Mergers, Welfare, and Bargaining Power[☆]

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Abstract

I study the impact of upstream mergers on prices in a model of vertically related markets where input prices are determined by negotiations. I show how the impact of bargaining power on price changes caused by a merger can be non-monotonic. This analysis has two implications. First, when conducting a merger simulation in a vertically related industry it is important to estimate bargaining weights. Second, the efficiency gains that are necessary for a merger to be welfare increasing are rather small when either sellers have little bargaining power or when buyers have little bargaining power and there are large differences in upstream efficiency. The identified non-monotonicity is more general and can be applied to any bargaining context where a buyer's disagreement payoff varies.

Keywords: Vertical relations, Nash-in-Nash bargaining solution, merger analysis, buyer power

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1. Introduction

The analysis of horizontal mergers is an important issue in economics of industrial organization. Both academic researchers and antitrust authorities are interested in the incentives leading to mergers and the resulting welfare implications. While the realization that mergers and their regulation are an important issue is rather old[2], it continues to be relevant in today's marketplace. According to Thomson Reuters (2018) the total number of announced merger and acquisition deals worldwide in 2017 was 49,448 resulting in a total transaction volume of $3.6 trillion. More importantly, mergers take place in key industries where concentrations are often already high, such as the health care market[3], and thus are likely to have a significant impact on market outcomes and social welfare.

Early contributions to the analysis of horizontal mergers consider markets where firms interact directly with final consumers and costs of inputs are given exogenously. This, however, may be an unfitting description of some markets found in practice. Rather than being determined by take-it-or-leave-it offers, input prices may be the result of negotiations between upstream sellers and intermediate downstream buyers - especially when the number of buyers and sellers is rather small or the intermediate good is specialized to fit a buyer's needs. For this reason, it is not surprising that in recent years the economic literature has seen the rise of more papers - both empirical and theoretical - that study the impact of mergers using models that explicitly account for negotiations in vertically related markets.

The present note advances the understanding of horizontal merger implications for prices and welfare in this class of models. All results are derived in a stylized model where a single (intermediate) buyer purchases a homogeneous good from N upstream sellers, that differ in their marginal costs, in order to resell it to final consumers. Wholesale prices are negotiated bilaterally which is implemented by applying the "Nash-in-Nash" bargaining solution which I discuss below. I show that the impact of an upstream merger on negotiated wholesale prices may be non-monotonic in the sellers' bargaining power. A

[2]The Clayton Act, that prohibits mergers and acquisitions which may substantially lessen competition, was enacted in 1914 in the US.

[3]Gaynor and Town (2011) report that the concentration in the US hospital markets has increased in the past decades due to horizontal mergers. This phenomenon has been also observed in other countries in recent years, such as Germany or UK, cf. OECD (2012, p. 53). The consolidation process affects also other participants in the health care system such as insurers (Gaynor et al., 2015).

sufficient condition for this is that the buyer's outside option is sufficiently small.

In equilibrium, all but the most efficient firms sell the product at marginal costs. The most efficient firm offers its product at a price that lies between its own marginal costs and the costs of its most efficient competitor. Clearly, the buyer will purchase from the most efficient firm. An upstream merger has a negative impact on the buyer's disagreement payoff in the negotiations with that seller. When the seller has no bargaining power, as captured by the bargaining weight, the outcome of the bilateral negotiation will be equivalent to the outcome where the buyer makes a take-it-or-leave-it offer. In this case changes in the buyer's outside option will have no impact on his equilibrium profits as he is already capturing all the surplus. If, in contrast, the seller has all the bargaining power, the bargaining outcome will resemble one where the seller makes a take-it-or-leave-it offer to the buyer that maximizes the seller's profits and makes the buyer (weakly) prefer the seller's offer to his outside option. If the buyer's profit from accepting the seller's offer is strictly higher than his outside option[4] then clearly a (small) variation of his outside option will have no impact on the negotiated outcome. In contrast, when the outside option is sufficiently high so that the buyer's profits are equal to the outside, any decrease in the outside option will result in a strict decrease of profits. When bargaining power is intermediate, a decrease in the buyer's disagreement payoff has a negative impact on his profits and the equilibrium input price increases. Taken together, this is sufficient to establish the non-monotonic effect of the bargaining weight on merger impact on prices and welfare.

In addition, I conduct a numerical merger analysis when the final consumer market exhibits either linear or logit demand. First, I show how the merger impact on negotiated input prices changes as the pre-merger disagreement payoff varies. Then, I study by how much the marginal costs of the merged firms have to decline (say due synergies or other forms of efficiency increases) in order to keep the wholesale prices after the merger at the pre-merger levels.

The analysis in this note has two implications. First, it is vital to estimate bargaining weights when performing merger simulations in real world cases in order to obtain reliable predictions of the merger impact. For example, if one assumes that negotiations always

[4] As I show below, this may happen if the difference in marginal costs is sufficiently high.

result in an even split while sellers have all the bargaining power a merger simulation may either over-estimate or under-estimate the impact of a merger – depending on the relative size of the outside option.[5] Second, the analysis indicates that the efficiency gains that are necessary to make the merger overall welfare enhancing are rather small in industries where sellers either have only little bargaining power or when they have a lot of bargaining power and there are large differences in upstream efficiency.

While the focus of this note lies on mergers, the identified non-monotonicity is more generally applicable. The impact of any change in the disagreement payoff, say due to changes in the bargaining power[6] or costs of concession in other matches, will be affected in the same way by bargaining power. Moreover, also other questions in Industrial Organization can be analyzed using this reasoning. Consider, for instance, the question of cost pass-through. Suppose the seller has all bargaining power and his marginal costs increase. Then, the relative size of the buyer's outside option determines the pass-through. If it is below his profits that he obtains before the price change, then the seller will pass-through all the costs to the buyer. Otherwise, if the buyer's outside option is binding, the seller cannot pass-through any of the increase.

