric pure-strategy equilibria. As the mixed-strategy equilibrium is both composed of a subgame with symmetric qualities, to which the discussion of the (symmetric) benchmark applies, and one with heterogeneous qualities, to streamline the discussion we focus on the pure-strategy equilibrium with heterogeneous qualities. Expressing consumer welfare again as the difference between total welfare and firm profits, $S = w_{HL} - \pi_{HL} - \pi_{LH}$, using the respective profits in Lemma 1 it now holds that

$$\frac{dS}{dh} = \frac{dw_{HL}}{dh} - (1-\lambda) - \lambda \left(1 - \frac{q_L}{q_H}\right). \tag{15}$$

The right-hand side of expression (15) can be interpreted as follows. The first term captures the, as we know, negative effect of higher shrouded charges on efficiency (via an increase of σ_L, cf. Proposition 1), which is fully borne by consumers. This is also the effect on total welfare. The second and the third terms additionally capture the welfare transfer from consumers to firms as h increases: the limits to the waterbed effect. Here, the term $-(1-\lambda)$ arises analogously to the (benchmark) case with symmetric qualities. But now, the waterbed effect is further subdued by the final term. In particular, in the limit as $\lambda \to 1$, the waterbed effect is no longer equal to one, but converges to $\frac{q_L}{q_H} < 1$, so that in the limit a one dollar increase in shrouded charges is only passed through into a $\frac{q_L}{q_H} < 1$ dollar reduction in headline prices. The intuition follows immediately from consumers' choice criterion as follows. For this, suppose that the high-quality firm would choose a headline price of $c_H - h$, so that its margin becomes zero. Given consumers' choice criterion, to attract market-savvy consumers the low-quality firm needs to ensure that the ratio $\frac{p_L}{p_H}$ lies (just) below $\frac{q_L}{q_H}$ and thus that p_L lies (just) below $\frac{q_L}{q_H}(c_H - h)$. If now the high-quality firm reduced its headline price by one dollar, following the same increase in shrouded charges, to still capture the whole market when $\lambda \to 1$, the low-quality firm would thus need to lower its headline price by only $\frac{q_L}{q_H} < 1$.

We summarize our discussion of consumer welfare as follows.

Corollary 2 *Suppose that firms choose different qualities (requiring $h > h^*$). When now shrouded charges (further) increase, consumers bear the full burden of the reduced efficiency resulting from a shift towards low-quality products. In addition, the waterbed effect, which limits the direct transfer to firms, is strictly smaller than with symmetric qualities, and it remains incomplete even as $\lambda \to 1$.*

Imposing a limit on hidden charges thus protects consumers in two ways, both by shielding them from a direct price effect, as in our model the waterbed effect is never complete when there are heterogeneous qualities in the market, and by limiting the provision and purchase of inferior low-quality products, as the resulting inefficiency is fully borne by consumers. Relying on market forces alone is instead not sufficient, and, as already noted, without constraining hidden charges, the resulting excessive competition on headline prices can actually hurt consumers. This is particularly true for those who actively compare offers. In fact, while with standard preferences market-savvy consumers with a larger consideration set are always (weakly) better off, even from an ex-ante perspective they may be worse off under salient or relative thinking. To show this through an example, we consider the case where $\lambda \to 1$, as then expressions become more tractable. Around this limit, the parameter region where savvy consumers are worse off (than the average non-savvy consumer) is non-empty. In the proof of Observation 1 in the Appendix, we also derive explicitly the expected welfare for both types of consumers.

Observation 1 *With different qualities in the market, there is a parameter range of strictly positive measure so that market-savvy consumers are strictly worse off also from an ex-ante perspective than consumers with a smaller consideration set.*

When not anticipating their potentially erroneous decisions, market-savvy consumers will overestimate their expected surplus. For future work, it would seem interesting to explore this insight further when endogenizing consumers' decision to become informed about more offers in the market. We conjecture, for instance, that this may frequently lead to overinvestment into the associated activities such as shopping, paying attention to offers, or memorizing different offers. Policies that encourage such activities to "make the market work", e.g., by providing or sponsoring comparison websites, could then backfire.

7 Conclusion

Consumer protection policy and its enforcement aim at protecting consumers from unfair trading practices and thereby, notably, also from the imposition of hidden charges. This topic features prominently in the recent survey of Behavioral Industrial Organization in Heidhues and Kőszegi (2018). It is there rightly observed that the market provides a first layer of protection, as when competition is intense, this will result in lower headline prices through a waterbed effect. A core insight of the present analysis is however that such competition can be excessive and reduce both total and consumer welfare when consumers are prone to salient or relative thinking. As perceived (headline) prices thereby become artificially low, this makes quality differences relatively less important, distorting both the provision and competitive position of higher-quality products. Competition is thus not a substitute to consumer protection policy, but without adequate consumer protection, it can even exacerbate consumer detriment.

On the other hand, we show how competition can work when it generates sufficient incentives for high-quality firms to unshroud theirs as well as rivals' hidden charges so as to eliminate a competitive disadvantage. This effect is not direct, but it works through an increase in headline prices following a reduction of hidden charges (to zero), which renders quality differences relatively more important in the eyes of consumers. While in equilibrium such unshrouding would not be observed in our model, it disciplines firms' choice of qualities, but only when the extent to which charges can be maximally shrouded is sufficiently restricted by consumer protection policy. In this case, consumer protection policy and competition can jointly ensure that the market works efficiently.

The relevance of our model and its implications hinge crucially on the importance of the specific consumer decision bias that we harnessed for our analysis, i.e., that of salient or relative thinking. There is some empirical and experimental evidence that the relative importance of attributes changes with consumers' reference point, as derived from all observed offers in the market (cf. Hastings and Shapiro (2013); Dertwinkel-Kalt et al. (2017)). In the Introduction, we already noted that such a bias should also be more important when the experience of quality does not immediately derive from (physical) interaction with the product and when, e.g., through frequent promotions, consumers constantly need to reassess the relative positioning of offers. A similar reassessment may also be triggered when consumer protection policy or unshrouding lead to drastic changes in headline prices. Such a drastic increase in headline prices may also arise when firms can no longer secretly shift some costs towards consumers, such as those arising from the malfunctioning of a product. This all speaks in favor of a wider applicability of our model.

While some of the undertaken assumptions are admittedly stark, one of our model's key benefits is its tractability, despite the endogenization of product and pricing choices as well as potential unshrouding. As we mentioned earlier, future work may also endogenize the size of consumers' consideration sets and thereby both the competitiveness of the market and the extent to which salient or relative thinking becomes effective. This would allow to assess policies that intend to encourage such shopping so as to "make the market work", which may however backfire in light of a biased consumer choice.

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Appendix: Proofs

Proof of Lemma 1. We first state more explicitly the characterization of the pricing equilibrium, as we will refer to this also in subsequent proofs:

Claim: *For $h \leq \widetilde{h}$: Firm H randomizes over $[\underline{p}_H, \overline{p}_H)$, where $\underline{p}_H = \frac{q_H}{q_L}[c_L - h + \frac{1-\lambda}{1+\lambda}(q_L - c_L + h)]$ and $\overline{p}_H = q_H$, according to the CDF*

$$F_H(p_H) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_L - c_L + h}{p_H \frac{q_L}{q_H} - c_L + h}\right).$$

Firm L randomizes over $[\underline{p}_L, \overline{p}_L]$, where $\underline{p}_L = c_L - h + \frac{1-\lambda}{1+\lambda}(q_L - c_L + h)$ and $\overline{p}_L = q_L$, according to the CDF

$$F_L(p_L) = \frac{1+\lambda}{2\lambda} - \frac{\frac{1-\lambda}{2\lambda}(q_H - c_H + h) + [\frac{q_H}{q_L}(c_L - h) - (c_H - h)]}{p_L \frac{q_H}{q_L} - c_H + h} \text{ for } p_L < \overline{p}_L,$$

and with a mass point at $\overline{p}_L$ of size $m_L = \frac{\frac{q_H}{q_L}(c_L - h) - (c_H - h)}{q_H - c_H + h}$ (which is zero if $h = \widetilde{h}$).

For $h > \widetilde{h}$: Firm L randomizes over $[\underline{p}_L, \overline{p}_L)$, where $\underline{p}_L = \frac{q_L}{q_H}[c_H - h + \frac{1-\lambda}{1+\lambda}(q_H - c_H + h)]$ and $\overline{p}_L = q_L$, according to the CDF

$$F_L(p_L) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_H - c_H + h}{p_L \frac{q_H}{q_L} - c_H + h}\right).$$

Firm H randomizes over $[\underline{p}_H, \overline{p}_H)$, where $\underline{p}_H = c_H - h + \frac{1-\lambda}{1+\lambda}(q_H - c_H + h)$ and $\overline{p}_H = q_H$, according to the CDF

$$F_H(p_H) = \frac{1+\lambda}{2\lambda} - \frac{\frac{1-\lambda}{2\lambda}(q_L - c_L + h) + [\frac{q_L}{q_H}(c_H - h) - (c_L - h)]}{p_H \frac{q_L}{q_H} - c_L + h} \text{ for } p_H < \overline{p}_H,$$

and with a mass point at $\overline{p}_H$ of size $m_H = \frac{\frac{q_L}{q_H}(c_H - h) - (c_L - h)}{q_L - c_L + h}$.

We prove this claim together with the respective expressions for profits. For this we treat separately the cases $h \leq \widetilde{h}$ (Case A) and $h > \widetilde{h}$ (Case B) in a series of assertions.

Case A: $h \leq \widetilde{h}$.

Assertion (i): Supports are convex and cannot contain mass points in the interior or at the lower boundary, while upper boundaries are given by q_i.

Proof of Assertion (i): This follows from standard arguments, see e.g. Varian (1980). **Q.E.D.**

Assertion (ii): $\pi_L = (q_L - c_L + h)\frac{1-\lambda}{2}$.

Proof of Assertion (ii): As this is what the firm can realize by choosing $p_L = q_L$, we only need to show that this is also an upper boundary. We argue to a contradiction and suppose that π_L was higher. Then, denoting L's upper support bound by $\overline{p}_L \leq q_L$, it must hold that L then attracts more consumers than $\frac{1-\lambda}{2}$, so that H must have positive probability mass at or above $\overline{p}_L \frac{q_H}{q_L} \leq q_H$, which further implies that $\pi_H \leq (q_H - c_H + h)\frac{1-\lambda}{2}$ (this is true in particular since it cannot be the case that both L has a mass point at $\overline{p}_L$ and H has a mass point at $\overline{p}_L \frac{q_H}{q_L}$). We now obtain a contradiction as H can realize strictly higher profits by choosing a price constructed as follows: Since $\pi_L > (q_L - c_L + h)\frac{1-\lambda}{2}$ by assumption, L's pricing is bounded below by p'_L that solves $(p_L - c_L + h)\frac{1+\lambda}{2} = \pi_L$, so that when H chooses $p'_L \frac{q_H}{q_L}$, from $h \leq \widetilde{h}$ profits indeed exceed $(q_H - c_H + h)\frac{1-\lambda}{2}$. **Q.E.D.**

Assertion (iii): $\pi_H = \frac{1-\lambda}{2}(q_H - c_H + h) + \lambda \left[\frac{q_H}{q_L}(c_L - h) - (c_H - h)\right]$.

Proof of Assertion (iii): H can ensure at least this profit by pricing at L's lower boundary $\underline{p}_L$, which solves $(p_L - c_L + h)\frac{1+\lambda}{2} = \pi_L$ (using Assertion (ii)). Suppose next to the contrary that H's profits strictly exceeded π_H, from which (for the respective equilibrium) it must hold that $\underline{p}_H > \underline{p}_L \frac{q_H}{q_L}$. But then, by pricing at $\underline{p}_H \frac{q_L}{q_H}$, L could realize strictly more than π_L, as given in Assertion (ii). **Q.E.D.**

With Assertions (i)-(iii) at hands, the respective characterizations of F_i are now immediate from the indifference condition (4). Note finally that these CDFs are indeed well-behaved with $F_H(\underline{p}_H) = 0$ and $\lim_{p_H \to q_H} F_H(p_H) = 1$, whereas $F_L(\underline{p}_L) = 0$ and $F_L(q_L) = \frac{q_L - c_L + h}{q_L - \frac{q_L}{q_H}(c_H - h)} \in (0, 1]$ due to $h \leq \widetilde{h}$.

Case B: $h > \widetilde{h}$.
Assertion (i): Supports are convex and cannot contain mass points in the interior or at the lower boundary, while upper boundaries are given by q_i.
Proof of Assertion (i): Again, this follows from standard arguments, see e.g. Varian (1980). **Q.E.D.**

Assertion (ii) and (iii): $\pi_H = (q_H - c_H + h)\frac{1-\lambda}{2}$ and $\pi_L = \frac{1-\lambda}{2}(q_L - c_L + h) + \lambda \left[\frac{q_L}{q_H}(c_H - h) - (c_L - h) \right]$.
Proof of Assertions (ii) and (iii): This is analogous to the proof of Assertions (ii) and (iii) in Case A above when swapping firm indices. **Q.E.D.**

With Assertions (i)-(iii) at hands, the respective characterizations of F_i are now immediate from the indifference condition (4). Note finally that these CDFs are indeed well behaved with $F_L(\underline{p}_L) = 0$ and $\lim_{p_L \to q_L} F_L(p_L) = 1$, whereas $F_H(\underline{p}_H) = 0$ and $F_H(q_H) = \frac{q_H - c_H + h}{q_H - \frac{q_H}{q_L}(c_L - h)} \in (0, 1)$ due to $h > \widetilde{h}$.

Having analyzed both Case A and B, this concludes the proof of Lemma 1. **Q.E.D.**

Proof of Proposition 1. There are again two cases, as in Lemma 1.

Case A: $h \leq \widetilde{h}$.
Assertion (i): Savvy consumers' probability of purchasing at L is given by

$$\sigma_L = \int_0^1 \frac{k}{1 + \frac{1+\lambda-2\lambda k}{1-\lambda}\left[\frac{c_L - h - \frac{q_L}{q_H}(c_H - h)}{q_L - c_L + h}\right]} dk.$$

Proof of Assertion (i): We first integrate over firms' price realizations in order to express firm L's probability of attracting savvy consumers, which yields

$$\sigma_L = \int_{\underline{p}_H}^{\overline{p}_H} F_L\Big(p_H \frac{q_L}{q_H}\Big) dF_H(p_H),$$

noting that $F_L\Big(p_H \frac{q_L}{q_H}\Big)$ is defined over the same support as $F_H(p_H)$. We now introduce the following substitution of variables: $k = F_H(p_H)$, so that

$$p_H(k) = F_H^{-1}(k) = \frac{q_H}{q_L}\left[\frac{q_L(1-\lambda) + 2\lambda(1-k)(c_L - h)}{1 + \lambda - 2\lambda k}\right],$$

and suppressing the dependency $p_H(k)$, we can rewrite σ_L as

$$\sigma_L = \int_0^1 F_L\Big(p_H \frac{q_L}{q_H}\Big) dk. \tag{16}$$

Comparing $F_L\Big(p_H \frac{q_L}{q_H}\Big)$ with $F_H(p_H)$, we can furthermore rewrite $F_L\Big(p_H \frac{q_L}{q_H}\Big)$ as

$$F_L\Big(p_H \frac{q_L}{q_H}\Big) = F_H(p_H) \frac{\frac{q_L}{q_H} p_H - c_L + h}{\frac{q_L}{q_H} p_H - \frac{q_L}{q_H}(c_H - h)}.$$

Substituting now $p_H(k)$ yields, after various transformations,

$$F_L\Big(p_H \frac{q_L}{q_H}\Big) = \frac{k}{1 + \frac{1+\lambda-2\lambda k}{1-\lambda}\left[\frac{c_L - h - \frac{q_L}{q_H}(c_H - h)}{q_L - c_L + h}\right]}.$$

Inserting this back into (16) yields σ_L as stated in the assertion. **Q.E.D.**

Assertion (ii): σ_L is strictly increasing in h.

Proof of Assertion (ii): Since $\frac{c_L - h - \frac{q_L}{q_H}(c_H - h)}{q_L - c_L + h}$ is strictly decreasing in h, as is easy to show, it follows that σ_L is strictly increasing in h. **Q.E.D.**

Assertion (iii): $\lim_{h \uparrow \widetilde{h}} \sigma_L = 1/2$.

Proof of Assertion (iii): This is obvious when noting that σ_L collapses to $\int_0^1 k dk$ for $h = \widetilde{h}$. **Q.E.D.**

Case B: $h > \widetilde{h}$.