Literature. In general, a (horizontal) merger will have two effects. On the one hand, the merger will reduce competition by improving coordination among the merging parties and lead ultimately to anticompetitive price effects. On the other hand, a merger may result in efficiency gains, e.g. due to economies of scale or synergies. For the assessment of a merger it is thus important to understand whether the former effect outweighs the latter. This trade-off is known as the *Williamson (1968) trade-off*.[7]

Early works in the field such as Salant et al. (1983) or Perry and Porter (1985) have studied this trade-off in a variety of models. However, they initially considered industries where firms interact directly with final consumers and where input prices are exogenously given. While these models may be a fitting representation of markets where intermediate

[5] Clearly, the misspecification of bargaining weights will have other detrimental effect in an empirical analysis. For example, Gowrisankaran et al. (2015) point out that assuming that sellers make take-it-or-leave-it offers in markets where negotiations are conducted may result in negative estimated marginal costs.

[6] Grennan (2014) argues that bargaining power may change over time because of learning.

[7] See for example the 2010 U.S. Horizontal Merger Guidelines `https://www.justice.gov/atr/horizontal-merger-guidelines-08192010`.

firms are price-takers, they may be misleading in others where input prices are likely to be the result of negotiations.

The seminal work of Horn and Wolinsky (1988) provides a framework that models negotiations in vertically related markets and allows to study horizontal mergers both in the upstream and the downstream market. In this bilateral monopoly framework the outcome of negotiations about an input price between a buyer and a seller is given as the solution to the respective Nash bargaining problem[8] - taking the solutions of the Nash bargaining problems of all other buyer-seller pairs as given. Thus, this concept nests the (cooperative) Nash bargaining into the (non-cooperative) Nash equilibrium. For this reason the literature often refers to this framework as the "Nash-in-Nash" solution.[9] It is critical for finding the equilibrium outcome to determine what happens in the case of a breakdown of negotiations between a pair. The Nash-in-Nash solution generally assumes that negotiations take place simultaneously and thus a breakdown between a buyer and a seller has no impact on the bargaining outcomes of the remaining pairs. This is sometimes justified by stipulating that each firm sends a separate agent to each of its counterparties who bargain simultaneously and assume that all negotiations will result in the equilibrium outcome.[10]

More recently, there is an emerging empirical literature that studies the impact of horizontal mergers in vertically related markets with structural models by applying the Nash-in-Nash bargaining solution. Grennan (2013) studies bargaining between hospitals and producers of coronary stents and shows that bargaining power and changes in it due to a merger have a significant impact on the profitability of downstream mergers. Another example is Gowrisankaran et al. (2015) who study a bargaining model of competition between hospitals and managed care organizations (MCOs). They find that MCO bargaining

[8]Nash (1950, 1953) describes a solution to a two person bargaining game that satisfies certain axioms like Pareto efficiency and independence of irrelevant alternatives. Rubinstein (1982) and Binmore et al. (1986) provide a (non-cooperative) foundation in a game of alternating offers in a bilateral setting for the Nash bargaining solution.

[9] Two foundations for the Nash-in-Nash solution are given by the following papers. First, Inderst and Montez (2016) consider a discrete-time model where sellers and buyers play a non-cooperative "Nash followed by Rubinstein" game. In the first period, parties make simultaneous offers in the sense of Nash (1953). If an agreement is not reached, they continue to bargain by making alternating offers as in Rubinstein (1982). Second, Collard-Wexler et al. (2014) extend the model by Rubinstein (1982) to multiple downstream and upstream firms and provide conditions such that prices in their model come arbitrarily close to the "Nash-in-Nash" prices as the time between offers becomes sufficiently small.

[10]See, for example, footnote 16 in Dobson and Waterson (2007).

restrains hospital prices significantly and they argue that even the sign of price changes due to a merger may change depending on whether one supposes that prices are determined under Bertrand hospital price setting or with bargaining. Further examples of structural papers that investigate the impact of mergers, or more generally market concentration, on prices are Chipty and Snyder (1999), Crawford and Yurukoglu (2012), Lewis and Pflum (2015), Dafny et al. (2016), and Ho and Lee (2017).

The rest of the paper is organized as follows. I present the basic model in the next Section and analyze in Section 3 the impact of an upstream merger. This is followed by a numerical analysis with linear demand and logit demand in Section 4. Section 5 discusses several issues regarding the generality of the results and Section 6 concludes.

2. The Basic Model

I consider an industry in which a single downstream buyer, the "retailer", purchases intermediate goods from N upstream sellers, the "manufacturers", that are denoted by M_1, M_2, ..., M_N, and sells these goods to final consumers. A manufacturer M_i produces his product at constant marginal costs c_i and I assume, without loss of generality, that $c_1 \leq c_2 \leq ... \leq c_N$. The retailer and manufacturer terminology is purely for expositional convenience, the results can be applied to any vertically related market, e.g. insurers and hospitals.

The manufacturers produce a homogeneous product. Denote with $q(p)$ the direct demand function in the final consumer market and suppose that it is decreasing and thrice continuously differentiable. Moreover, it is convenient to assume that $2\partial q(p)/\partial p + (p - w)\partial^2 q(p)/\partial p^2 < 0$ so that $(p - w)q(p)$ has a unique maximum p^* for all $w \geq 0$. When deriving quantitative implications of mergers, I will use specific demand functions, notably linear demand and logit demand with an outside good.

The timing of the industry is as follows. At the first stage, the retailer negotiates with each manufacturer i individually and simultaneously about the corresponding linear wholesale price w_i.[11] Negotiations are modeled by applying the Nash-in-Nash bargaining solution which I describe in more detail below. In the second stage, the retailer chooses

[11]The assumption of linear wholesale prices is common in the applied literature. For further details see the discussion in Iozzi and Valletti (2014).

optimally prices p_1, ..., p_N given the wholesale prices negotiated in the first stage. Lastly, consumer demand is realized.

Bargaining. The negotiations in the first stage of the game between the retailer and all manufacturers take place simultaneously. This implies that in each negotiation the participating agents take the equilibrium wholesale prices from the other negotiations as given. The negotiation outcome between the retailer and any single manufacturer is obtained by applying the Nash bargaining solution. Thus, the equilibrium is a set of wholesale prices $w_1^*, w_2^*, ..., w_N^*$ that constitute a Nash equilibrium in Nash bargains.

Denote with $\pi_R(w_i, w_{-i})$ the retailer's profit that results from choosing retail prices optimally given wholesale prices w_i and w_{-i} and denote with $\pi_{M_i}(w_i, w_{-i})$ the corresponding profits of manufacturer i where w_i is M_i's input price and w_{-i} are the input prices from all the other manufacturers. When negotiations break down, the retailer and the manufacturer obtain their respective disagreement payoffs. Clearly, in this case the manufacturer has no other opportunity to sell his product so that his disagreement payoff is equal to zero. The retailer, however, still can purchase - at negotiated equilibrium prices - from the other manufacturers so that his disagreement payoff is given by $\pi_R^o(w_{-i})$.[12]

Since N manufacturers and one retailer are in the industry, there are N bargains. The outcome of each bargain is determined by the Nash bargaining solution which requires to find the wholesale price w_i that maximizes Nash product of the retailer and the manufacturer M_i

$$w_i^* = \arg\max_{w_i} \ \pi_{M_i}(w_i, w_{-i})^b(\pi_R(w_i, w_{-i}) - \pi_R^o(w_{-i}))^{1-b} \ \text{ for } i = 1, ..., N, \tag{1}$$

where $0 \leq b \leq 1$ is the bargaining weight of the manufacturer and $1 - b$ is the bargaining weight of the retailer. The equilibrium given by the Nash-in-Nash bargaining solution is then a set of prices $w_1^*, ..., w_N^*$ such that each input price w_i^* maximizes the respective Nash product given all other prices w_{-i}^*.

Equilibrium. Next, I characterize the equilibrium outcome of this game. To streamline the exposition impose that $c_1 < c_2 < c_3$. In the second stage, the retailer can purchase from

[12]Note that after the negotiations are conducted a breakdown is observable by the retailer so that he can adjust the retail prices to the break-down in stage two.

any of the firms at wholesale prices that were negotiated in the first stage of the game. Clearly, since products are homogeneous the retailer will chose to sell the product with the lowest wholesale price, $\underline{w} = \min(w_1, ..., w_N)$. In case of a tie the retailer chooses the manufacturer with the lower marginal costs c_i. For given $\underline{w}$, the retailer chooses a retail price $p^*(\underline{w})$ that maximizes his profits:

$$p^*(\underline{w}) = \arg\max_p (p - \underline{w})q(p),$$

and thus, with a slight abuse of notation, the first stage profits of the retailer are given by

$$\pi_R(\underline{w}) = (p^*(\underline{w}) - \underline{w})q(p^*(\underline{w})).$$

It is immediate that the retailer's profits are decreasing in $\underline{w}$. When manufacturer M_i has the lowest negotiated input price, $\underline{w} = w_i$, his profits are given by

$$\pi_{M_i}(\underline{w}) = (\underline{w} - c_i)q(p^*(\underline{w}))$$

and the remaining manufacturers make zero profits.

Next, I turn to the negotiations in the first stage of the game and argue that a possible equilibrium has the following shape. All retailer-manufacturer negotiations except the one with the most efficient manufacturer result in a negotiated wholesale price that is equal to the respective manufacturer's marginal costs: $w_i^* = c_i$ for all $i \geq 2$. The wholesale price between the retailer and the most efficient manufacturer is determined by taking the first-order condition of the corresponding Nash product with $c_1 \leq w_1^* \leq c_2$. I impose that the Nash product has a unique maximum w_1^* for all $0 \leq b \leq 1$[13] and show that this is the case with linear demand and logit demand in Section 4. I streamline the notation by setting $\pi_M = \pi_{M_1}$. The following Lemma summarizes the equilibrium.

Lemma 1. *Consider a homogeneous goods market where one retailer and N manufacturers, that are heterogeneous in their efficiency, negotiate simultaneously over input prices. The Nash-in-Nash bargaining solution in this market is given by* $w_i^* = c_i$ *for* $i \geq 2$ *and*

[13]Note that this implies that $\pi_{M_i}(w)$ has a unique maximum.

$c_1 \leq w_1^* \leq c_2$ *is characterized by*

$$\frac{\pi_M(w_1^*)}{\pi_R(w_1^*) - \pi_R^o} = -\frac{b}{1-b}\frac{\partial \pi_M(w_1^*)/\partial w}{\partial \pi_R(w_1^*)/\partial w} \tag{2}$$

where $\pi_R^o = \pi_R(c_2)$.

Proof. See Appendix.

Rewriting the condition in (2) allows us to discuss factors that impact the division of surplus between seller and buyer

$$w_1^* = c_1 + b\left[(w_1^* - c_1) + \frac{\pi_R(w_1^*) - \pi_R^o}{q^*(w_1^*)}\frac{\partial \pi_M(w_1^*)/\partial w}{q^*(w_1^*)}\right] \tag{3}$$

where $q^*(w) = q(p^*(w))$ and $\pi_R^o = \pi_R(c_2)$.[14] The equilibrium input price is given by the manufacturer's marginal costs plus a margin that is determined by the split of the surplus that is generated through the bargain. The term $(\partial \pi_M(w_1^*)/\partial w)/q^*$ accounts for the fact that utility is not fully transferable in this model as the input price has an impact on the retail price. If utility was fully transferable, the sum of profits $\pi_R(w) + \pi_M(w)$ would be constant and it would thus hold that

$$-\frac{\partial \pi_M(w_1^*)/\partial w}{\partial \pi_R(w_1^*)/\partial w} = \frac{\partial \pi_M(w_1^*)/\partial w}{q^*(w_1^*)} = 1$$

so that the manufacturer's profits would be equal to a fraction b of the total surplus generated in the bargain.