Assertion (i): Savvy consumers' probability of purchasing at L is given by

$$\sigma_L = 1 - \int_0^1 \frac{k}{1 + \frac{1+\lambda-2\lambda k}{1-\lambda}\left[\frac{c_H - h - \frac{q_H}{q_L}(c_L - h)}{q_H - c_H + h}\right]} dk.$$

Proof of Assertion (i): We first integrate over firms' price realizations in order to express firm L's probability of attracting savvy consumers, which yields

$$\sigma_L = \int_{\underline{p}_L}^{\overline{p}_L} \left[1 - F_H\left(p_L \frac{q_H}{q_L}\right)\right] dF_L(p_L) = 1 - \int_{\underline{p}_L}^{\overline{p}_L} F_H\left(p_L \frac{q_H}{q_L}\right) dF_L(p_L),$$

noting that $F_H\left(p_L \frac{q_H}{q_L}\right)$ is defined over the same support as $F_L(p_L)$. We now introduce the following substitution of variables: $k = F_L(p_L)$, so that

$$p_L(k) = F_L^{-1}(k) = \frac{q_L}{q_H}\left[\frac{q_H(1-\lambda) + 2\lambda(1-k)(c_H - h)}{1 + \lambda - 2\lambda k}\right],$$

and, suppressing the dependency $p_L(k)$, we can rewrite σ_L as

$$\sigma_L = 1 - \int_0^1 F_H\left(p_L \frac{q_H}{q_L}\right) dk. \tag{17}$$

Comparing $F_H\left(p_L \frac{q_H}{q_L}\right)$ with $F_L(p_L)$, we can furthermore rewrite $F_H\left(p_L \frac{q_H}{q_L}\right)$ as

$$F_H\left(p_L \frac{q_H}{q_L}\right) = F_L(p_L) \frac{\frac{q_H}{q_L} p_L - c_H + h}{\frac{q_H}{q_L} p_L - \frac{q_H}{q_L}(c_L - h)}.$$

Substituting now $p_L(k)$ yields, after various transformations,

$$F_H\left(p_L \frac{q_H}{q_L}\right) = \frac{k}{1 + \frac{1+\lambda-2\lambda k}{1-\lambda}\left[\frac{c_H - h - \frac{q_H}{q_L}(c_L - h)}{q_H - c_H + h}\right]}.$$

Inserting this back into (17) yields σ_L as stated in the assertion. **Q.E.D.**

Assertion (ii): σ_L is strictly increasing in h.
Proof of Assertion (ii): Since $\frac{c_H - h - \frac{q_H}{q_L}(c_L - h)}{q_H - c_H + h}$ is strictly increasing in h, as is easy to show, it follows that σ_L is strictly increasing in h. **Q.E.D.**

Assertion (iii): $\lim_{h \downarrow \widetilde{h}} \sigma_L = 1/2$.
Proof of Assertion (iii): This is obvious when noting that σ_L collapses to $1 - \int_0^1 k dk$ for $h = \widetilde{h}$. **Q.E.D.**

Having analyzed both Case A and B, this concludes the proof of Proposition 1. **Q.E.D.**

Proof of Proposition 2. We consider various cases, depending on whether (9) holds as well as on the size of h.

(i) If $q_L - c_L \geq q_H - c_H$ (the converse of (9) holds), this implies $\frac{q_L}{c_L} > \frac{q_H}{c_H}$ and thus $\widetilde{h} < 0 \leq h$, so that from Lemma 1 $\pi_{H,L} = \pi_{H,H} = (q_H - c_H + h)\frac{1-\lambda}{2}$, $\pi_{L,H} = (q_L - c_L + h)\frac{1-\lambda}{2} + \lambda \left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right]$, and $\pi_{L,L} = (q_L - c_L + h)\frac{1-\lambda}{2}$. Direct comparison reveals that (q_L, q_L) constitutes an equilibrium in product choice, as $\pi_{L,L} \geq \pi_{H,L}$, and that, unless $q_L - c_L = q_H - c_H$, no other equilibrium exists, as a firm with q_H would strictly prefer to deviate, regardless of its rival's choice. When $q_L - c_L = q_H - c_H$, also an asymmetric equilibrium exists where one firm chooses q_L and the other q_H.

(ii) If (9) holds and $h \leq \widetilde{h}$, we have from Lemma 1 that $\pi_{L,H} = (q_L - c_L + h)\frac{1-\lambda}{2}$, $\pi_{H,L} = (q_H - c_H + h)\frac{1-\lambda}{2} + \lambda \left[\frac{q_H}{q_L}(c_L - h) - (c_H - h)\right] \geq (q_H - c_H + h)\frac{1-\lambda}{2} > \pi_{L,H}$, $\pi_{L,L} = (q_L - c_L + h)\frac{1-\lambda}{2}$, and $\pi_{H,H} = (q_H - c_H + h)\frac{1-\lambda}{2} > \pi_{L,L}$. Direct comparison reveals that (q_H, q_H) constitutes an equilibrium in product choice, as deviating to q_L is strictly inferior, and that no other equilibrium exists, as a firm with q_L would strictly prefer to deviate, regardless of its rival's choice.

(iii) If (9) holds and $h > \widetilde{h}$, we have from Lemma 1 that $\pi_{L,H} = (q_L - c_L + h)\frac{1-\lambda}{2} + \lambda\left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right]$, $\pi_{H,L} = (q_H - c_H + h)\frac{1-\lambda}{2}$, $\pi_{H,H} = (q_H - c_H + h)\frac{1-\lambda}{2}$, and $\pi_{L,L} = (q_L - c_L + h)\frac{1-\lambda}{2}$. We have that $\pi_{H,H} \geq \pi_{L,H}$ holds if and only if $h \leq h^*$, where $h^* > \widetilde{h}$, so that for $h \leq h^*$, (q_H, q_H) constitutes an equilibrium. It is also the unique equilibrium for $h < h^*$, as it holds that $\pi_{L,L} < \pi_{H,L}$, and $\pi_{L,H} < \pi_{H,H}$ for $h < h^*$. Next, for $h > h^*$, no high-quality equilibrium exists and also no low-quality equilibrium, since $\pi_{L,L} < \pi_{H,L}$. A symmetric equilibrium must therefore be in mixed strategies. The characterization of $\gamma \in (0, 1)$ then follows from the equal-expected-profit condition $\gamma\pi_{H,H} + (1-\gamma)\pi_{H,L} = \gamma\pi_{L,H} + (1-\gamma)\pi_{L,L}$, which gives $\gamma = \frac{\pi_{H,H} - \pi_{L,L}}{\pi_{L,H} - \pi_{L,L}}$ and thereby (12) after substitution. The asymmetric equilibria exist for $h \geq h^*$ since then $\pi_{L,H} \geq \pi_{H,H}$, and $\pi_{H,L} < \pi_{L,L}$. **Q.E.D.**

Proof of Proposition 4. We now solve for stage $t = 0.5$, given firms' choices of qualities. Since the statement for homogeneous qualities is obvious, we turn directly to heterogeneous qualities. We distinguish between the following cases:

(i) Condition (9) does not hold. As then from $\widetilde{h} < 0$ it holds that $h > \widetilde{h}$ for all $h \geq 0$, firm H's profit under shrouding is always given by $\pi_H^S = \frac{1-\lambda}{2}(q_H - c_H + h)$, while after unshrouding it is always given by $\pi_H^U = \frac{1-\lambda}{2}(q_H - c_H) < \pi_H^S$. Firm L's profit under shrouding is always given by $\pi_L^S = \frac{1-\lambda}{2}(q_L - c_L + h) + \lambda\left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right]$, while under unshrouding it is always given by $\pi_L^U = \frac{1-\lambda}{2}(q_L - c_L) + \lambda\left[\frac{q_L}{q_H}c_H - c_H\right] < \pi_L^S$. Hence, no firm unshrouds.

(ii) Condition (9) holds and $\widetilde{h} \leq 0$. Again this implies $h \geq \widetilde{h}$ for all $h \geq 0$, so that the results from (i) apply as well.

(iii) Condition (9) holds, $\widetilde{h} > 0$ (i.e., $\frac{q_H}{c_H} > \frac{q_L}{c_L}$), and $h \leq \widetilde{h}$. While it is again immediate that firm L does not unshroud, now firm H's profit with shrouding is $\pi_H^S = \frac{1-\lambda}{2}(q_H - c_H + h) + \lambda\left[\frac{q_H}{q_L}(c_L - h) - (c_H - h)\right]$, while after unshrouding it is $\pi_H^U = \frac{1-\lambda}{2}(q_H - c_H) + \lambda\left[\frac{q_H}{q_L}c_L - c_H\right]$. Comparison reveals that H finds it optimal to unshroud if $\frac{1-\lambda}{2\lambda} \leq \frac{q_H - q_L}{q_L}$, which holds if and only if $\lambda \geq \frac{q_L}{2q_H - q_L} = \underline{\lambda}$.

(iv) Condition (9) holds, $\widetilde{h} > 0$, and $h > \widetilde{h}$. Focusing again on firm H, we have $\pi_H^S = \frac{1-\lambda}{2}(q_H - c_H + h)$ and $\pi_H^U = \frac{1-\lambda}{2}(q_H - c_H) + \lambda \left[\frac{q_H}{q_L} c_L - c_H \right]$, so that firm H finds it optimal to unshroud if and only if $h \leq \frac{2\lambda}{1-\lambda} \left(\frac{q_H c_L - c_H q_L}{q_L} \right) = \overline{h}$.

We now sum up the different cases. We have that only H has an incentive to unshroud and that this is the case only in (iii) and (iv). What is then required, next to $\widetilde{h} > 0$, is that either $h \leq \widetilde{h}$ and $\lambda \geq \underline{\lambda}$, or $h \in (\widetilde{h}, \overline{h}]$, where the latter is only possible (as then $\overline{h} > \widetilde{h}$) if $\lambda > \underline{\lambda}$. **Q.E.D.**

Proof of Proposition 5. Proposition 4 shows that unshrouding occurs (by firm H) only with heterogeneous qualities and when, next to $\widetilde{h} > 0$, $\lambda \geq \underline{\lambda}$ and $h \leq \overline{h}(\lambda)$. When it occurs in this case, the low-quality firm is strictly worse off than if it had chosen high quality instead, so that then q_H is chosen by both firms. This represents a change in the equilibrium outcome, compared to when shrouding is not feasible, only when $h \in (h^*(\lambda), \overline{h}(\lambda)]$ (where instead of an asymmetric or mixed-strategy equilibrium in product choice the possibility of shrouding leads to the deterministic choice of q_H). Since $h^*(\lambda)$ is continuous and strictly decreasing, $\overline{h}(\lambda)$ is continuous and strictly increasing (given $\widetilde{h} > 0$, as assumed) and $\overline{h}(\underline{\lambda}) = h^*(1) = \widetilde{h}$, it follows that there must be a unique $\hat{\lambda} \in (\underline{\lambda}, 1)$ satisfying $h^*(\hat{\lambda}) = \overline{h}(\hat{\lambda})$, such that $\overline{h}(\lambda) > h^*(\lambda)$ if and only if $\lambda > \hat{\lambda}$. Hence, unshrouding may only affect the equilibrium outcome if $\lambda > \hat{\lambda} \in (\underline{\lambda}, 1)$. **Q.E.D.**

Proof of Corollary 2. It remains to derive that, when the expected price paid by consumers is $E[p]$, $\lim_{\lambda \to 1} \frac{dE[p]}{dh} = -\frac{q_L}{q_H}$ in the subgame with different qualities. It is first straightforward to check that firm L prices at $\frac{q_L}{q_H}(c_H - h)$ deterministically in the limit as $\lambda \to 1$, so that from continuity it follows that $\lim_{\lambda \to 1} E[p_L] = \frac{q_L}{q_H}(c_H - h)$. Note finally that we can focus on firm L as in the limit savvy consumers purchase at firm L with probability one and as then there are only savvy consumers. **Q.E.D.**

Proof of Observation 1. In what follows, denote an individual savvy (non-savvy) consumer's expected surplus by ω_S (ω_{NS}). Recall that we consider first the limit $\lambda \to 1$ in a subgame where firms offer different qualities. Since savvy consumers purchase at firm L with probability 1 and in the limit firm L deterministically charges $p_L = \frac{q_L}{q_H}(c_H - h)$, it follows that $\omega_S = q_L - \frac{q_L}{q_H}(c_H - h) - h$. On the other hand, using that

$$\omega_{NS} = \frac{1}{2}\left[\int_{\underline{p}_L}^{q_L} F_L(p_L)dp_L + \int_{\underline{p}_H}^{q_H} F_H(p_H)dp_H\right] - h,$$

it is easily confirmed in the limit that

$$\omega_{NS} = \frac{1}{2}\left\{q_L - \frac{q_L}{q_H}(c_H - h)\right\} + \\ \frac{1}{2}\left\{\left(q_H - c_H + h\right) - \left[c_H - h - \frac{q_H}{q_L}(c_L - h)\right]\log\left(\frac{q_H - \frac{q_H}{q_L}(c_L - h)}{c_H - h - \frac{q_H}{q_L}(c_L - h)}\right)\right\} - h$$

and that $\omega_{NS} > \omega_S$ holds if

$$(q_H - c_H + h) - \frac{q_L}{q_H}[q_H - c_H + h] > \left[c_H - h - \frac{q_H}{q_L}(c_L - h)\right]\log\left(\frac{q_H - \frac{q_H}{q_L}(c_L - h)}{c_H - h - \frac{q_H}{q_L}(c_L - h)}\right).$$

Note that the LHS of the above inequality is strictly positive and independent of c_L. At the same time, one can check that the limit of the RHS as c_L tends to $h + \frac{q_L}{q_H}(c_H - h)$ (the highest value of c_L that is compatible with $h > \widetilde{h}$) is zero. Hence, by continuity, if both λ is sufficiently close to 1 and c_L is sufficiently large, it follows that $\omega_{NS} > \omega_S$. **Q.E.D.**

Online Appendix for "Excessive Competition on Headline Prices"

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Organization:

1. Omitted Analysis of the Salience Interpretation (General δ)
2. Omitted Analysis for a General Number of Firms
3. Model where Only Some Consumers are Salient or Relative Thinkers

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1. Omitted Analysis of the Salience Interpretation (General δ)

We return to our definition of the consumer choice criterion, precisely to the applied notion of salience. In the subsequent extension, the non-salient attribute is no longer completely ignored. For this we adopt from Bordalo et al. (2013) the concept of (gradually) discounting the respective importance, while we still follow the overall approach as in Inderst and Obradovits (2020). As discussed in the main text, we stipulate that salient thinking only affects consumers' ordering of options that can be compared along the described attributes, in our case price and quality. Alternatives outside this category, that is in our case the alternative of not buying, can not be compared along the same attributes, which is why we still posit that the comparison between firms' offers and the outside alternative is not affected by salient thinking. This hierarchical model of decision-making thus has consumers to first select their preferred choice among comparable product offers (i.e., within a "category"), in our case (p_i, q_i), before making another comparison across "categories", which in our case boils down to the decision whether to make a purchase or whether to choose the outside option of not buying. This motivates why in what follows, the respective reference point, relative to which attributes price and quality are assessed, is determined solely from the offers (p_i, q_i), that is, without reference to the value of the outside option of not buying.[1]

[1] In our view, the non-purchase decision, which is the source of consumers' reservation value, can not be meaningfully compared along the attributes of price and quality. Therefore, we posit that the choice between the option of no purchase and that of purchasing a given product is not distorted by the invoked relative or salient thinking. In models of imperfect competition, such as ours, the comparison with the outside option of not buying represents a key ingredient by constraining firms' pricing (which is not needed under perfect competition). This is different from the specification of salience in Bordalo et al. (2016). There, when a firm for instance deviates to a low price and when this makes price salient, rather than quality, this may reduce consumers' overall perceived surplus relative to the outside option, implying that while the firm diverts business from its rival, its overall quantity may decrease, as more consumers select the outside option. This is not the case in our hierarchical model and could thus constitute a testable difference in implications.

As in the main text, price (quality) is still salient if

$$\frac{q_L}{p_L} > \frac{q_H}{p_H} \tag{1}$$

holds (the strict converse of (1) holds), though now we stipulate that market-savvy consumers no longer compare products only w.r.t. the salient attribute, but still take both attributes into consideration as follows. The non-salient attribute is now discounted by some factor $\delta \in [0, 1)$, which thus measures the importance of salience. When, for instance, price is salient and when one product has high and the other low quality, low quality is strictly preferred if $p_H - p_L > \delta(q_H - q_L)$, while when quality is salient instead, the respective condition becomes $\delta(p_H - p_L) > q_H - q_L$. With respect to the outside option, which can not be compared with firms' offers along the attributes of price and quality, when i is preferred among firms' offers, consumers strictly prefer to make a purchase when $q_i - p_i > 0$.

We completely retrieve our previous analysis when $\delta = 0$. While we now extend results, it will also become immediate how expressions become more complex, though they still prove tractable.

Clearly, regardless of the size of δ, salient thinking only matters when firms offer different qualities, which is the subgame on which we now focus. Also, we restrict ourselves to parameter values for which, in the full game, the respective subgame with heterogeneous qualities indeed arises with positive probability. From the results in the main text, we can already conjecture that this requires that

$$q_H - c_H > q_L - c_L \tag{2}$$

holds as well as $h > \widetilde{h} = \frac{q_H c_L - q_L c_H}{q_H - q_L}$. We provide a formal proof of this once we endogenize firms' choice of product quality below.

When $\delta > 0$, to ensure that savvy consumers choose low quality, two conditions have now to be satisfied: First, price has to be salient with $\frac{p_L}{p_H} < \frac{q_L}{q_H}$ and, second, given that price is salient, a consumer must indeed prefer low quality as $p_H - p_L \geq \delta(q_H - q_L)$. We ask now when the condition that price is salient implies the condition that the price difference indeed outweighs the (perceived) quality difference. After substitution, this is the case if and only if $p_H \geq \delta q_H$. Hence, we can indeed neglect the second "value constraint", as this is implied by the first "salience constraint" for all price realizations if also the lower support of $F_H(p_H)$ lies above δq_H, i.e., if $\underline{p}_H \geq \delta q_H$. We denote the respective threshold by $\underline{\delta} = \underline{p}_H / q_H$, so that for $\delta \leq \underline{\delta}$ the preceding characterization for $\delta = 0$ still fully applies. But this is obviously no longer the case when δ becomes larger. Then, at least for some realizations (p_L, p_H), even though price is salient, savvy consumers will still choose the high-quality product. This adds an additional complication to the characterization of the equilibrium price distributions: these are then directly affected by δ. We will explore this further after we have provided a full characterization, which we do next.

Before we state the respective result, note the following. For sufficiently high values of δ, obviously the low-quality firm no longer enjoys a competitive advantage vis-à-vis savvy consumers, given that we presently suppose that (2) holds. We know that this then manifests itself in a lower likelihood of promotions compared to the high-quality firm. As we show next, this is the case when δ lies above the threshold defined as

$$\overline{\delta} = \left(\frac{c_H - c_L}{q_H - q_L} \right) \frac{2\lambda}{1+\lambda} + \frac{1-\lambda}{1+\lambda}. \tag{3}$$

We next provide a full characterization. A proof of this and of all following results with general δ is contained at the end of this section.

Proposition 1 *Suppose that firms have different qualities in $t = 1$ and that (2) holds together with $h > \widetilde{h}$ (which will be a prerequisite for that different qualities are indeed offered in an equilibrium of the full game). Then, now for any δ, there is a unique pricing equilibrium as follows:*
i) When $\delta \leq \underline{\delta}$, it is given by Case ii) in Lemma 1 from the main text (where we have set $\delta = 0$).
ii) When $\underline{\delta} < \delta < \overline{\delta}$, L chooses prices $p_L \in [\underline{p}_H - \delta(q_H - q_L), q_L)$, where $\underline{p}_H = c_H - h + (q_H - c_H + h)\frac{1-\lambda}{1+\lambda}$, according to the CDF

$$F_L(p) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda} \left(\frac{q_H - c_H + h}{\max\{p + \delta(q_H - q_L), p\frac{q_H}{q_L}\} - c_H + h} \right), \tag{4}$$

while H chooses prices $p_H \in [\underline{p}_H, q_H)$ according to the CDF

$$F_H(p) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_H - \delta(q_H - q_L) - c_L + h}{\min\{p - \delta(q_H - q_L), p\frac{q_L}{q_H}\} - c_L + h}\right) \quad (5)$$

and the non-discounted price $p_H = q_H$ with probability

$$m_H = 1 - \frac{q_H - c_H + h}{q_L - c_L + h} + \frac{1+\lambda}{2\lambda}\left[\frac{(q_H - q_L)(1-\delta)}{q_L - c_L + h}\right]. \quad (6)$$

iii) When $\delta \geq \overline{\delta}$, L chooses prices $p_L \in [\underline{p}_L, q_L)$, where $\underline{p}_L = c_L - h + (q_L - c_L + h)\frac{1-\lambda}{1+\lambda}$, according to the CDF

$$F_L(p) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_L + \delta(q_H - q_L) - c_H + h}{\max\{p + \delta(q_H - q_L), p\frac{q_H}{q_L}\} - c_H + h}\right) \quad (7)$$

and the non-discounted price $p_L = q_L$ with probability

$$m_L = 1 - \frac{q_L - c_L + h}{q_H - c_H + h} - \frac{1+\lambda}{2\lambda}\left[\frac{(q_H - q_L)(1-\delta)}{q_H - c_H + h}\right],$$

while H chooses prices $p_H \in [\underline{p}_L + \delta(q_H - q_L), q_H)$ according to the CDF

$$F_H(p) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_L - c_L + h}{\min\{p - \delta(q_H - q_L), p\frac{q_L}{q_H}\} - c_L + h}\right).$$

Proposition 1 reveals how firms' pricing strategies change as consumers discount non-salient attributes to a greater extent (lower δ). There is an interesting difference between the case where δ is high ($\delta \geq \overline{\delta}$) and that where δ is low ($\underline{\delta} < \delta < \overline{\delta}$), noting that price strategies remain unchanged for $\delta \leq \underline{\delta}$: When δ is still low, an increase shifts both price distributions F_L and F_H "downwards" in the sense of strict First-Order-Stochastic-Dominance (implying, in particular, that expected prices for both firms decrease), while the opposite holds when δ is high, as then a further increase in δ shifts price distributions "upwards" in the sense of strict First-Order-Stochastic-Dominance (implying now that expected prices for both firms increase).[2] This non-monotonic, though clearly signed, change in expected prices has the following intuition. When $\delta = 1$, obviously the high-quality product has a clear advantage in the market, and the same applies for $\delta \leq \underline{\delta}$ with respect to the low-quality product. In this sense, the two firms with different qualities compete "on more equal grounds" for intermediate values of δ, which then leads to the lowest (expected) prices. We can unambiguously sign how the likelihood with which savvy consumers purchase high or low quality changes in δ, given firms' equilibrium pricing.[3]

Corollary 1 *Suppose that firms have different qualities in $t = 1$ and that (2) holds together with $h > \widetilde{h}$. Then, the likelihood that the low-quality product is chosen by savvy consumers, σ_L, changes in δ as follows:*
i) For $\delta \leq \underline{\delta}$, σ_L remains constant, with $\sigma_L > 0.5$.
ii) For $\delta > \underline{\delta}$, σ_L is strictly decreasing in δ, with $\sigma_L < 0.5$ as δ approaches 1.

Corollary 1 summarizes the impact that δ has on the equilibrium outcome with heterogeneous qualities. As the extent to which salience affects savvy consumers' choice decreases (δ increases), savvy consumers become less likely to choose low quality. Note that this captures the combined effect of both an increase in δ and the thereby induced change in equilibrium prices.

Recall next that when $\delta > 0$, the determination of the salient attribute no longer automatically determines the choice of savvy consumers, as was the case when $\delta = 0$ (and as is still the case if and only if $\delta \leq \underline{\delta}$). As we have observed in the main text, a savvy consumer may show ex-post regret if price is salient, but not if quality is salient. As now δ increases, such incidences of

[2] To make this formal, denote for two values $\delta < \widehat{\delta}$ the respective distributions by F_i and $\widehat{F}_i$. Then, we have in the first case with $\underline{\delta} < \delta < \widehat{\delta} < \overline{\delta}$ that $\widehat{F}_i(p) \geq F_i(p)$ holds everywhere and strictly so for a positive interval, while for $\overline{\delta} < \delta < \widehat{\delta}$ the converse holds. This follows immediately from differentiation of the respective CDFs and noting that obviously $p \geq \underline{p}_i$ over the respective support.

[3] A first indication of this is provided by the following observation from Proposition 1. Note that when we proceed from case ii) to case iii), as δ increases, the identity of the firm that promotes more often changes: When still $\delta < \overline{\delta}$, the low-quality product is promoted more often, while when $\delta > \overline{\delta}$, it is now the low-quality firm's price distribution that has a mass point at the non-discounted price $p_L = q_L$, while this is then no longer the case for the high-quality firm's price distribution.

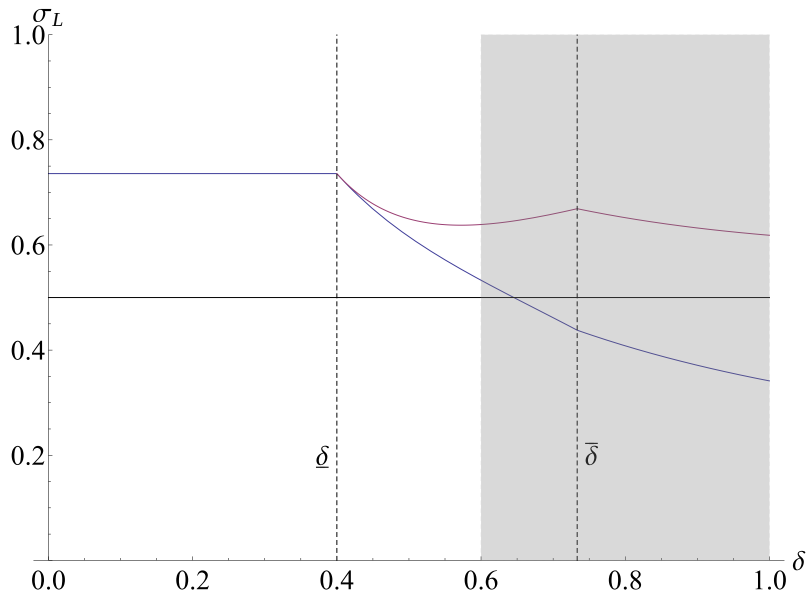

Figure 1: Illustration of savvy consumers' probability of choosing q_L (σ_L, lower line) and the probability that price is salient (upper line), given δ. The parameters used are $q_H = 1$, $c_H = 0.7$, $q_L = 0.5$, $c_L = 0.4$, $h = 0.6$, $\lambda = 0.5$.

ex-post regret become increasingly less likely, so that notably price becomes salient but savvy consumers still decide to purchase the more expensive high-quality product. Incidentally, the likelihood with which quality or price becomes salient in equilibrium proves not to be monotonic in δ. This is illustrated by the example in Figure 1. Note that we have already shaded in Figure 1 the area (that is, all sufficiently high values of δ) for which, as shown below, no longer an equilibrium with heterogeneous qualities exists.

We conclude our discussion of the pricing equilibrium with an additional observation that may be informative for empirical researchers. We have shown that (at least when $h > \widetilde{h}$ and $\delta > \underline{\delta}$) the extent to which consumers discount non-salient attributes has a distinct impact on the likelihood with which low or high-quality products are purchased in equilibrium. We use next the fact that δ does not affect the behavior of non-savvy consumers, but only that of consumers who are savvy to all offers. The difference between their behavior under firms' equilibrium pricing strategies can thus be informative regarding the extent of salient thinking (i.e., regarding the extent to which non-salient attributes are discounted). Precisely, the ratio $\frac{\sigma_L}{1/2}$ measures the propensity of savvy consumers to purchase low quality relative to that of those who do not compare offers (and thus end up purchasing always the same product, regardless of whether, for instance, one or the other product is promoted). In fact, to empiricists consumers' propensity to compare offers and switch (λ) may be readily observable in data. By the preceding observations, ceteris paribus, differences in the decisions of individual consumers may then provide information on the extent to which salient thinking leads to a discounting of non-salient attributes.

We finally solve for the unique symmetric equilibrium in product choice (at $t = 0$).[4] Note that for brevity's sake we now no longer consider the possibility of unshrouding (in $t = 0.5$).

Proposition 2 *If the converse of (2) holds, then for all $\delta \in [0, 1]$ both firms choose low quality in the unique equilibrium. If (2) holds and either*

$$h \leq h^* = \widetilde{h} + \frac{1-\lambda}{2\lambda} \frac{q_H}{q_H - q_L} [(q_H - c_H) - (q_L - c_L)]$$

or $\delta \geq \widetilde{\delta} = \frac{c_H - c_L}{q_H - q_L}$, both firms choose high quality in the unique equilibrium. Otherwise, that is if (2) holds, $h > h^*$, and $\delta < \widetilde{\delta}$, both low and high quality are offered with positive probability in the unique symmetric equilibrium. Precisely, when $\delta \leq \underline{\delta}$, the characterization of Proposition 2 from the main text applies, so that the choice of high quality γ is independent of δ, while when $\delta \in (\underline{\delta}, \widetilde{\delta})$, we have that

$$\gamma = \Pr(q_i = q_H) = \frac{\frac{1-\lambda}{1+\lambda} [(q_H - c_H) - (q_L - c_L)]}{\frac{1-\lambda}{1+\lambda} [(q_H - c_H) - (q_L - c_L)] + (c_H - c_L) - \delta(q_H - q_L)},$$

which strictly increases in δ (with $\gamma(\widetilde{\delta}) = 1$).

There are thus again two channels through which the extent of salient thinking affects the likelihood with which consumers end up buying low-quality goods even when this is inefficient: When δ is lower, this negatively affects, first, the

[4] For the characterization, it is instructive to note that $h > h^*$ is equivalent to $\widetilde{\delta} > \underline{\delta}$.

likelihood with which each firm provides high quality, γ, and, second, the likelihood $\sigma_H = 1 - \sigma_L$ with which, provided that different qualities are offered, savvy consumers purchase high quality under the resulting equilibrium prices. Consequently, the ex-ante likelihood is strictly lower when also δ is lower. We summarize these observations as follows:

Corollary 2 *When (2) holds and firms can shroud charges sufficiently ($h > h^*$), as δ decreases, this tilts firms' product choice towards the low-quality product (lower γ) and, when products of different qualities are offered, makes it more likely that market-savvy consumers purchase low-quality (higher σ_L).*

Proof of Proposition 1. We treat separately the three cases with $\delta \in [0, \underline{\delta}]$, $\delta \in (\underline{\delta}, \overline{\delta})$, and $\delta \in [\overline{\delta}, 1]$, noting that $0 < \underline{\delta} < \overline{\delta} < 1$ (using that $h > \widetilde{h}$ for $\underline{\delta} < \overline{\delta}$). Before doing so, we make however some observations that apply generally.

In the pricing equilibria, we know from standard arguments, see e.g. Varian (1980) and Narasimhan (1988), that supports are convex and cannot contain mass points in the interior or at the lower boundary, while upper boundaries are given by q_i. Further, for either firm the minimum equilibrium profits are clearly given by $(q_i - c_i + h)\frac{1-\lambda}{2}$. Our final observation now relates to the behavior of savvy consumers. Note first that also for positive δ, the firm with quality q_L can only attract savvy consumers if price is salient. To see this, suppose that quality is salient instead, $p_L > p_H \frac{q_L}{q_H}$. Consumers would then still be attracted by L if $q_H - \delta p_H < q_L - \delta p_L$, i.e. $p_L < p_H - \frac{q_H - q_L}{\delta}$. Hence, the two inequalities for p_L can only be satisfied simultaneously if $p_H - \frac{q_H - q_L}{\delta} > p_H \frac{q_L}{q_H}$, which reduces to $p_H \overset{!}{>} \frac{q_H}{\delta} > q_H$ and thus cannot occur in equilibrium. As a consequence, p_L needs to satisfy two conditions in order to attract savvy consumers: (1) $p_L < p_H \frac{q_L}{q_H}$ (ensuring that price is salient) and (2) $\delta q_H - p_H < \delta q_L - p_L$, i.e. $p_L < p_H - \delta(q_H - q_L)$ (ensuring that firm L offers a higher perceived utility than H, given that price is salient). Taken together, it must hold that $p_L < \min\{p_H \frac{q_L}{q_H}, p_H - \delta(q_H - q_L)\}$, while when we change perspective, for H to attract savvy consumers it must hold that $p_H < \max\{p_L \frac{q_H}{q_L}, p_L + \delta(q_H - q_L)\}$. These conditions will be used in what follows.

Case i): $\delta \in [0, \underline{\delta}]$.

Assertion (i): $\pi_H = (q_H - c_H + h)\frac{1-\lambda}{2}$.

Proof of Assertion (i): It thus remains to contradict that $\pi'_H > \pi_H$ constitutes H's equilibrium profit. In that case, if H's upper pricing bound is denoted by $\overline{p}_H \leq q_H$, H must attract savvy consumers with positive probability when pricing at $\overline{p}_H$. It follows that L must have positive probability mass at or above $\min\{\overline{p}_H \frac{q_L}{q_H}, \overline{p}_H - \delta(q_H - q_L)\}$, which further implies that L's equilibrium profit is bounded above by $\overline{\pi}_L = (q_L - c_L + h)\frac{1-\lambda}{2}$ (this is true in particular since it cannot be the case that both H has a mass point at $\overline{p}_H$ and L has a mass point at $\min\{\overline{p}_H \frac{q_L}{q_H}, \overline{p}_H - \delta(q_H - q_L)\}$). Now since $\pi'_H > (q_H - c_H + h)\frac{1-\lambda}{2}$ by assumption, H's pricing is bounded below by $\underline{p}'_H$ which solves $(p_H - c_H + h)\frac{1+\lambda}{2} = \pi'_H$. Clearly, $\underline{p}'_H > \underline{p}_H = c_H - h + \frac{1-\lambda}{1+\lambda}(q_H - c_H + h)$ due to $\pi'_H > (q_H - c_H + h)\frac{1-\lambda}{2}$. Hence, by pricing at $\min\{\underline{p}'_H \frac{q_L}{q_H}, \underline{p}'_H - \delta(q_H - q_L)\} = \underline{p}'_H \frac{q_L}{q_H}$ (where the equality follows from $\delta < \frac{\underline{p}'_H}{q_H}$, which is true since $\delta \leq \underline{\delta} = \frac{\underline{p}_H}{q_H}$ by assumption, and $\underline{p}'_H > \underline{p}_H$), L could guarantee a profit of $(\underline{p}'_H \frac{q_L}{q_H} - c_L + h)\frac{1+\lambda}{2} > \overline{\pi}_L$ due to $h > \widetilde{h}$. Thus we obtain a contradiction. **Q.E.D.**

Assertion (ii): $\pi_L = \frac{1-\lambda}{2}(q_L - c_L + h) + \lambda\left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right]$.

Proof of Assertion (ii): From Assertion (i) above, we know H's equilibrium profit π_H, so that p_H is not below the value that solves $(p_H - c_H + h)\frac{1+\lambda}{2} = \pi_H$, i.e. $p_H \geq \underline{p}_H = c_H - h + \frac{1-\lambda}{1+\lambda}(q_H - c_H + h)$. Thus, by pricing at $\min\{\underline{p}_H \frac{q_L}{q_H}, \underline{p}_H - \delta(q_H - q_L)\} = \underline{p}_H \frac{q_L}{q_H}$, where the equality follows from $\delta \leq \underline{\delta}$, L can guarantee a profit of $(\underline{p}_H \frac{q_L}{q_H} - c_L + h)\frac{1+\lambda}{2} = \pi_L$. We proceed to show that L cannot make a higher profit. Suppose it did, such that $\pi'_L > \pi_L$. In turn, the lowest price L may ever charge in the respective candidate equilibrium strictly exceeds $\underline{p}_H \frac{q_L}{q_H}$, i.e. $\underline{p}_L > \underline{p}_H \frac{q_L}{q_H}$. Hence, firm H could guarantee to attract all savvy consumers by pricing at $\underline{p}_L \frac{q_H}{q_L} > \underline{p}_H$, making an expected profit that strictly exceeds π_H. Clearly, this contradicts Assertion (i). **Q.E.D.**

For $\delta \in [0, \underline{\delta}]$ profits as well as equilibrium price distributions are thus as for $\delta = 0$. Note in particular that as firm H will never price below $\underline{p}_H$, it was already argued earlier in this section that for $\delta \leq \underline{\delta}$, consumers' choice remains identical to the case where $\delta = 0$.