Note that the input price is increasing in the bargaining weight b. When $b = 0$ the

[14] Another common way in the literature how to write down the first order conditions is the following

$$\frac{w_1^* - c_1}{w_1^*} = \left[-\frac{w_1^*}{q^*(w_1^*)}\frac{dq^*(w_1^*)}{dw} + \frac{1-b}{b}\frac{w_1^* q^*(w_1^*)}{\pi_R(w_1^*) - \pi_R(c_2)}\right]^{-1}$$

which illuminates the connection to the Lerner index. When $b = 1$, the Lerner index on the left is equal to the inverse price elasticity of the "effective" demand, that accounts for retailer pricing in the consumer market, faced by the manufacturer and thus the price is the same as in the case of Bertrand pricing. Moreover, this representations shows that misspecification of bargaining weights may result in biased marginal cost estimates. For instance, Gowrisankaran et al. (2015) show in their model of hospital and MCO competition that the assumption of Bertrand competition in the upstream market implies negative marginal costs.

manufacturer sells the product at marginal costs and when $b = 1$ the equilibrium input price w_1^* has to satisfy

$$[\pi_R(w_1^*) - \pi_R^o] \frac{\partial \pi_M(w_1^*)}{\partial w} = 0$$

which is satisfied when either of the factors is equal to zero. Recall that $\pi_R(w)$ is decreasing in w so that $w_1^* \leq c_2$. This implies that when c_2 is sufficiently high it will hold that $\pi_R(w_1^*) > \pi_R^o$ even when $b = 1$ which is key for the non-monotonic effect of the bargaining weight derived in the next Section.

3. Welfare impact of mergers

For a merger to have any impact on the equilibrium, I assume that the two most efficient manufacturers M_1 and M_2 merge and can produce the product at marginal costs c_1 after the merger.[15] Denote this newly formed manufacturer with M_m. Clearly, in the post-merger equilibrium it continues to hold that $w_i^* = c_i$ with $i \geq 3$ and that the retailer will not sell the products of those firms. Instead, he will sell the good of the most efficient manufacturer, here M_m, and the respective input price w_m^* will be determined by the FOC of the respective Nash product. What changes is, however, the retailer's disagreement payoff. As manufacturer M_2 is no longer a viable outside option in the case negotiations break down, his new disagreement point is given by the offer of manufacturer M_3. That is, post-merger the disagreement point is $\pi_R^o = \pi_R(c_3) < \pi_R(c_2)$.

As a first step towards a fully fledged surplus analysis, I study how the equilibrium payoff of the retailer changes when his disagreement payoff π_R^0 varies marginally. In particular, I am interested in how this marginal effect depends on the bargaining weight b. Recall that b is the bargaining weight of the manufacturer and $1 - b$ is the weight of the retailer. Denote $\pi_R^* = \pi_R(w_1^*)$. By applying implicit differentiation on (2), that defines the outcome of the negotiations between manufacturer M_1 and the retailer, I obtain with

[15] This can be justified through the adoption of the more efficient technology of M_1 in the production sites of manufacturer M_2.

$d\pi_R^*/d\pi_R^o = \partial\pi_R(w_1^*)/\partial w \cdot dw_1^*/d\pi_R^o$ that

$$\frac{d\pi_R^*}{d\pi_R^o} = \left[\frac{1}{b} + (\pi_R^* - \pi_R^o)\left(\frac{\partial^2\pi_M(w_1^*)/\partial w^2}{\partial\pi_M(w_1^*)/\partial w \cdot \partial\pi_R(w_1^*)/\partial w} - \frac{\partial^2\pi_R(w_1^*)/\partial w^2}{(\partial\pi_R(w_1^*)/\partial w)^2}\right)\right]^{-1}. \quad (4)$$

Based on this I can characterize how $d\pi_R^*/d\pi_R^o$ changes in b. Denote with $\underline{\pi}_R$ the profits that the retailer makes under an input price $\overline{w} = \arg\max_w \pi_M(w)$, that is the input price that maximizes the manufacturer's profits irrespective of the retailer's participation constraint while still taking into account the retailer's pricing in the final consumer market. Solely to avoid case distinctions suppose that $\underline{\pi}_R > 0$.

Lemma 2. *Consider Nash bargaining between the retailer and the most efficient manufacturer M_1. Then $d\pi_R^*/d\pi_R^0$ is given by (4) and it is continuous and non-negative for all $0 \leq b \leq 1$. Moreover, when $\pi_R^0 < \underline{\pi}_R$ it holds that $d\pi_R^*/d\pi_R^0 = 0$ both when $b = 0$ and $b = 1$, and when $\pi_R^0 \geq \underline{\pi}_R$ it holds that $d\pi_R^*/d\pi_R^0 = 0$ when $b = 0$ and $d\pi_R^*/d\pi_R^0 = 1$ when $b = 1$.*

Proof. See Appendix.

The intuition for the changes in the impact of the disagreement payoff is immediate. When the retailer has all bargaining power ($b = 0$) he obtains the maximum profits and thus changes in his disagreement payoff will have no impact on his equilibrium profits. If the manufacturer has all bargaining power ($b = 1$) then the impact of the disagreement payoff depends starkly on the relation between the retailer's disagreement payoff π_R^o and the retailer's profit when offered $\overline{w}$. When π_R^o is smaller than $\underline{\pi}_R = \pi_R(\overline{w})$ then clearly changes in π_R^o will have no impact on retailer's profits. If, in contrast, π_R^o is larger than $\underline{\pi}_R$ then any change in the disagreement payoff result in an equivalent increase in the retailer's equilibrium profits.

Note that a necessary condition for the non-monotonicity is that bargaining is inefficient as it is the case with negotiations over a linear input price. This is the reason why the retailer may obtain a positive surplus above his outside option even when the manufacturer has all the bargaining power $b = 1$. If bargaining is efficient, the seller can always obtain all surplus generated in the bargain when $b = 1$ and generally it holds that $d\pi_R^*/d\pi_R^o = b$.[16]

[16]Under efficient trading we would have $\partial\pi_M(w)/\partial w = -\partial\pi_R(w)/\partial w$ for all w and thus $\partial^2\pi_M(w)/\partial w^2 =$

Lemma 2 also shows that in order to provide a comprehensive merger analysis in practice, it is vital to have reliable estimates of both the bargaining power and the disagreement payoffs as failing to do so may result in either an upward or a downward bias in the simulated post-merger prices. For example, if the empirical analysis correctly reveals that the bargaining weight b is close to one but misspecifies the retailer's disagreement payoff, then a merger simulation will overestimate the merger impact if the estimated value of π_R^o is larger than $\underline{\pi}_R$, while the true value of π_R^o is lower than $\underline{\pi}_R$, and it will underestimate the merger impact if the misspecification is reversed.