Case ii): $\delta \in (\underline{\delta}, \overline{\delta})$.

Assertion (i): $\pi_H = (q_H - c_H + h)\frac{1-\lambda}{2}$.

Proof of Assertion (i): It thus remains to contradict that $\pi'_H > \pi_H$ constitutes H's equilibrium profit. By an analogous argument as in Case i), in this case L's equilibrium profit would be bounded above by $\overline{\pi}_L = (q_L - c_L + h)\frac{1-\lambda}{2}$. But as $\pi'_H > (q_H - c_H + h)\frac{1-\lambda}{2}$, we know that H's pricing is bounded below by $\underline{p}'_H$ which solves $(p_H - c_H + h)\frac{1+\lambda}{2} = \pi'_H$. Clearly, $\underline{p}'_H > \underline{p}_H = c_H - h + \frac{1-\lambda}{1+\lambda}(q_H - c_H + h)$ due to $\pi'_H > (q_H - c_H + h)\frac{1-\lambda}{2}$. Hence, by pricing at $\min\{\underline{p}'_H \frac{q_L}{q_H}, \underline{p}'_H - \delta(q_H - q_L)\}$, L could guarantee to make a profit of $\pi'_L = (\min\{\underline{p}'_H \frac{q_L}{q_H}, \underline{p}'_H - \delta(q_H - q_L)\} - c_L + h)\frac{1+\lambda}{2}$. Note that no matter whether $\underline{p}'_H \frac{q_L}{q_H}$ or $\underline{p}'_H - \delta(q_H - q_L)$ is the relevant expression under the minimum operator, π'_L strictly exceeds $\overline{\pi}_L$ due to $\underline{p}'_H > \underline{p}_H$, $h > \widetilde{h}$, and $\delta < \overline{\delta}$. Thus we obtain a contradiction. **Q.E.D.**

Assertion (ii): $\pi_L = [\underline{p}_H - \delta(q_H - q_L) - c_L + h]\frac{1+\lambda}{2}$.

Proof of Assertion (ii): From Assertion (i) above, we know H's equilibrium profit π_H, so that p_H is not below the value that solves $(p_H - c_H + h)\frac{1+\lambda}{2} = \pi_H$, i.e. $p_H \geq \underline{p}_H = c_H - h + \frac{1-\lambda}{1+\lambda}(q_H - c_H + h)$. Thus, by pricing at $\min\{\underline{p}_H \frac{q_L}{q_H}, \underline{p}_H - \delta(q_H - q_L)\} = \underline{p}_H - \delta(q_H - q_L)$, where the equality follows from $\delta > \underline{\delta}$, L can guarantee to make a profit of $(\underline{p}_H - \delta(q_H - q_L) - c_L + h)\frac{1+\lambda}{2} = \pi_L$. We proceed to show that L cannot make a higher profit. Suppose it did, such that $\pi'_L > \pi_L$. In turn, the lowest price L may ever charge in the respective candidate equilibrium strictly exceeds $\underline{p}_H - \delta(q_H - q_L)$, i.e. $\underline{p}_L > \underline{p}_H - \delta(q_H - q_L)$. Hence, firm H could guarantee to attract all savvy consumers by pricing at $\underline{p}_L + \delta(q_H - q_L) > \underline{p}_H$, making an expected profit that strictly exceeds π_H. Clearly, this contradicts Assertion (i). **Q.E.D.**

Given that $h > \widetilde{h}$ and $\delta \in (\underline{\delta}, \overline{\delta})$, each candidate pricing equilibrium must satisfy the following conditions: (a) $\pi_L = [\underline{p}_H - \delta(q_H - q_L) - c_L + h]\frac{1+\lambda}{2}$ (b) $\pi_H = \frac{1-\lambda}{2}(q_H - c_H + h)$ (c) $\underline{p}_H = \underline{p}_L + \delta(q_H - q_L)$ (d) $\overline{p}_H = \overline{p}_L \frac{q_H}{q_L} = q_H$ (e) both $F_H(p_H)$ and $F_L(p_L)$ are continuous and strictly increasing over their support. We complete Case ii) by solving for firms' (unique) pricing equilibrium in a constructive manner from the respective indifference condition, which for H is

$$(p_H - c_H + h)\left(\frac{1-\lambda}{2} + \lambda\left[1 - F_L\Big(\min\{p_H\frac{q_L}{q_H}, p_H - \delta(q_H - q_L)\}\Big)\right]\right) \overset{!}{=} \pi_H.$$

Note now that $\min\{p_H\frac{q_L}{q_H}, p_H - \delta(q_H - q_L)\}$ is given by $p_H - \delta(q_H - q_L)$ for $p_H < \delta q_H$, and by $p_H\frac{q_L}{q_H}$ for $p_H > \delta q_H$. Isolating for $F_L(.)$ in both cases yields $F_L(p_H - \delta(q_H - q_L)) = \frac{1+\lambda}{2\lambda} - \frac{\pi_H/\lambda}{p_H - c_H + h}$ for $p_H < \delta q_H$ and $F_L\big(p_H\frac{q_L}{q_H}\big) = \frac{1+\lambda}{2\lambda} - \frac{\pi_H/\lambda}{p_H - c_H + h}$ for $p_H > \delta q_H$. After inserting π_H and substitution we thus obtain $F_L(p_L) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_H - c_H + h}{p_L + \delta(q_H - q_L) - c_H + h}\right)$ for $p_L < \delta q_L$ and $F_L(p_L) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_H - c_H + h}{p_L\frac{q_H}{q_L} - c_H + h}\right)$ for $p_L > \delta q_L$. Taken together, we can also write $F_L(p_L)$ as in (5). Proceeding likewise for L, we have

$$(p_L - c_L + h)\left(\frac{1-\lambda}{2} + \lambda\left[1 - F_H\Big(\max\{p_L\frac{q_H}{q_L}, p_L + \delta(q_H - q_L)\}\Big)\right]\right) \overset{!}{=} \pi_L.$$

Note now that $\max\{p_L\frac{q_H}{q_L}, p_L + \delta(q_H - q_L)\}$ is given by $p_L + \delta(q_H - q_L)$ for $p_L < \delta q_L$, and by $p_L\frac{q_H}{q_L}$ for $p_L > \delta q_L$. Isolating for $F_H(.)$ in both cases yields $F_H(p_L + \delta(q_H - q_L)) = \frac{1+\lambda}{2\lambda} - \frac{\pi_L/\lambda}{p_L - c_L + h}$ for $p_L < \delta q_L$ and $F_H\big(p_L\frac{q_H}{q_L}\big) = \frac{1+\lambda}{2\lambda} - \frac{\pi_L/\lambda}{p_L - c_L + h}$ for $p_L > \delta q_L$. After inserting π_H and substitution we thus obtain $F_H(p_H) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_H - \delta(q_H - q_L) - c_L + h}{p_H - \delta(q_H - q_L) - c_L + h}\right)$ for $p_H < \delta q_H$ and $F_H(p_H) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_H - \delta(q_H - q_L) - c_L + h}{p_H\frac{q_L}{q_H} - c_L + h}\right)$ for $p_H > \delta q_H$. Taken together, we can also write $F_H(p_H)$ as in (5).

We finally show that the equilibrium CDFs are well-behaved. Clearly, it follows directly from their respective indifference conditions that the CDFs are strictly increasing. Note also that $F_L(\underline{p}_L) = 0$ and $\lim_{p_L \to q_L} F_L(p_L) = 1$, whereas $F_H(\underline{p}_H) = 0$ and $F_H(q_H) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_H - \delta(q_H - q_L) - c_L + h}{q_L - c_L + h}\right) = \frac{q_H - c_H + h}{q_L - c_L + h} - \frac{1+\lambda}{2\lambda}\left[\frac{(q_H - q_L)(1-\delta)}{q_L - c_L + h}\right] < 1$ due to $\delta < \overline{\delta}$. Hence, also firm H's mass point at q_H, $m_H = 1 - F_H(q_H)$, is well-behaved.

Case iii): $\delta \in [\overline{\delta}, 1]$.
Assertion (i): $\pi_L = (q_L - c_L + h)\frac{1-\lambda}{2}$.

Proof of Assertion (i): It thus remains to contradict that $\pi'_L > \pi_L$ constitutes L's equilibrium profit. In that case, if L's upper pricing bound is denoted by $\overline{p}_L \leq q_L$, L must attract savvy consumers with positive probability when pricing at $\overline{p}_L$. It follows that firm H must have positive probability mass at or above $\max\{\overline{p}_L \frac{q_H}{q_L}, \overline{p}_L + \delta(q_H - q_L)\}$, which further implies that H's equilibrium profit is bounded above by $\overline{\pi}_L = (q_H - c_H + h)\frac{1-\lambda}{2}$ (this is true in particular since it cannot be the case that both L has a mass point at $\overline{p}_L$ and H has a mass point at $\max\{\overline{p}_L \frac{q_H}{q_L}, \overline{p}_L + \delta(q_H - q_L)\}$). Now since $\pi'_L > (q_L - c_L + h)\frac{1-\lambda}{2}$ by assumption, L's pricing is bounded below by $\underline{p}'_L$ which solves $(p_L - c_L + h)\frac{1+\lambda}{2} = \pi'_L$. Clearly, $\underline{p}'_L > \underline{p}_L = c_L - h + \frac{1-\lambda}{1+\lambda}(q_L - c_L + h)$ due to $\pi'_L > (q_L - c_L + h)\frac{1-\lambda}{2}$. Hence, by pricing at $\max\{\underline{p}'_L \frac{q_H}{q_L}, \underline{p}'_L + \delta(q_H - q_L)\} = \underline{p}'_L + \delta(q_H - q_L)$ (where the equality follows from $\delta > \frac{\underline{p}'_L}{q_L}$, which is true since (i) $\delta \geq \overline{\delta}$ by assumption, (ii) $\overline{\delta} > \frac{\underline{p}_L}{q_L}$ is equivalent to $h > \widetilde{h}$, as assumed, and (iii) $\underline{p}'_L > \underline{p}_L$), H could guarantee to make a profit of $\pi'_H = (\underline{p}'_L + \delta(q_H - q_L) - c_H + h)\frac{1+\lambda}{2} > \overline{\pi}_H$ due to $\underline{p}'_L > \underline{p}_L$ and $\delta > \overline{\delta}$. Thus we obtain a contradiction. **Q.E.D.**

Assertion (ii): $\pi_H = (\underline{p}_L + \delta(q_H - q_L) - c_H + h)\frac{1+\lambda}{2}$.
Proof of Assertion (ii): From Assertion (i) above, we know L's equilibrium profit π_L, so that p_L is not below the value that solves $(p_L - c_L + h)\frac{1+\lambda}{2} = \pi_L$, i.e. $p_L \geq \underline{p}_L = c_L - h + \frac{1-\lambda}{1+\lambda}(q_L - c_L + h)$. Thus, by pricing at $\max\{\underline{p}_L \frac{q_H}{q_H}, \underline{p}_L + \delta(q_H - q_L)\} = \underline{p}_L + \delta(q_H - q_L)$—where the equality follows from $\delta > \frac{\underline{p}_L}{q_L}$, which is true by the same reasons as given in the proof of Assertion 1 above—H can guarantee to make a profit of $(\underline{p}_L + \delta(q_H - q_L) - c_H + h)\frac{1+\lambda}{2} = \pi_H$. We proceed to show that H cannot make a higher profit. Suppose it did, such that $\pi'_H > \pi_H$. In turn, the lowest price H may ever charge in the respective candidate equilibrium strictly exceeds $\underline{p}_L + \delta(q_H - q_L)$, i.e. $\underline{p}_H > \underline{p}_L + \delta(q_H - q_L)$. Hence, firm L could guarantee to attract all savvy consumers by pricing at $\min\{\underline{p}_H - \delta(q_H - q_L), \underline{p}_H \frac{q_L}{q_H}\} > \underline{p}_L$, making a profit that strictly exceeds π_L. Clearly, this contradicts Assertion (i). **Q.E.D.**

Given that $h > \widetilde{h}$ and $\delta \in [\overline{\delta}, 1]$, each candidate pricing equilibrium must satisfy the following conditions: (a) $\pi_L = (q_L - c_L + h)\frac{1-\lambda}{2}$ (b) $\pi_H = (\underline{p}_L +$

$\delta(q_H - q_L) - c_H + h)\frac{1+\lambda}{2}$ (c) $\underline{p}_H = \underline{p}_L + \delta(q_H - q_L)$ (d) $\overline{p}_H = \overline{p}_L \frac{q_H}{q_L} = q_H$ (e) both $F_H(p_H)$ and $F_L(p_L)$ are continuous and strictly increasing over their support. We complete Case iii) by solving for firms' (unique) pricing equilibrium in a constructive manner from the indifference condition. Then, by the same steps as in Case ii) above, it follows that $F_L(p_H - \delta(q_H - q_L)) = \frac{1+\lambda}{2\lambda} - \frac{\pi_H/\lambda}{p_H - c_H + h}$ for $p_H < \delta q_H$ and $F_L\big(p_H \frac{q_L}{q_H}\big) = \frac{1+\lambda}{2\lambda} - \frac{\pi_H/\lambda}{p_H - c_H + h}$ for $p_H > \delta q_H$. After inserting π_H and substitution we thus obtain $F_L(p_L) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_L + \delta(q_H - q_L) - c_H + h}{p_L + \delta(q_H - q_L) - c_H + h}\right)$ for $p_L < \delta q_L$ and $F_L(p_L) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_L + \delta(q_H - q_L) - c_H + h}{p_L \frac{q_H}{q_L} - c_H + h}\right)$ for $p_L > \delta q_L$. Taken together, we can also write $F_L(p_L)$ as (6). Using next firm L's indifference condition, once again by the same steps as in Case ii) above, it follows that $F_H(p_L + \delta(q_H - q_L)) = \frac{1+\lambda}{2\lambda} - \frac{\pi_L/\lambda}{p_L - c_L + h}$ for $p_L < \delta q_L$ and $F_H\big(p_L \frac{q_H}{q_L}\big) = \frac{1+\lambda}{2\lambda} - \frac{\pi_L/\lambda}{p_L - c_L + h}$ for $p_L > \delta q_L$. After inserting π_H and substitution we thus obtain $F_H(p_H) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_L - c_L + h}{p_H - \delta(q_H - q_L) - c_L + h}\right)$ for $p_H < \delta q_H$ and $F_H(p_H) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_L - c_L + h}{p_H \frac{q_L}{q_H} - c_L + h}\right)$ for $p_H > \delta q_H$. Taken together, we can also write $F_H(p_H)$ as in (7). To finally see that the CDFs are well-behaved, note in particular that $F_H(\underline{p}_H) = 0$ and $\lim_{p_H \to q_H} F_H(p_H) = 1$, whereas $F_L(\underline{p}_L) = 0$ and $F_L(q_L) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_L + \delta(q_H - q_L) - c_H + h}{q_H - c_H + h}\right) = \frac{q_L - c_L + h}{q_H - c_H + h} + \frac{1+\lambda}{2\lambda}\left[\frac{(q_H - q_L)(1-\delta)}{q_H - c_H + h}\right] \leq 1$ due to $\delta \geq \overline{\delta}$, so that firm L's mass point at q_L, $m_L = 1 - F_L(q_L)$, is well-behaved. **Q.E.D.**

Proof of Corollary 1. We treat separately the cases as in the assertions in Proposition 1.

(i) Since for $\delta \leq \underline{\delta}$ firms' equilibrium strategies are independent of δ, changes in δ clearly do not affect the probability σ_L that savvy consumers choose q_L. The assertion that $\sigma_L > 1/2$ follows from $h > \widetilde{h}$ and Proposition 1 in the main text.

(ii) For $\delta \in (\underline{\delta}, \overline{\delta})$, π_H does not depend on δ. Since firm H must be indifferent between each price in its support, the probability that firm L wins the savvy consumers when firm H chooses some price p_H is defined implicitly by

$$\pi_H(p_H) = (p_H - c_H + h)\left[\frac{1+\lambda}{2} - \lambda \Pr\{\text{L wins} \mid p_H\}\right] \overset{!}{=} (q_H - c_H + h)\frac{1-\lambda}{2},$$

which becomes

$$\Pr\{\text{L wins} \mid p_H\} = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_H - c_H + h}{p_H - c_H + h}\right).$$

Hence, the unconditional probability that firm L wins the savvy consumers is given by

$$\sigma_L = \int_{\underline{p}_H}^{q_H} \Pr\{\text{L wins} \mid p_H\} f_H(p_H) dp_H + m_H,$$

where m_H denotes the probability that firm H chooses its mass point at q_H. Using integration by parts, this can be rewritten as

$$\begin{aligned}\sigma_L &= \Pr\{\text{L wins} \mid p_H\} F_H(p_H)\Big|_{\underline{p}_H}^{q_H} - \int_{\underline{p}_H}^{q_H} \frac{d\Pr\{\text{L wins} \mid p_H\}}{dp_H} F_H(p_H) dp_H + m_H \\ &= 1 - \int_{\underline{p}_H}^{q_H} \left[\frac{1-\lambda}{2\lambda}\left(\frac{q_H - c_H + h}{(p_H - c_H + h)^2}\right)\right] F_H(p_H) dp_H.\end{aligned}$$

That σ_L is strictly decreasing in δ for $\delta \in (\underline{\delta}, \overline{\delta})$ now follows because the relevant expression for $F_H(p_H)$ in (5) is strictly increasing in δ.

(iii) For $\delta \geq \overline{\delta}$, π_L does not depend on δ. Similar to above, since firm L must be indifferent between each price in its pricing support, we have from

$$\pi_L(p_L) = (p_L - c_L + h)\left[\frac{1-\lambda}{2} + \lambda \Pr\{\text{L wins} \mid p_L\}\right] \stackrel{!}{=} (q_L - c_L + h)\frac{1-\lambda}{2}$$

that

$$\Pr\{\text{L wins} \mid p_L\} = \frac{1-\lambda}{2\lambda}\left(\frac{q_L - c_L + h}{p_L - c_L + h}\right)$$

and thus

$$\sigma_L = \int_{\underline{p}_L}^{q_L} \Pr\{\text{L wins} \mid p_L\} f_L(p_L) dp_L,$$

noting that it cannot attract the savvy consumers if it chooses its mass point at q_L. Using integration by parts, this can be rewritten as

$$\begin{aligned}\sigma_L &= \Pr\{\text{L wins} \mid p_L\} F_L(p_L)\Big|_{\underline{p}_L}^{q_L} - \int_{\underline{p}_L}^{q_L} \frac{d\Pr\{\text{L wins} \mid p_L\}}{dp_L} F_L(p_L) dp_L \\ &= \int_{\underline{p}_L}^{q_L} \left[\frac{1-\lambda}{2\lambda}\left(\frac{q_L - c_L + h}{(p_L - c_L + h)^2}\right)\right] F_L(p_L) dp_L.\end{aligned}$$

That σ_L is strictly decreasing in δ for $\delta \in [\underline{\delta}, 1]$ now follows because the relevant expression for $F_L(p_L)$ in (6) is strictly decreasing in δ.

In the rest of the proof we show that $\sigma_L < 1/2$ for $\delta \to 1$. To see this, we first calculate the limit CDFs as $\delta \to 1$, which yields

$$F_H(p_H) = \frac{1+\lambda}{2\lambda} - \frac{1-\lambda}{2\lambda}\left(\frac{q_L - c_L + h}{p_H - (q_H - q_L) - c_L + h}\right)$$

and

$$F_L(p_L) = \frac{1+\lambda}{2\lambda} - \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_L + (q_H - q_L) - c_H + h}{p_L + (q_H - q_L) - c_H + h}\right).$$

Note now that for $\delta \to 1$, savvy consumers choose the product with quality q_L if and only if $u_L = q_L - p_L > q_H - p_H = u_H$. Substituting appropriately, we may reinterpret firms' pricing CDFs as random utility draws, where

$$F_L^u(u_L) = \Pr\{\widetilde{u}_L \le u_L\} = 1 - F_L(q_L - u_L) = \frac{1+\lambda}{2\lambda}\left(\frac{\underline{p}_L + (q_H - q_L) - c_H + h}{q_H - u_L - c_H + h}\right) - \frac{1-\lambda}{2\lambda}$$

and

$$F_H^u(u_H) = \Pr\{\widetilde{u}_H \le u_H\} = 1 - F_H(q_H - u_H) = \frac{1-\lambda}{2\lambda}\left(\frac{q_L - c_L + h}{q_L - u_H - c_L + h}\right) - \frac{1-\lambda}{2\lambda}.$$

Observe moreover that since $\underline{p}_H = \underline{p}_L + (q_H - q_L)$ and $\overline{p}_H = q_H = \overline{p}_L + (q_H - q_L)$, $F_L^u(u_L)$ and $F_H^u(u_H)$ are defined over the same supports. It is then apparent that $\sigma_L < 1/2$ for $\delta \to 1$ if $F_H^u(u) \le F_L^u(u)$ for all u in firms' joint supports (with strict inequality for some u), i.e. if $F_H^u(u)$ first-order stochastically dominates

$F_L^u(u)$. After simplifying $F_H^u(u) \leq F_L^u(u)$ and rearranging, we thus ask if

$$\frac{1-\lambda}{2}(q_L - c_L + h) \leq \frac{1+\lambda}{2}(\underline{p}_L + (q_H - q_L) - c_H + h)\left(\frac{q_L - u - c_L + h}{q_H - u - c_H + h}\right).$$

As by definition $\frac{1-\lambda}{2}(q_L - c_L + h) = \frac{1+\lambda}{2}(\underline{p}_L - c_L + h)$, after some manipulation the above condition can further be simplified to

$$u \leq (q_L - c_L + h)\left(1 - \frac{1-\lambda}{1+\lambda}\right).$$

Considering firm L, this is equivalent to asking whether $q_L - p_L \leq (q_L - c_L + h)\left(1 - \frac{1-\lambda}{1+\lambda}\right)$ for all p_L, i.e. $p_L \geq q_L - (q_L - c_L + h)\left(1 - \frac{1-\lambda}{1+\lambda}\right) = c_L - h + \frac{1-\lambda}{1+\lambda}(q_L - c_L + h) = \underline{p}_L$. This is indeed satisfied for all p_L, and with strict inequality for $p_L > \underline{p}_L$. This completes the proof. **Q.E.D.**

Proof of Proposition 2. We first prove that (q_L, q_L) is the unique equilibrium in product choice if $q_H - c_H < q_L - c_L$. Note here that the analysis in Proposition 1 has not yet treated this case. However, the following assertion follows from arguments that are fully analogous.

Assertion (i): If $q_H - c_H < q_L - c_L$, firm H's equilibrium profit is bounded above by $\pi_H = (q_H - c_H + h)\frac{1-\lambda}{2}$ for all $\delta \in [0, 1]$.
Proof of Assertion (i): Suppose to the contrary that $\pi'_H > \pi_H$ constitutes H's equilibrium profit. Then, if H's upper pricing bound is denoted by $\overline{p}_H \leq q_H$, H must attract savvy consumers with positive probability when pricing at $\overline{p}_H$. It follows that L must have positive probability mass at or above $\min\{\overline{p}_H \frac{q_L}{q_H}, \overline{p}_H - \delta(q_H - q_L)\}$, which further implies that L's equilibrium profit is bounded above by $\overline{\pi}_L = (q_L - c_L + h)\frac{1-\lambda}{2}$ (this is true in particular since it cannot be the case that both H has a mass point at $\overline{p}_H$ and L has a mass point at $\min\{\overline{p}_H \frac{q_L}{q_H}, \overline{p}_H - \delta(q_H - q_L)\}$). Now since $\pi'_H > (q_H - c_H + h)\frac{1-\lambda}{2}$ by assumption, H's pricing is bounded below by $\underline{p}'_H$ which solves $(p_H - c_H + h)\frac{1+\lambda}{2} = \pi'_H$. Clearly, $\underline{p}'_H > \underline{p}_H = c_H - h + \frac{1-\lambda}{1+\lambda}(q_H - c_H + h)$ due to $\pi'_H > (q_H - c_H + h)\frac{1-\lambda}{2}$. Hence, for all $\delta \in [0, 1]$, by pricing at $\underline{p}'_H - (q_H - q_L) < \underline{p}_H \frac{q_L}{q_H}$, L could guarantee to make a

profit of $\pi'_L = (\underline{p}'_H - (q_H - q_L) - c_L + h)\frac{1+\lambda}{2}$. Note that this strictly exceeds $\overline{\pi}_L$ due to $\underline{p}'_H > \underline{p}_H$ and $q_H - c_H < q_L - c_L$. Thus we obtain a contradiction. **Q.E.D.**

With Assertion (i), $q_H - c_H < q_L - c_L$, and as we know that a firm's profit is always bounded from below by $(q_i - c_i + h)\frac{1-\lambda}{2}$, for all δ choosing L is thus a dominant strategy for both firms. In the rest of the proof we thus consider the case where (2) holds, where we first suppose that $h \leq \widetilde{h}$.

Assertion (ii): If $q_H - c_H > q_L - c_L$ and $h \leq \widetilde{h}$, firm L's equilibrium expected profit is bounded above by $\pi_L = (q_L - c_L + h)\frac{1-\lambda}{2}$ for all $\delta \in [0, 1]$.
Proof of Assertion (ii): Assuming to the contrary that $\pi'_L > \pi_L$ constitutes L's equilibrium profit, as previously we can then argue that now H's equilibrium profit is bounded above by $\overline{\pi}_H = (q_H - c_H + h)\frac{1-\lambda}{2}$. As $\pi'_L > (q_L - c_L + h)\frac{1-\lambda}{2}$, we know that L's pricing is bounded below by $\underline{p}'_L$ which solves $(p_L - c_L + h)\frac{1+\lambda}{2} = \pi'_L$. Clearly, $\underline{p}'_L > \underline{p}_L = c_L - h + \frac{1-\lambda}{1+\lambda}(q_L - c_L + h)$ due to $\pi'_L > (q_L - c_L + h)\frac{1-\lambda}{2}$. Hence, for all $\delta \in [0, 1]$, by pricing at $\underline{p}'_L \frac{q_H}{q_L}$, H could guarantee to make a profit of $\pi'_H = (\underline{p}'_L \frac{q_H}{q_L} - c_H + h)\frac{1+\lambda}{2} > \overline{\pi}_H$ due to $\underline{p}'_L > \underline{p}_L$ and $h > \widetilde{h}$. Thus we obtain a contradiction. **Q.E.D.**

With Assertion (ii), $q_H - c_H > q_L - c_L$, and as we know that a firm's profit is always bounded from below by $(q_i - c_i + h)\frac{1-\lambda}{2}$, for all δ choosing H is thus a dominant strategy for both firms in this case. In the rest of the proof we now suppose that $q_H - c_H > q_L - c_L$ and $h > \widetilde{h}$, for which we can now fully rely on our characterization of the unique pricing equilibria for arbitrary δ, as given in Proposition 1. In particular, we employ firms' equilibrium profits as specified in the proof of this proposition. We distinguish between two cases, according to the size of h.

Case A: $h \leq h^*$. Note that this is equivalent to $\widetilde{\delta} \leq \underline{\delta}$. Consider first $\delta \leq \underline{\delta}$ and observe that then $\pi_{L,H} = (\underline{p}_H \frac{q_L}{q_H} - c_L + h)\frac{1+\lambda}{2}$ and $\pi_{H,H} = (q_H - c_H + h)\frac{1-\lambda}{2} = (\underline{p}_H - c_H + h)\frac{1+\lambda}{2}$, so that $\pi_{L,H} \leq \pi_{H,H}$ if and only if $\widetilde{\delta} \leq \underline{\delta}$, hence (q_H, q_H) indeed constitutes an equilibrium. For $\delta \in (\underline{\delta}, \overline{\delta})$, we have that $\pi_{L,H} = (\underline{p}_H - \delta(q_H - q_L) - c_L + h)\frac{1+\lambda}{2}$, so that now $\pi_{L,H} \leq \pi_{H,H}$ if and only if $\delta \geq \widetilde{\delta}$, which is the case because by assumption $\delta > \underline{\delta}$ and $\underline{\delta} \geq \widetilde{\delta}$ (as follows from $h \leq h^*$). If $\delta \geq \overline{\delta}$, we have that $\pi_{L,H} = (q_L - c_L + h)\frac{1-\lambda}{2} < \pi_{H,H}$. To finally prove uniqueness, note that (L, L) cannot be an equilibrium as $\pi_{L,L} = (q_L - c_L + h)\frac{1-\lambda}{2} < (q_H - c_H + h)\frac{1-\lambda}{2} \leq \pi_{H,L}$, while we have already shown that deviations from (H, L) are (strictly) profitable for L if $h \leq h^*$ $(h < h^*)$.

Case B: $h > h^*$. Note that this is equivalent to $\widetilde{\delta} > \underline{\delta}$. Now for $\delta \in [0, \underline{\delta}]$ the same comparison as in Case A reveals that $\pi_{L,H} > \pi_{H,H}$, since $\widetilde{\delta} > \underline{\delta}$. Hence, as neither (H, H) nor (L, L) constitutes an equilibrium (see the last part of Case A for the latter), any symmetric equilibrium must be in mixed strategies and we obtain γ from indifference (as in the case where $\delta = 0$). When $\delta \in (\underline{\delta}, \widetilde{\delta})$, the same comparison as in Case A reveals again that $\pi_{L,H} > \pi_{H,H}$, since $\delta < \widetilde{\delta}$ by assumption. As again neither (H, H) nor (L, L) constitutes an equilibrium, γ is obtained from indifference. Finally, take $\delta \geq \widetilde{\delta}$. When in addition $\delta \in [\widetilde{\delta}, \overline{\delta})$, (H, H) constitutes an equilibrium, as we know from Case A that here $\pi_{L,H} \leq \pi_{H,H}$ due to $\delta \geq \widetilde{\delta}$. If $\delta \geq \overline{\delta}$, it holds that $\pi_{L,H} = (q_L - c_L + h)\frac{1-\lambda}{2} < \pi_{H,H}$, so (H, H) also constitutes an equilibrium. In both cases (H, H) is also the unique equilibrium, since again (L, L) can be ruled out and as we have already shown that in this case $\pi_{L,H} < \pi_{H,H}$. **Q.E.D.**

2. Omitted Analysis for a General Number of Firms

We extend our baseline analysis to $I > 2$ firms. The subsequent derivations provide both robustness and are potentially of independent theoretical interest.

With $I > 2$ firms, the consideration set of market-savvy consumers is larger. We show first that our simple choice rule, which allows for various interpretations, survives when we make the following specification: We suppose that for the construction of the reference point only non-dominated options are considered, excluding those where there exists another offer which has both (weakly) higher quality and (weakly) lower price (with one of these strict). Market-savvy consumers thus simplify or edit their potentially larger consideration set in this way. Still, there could be more than one equally priced high- or low-quality offers. Denote the respective numbers by $J_H \geq 1$ and $J_L \geq 1$, so that, with $J = J_L + J_H$, $Q = (J_H q_H + J_L q_L)/J$ and $P = (J_H p_H + J_L p_L)/J$. For a given low-quality product its (low) price (and not its (low) quality) is thus salient if $\frac{p_L}{P} < \frac{q_L}{Q}$. It is easy to confirm that this transforms again to the requirement that $\frac{p_L}{p_H} < \frac{q_L}{q_H}$ and that, in this case, price is also salient when consumers assess a high-quality product. Consequently, we indeed obtain the same choice criterion as with $I = 2$ firms.

Note also that in our simple context, market-savvy consumers still make the same choice when they compare products pairwise and select that with the highest "quality-per-dollar", $\frac{q_i}{p_i}$.

We next turn to pricing and product choice, noting again that for brevity's sake we omit the possibility of unshrouding ($t = 0.5$).

Again, we confine our characterization to the case where (2) holds, as otherwise only the subgame with only low-quality products arises, regardless of the size of h. Still, the number of subgames is now much larger (as any of the I firms could have high or low quality) and their analysis more complex. The latter is already evident when all firms have the same quality $q_i = q$, where we know from the literature that, despite symmetry, the pricing game has multiple equilibria (Baye et al. 1992). In light of our subsequent characterization of all other subgames, we pick the following pricing equilibrium in case of symmetry: Out of the presently considered I firms with the same quality q, $I - 2$ firms abstain from competition and choose the highest feasible price $p_i = q$, leaving two firms to compete actively (by pricing below q). Still, all (presently homogeneous) firms make the same profits of $\frac{1-\lambda}{I}(q-c+h)$, and the two firms that compete for savvy consumers choose $p_i \in [c-h+\frac{1-\lambda}{1-\lambda+\lambda I}(q-c+h), q]$ according to the CDF[5]

$$F(p_i) = 1 - \frac{1-\lambda}{\lambda I}\left(\frac{q-c+h}{p-c+h} - 1\right). \tag{8}$$

We can next extend this equilibrium characterization to other subgames, recalling that the size of shrouded charges determines whether low-quality or high-quality firms are more "competitive" when (2) holds. Intuitively, when from $h > \widetilde{h}$ low-quality firms are more competitive, the two firms that compete actively must be low-quality firms, provided that there are at least two such firms. When there is only one low-quality firm in the considered subgame, then this firm competes with a high-quality firm. When high-quality firms are more "competitive" as shrouded charges remain sufficiently low from $h \leq \widetilde{h}$, the picture is reversed. Importantly, when a low-quality firm and a high-quality firm compete, we can (largely) rely on our preceding characterization for $I = 2$ (and heterogeneous qualities), as then the characterization from Lemma 1 in the main text needs to be adjusted only with respect to the mass of non-savvy consumers that frequent either firm, i.e., from $\frac{1-\lambda}{2}$ down to $\frac{1-\lambda}{I}$ when $I > 2$. Proofs of all subsequent results with $I > 2$ firms are relegated to the end of this section. There, for completeness, we also characterize the pricing equilibrium where all firms with the same quality choose symmetric pricing strategies, which proves to be more involved. We note below that our subsequent result on product choice is however independent of the choice of pricing equilibrium, given that we show that profits are always pinned down uniquely.

Proposition 3 *Take the case with $I > 2$ firms and suppose that (2) holds. For the subgame in $t = 1$, denote the number of firms with high quality by I_H and that with low quality by I_L. Then there exists the following pricing equilibrium. First, when all I firms have the same quality $q \in \{q_H, q_L\}$, then $I - 2$ firms choose $p_i = q$ and two firms choose prices according to (8). Second, when there*

[5] This is derived formally in the proof of Proposition 3.

are different qualities in the market, the following case distinction applies, based on the maximum level of shrouded charges, h:
i) $h \leq \widetilde{h}$*: When at least* $I_H \geq 2$*, then all low-quality firms choose* $p_i = q_L$ *and* $I_H - 2$ *high-quality firms choose* $p_i = q_H$*, while two high-quality firms choose prices according to (8) (for* $q = q_H$ *and* $c = c_H$*). When* $I_H = 1$*, then* $I_L - 1$ *low-quality firms choose* $p_i = q_L$*, while one low-quality firm and the single high-quality firm choose prices as characterized in Case i) of Lemma 1 in the main text, adjusted for the measure of each firm's non-savvy consumers* $\frac{1-\lambda}{I}$*.*
ii) $h > \widetilde{h}$*: When at least* $I_L \geq 2$*, then all high-quality firms choose* $p_i = q_H$ *and* $I_L - 2$ *low-quality firms choose* $p_i = q_L$*, while two low-quality firms choose prices according to (8) (for* $q = q_L$ *and* $c = c_L$*). When* $I_L = 1$*, then* $I_H - 1$ *high-quality firms choose* $p_i = q_H$*, while one high-quality firm and the single low-quality firm choose prices as characterized in Case ii) of Lemma 1 in the main text, adjusted for the measure of each firm's non-savvy consumers* $\frac{1-\lambda}{I}$*.*

Based on this characterization, together with the preceding analysis for $I = 2$, we obtain the following analogous result to that in Proposition 1 from the main text.