I can now use Lemma 2 to establish a link between the differences in efficiency in an industry and the implications of a merger for prices and welfare. For this I denote with $\Delta c \equiv c_2 - c_1$ the difference in marginal costs between the most and the second most efficient manufacturer M_1 and M_2.

Proposition 1. *Consider a homogeneous goods market where one retailer and N manufacturers, that are heterogeneous in their efficiency, negotiate simultaneously over input prices. When the bargaining weight b approaches zero, so that the retailer obtains all the bargaining power, the impact of an upstream merger on wholesale prices, retailer profits, and social welfare approaches zero. When b approaches 1, so that manufacturers have all the bargaining power, then the impact of a merger depends on the cost difference Δc. If the cost difference Δc is rather large, so that $\pi_R^o = \pi_R(c_2) < \underline{\pi}_R$, then the merger's impact goes to zero. If the cost difference is rather small, so that $\pi_R^o \geq \underline{\pi}_R$, then input prices will strictly increase while retailer profits and social welfare will strictly decrease after the merger.*

Proposition 1 follows directly from Lemma 2. In this simple model the equilibrium quantity is a proxy for social welfare as firms will always cover their costs. Note that the impact of a merger in cases when b is intermediate will depend on further factors such as the curvature of the retailer's and manufacturer's profit functions as will be illustrated in the next Section.

$-\partial^2\pi_R(w)/\partial w^2$ for all w. This implies that in this case the second term in the square brackets in (4) is zero.

4. Numerical Analysis

In this section I conduct a merger analysis with two examples: linear demand and logit demand. In particular, I am interested in the impact of an upstream merger on the wholesale price and how this depends on the size of the disagreement payoff of the retailer relative to the profits he would realize if the manufacturer was to make a take-it-or-leave-it-offer. In this model the merger results always in an increase of the input price and thus in a decrease in social welfare. I illustrate how the magnitude of price increases varies depending on the bargaining weights and the disagreement payoff. In practice a common merger defense is to claim that the merger may result in greater efficiency. Thus, in a second step, I evaluate the magnitude of efficiency gains that are necessary to keep the wholesale price at the pre-merger level. This provides minimum requirements for efficiency gains so that the merger is just neutral to social welfare. As in general the Nash bargaining solution does not result in closed form solutions I conduct numerical simulations to derive the results.

Demand specifications. The first specification that I consider is linear demand. In this case, the demand function is given by

$$q(p) = a - dp \tag{5}$$

with $a > 0$ and $d > 0$. The linear demand is derived from a representative consumer who has a quasi-linear quadratic utility function

$$U = \frac{a}{d}q - \frac{1}{2d}q^2 + q_0$$

where q_0 is the numeraire good and captures the consumption of all other goods.

The second specification is logit demand which is a discrete choice model that is common in the empirical industrial organization literature. Suppose there is a unit mass of consumers who are heterogeneous with respect to their tastes and who purchase either one or zero units of the product offered by the retailer. The utility that a consumer j obtains of purchasing the product is given by

$$U_j = a - dp + \varepsilon_j$$

where ε_j captures taste differences among consumers. The utility from not purchasing the product is given by ε_j^o. Assuming that both ε_j and ε_j^o are identically and independently type I extreme value distributed across consumers, the demand faced by the retailer is given by

$$q(p) = \frac{\exp(a - dp)}{1 + \exp(a - dp)}. \tag{6}$$

I show next that both demand specifications result in a unique Nash bargaining solution for a given retailer disagreement payoff $\pi_R^o = \pi_R(c_2)$.

Lemma 3. *Consider Nash bargaining between the retailer and the most efficient manufacturer. A unique negotiated input price w_1^* exists when demand is linear and costs are $c_1 < a/d$ or when demand is logit with a single outside option.*

Proof. See Appendix.

Results. I present first the results with linear demand. Throughout the analysis, I set $a = d = 1$ and suppose that the most efficient manufacturer produces at costs $c_1 = 0.25$. Conditional on these costs, the highest input price this manufacturer would set is $\overline{w} = 0.625$ which results in retailer profits $\underline{\pi}_R \approx 0.0352$. Suppose that $c_3 = 1$ so that the retailer's outside option after the merger is $\pi_R^o = 0$.

I study first how input prices change after a merger. In order to illustrate the impact of the disagreement payoff, I consider two cases: First, when $c_2 = 0.725 > \overline{w}$ so that $\pi_R^o < \underline{\pi}_R$ and second, when $c_2 = 0.525 < \overline{w}$ so that $\pi_R^o > \underline{\pi}_R$. As expected, the wholesale price increases in the manufacturers' bargaining power irrespective of the magnitude of the retailer's disagreement payoff. The input price ranges from $c_1 = 0.25$ for $b = 0$ to the smaller of $\overline{w}$ and the disagreement input price $w_2^* = c_2$, for $b = 1$. Thus, before the merger, when $b = 1$ and the disagreement payoff is small ($c_2 = 0.725$) the input price is $w_1^* = \overline{w} = 0.625$ and when the disagreement payoff is rather large ($c_2 = 0.525$) it is $w_1^* = 0.525$. Note that in both cases the difference between w_m^* and w_1^* is rather small when b is small. However, this does no longer hold when b approaches 1. When the outside option is small, then the price difference approaches 0 whereas when the outside is large ($c_2 = 0.525$) the difference is strictly positive and economically significant as in this case the merger results in a price increase of over 19%.