Proposition 4 *Take the case with* $I > 2$ *firms and suppose that not all firms have the same quality in* $t = 1$*, while (2) holds (which will be a prerequisite for that different qualities are indeed offered in an equilibrium of the full game). Then the pricing equilibrium characterized in Proposition 3 exhibits the following comparative results:*
i) If the maximum shrouded charges are sufficiently large with $h > \widetilde{h}$*, in the resulting pricing equilibrium market-savvy consumers choose low quality with probability one (*$\sigma_L = 1$*) when there are at least two firms with low quality. Otherwise, when there is only one firm with low quality,* $I_L = 1$*, it still holds that* $\sigma_L > \sigma_H$ *and* σ_L *strictly increases in h.*
ii) If the maximum shrouded charges remain relatively small with $h \leq \widetilde{h}$*, market-savvy consumers choose high quality with probability one (*$\sigma_H = 1$*) when there are at least two firms with high quality. Otherwise, when there is only one firm with high quality,* $I_H = 1$*, it still holds that* $\sigma_H \geq \sigma_L$ *and* σ_H *strictly decreases in h.*

We next proceed to a characterization of product choice in $t = 0$. For this we show in the proof of Proposition 5 that, for all parameter constellations, firms' profits are uniquely determined for any given vector of firm qualities (q_i for $i = 1, \ldots, I$).[6] Intuitively, a firm always realizes exactly $\frac{1-\lambda}{I}(q_i - c_i + h)$ when there is at least one rival firm with the same quality. It also does not realize more when it has quality $q_i = q_H$ in case $h > \widetilde{h}$ and quality $q_i = q_L$ in case $h \leq \widetilde{h}$, irrespective of other firms' choices. Put differently, to realize strictly higher profits, the respective quality q_i must not be chosen as well by any other firm *and* it must give the firm a "competitive advantage" in attracting savvy consumers. This observation can then be used for the following characterization of the unique symmetric equilibrium in product choice.

Proposition 5 *Consider $I > 2$ firms' choice of products at $t = 0$. There is a unique symmetric equilibrium, where firms' probability of choosing high quality, $\gamma_i = \gamma$, is characterized as follows:*
i) If the converse of condition (2) holds, all firms always choose low quality ($\gamma = 0$).
ii) If (2) holds, then the size of firms' maximum shrouded charges h determines the provision of qualities. When $h \leq h_I^*$, with

$$h_I^* = \widetilde{h} + \frac{1-\lambda}{\lambda I}\frac{q_H}{q_H - q_L}[(q_H - c_H) - (q_L - c_L)],$$

only high quality is provided ($\gamma = 1$). When $h > h_I^*$, then both qualities are offered with positive probability, where $0 < \gamma < 1$ satisfies

$$\gamma = \sqrt[I-1]{\frac{1-\lambda}{\lambda I}\left[\frac{(q_H - c_H) - (q_L - c_L)}{\frac{q_L}{q_H}(c_H - h) - (c_L - h)}\right]},$$

which is strictly decreasing in h. Hence, also in this case low quality is provided with positive probability, provided that firms can shroud charges sufficiently, $h > h^*$, and this is more likely (γ lower) when h is still higher.

[6] This implies that the subsequent characterization for $t = 0$ applies independently of the choice of pricing equilibrium in $t = 1$.

This fully extends the characterization to the case with $I > 2$ firms. When firms can shroud charges to a larger extent, this unambiguously shifts both product provision and, also for given product provision, sales towards low quality.

To conclude, also the implications for welfare are then the same as with $I = 2$ firms. In addition, we can observe now that an inefficient provision of low quality (for the chosen criterion of consumer welfare), which obtains when $h > h^*$, becomes more likely (that is, γ decreases) when the number of firms I increases. This holds as then each individual firm can attract only a smaller fraction of non-savvy consumers, so that for each firm the (competitive) fraction of savvy consumers, who compare all offers, becomes more relevant.

Proof of Proposition 3. Our proof proceeds as follows. First, we consider an auxiliary setup in which only two firms (w.l.o.g. $i = 1, 2$, with $q_1 \geq q_2$) are in the market but where each has a non-savvy consumer fraction of size $\frac{1-\lambda}{I}$ (while there remains a mass λ of savvy consumers). For this auxiliary setup, we fully characterize the unique pricing equilibrium. Second, we build on our results from the auxiliary game and characterize an equilibrium of the full game with $I > 2$ firms. In this equilibrium, only two firms compete for savvy consumers, while all other firms set $p_i = q_i$.

Step 1: Auxiliary Setting. Although the described auxiliary setting is a straightforward extension from our main analysis with $I = 2$ firms and a non-savvy consumer "base" of $\frac{1-\lambda}{2}$, we provide a full characterization.

Lemma 1 *Consider a game with $I = 2$ firms, each of which has a non-savvy consumer fraction of mass $\frac{1-\lambda}{I}$. Then we have the following characterization:*

Case (i) If $q_1 = q_2 = q \in \{q_H, q_L\}$ and $c_1 = c_2 = c \in \{c_H, c_L\}$, in the unique pricing equilibrium firms $i = 1$ and 2 draw prices from the CDF

$$F(p) = 1 - \frac{1-\lambda}{\lambda I}\left(\frac{q-c+h}{p-c+h} - 1\right)$$

with support $[c - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q - c + h), q]$. Both firms make an expected profit of $\frac{1-\lambda}{I}(q - c + h)$.

Case (ii) If $q_1 = q_H$, $q_2 = q_L$, and $h < \widetilde{h}$, in the unique pricing equilibrium firm 1 draws prices from the CDF

$$F_1(p) = 1 - \frac{1-\lambda}{\lambda I}\left(\frac{q_H - p}{p - \frac{q_H}{q_L}(c_L - h)}\right)$$

over the support $[\frac{q_H}{q_L}\left(c_L - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_L - c_L + h)\right), q_H)$*, whereas firm 2 draws prices from the CDF*

$$F_2(p) = 1 - \frac{\frac{1-\lambda}{\lambda I}(q_L - p) + (c_L - h) - \frac{q_L}{q_H}(c_H - h)}{p - \frac{q_L}{q_H}(c_H - h)}$$

over the support $[c_L - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_L - c_L + h), q_L)$*, with a mass point of size* $m_L = \frac{c_L - h - \frac{q_L}{q_H}(c_H - h)}{q_L - \frac{q_L}{q_H}(c_H - h)}$ *at* $p = q_L$*. Firm 1 makes an expected profit of* $\frac{1-\lambda}{I}(q_H - c_H + h) + \lambda\left[\frac{q_H}{q_L}(c_L - h) - (c_H - h)\right]$*, while firm 2 makes an expected profit of* $\frac{1-\lambda}{I}(q_L - c_L + h)$*.*

Case (iii) If $q_1 = q_H$, $q_2 = q_L$, and $h > \widetilde{h}$, in the unique pricing equilibrium firm 1 draws prices from the CDF

$$F_1(p) = 1 - \frac{\frac{1-\lambda}{\lambda I}(q_H - p) + (c_H - h) - \frac{q_H}{q_L}(c_L - h)}{p - \frac{q_H}{q_L}(c_L - h)}$$

over the support $[c_H - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_H - c_H + h), q_H)$*, with a mass point of size* $m_H = \frac{c_H - h - \frac{q_H}{q_L}(c_L - h)}{q_H - \frac{q_H}{q_L}(c_L - h)}$ *at* $p = q_H$*. Firm 2 draws prices from the CDF*

$$F_2(p) = 1 - \frac{1-\lambda}{\lambda I}\left(\frac{q_L - p}{p - \frac{q_L}{q_H}(c_H - h)}\right)$$

over the support $[\frac{q_L}{q_H}\left(c_H - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_H - c_H + h)\right), q_L)$*. Firm 1 makes an expected profit of* $\frac{1-\lambda}{I}(q_H - c_H + h)$*, while firm 2 makes an expected profit of* $\frac{1-\lambda}{I}(q_L - c_L + h) + \lambda\left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right]$*.*

Proof of Lemma 1. Case (i) is standard, see e.g. Baye et al. (1992) for a detailed proof. Cases (ii) and (iii) follow directly from our proof of Lemma 1 from the main text, with the slight change that firms' non-savvy consumer base is $\frac{1-\lambda}{I}$ instead of $\frac{1-\lambda}{2}$. **Q.E.D.**

Step 2: Full Game with $I > 2$ Firms.
We now reintroduce all $I > 2$ firms. Building on Cases (i)-(iii) of Lemma 1 above, we will show that the characterization in Proposition 3 indeed constitutes an equilibrium. For this, we again split up the analysis into the three corresponding cases.

Case (i): Homogeneous qualities. The claim is now that it constitutes an equilibrium that $I - 2$ firm price at $p_i = q$, while two firms sample prices from a CDF as given by equation (8). To show this, we proceed in two steps. First, it is clear from Lemma 1 that the two randomizing firms do not have a profitable deviation. Second, we check whether the non-randomizing firms might have a profitable deviation. Clearly, this cannot be the case, since even when only competing against one (and not both) randomizing firms, by construction each price in the interval $[c-h+\frac{1-\lambda}{1-\lambda+\lambda I}(q-c+h), q]$ yields the same expected profit of $\frac{1-\lambda}{I}(q-c+h)$ (while prices below that interval yield a strictly lower profit, as savvy consumers are already attracted for sure at the lower bound). Since a deviating firm's expected profit is weakly lower when competing against both randomizing rivals (and strictly so when not pricing at the lower bound), the non-randomizing firms do not have a profitable deviation.

Case (ii): Heterogeneous qualities with $h \leq \widetilde{h}$.
Subcase (iia): Two or more firms with $q_i = q_H$. The claim is now that it constitutes an equilibrium that all firms with quality q_L choose $p_i = q_L$, two firms with quality q_H sample prices from a CDF as given by equation (8), while all other firms with quality q_H choose $p_i = q_H$. Note first that in this candidate equilibrium, only the better of the two randomizing high-quality firms' offers will be considered by savvy consumers, which compare it to all low-quality firms' offers of $p_i = q_L$. It is then easily confirmed that quality will always be salient and savvy consumers will always purchase at the lowest-priced randomizing

high-quality firm. Given this, it is clear from Lemma 1 that the two randomizing high-quality firms do not have a profitable deviation.

We now check that neither non-randomizing high-quality firms, nor low-quality firms may have a profitable deviation. The former is clear by construction, see Case (i) above. For the latter, note first that a deviating low-quality firm always beats all other low-quality firms' offers. It thus attracts savvy consumers if it renders price salient, which, when pricing at p_L, is the case if $\frac{p_L}{q_L} < \frac{p_H^{min}}{q_H}$, where p_H^{min} is the minimum price offered by high-quality firms. Suppose now that a deviating low-quality firm only needed to beat one (and not both) randomizing high-quality firms, which clearly gives an upper bound on such a firms deviation profit. We can then write its expected profit as

$$\begin{aligned}\pi_L(p_L) &= (p_L - c_L + h)\left[\frac{1-\lambda}{I} + \lambda\Big(1 - F\big(p_L\frac{q_H}{q_L}\big)\Big)\right] \\ &= \frac{1-\lambda}{I}(q_H - c_H + h)\left(\frac{p_L - c_L + h}{p_L\frac{q_H}{q_L} - c_H + h}\right).\end{aligned}$$

It is easy to check that this expression is maximized for $p_L = q_L$ given that $h \leq \widetilde{h}$, for a profit of $\frac{1-\lambda}{I}(q_L - c_L + h)$ that is not higher than in the candidate equilibrium. Hence, no firm has a profitable deviation.

Subcase (iib): One firm with $q_i = q_H$. The claim is now that it constitutes an equilibrium that the single firm with quality q_H and one firm with quality q_L draw prices randomly from CDFs as characterized in Case (ii) of Lemma 1 above. All other firms with low quality price at $p_i = q_L$. To see this, note first that it follows immediately from Lemma 1 that neither the firm with quality q_H, nor the single randomizing firm with quality q_L have a profitable deviation. Second, it is clear that no low-quality firm pricing at q_L has a profitable deviation, as it would have to beat the high-quality firm's offer (providing a higher quality-per-price ratio) and price lower than the single randomizing low-quality firm. But even without needing to beat the randomizing low-quality rival, by construction a (deviating) low-quality firm would be indifferent between pricing at q_L or any lower price down to the level where savvy consumers are attracted for sure. Hence, no firm has a profitable deviation.

Case (iii): Heterogeneous qualities with $h > \widetilde{h}$.
Subcase (iiia): Two or more firms with $q_i = q_L$. The claim is now that it constitutes an equilibrium that all firms with quality q_H choose $p_i = q_H$, two firms with quality q_L sample prices from a CDF as given by equation (8), while all other firms with quality q_L choose $p_i = q_L$. Note first that in this candidate equilibrium, only the better of the two randomizing low-quality firms' offers will be considered by savvy consumers, which compare it to all high-quality firms' offers of $p_i = q_H$. It is easily confirmed that the price-per-quality ratio for all high-quality firms is always lower and savvy consumers will thus always purchase at the lowest-priced randomizing low-quality firm. Given this, it is clear from Lemma 1 that the two randomizing low-quality firms do not have a profitable deviation.

We now check that neither non-randomizing low-quality firms, nor high-quality firms may have a profitable deviation. The former is clear by construction, see Case (i) above. For the latter, note first that a deviating high-quality firm always beats all other high-quality firms' offers. It thus attracts savvy consumers if, when pricing at p_H, $\frac{q_H}{p_H} > \frac{q_L}{p_L^{min}}$, where p_L^{min} is the minimum price offered by low-quality firms.

Suppose now that a deviating high-quality firm only needed to beat one (and not both) randomizing low-quality firms, which clearly gives an upper bound on such a firms deviation profit. We can then write its expected profit as

$$\begin{aligned}\pi_H(p_H) &= (p_H - c_H + h)\left[\frac{1-\lambda}{I} + \lambda\Big(1 - F\big(p_H \frac{q_L}{q_H}\big)\Big)\right] \\ &= \frac{1-\lambda}{I}(q_L - c_L + h)\left(\frac{p_H - c_H + h}{p_H \frac{q_L}{q_H} - c_L + h}\right).\end{aligned}$$

It is easy to check that this expression is maximized for $p_H = q_H$ given that $h > \widetilde{h}$, for a profit of $\frac{1-\lambda}{I}(q_H - c_H + h)$ that is not higher than in the candidate equilibrium. Hence, no firm has a profitable deviation.

Subcase (iiib): One firm with $q_i = q_L$. The claim is now that it constitutes an equilibrium that the single firm with quality q_L and one firm with quality q_H draw prices randomly from CDFs as characterized in Case (iii) of Lemma 1 above. All other firms with high quality price at $p_i = q_H$. To see this, note first that it follows immediately from Lemma 1 that neither the firm with quality q_L, nor the single randomizing firm with quality q_H have a profitable deviation. Second, it is clear that no high-quality firm pricing at q_H has a profitable deviation, as it would have to beat the low-quality firm's offer and also price lower than the single randomizing high-quality firm. But even without needing to beat the randomizing high-quality rival, by construction a (deviating) high-quality firm would be indifferent between pricing at q_H or any lower price down to the level where savvy consumers are attracted for sure. Hence, no firm has a profitable deviation.

This completes the proof of Proposition 3. **Q.E.D.**

Proof of Corollary 4. *Part (i):* Given the characterized equilibria for heterogeneous qualities and $h > \widetilde{h}$ (compare with Proposition 3 and its proof), it is first easy to check that q_i/p_i will be highest for a low-quality firm if two or more firms have quality q_L. Hence, in this case savvy consumers surely choose low quality. If there is just one firm with low quality, an analogous proof as for $I = 2$ in the main text, adjusted for firms' modified share of non-savvy consumers, reveals that savvy consumers' probability of choosing low quality, σ_L, can now be written as

$$\sigma_L = 1 - \int_0^1 \frac{k}{1 + \frac{1-\lambda+I\lambda(1-k)}{1-\lambda}\left[\frac{c_H - h - \frac{q_H}{q_L}(c_L - h)}{q_H - (c_H - h)}\right]} dk.$$

Since $\frac{c_H - h - \frac{q_H}{q_L}(c_L - h)}{q_H - (c_H - h)}$ is strictly increasing in h given that (2) holds, it again follows that σ_L is strictly increasing in h. Moreover, one can easily check that $\sigma_L = 1/2$ for $h \downarrow \widetilde{h}$.

Part (ii): Again, given the characterized equilibria for heterogeneous qualities and $h \leq \widetilde{h}$ (compare once more with Proposition 3 and its proof), it can easily be verified that q_i/p_i is highest for a high-quality firm if two or more firms have quality q_H. Hence, in this case savvy consumers surely choose high quality.

If there is just one firm with high quality, an analogous proof as for $I = 2$ in the main text, adjusted for firms' modified share of non-savvy consumers, reveals that σ_L can now be written as

$$\sigma_L = \int_0^1 \frac{k}{1 + \frac{1-\lambda+I\lambda(1-k)}{1-\lambda}\left[\frac{c_L-h-\frac{q_L}{q_H}(c_H-h)}{q_L-(c_L-h)}\right]} dk.$$

Since $\frac{c_L-h-\frac{q_L}{q_H}(c_H-h)}{q_L-(c_L-h)}$ is strictly decreasing in h given that (2) holds, it again follows that σ_L is strictly increasing in h. Finally, one can easily check that $\sigma_L = 1/2$ for $h \uparrow \widetilde{h}$. **Q.E.D.**

Proof of Proposition 5. As noted above, observe first that the statement of the proposition is independent of the subsequently chosen pricing equilibrium (at $t = 2$) and therefore does not rely on the specific characterization in Proposition 3. We now first derive the unique equilibrium profits for any given choice of qualities in $t = 1$.

Step 1: Auxiliary Lemma.

Lemma 2 *Given $I \geq 2$ firms, out of which $I_H \geq 0$ have chosen high quality and $I_L \geq 0$ have chosen low quality, any pricing equilibrium must give rise to the following unique equilibrium profits:*
(i) If firms' qualities are homogeneous, such that $I_H = I$ and $I_L = 0$, or $I_L = I$ and $I_H = 0$, each firm makes a profit of $\frac{1-\lambda}{I}(q_i - c_i + h)$.
(ii) If qualities differ and if $h \leq \widetilde{h}$, then

(iia) if $I_H = 1$ and $I_L = I - 1$, the single firm with quality q_H makes a profit of $\frac{1-\lambda}{I}(q_H - c_H + h) + \lambda\left[\frac{q_H}{q_L}(c_L - h) - (c_H - h)\right]$, while all other (low-quality) firms make a profit of $\frac{1-\lambda}{I}(q_L - c_L + h)$.

(iib) If instead $I_H \geq 2$, each firm makes a profit of $\frac{1-\lambda}{I}(q_i - c_i + h)$.
(iii) If $h > \widetilde{h}$, then

(iiia) if qualities differ and if $I_L = 1$ and $I_H = I-1$, the single firm with quality q_L makes a profit of $\frac{1-\lambda}{I}(q_L - c_L + h) + \lambda\left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right]$, while all other (high-quality) firms make a profit of $\frac{1-\lambda}{I}(q_H - c_H + h)$.

(iiib) If instead $I_L \geq 2$, each firm makes a profit $\frac{1-\lambda}{I}(q_i - c_i + h)$.

Proof of Lemma 2. We deal with cases (i) - (iii) in turn.
(i) This follows directly from Baye et al. (1992).
(iia) We prove this in four steps. Note however first that we use throughout once again that a lower bound on profits is always $\frac{1-\lambda}{I}(q_i - c_i + h)$.
Assertion 1: $\pi_L = \frac{1-\lambda}{I}(q_L - c_L + h)$.
Proof of Assertion (i): We thus only need to contradict that there is at least one low-quality firm with profits $\pi'_L > \pi_L$. Denote this firm by L. Then, if L's upper pricing bound is given by $\overline{p}_L \leq q_L$, L must attract savvy consumers with positive probability when pricing at $\overline{p}_L$. It follows that the single firm with quality q_H must have positive probability mass at or above $\overline{p}_L\frac{q_H}{q_L} \leq q_H$, which further implies that this firm's equilibrium profit is bounded above by $\overline{\pi}_H = (q_H - c_H + h)\frac{1-\lambda}{I}$ (this is true in particular since it cannot be the case that both L has a mass point at $\overline{p}_L$ and the high-quality firm has a mass point at $\overline{p}_L\frac{q_H}{q_L}$). Note now that since every low-quality firm can guarantee at least π_L, the lowest price solves $(p_L - c_L + h)\left(\frac{1-\lambda}{I} + \lambda\right) = \pi_L$, that is $p_L \geq \underline{p}_L = c_L - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_L - c_L + h)$ for all firms with q_L. But then, by pricing marginally below $\frac{q_H}{q_L}\underline{p}_L$, firm H could guarantee to attract all savvy consumers and make a profit that is arbitrarily close to $\left(\frac{q_H}{q_L}\underline{p}_L - c_H + h\right)\left(\frac{1-\lambda}{I} + \lambda\right) = \frac{1-\lambda}{I}(q_H - c_H + h) + \lambda\left[\frac{q_H}{q_L}(c_L - h) - (c_H - h)\right] > \overline{\pi}_H$, where the last inequality is due to $h < \widetilde{h}$. Thus we obtain a contradiction. **Q.E.D.**

Assertion 2: At least one low-quality firm's lower support bound must be given by $c_L - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_L - c_L + h) = \underline{p}_L$.
Proof of Assertion 2: Clearly, no low-quality firm's lower support bound may lie below $\underline{p}_L$ (see the proof of Assertion 1 above). We now show that not all low-quality firms' lower support bounds may lie strictly above $\underline{p}_L$. To see this, suppose they did, such that $p_L \geq \underline{p}'_L > \underline{p}_L$ for all firms with q_L. Then it is obvious that the single high-quality firm would never price below $\underline{p}'_L\frac{q_H}{q_L}$ in

the respective candidate equilibrium, as this price already beats all low-quality firms' offers. But then any low-quality firm could achieve a profit that strictly exceeds $(q_L - c_L + h)\frac{1-\lambda}{I}$ by pricing arbitrarily close below $\underline{p}'_L$, which contradicts Assertion 1 above. **Q.E.D.**

Assertion 3: The union of all low-quality firms' supports is $[\underline{p}_L, q_L]$.
Proof of Assertion 3: We know from Assertion 2 that at least one low-quality firm must price down to $\underline{p}_L$, while from Assertion 1 it is clear that no low-quality firm may ever price below $\underline{p}_L$. What remains to show is that the union of all low-quality firms' supports does not contain a hole somewhere in $(\underline{p}_L, q_L]$. Suppose to the contrary that this was the case, such that no low-quality firm spreads any probability mass on some interval $(a, b) \subset (\underline{p}_L, q_L]$. Then clearly, the single high-quality firm would not put any probability mass in the interval $(\frac{q_H}{q_L}a, \frac{q_H}{q_L}b)$. In turn, this implies that the low-quality firm which samples the highest price below a in equilibrium (and there must exist such a firm due to Assertion 2) would have a strictly profitable deviation to pricing at b. **Q.E.D.**

Assertion 4: $\pi_H = \frac{1-\lambda}{I}(q_H - c_H + h) + \lambda \left[\frac{q_H}{q_L}(c_L - h) - (c_H - h)\right]$.
Proof of Assertion 4: From Assertion 1 we know any low-quality firm's equilibrium profit π_L. Hence, such firms' pricing is bounded below by the value of p_L that solves $(p_L - c_L + h)\left(\frac{1-\lambda}{I} + \lambda\right) = \pi_L$, i.e. $p_L \geq \underline{p}_L = c_L - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_L - c_L + h)$. Thus, by pricing arbitrarily close below $\underline{p}_L \frac{q_H}{q_L}$, the high-quality firm can guarantee to make a profit of that is arbitrarily close to $(\underline{p}_L \frac{q_H}{q_L} - c_H + h)\left(\frac{1-\lambda}{I} + \lambda\right) = \pi_H$. We proceed to show that the high-quality firm cannot make a higher expected profit in equilibrium. Suppose it did, such that $\pi'_H > \pi_H$. In turn, the lowest price the high-quality firm may ever charge in the respective candidate equilibrium strictly exceeds $\underline{p}_L \frac{q_H}{q_L}$. Denote this price by $\underline{p}'_H > \underline{p}_L \frac{q_H}{q_L}$. From Assertion 3 above, we now know that there must exist at least one low-quality firm, say L, whose pricing support contains the price $p_L = \underline{p}'_H \frac{q_L}{q_H} > \underline{p}_L$. Since by Assertion 1, firm L makes an expected profit of $\frac{1-\lambda}{I}(q_L - c_L + h)$ for each price in its support, we can calculate L's expected demand at $\underline{p}'_H \frac{q_L}{q_H}$, which is given by $\frac{1-\lambda}{I}\left(\frac{q_L - c_L + h}{\underline{p}'_H \frac{q_L}{q_H} - c_L + h}\right)$. Note moreover that at $p_L = \underline{p}'_H \frac{q_L}{q_H}$, L may only lose

savvy consumers to other low-quality firms, and not to firm H. Instead, when firm H prices at $\underline{p}'_H$, it may also lose the savvy consumers to firm L. Hence it follows that the expected demand of H at $\underline{p}'_H$ does not exceed that of L at $\underline{p}'_H$, such that, when we now write this as a function of $\underline{p}'_H$, we have

$$\pi_H(\underline{p}'_H) \leq (\underline{p}'_H - c_H + h)\frac{1-\lambda}{I}\left(\frac{q_L - c_L + h}{\underline{p}'_H \frac{q_L}{q_H} - c_L + h}\right).$$

It is now straightforward to derive that the expression on the RHS of the above inequality is (strictly) decreasing in $\underline{p}'_H$ for all $h \leq \widetilde{h}$ ($h < \widetilde{h}$), with $\pi_H(\underline{p}_H) = \pi_H$ (where the latter follows from $\frac{1-\lambda}{I}(q_L - c_L + h) = \left(\frac{1-\lambda}{I} + \lambda\right)(\underline{p}_L - c_L + h)$ and $\underline{p}_H = \underline{p}_L \frac{q_H}{q_L}$). Hence we obtain a contradiction. **Q.E.D.**

(iib) We prove this in two steps:
Assertion 1: $\pi_H = \frac{1-\lambda}{I}(q_H - c_H + h)$.
Proof of Assertion 1: It remains to show that this is also an upper bound. To argue to a contradiction, suppose thus that at least one high-quality firm has profits of $\pi'_H > \pi_H$ in equilibrium. Denote this firm by H. Then, if H's upper pricing bound is denoted by $\overline{p}_H \leq q_H$, it must attract savvy consumers with positive probability when pricing at $\overline{p}_H$. Hence, all other high-quality firms must have positive probability mass at or above $\overline{p}_H$, while all low-quality firms must have positive probability mass at or above $\overline{p}_H \frac{q_L}{q_H}$. This in turn implies that all firms other than H, and in particular all other firms with high quality, can at most make their maximum profit with non-savvy consumers $\frac{1-\lambda}{I}(q_i - c_i + h)$ (this is so because they certainly lose the savvy consumers to firm H when pricing in the aforementioned upper part of their pricing support). But then each other firm with quality q_H would have a profitable deviation when pricing at or marginally below firm H's lower support bound, as this realizes $\pi'_H > \pi_H$. **Q.E.D.**

Assertion 2: $\pi_L = \frac{1-\lambda}{I}(q_L - c_L + h)$.
Proof of Assertion 2: Analogous to the proof of Assertion 1 above. **Q.E.D.**

The proof of cases (iiia) and (iiib) is analogous to the proof of cases (iia) and (iib) above. This completes the proof of Lemma 2. **Q.E.D.**

We now proceed with the proof of Proposition 5, building on the profits derived in Lemma 2. When $q_L - c_L \geq q_H - c_H$, which implies $\widetilde{h} < 0$ and therefore $h > \widetilde{h}$ for all $h \geq 0$, choosing q_L realizes $\frac{1-\lambda}{I}(q_H - c_H + h)$, while choosing q_L yields at least $\frac{1-\lambda}{I}(q_L - c_L + h)$. Hence, a choice of low quality by all firms is the unique equilibrium outcome. If $q_H - c_H > q_L - c_L$ and $h \leq \widetilde{h}$, choosing q_L realizes $\frac{1-\lambda}{I}(q_L - c_L + h)$, while choosing q_H yields at least $\frac{1-\lambda}{I}(q_H - c_H + h)$. Hence, a choice of high quality by all firms is the unique equilibrium outcome. Suppose finally that $q_H - c_H > q_L - c_L$ and $h > \widetilde{h}$, where we know that firm i realizes $(q_i - c_i + h)\frac{1-\lambda}{I}$ *unless* it is the unique firm with q_L. Direct comparison of the respective profits yields that the latter is higher if and only if $h \leq h_I^*$, where $h_I^* > \widetilde{h}$. This implies that for $h \leq h_I^*$, $q_i = q_H$ for all i indeed constitutes an equilibrium, as no firm has an incentive to deviate, and it is the unique equilibrium outcome when $h < h_I^*$. Finally, for $h > h_I^*$ no high-quality equilibrium exists and also no low-quality equilibrium as the respective profit of $(q_L - c_L + h)\frac{1-\lambda}{I}$ is strictly below that from deviating to H, $(q_H - c_H + h)\frac{1-\lambda}{I}$. The probability $\gamma \in (0,1)$ that characterizes the unique symmetric equilibrium is then obtained from equating $(q_H - c_H + h)\frac{1-\lambda}{I}$ for the choice of high quality with

$$(q_L - c_L + h)\frac{1-\lambda}{I} + \gamma^{I-1}\lambda\left[\frac{q_L}{q_H}(c_H - h) - (c_L - h)\right]$$

for the choice of low quality. This concludes the proof of Proposition 5. **Q.E.D.**

Recall finally that with I firms we chose a pricing equilibrium in which only two firms actively competed for savvy consumers. This allowed us to directly rely on our previous characterization with two firms. Note however as well that we showed for Proposition 5 that regardless of the choice of pricing equilibrium, in any subgame starting at $t = 1$ profits are uniquely determined. Hence, the characterization of the equilibrium in $t = 0$ did not rely on the chosen price equilibrium.

We now characterize as well the unique pricing equilibrium where all firms that offer the same quality choose the same (symmetric) pricing strategy, that is $F_L(p_L)$ or $F_H(p_H)$. For brevity, we focus on the interesting case where (2) holds and $h > \widetilde{h}$, which, by Proposition 5, are necessary conditions in order to reach subgames with heterogeneous product qualities. A proof of the following result can be obtained from the authors upon request. The analytical tractability of our model even with $I > 2$ thus does not rest on our particular choice of the pricing equilibrium.

Lemma 3 *Suppose that (2) holds and $h > \widetilde{h}$, and consider a pricing subgame in which I_H firms have chosen q_H and I_L firms have chosen q_L, with $I = I_H + I_L \geq 2$. Then there are two cases.*
(i) If $I_L \geq 2$, then for any $I_H \geq 0$ the following constitutes a symmetric pricing equilibrium: All firms with quality q_H set $p_H = q_H$ deterministically and all firms with quality $q_i = q_L$ sample prices randomly and without mass points from the support $[\underline{p}_L, q_L)$, where $\underline{p}_L = c_L - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_L - c_L + h)$, following the CDF

$$F_L(p) = 1 - \sqrt[I_L-1]{\frac{1-\lambda}{\lambda I}\left(\frac{q_L - c_L + h}{p - c_L + h} - 1\right)}.$$

(ii) If $I_L = 1$, then for any $I_H \geq 1$ the following constitutes a pricing equilibrium: All firms with quality q_H sample prices randomly from the support $[\underline{p}_H, q_H)$, where $\underline{p}_H = c_H - h + \frac{1-\lambda}{1-\lambda+\lambda I}(q_H - c_H + h)$, following the CDF

$$F_H(p) = 1 - \sqrt[I_H]{\frac{1-\lambda}{\lambda I}\left[\frac{q_L - c_L + h}{p\frac{q_L}{q_H} - c_L + h} - 1\right] + \frac{\frac{q_L}{q_H}(c_H - h) - (c_L - h)}{p\frac{q_L}{q_H} - c_L + h}}.$$

Each of these firms has a mass point of size

$$m_H = \sqrt[I_H]{\frac{\frac{q_L}{q_H}(c_H - h) - (c_L - h)}{q_L - c_L + h}}$$

at $p_H = q_H$*. The single firm with quality* $q_i = q_L$ *samples prices randomly and without mass points from the support* $[\underline{p}_H \frac{q_L}{q_H}, q_L)$ *following the CDF*

$$F_L(p) = 1 - \frac{\frac{1-\lambda}{\lambda I}\left(\frac{q_H - c_H + h}{p\frac{q_H}{q_L} - c_H + h} - 1\right)}{\left[\frac{1-\lambda}{\lambda I}\left(\frac{q_L - c_L + h}{p - c_L + h} - 1\right) + \frac{\frac{q_L}{q_H}(c_H - h) - (c_L - h)}{p - c_L + h}\right]^{\frac{I_H - 1}{I_H}}}.$$

3. Model where Only Some Consumers are Salient or Relative Thinkers

So far, we stipulated the presence of both savvy and non-savvy consumers in the market, where the former "shop" and compare different offers. However, all consumers shared the same proclivity to salient thinking. As we outlined in the main text, this approach has various benefits. For instance, for empirical researchers the fraction of consumers who actively search (and possibly switch over time) may be observable (e.g., from traditional homescan data or from online tracking data), while an individual consumer's proclivity to salient thinking may not be observable (and thus also not the respective fractions in the market, in case consumers should differ in this respect). Recall also that in our extension for a generalized importance of salience, based on the observed difference in savvy and non-savvy consumers' quality choice, we derived a measure that is indicative of δ. We also conducted various comparative analyses with respect to the fraction of savvy ("shopping") consumers, which competition authorities may take as an indicator for a well-functioning market.

Still, as this may be of independent interest, in this section we offer some insights into the analysis of the following variant of our model. We now suppose that *all* consumers are savvy to all offers ($\lambda = 1$). However only the fraction $\theta \in (0, 1)$ of consumers is now prone to relative or salient thinking. We analyze the case with two firms, one having high and the other low quality. To guarantee that the prices set are strictly positive, we moreover assume that there is a minimum level of consumer protection such that $h < c_L$. We refer to this setting as the "*salient/rational model*". For this model, we can first establish the existence of a unique pure-strategy equilibrium (in non-dominated strategies) whenever one of the two products has a sufficiently large competitive advantage.