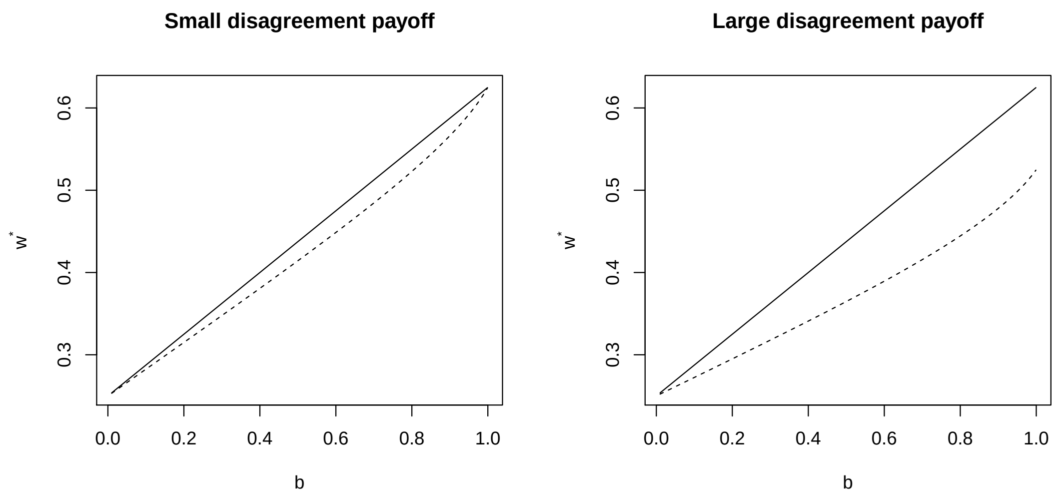

Figure 1: **Input prices with linear demand.** This figure depicts the pre-merger (dashed line) wholesale prices w_1^* and post-merger (solid line) wholesale prices w_m^* conditional on the manufacturer's bargaining power b when final demand is linear. In the left panel the retailer's pre-merger disagreement payoff π_R^o is determined by the outside wholesale price $w_2^* = c_2 = 0.725 > \overline{w}$ whereas in the right panel the disagreement payoff is determined by $w_2^* = c_2 = 0.525 < \overline{w}$.

Next, I calculate the costs that the merged firm must have after the merger (when $\pi_R^o = 0$) so that the negotiated input price after the merger equals the pre-merger input price for a given bargaining weight b. Denote these costs by $\widehat{c}_1$. In order to provide a comprehensive picture, I present the required cost level first as a function of bargaining weights b for different levels of the outside option in Figure 2 and as a function of the pre-merger level of the outside options input price $w_2^* = c_2$ for different levels of bargaining power b in Figure 3.[17]

Consider Figure 2. The non-monotonic effect of the bargaining weight on prices translates into a non-monotonic effect on the new cost $\widehat{c}_1$ when the disagreement payoff is small. Note that there is a non-monotonicity for b close to 1 even when the disagreement payoff is large which shows that non-monotonicities may arise due to other reasons such as the inefficiency of trade and the resulting curvature of the firms' profit functions. Again, notice the stark difference between the cases when π_R^o is small and large. When b approaches 1 the necessary reduction in marginal costs $c_1 - \widehat{c}_1$ approaches zero when π_R^o is small, whereas it continues to be strictly positive and highly significant when π_R^o is large. For

[17] Recall that the retailer's profit is strictly decreasing in the input price. Thus, a higher w_2^* results in a lower disagreement payoff π_R^o.

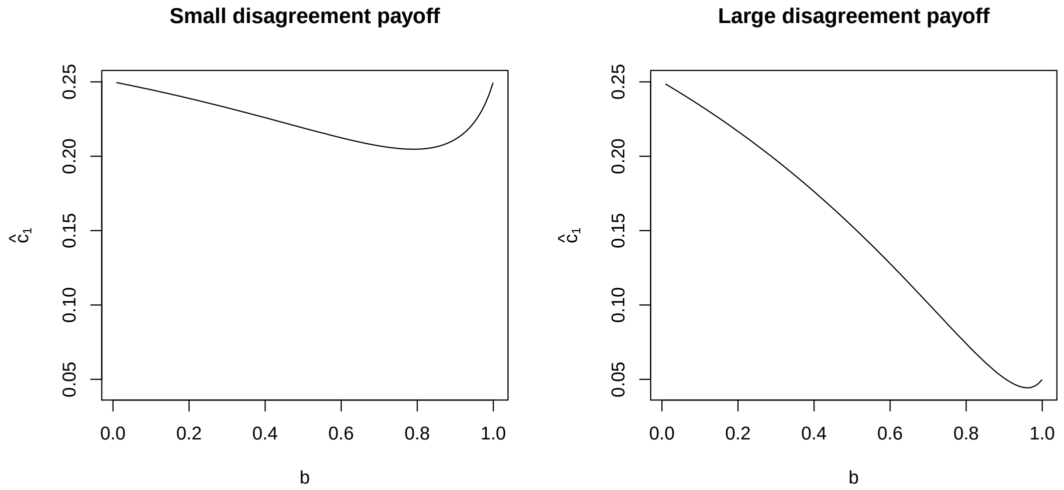

Figure 2: **Post-merger costs $\widehat{c}_1$ keeping prices constant conditional on b with linear demand.** This figure depicts the merged firm's marginal costs $\widehat{c}_1$ that keep, conditional on the bargaining weight b, input prices w_m^* after the merger equal to the pre-merger input prices w_1^*. In the left panel the retailer's pre-merger disagreement payoff is determined by the outside wholesale price $w_2^* = c_2 = 0.725 > \overline{w}$ whereas the in the right panel the disagreement payoff is determined by $w_2^* = c_2 = 0.525 < \overline{w}$.

example, when $b = 1$ the necessary reduction in costs is about 80.14% in order to keep prices constant.