Proposition 6 *Consider the salient/rational model. For the following parameters there exists a unique pricing equilibrium (when no firm chooses dominated strategies):*

(A) $q_H - c_H < q_L - c_L$ *and* $\theta \leq \frac{(q_L - c_L) - (q_H - c_H)}{\frac{q_L}{q_H}(c_H - h) - (c_L - h)} \in (0, 1)$*:* $p_H = c_H - h$ *and* $p_L = c_H - h - (q_H - q_L)$*, so that firms realize profits* $\pi_H = 0$ *and* $\pi_L =$

$(q_L - c_L) - (q_H - c_H)$*, while all consumers purchase the low-quality product.*

(B) $q_H - c_H > q_L - c_L$*,* $h < \widetilde{h}$ *and* $\theta \geq 1 - \frac{\frac{q_H}{q_L}(c_L - h) - (c_H - h)}{(q_H - c_H) - (q_L - c_L)} \in (0, 1)$*:* $p_L = c_L - h$ *and* $p_H = \frac{q_H}{q_L}(c_L - h)$*, so that firms realize profits* $\pi_L = 0$ *and* $\pi_H = \frac{q_H}{q_L}(c_L - h) - c_H + h$*, while all consumers purchase the high-quality product.*

We omit a proof of Proposition 6, which follows from standard arguments. Obviously, in these two cases (A and B), a marginal change in h has no impact on the fraction of high- or low-quality products that are purchased. Observe also that, albeit we do not state this formally, when product choice was endogenous, at least one firm would choose the efficient product (given our definition of consumer welfare), so that also with endogenous product choice only the efficient product would ultimately be purchased. But note already that whether Case B applies when $q_H - c_H > q_L - c_L$ (i.e., when condition (2) from the main text holds) depends on the level of shrouded charges, h: When these exceed the threshold $\widetilde{h}$, then there no longer exists a pure-strategy equilibrium where all consumers purchase for sure the high-quality product (the same is true even if $h \leq \widetilde{h}$, but the fraction of salient thinkers, θ, is sufficiently *low*[7]). Without having yet characterized a pricing equilibrium for this case, the following observation is immediate, once we note that in the presently considered salient/rational model both firms earn zero profits when they choose the same quality: With condition (2) but when h is high, so that the conditions of Case (B) do not hold, there no longer exists a pure-strategy equilibrium in product choice, and also in the equilibrium of the full game, including product choice, both high and low quality will be chosen with positive probability. Clearly, this would not be the case if all consumers were rational thinkers.

We now turn to the cases that are not covered by Cases (A) and (B) in Proposition 6. We however *do not* conduct a full analysis. In fact, our preliminary analysis already shows that, first, various cases need to be considered and that, second, we can not obtain an explicit characterization, which contrasts

[7] In this case, when the low-quality firm prices at its effective marginal cost, $p_L = c_L - h$, the high-quality rival strictly prefers to price at $p_L + (q_H - q_L) > \frac{q_H}{q_L} p_L$ and forgo consumption from the small share of salient thinkers. Because of this, both firms are active in equilibrium.

with our analysis in the main text. Next to revealing this complexity and the limitations in analytical tractability, by means of a numerical example, we illustrate as well the interaction between shrouding and salience in this model variant.

Before we proceed to deriving conditions for a characterization of a candidate equilibrium, when Cases (A) and (B) do not apply, we can relate this model variant as well as the one solved in the main text more closely to two seminal approaches in the literature. The model in the main text uses Varian (1980)'s seminal approach. Both for its intuitive underpinning and its tractability, this approach has received widespread application in the literature. This is arguably much less so for the approach pioneered by Shilony (1977), which also gives rise to mixed-strategy equilibria. The latter model resembles, both in its features and in the characterization of an equilibrium, much the model in this section: Also there, all consumers are free to observe and choose all offers without additional costs, but different (discrete) consumer groups differ in preferences; and, as in our subsequent characterization, this gives rise to mixed strategies that are piecewise defined, but that do not lend themselves to an immediate characterization (an explicit characterization is only obtained in case of symmetry).

We now provide a characterization of firms' mixed strategies in a candidate equilibrium and then summarize the implications that follow from this.

Firm L:

(i) Firm L samples prices continuously and without mass points over some range $[\underline{p}_L, \hat{p}_L]$ following a CDF $F_{L1}(p_L)$, while it samples prices continuously and without mass points over some range $[\hat{p}_L, \overline{p}_L]$ following a CDF $F_{L2}(p_L)$.

(ii) When sampling a price from F_{L1}, firm L attracts the fraction θ of salient thinkers deterministically, while it attracts the fraction $1-\theta$ of rational consumers with strictly positive probability. When sampling a price from F_{L2}, firm L does not receive any demand from rational consumers, while it attracts the salient thinkers with strictly positive probability (which is less than 1 for

$p_L > \hat{p}_L$). At $p_L = \hat{p}_L$, firm L attracts the salient thinkers with probability 1, while it cannot attract rational consumers.

Firm H:

(iii) Firm H samples prices continuously and without mass points over some range $[\underline{p}_H, \hat{p}_H]$ following a CDF $F_{H1}(p_H)$, while it samples prices continuously and without mass points over some range $[\hat{p}_H, \overline{p}_H]$ following a CDF $F_{H2}(p_H)$.

(iv) When sampling a price from F_{H1}, firm H attracts the fraction $1-\theta$ of rational consumers deterministically, while it attracts the fraction θ of salient thinkers with strictly positive probability. When sampling a price from F_{H2}, firm H does not receive any demand from salient thinkers, while it attracts the rational consumers with strictly positive probability (which is less than 1 for $p_H > \hat{p}_H$). At $p_H = \hat{p}_H$, firm H attracts the rational consumers with probability 1, while it cannot attract salient thinkers.

This characterization obviously contains various objects, such as the boundaries of the respective supports, which must be determined jointly. Again, the CDFs must be such that they ensure indifference by the rival. The tractability of Varian (1980)'s model, as well as ours in the main text, follows from the fact that one can relatively simply pin down equilibrium profits (which apply for *any* pricing equilibrium), which can then be plugged into these indifference conditions. This is now no longer the case, as we have no such equilibrium profits to start with. We next derive some implications that follow from the preceding characterization and optimality.

The characterization for firm L implies the following. First, since L deterministically attracts all salient thinkers if and only if it prices weakly below $\hat{p}_L$, it must hold that $\underline{p}_H = \hat{p}_L \frac{q_H}{q_L}$. Second, since L attracts rational consumers with positive probability if and only it prices strictly below $\hat{p}_L$, it must hold that $\overline{p}_H = \hat{p}_L + (q_H - q_L)$. Third, since L deterministically attracts all salient thinkers but cannot attract rational thinkers when pricing at $\hat{p}_L$, it must hold that $\pi_L = (\hat{p}_L - c_L + h)\theta$. Fourth, since L must be indifferent across all prices in the interval $[\underline{p}_L, \hat{p}_L]$, for this interval it must hold that

$$\pi_L(p_L) = (p_L - c_L + h)[\theta + (1-\theta)\mathbb{P}\{\tilde{p}_H > p_L + (q_H - q_L)\}] \stackrel{!}{=} \pi_L.$$

Denoting firm H's CDF when pricing above $\underline{p}_L + (q_H - q_L)$ by F_{H2}, the above condition can be rewritten as

$$\pi_L(p_L) = (p_L - c_L + h)[\theta + (1-\theta)(1 - F_{H2}(p_L + (q_H - q_L)))] \stackrel{!}{=} \pi_L,$$

which, after substituting $p_H = p_L + (q_H - q_L)$, implies

$$F_{H2}(p_H) = 1 - \frac{\frac{\pi_L}{p_H - (q_H - q_L) - c_L + h} - \theta}{1 - \theta}.$$

Fifth, since L must be indifferent across all prices in the interval $[\hat{p}_L, \overline{p}_L]$, for this interval it must hold that

$$\pi_L(p_L) = (p_L - c_L + h)\theta\mathbb{P}\{\tilde{p}_H > p_L \frac{q_H}{q_L}\} \stackrel{!}{=} \pi_L.$$

Denoting firm H's CDF when pricing below $\overline{p}_L \frac{q_L}{q_H}$ by F_{H1}, the above condition can be rewritten as

$$\pi_L(p_L) = (p_L - c_L + h)\theta(1 - F_{H1}(p_L \frac{q_H}{q_L})) \stackrel{!}{=} \pi_L,$$

which, after substituting $p_H = p_L \frac{q_H}{q_L}$, implies

$$F_{H1}(p_H) = 1 - \frac{\pi_L}{(p_H \frac{q_L}{q_H} - c_L + h)\theta}.$$

We turn next to analogous implications from the characterization for firm L. First, since H deterministically attracts all rational consumers if and only if it prices weakly below $\hat{p}_H$, it must hold that $\underline{p}_L = \hat{p}_H - (q_H - q_L)$. Second, since H attracts salient thinkers with positive probability if and only it prices strictly below $\hat{p}_H$, it must hold that $\overline{p}_L = \hat{p}_H \frac{q_L}{q_H}$. Third, since H deterministically attracts all rational consumers but cannot attract salient thinkers when pricing at $\hat{p}_H$, it must hold that $\pi_H = (\hat{p}_H - c_H + h)(1-\theta)$. Fourth, since H must be indifferent across all prices in the interval $[\underline{p}_H, \hat{p}_H]$, for this interval it must hold that

$$\pi_H(p_H) = (p_H - c_H + h)[(1-\theta) + \theta \mathbb{P}\{\tilde{p}_L > p_H \frac{q_L}{q_H}\}] \stackrel{!}{=} \pi_H.$$

Denoting firm L's CDF when pricing above $\underline{p}_H \frac{q_L}{q_H}$ by F_{L2}, the above condition can be rewritten as

$$\pi_H(p_H) = (p_H - c_H + h)[(1-\theta) + \theta(1 - F_{L2}(p_H \frac{q_L}{q_H}))] \stackrel{!}{=} \pi_H,$$

which, after substituting $p_L = p_H \frac{q_L}{q_H}$, implies

$$F_{L2}(p_L) = 1 - \frac{\frac{\pi_H}{p_L \frac{q_H}{q_L} - c_H + h} - (1-\theta)}{\theta}.$$

Fifth, since H must be indifferent across all prices in the interval $[\hat{p}_H, \overline{p}_H]$, for this interval it must hold that

$$\pi_H(p_H) = (p_H - c_H + h)(1-\theta)\mathbb{P}\{\tilde{p}_L > p_H - (q_H - q_L)\} \stackrel{!}{=} \pi_H.$$

Denoting firm L's CDF when pricing below $\overline{p}_H - (q_H - q_L)$ by F_{L1}, the above condition can be rewritten as

$$\pi_H(p_H) = (p_H - c_H + h)\theta(1 - F_{L1}(p_H - (q_H - q_L))) \stackrel{!}{=} \pi_H,$$

which, after substituting $p_L = p_H - (q_H - q_L)$, implies

$$F_{L1}(p_L) = 1 - \frac{\pi_H}{(p_L + (q_H - q_L) - c_H + h)(1-\theta)}.$$

Note finally that we have assumed throughout that firms sample prices continuously and without mass points over their respective CDFs. Of course, this is only satisfied if it both holds that (1) $F_{L1}(\hat{p}_L) \stackrel{!}{=} F_{L2}(\hat{p}_L)$ and (2) $F_{H1}(\hat{p}_H) \stackrel{!}{=} F_{H2}(\hat{p}_H)$. Expressing these conditions in terms of firms' candidate equilibrium profits, noting that $\hat{p}_L = \frac{\pi_L}{\theta} + c_L - h$ and $\hat{p}_H = \frac{\pi_H}{1-\theta} + c_H - h$, we finally obtain the (consistency) requirements

$$\frac{\theta\pi_H}{\frac{\pi_L}{\theta}+[(q_H-c_H)-(q_L-c_L)]}=\frac{(1-\theta)\pi_H}{\frac{\pi_L}{\theta}\frac{q_H}{q_L}+[\frac{q_H}{q_L}(c_L-h)-(c_H-h)]}-(1-\theta)^2 \quad (9)$$

and

$$\frac{(1-\theta)\pi_L}{\frac{\pi_H}{1-\theta}\frac{q_L}{q_H}+[\frac{q_L}{q_H}(c_H-h)-(c_L-h)]}=\frac{\theta\pi_L}{\frac{\pi_H}{1-\theta}-[(q_H-q_L)-(c_H-c_L)]}-\theta^2. \quad (10)$$

Note that these are only in terms of the unknown firm profits, π_L and π_H. Whenever a (non-negative) solution exists, we have from this and the preceding implications a fully specified equilibrium candidate that satisfies our conjectured equilibrium structure. The respective pricing strategies then indeed constitute an equilibrium if no firm finds it strictly profitable to sample prices *outside* its specified support.

Unfortunately, it is in general not feasible to report (or even work with) an explicit solution of the above non-linear system of equations (9) and (10). While this can be reduced to solving a single fourth-order polynomial in π_H or π_L, numerical checks with *Mathematica* suggest that the resulting polynomial is generally irreducible. This implies that, although a closed-form solution can in principle be given, it is typically extremely unwieldy. (In our case, the resulting output is multiple pages long.) We therefore proceed using numerical methods.

First, in order to give some support for our conjectured equilibrium, we take the following example parameters: $q_H = 1$, $c_H = 0.6$, $q_L = 0.75$, $c_L = 0.4$, $h = 0$, $\theta = 0.4$. Numerically, we find that the only non-negative solution to the above system of equations is given by $\pi_H \approx 0.057$ and $\pi_L \approx 0.04$. Plugging these values into our specified equilibrium CDFs, we obtain Figure 2.

In Figure 2, firm L's (H's) equilibrium CDF is depicted in purple (orange). It can clearly be discerned that each of the firms' CDFs is comprised of two parts, as specified in our characterization. In order to give a visual indication that the shown CDFs do indeed constitute an equilibrium, we have also plotted firms' expected profits (blue for L, red for H). One can observe that these are maximized (and identical) precisely for those prices that belong to firms'

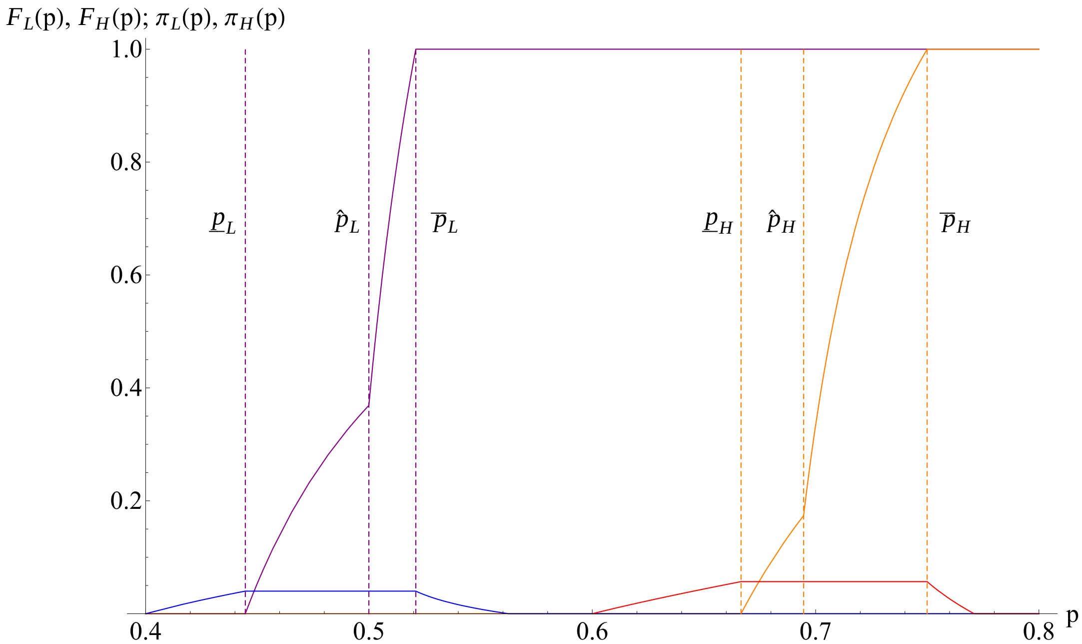

Figure 2: Example of a pricing equilibrium in the salient/rational model.

supports. Outside their specified supports, firms would make a strictly lower profit in expectation.

As a final step, we have checked whether the following qualitative result carries over from our analysis in the main text: firm L's expected demand strictly increases as h increases. Once more, we can show this through examples. Taking again the parameter combination that we used for Figure 2, in Figure 3 we plot firm L's expected demand as a function of h (where we vary h between 0 and c_L). We observe that L's expected demand strictly increases as the maximum amount of feasible shrouding h increases.

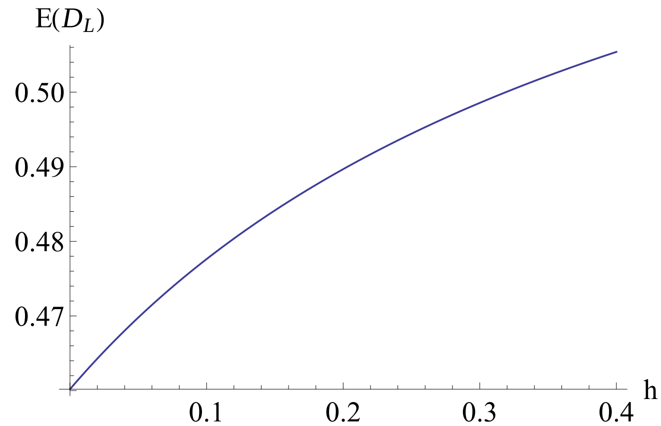

Figure 3: Firm L's expected demand in the salient/rational model as a function of h. The parameters used are $q_H = 1$, $c_H = 0.6$, $q_L = 0.75$, $c_L = 0.4$, $\theta = 0.4$.

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