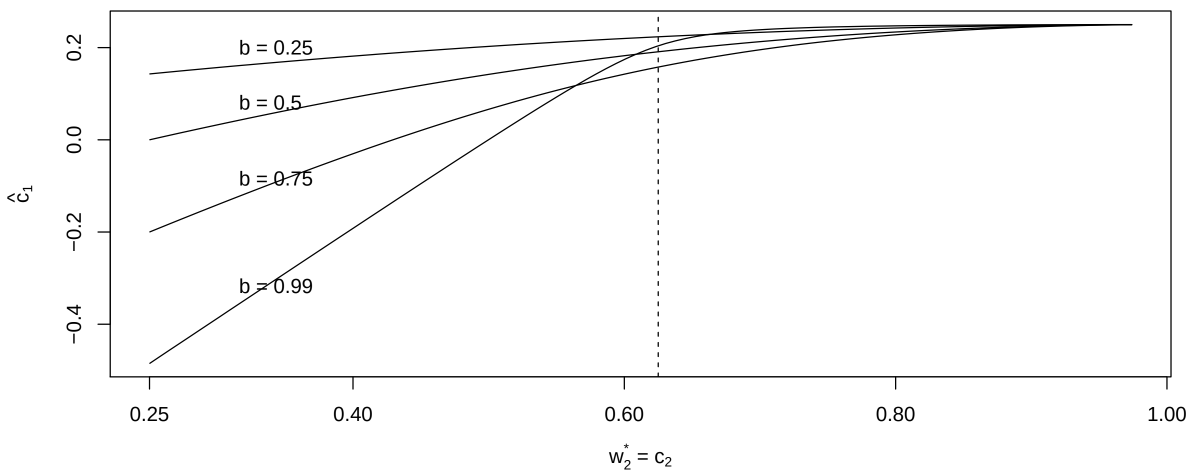

Figure 3: **Post-merger costs $\widehat{c}_1$ keeping prices constant conditional on $w_2^* = c_2$ with linear demand.** This figure depicts the merged firm's marginal costs $\widehat{c}_1$ that keep, conditional on the input price $w_2^* = c_2$, input prices w_m^* after the merger equal to the pre-merger input prices w_1^*. The vertical dashed line identifies $\overline{w}$, the highest price that the most efficient manufacturer would charge.

Next, consider Figure 3 which shows how $\widehat{c}_1$ changes in the retailer's outside input price $w_2^* = c_2$ that is varied between $c_1 = 0.25$ and $c_3 = 1$. It is remarkable that for cases when w_2^* is rather low and the manufacturer has a lot of bargaining power (e.g. when $w_2^* = 0.4$ and $b = 0.75$) then only *negative* marginal costs $\widehat{c}_1$ can result in constant input prices. In these cases it is unlikely that a merger will result in an improvement of social welfare. Next, observe that when $w_2^* > \overline{w}$ the differences between the cost functions $\widehat{c}_1$ across the different levels of bargaining power are rather small compared to the case when $w_2^* < \overline{w}$. Interestingly and in line with the observations from Figure 2, for larger values of w_2^* the necessary reduction in costs $c_1 - \widehat{c}_1$ is smaller for rather large values of b. This can be seen, for example, very clearly around $w_2^* = \overline{w}$ where $\widehat{c}_1$ when $b = 0.99$ is larger than $\widehat{c}_1$ both when $b = 0.75$ and $b = 0.5$.

With logit demand I have $a = 1$ and $d = 1$ and I set the costs $c_1 = 1$ and $c_3 = 5$ so that the post-merger outside option is given by $\pi_R^o = \pi_R(5) \approx 0.0067$. The highest price charged by the manufacturer is $\overline{w} \approx 2.21$. In order to generate the case with the small disagreement payoff in I set $c_2 = 1.9$ and I generate the case with the large disagreement payoff by setting $c_2 = 2.5$. Qualitatively, the results are analogous to those in the case of linear demand, however the non-monotonic impact of b on input prices when the disagreement payoff is large is more pronounced in Figure 4 than in Figure 1. The same holds with Figure 5 and Figure 6.

This analysis emphasizes the importance of bargaining power and the disagreement payoff for the analysis of merger implications for prices and welfare and illuminates that both components have an economically significant impact on outcomes. From the perspective of a regulatory body, mergers are likely to be welfare increasing when either the manufacturers' bargaining power is rather small or when this bargaining power is high and at the same time the disagreement payoff of the retailer is rather low.

5. Discussion

The model considered in this paper is stylized which allows me to identify the non-monotonic effect of bargaining weights in a transparent form. When a differentiated goods market is studied, the analysis becomes more involved and the impact of the non-monotonicity, while still being present, will be affected by additional factors. First, in the pre-merger equilibrium the retailer will, in general, prefer to buy the products of more

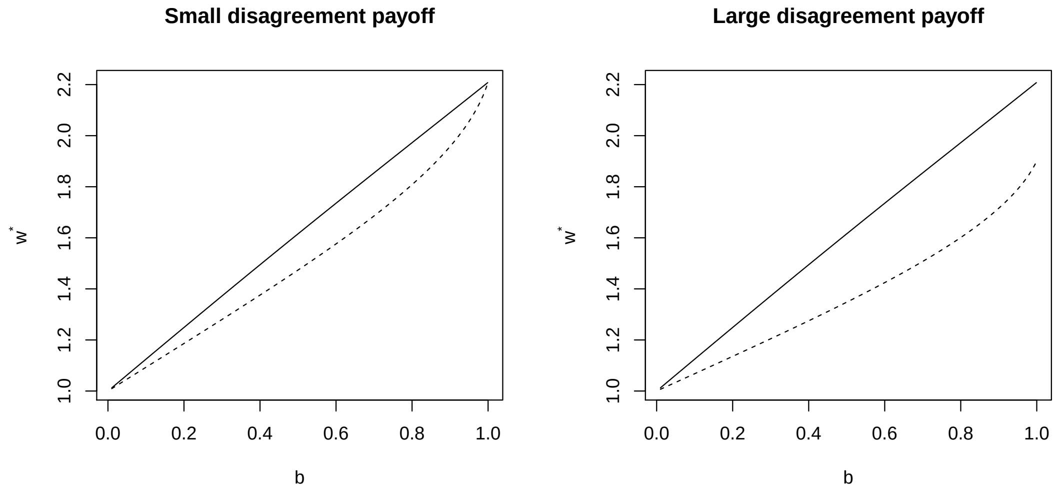

Figure 4: **Input prices with logit demand.** This figure depicts the pre-merger (dashed line) and post-merger (solid line) wholesale price conditional on the manufacturer's bargaining power b when final demand is logit. In the left panel the retailer's pre-merger disagreement payoff is determined by the outside wholesale price $w_2^* = c_2 = 2.5 > \overline{w}$ whereas the in the right panel the disagreement payoff is determined by $w_2^* = c_2 = 1.9 < \overline{w}$.

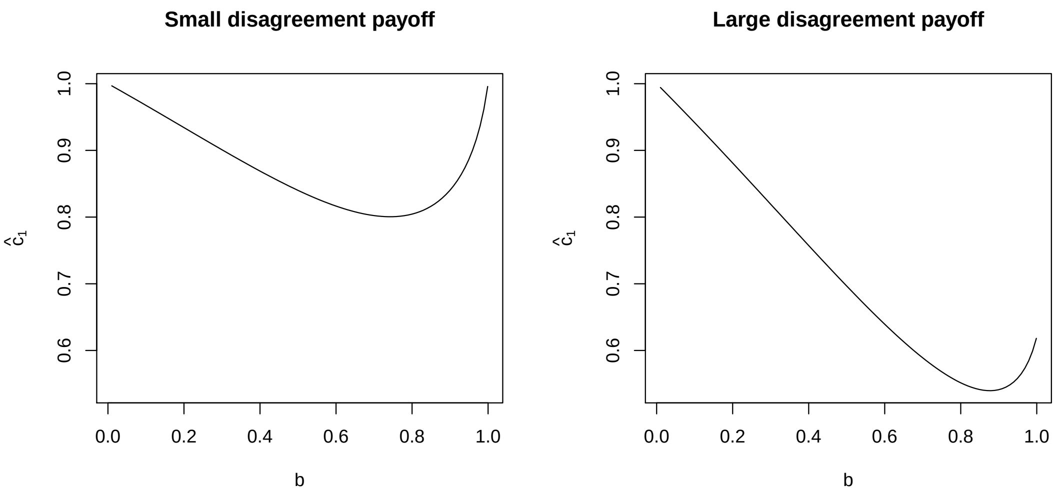

Figure 5: **Post-merger costs $\widehat{c}_1$ keeping prices constant conditional on b with logit demand.** This figure depicts the merged firm's marginal costs $\widehat{c}_1$ that keep, conditional on the bargaining weight b, input prices w_m^* after the merger equal to the pre-merger input prices w_1^*. In the left panel the retailer's pre-merger disagreement payoff is determined by the outside wholesale price $w_2^* = c_2 = 2.5 > \overline{w}$ whereas the in the right panel the disagreement payoff is determined by $w_2^* = c_2 = 1.9 < \overline{w}$.

than one firm and thus a merger will no longer simply reduce the retailer's outside option in a single negotiation but will more likely have complex repercussions on all negotiations. Moreover, bargaining between the merged firm and the retailer becomes more involved as

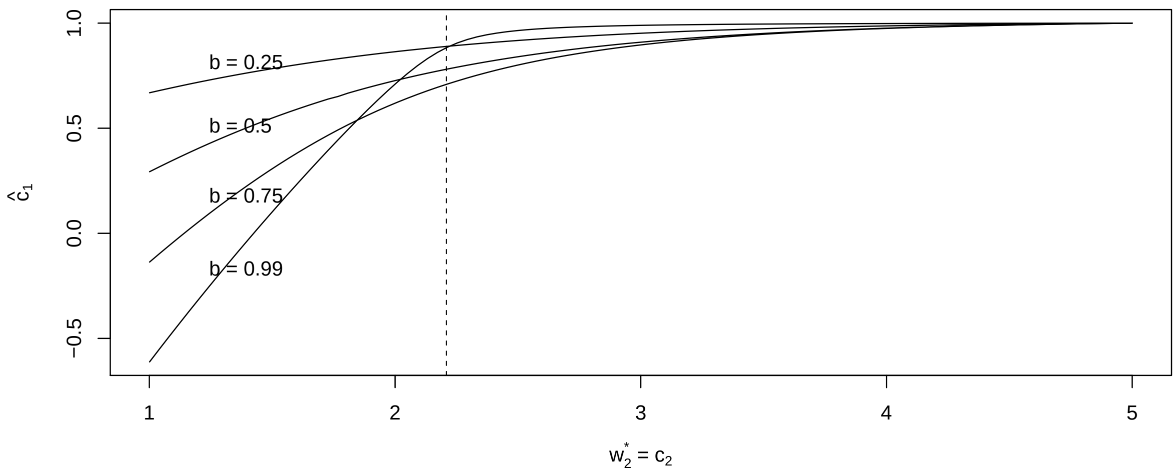

Figure 6: **Post-merger costs $\widehat{c}_1$ keeping prices constant conditional on $w_2^* = c_2$ with logit demand.** This figure depicts the merged firm's marginal costs $\widehat{c}_1$ that keep, conditional on the input price $w_2^* = c_2$, input prices w_m^* after the merger equal to the pre-merger input prices w_1^*. The vertical dashed line identifies $\overline{w}$, the highest price that the most efficient manufacturer would charge.

now the parties have to negotiate simultaneously about the input prices for two goods.[18] In order to make such an analysis comparable to the one in this note, it is important to keep in mind that in the model in Section 2 the bargaining weight b has essentially no impact on the outcome of negotiations between the retailer and any manufacturer M_i with $i \geq 2$. This is no longer the case if differentiated products are considered. Thus, the analysis should allow for heterogeneous bargaining weights so that one can study the impact of a bargaining weight b_i in a single bargain on changes of the retailer profits in the disagreement payoff.

While clearly the focus of this note was on the impact of mergers, the identified non-monotonicity is not limited to mergers. There is a variety of reasons why the disagreement payoff of a party can shift, such as changes in the bargaining power or the costs of concession in other negotiations. These changes maybe either the result of changes in the market structure, e.g. due to entry or exit, technological shocks (changes in the production costs) or regulation. The impact of all these structural changes can be non-monotonic in the

[18] See O'Brien and Shaffer (2005) who consider mergers in differentiated upstream markets when manufacturers and retailers negotiate over non-linear contracts.

bargaining weights of the parties.

An immediate question is whether this non-monotonic impact of the bargaining weight b on the (marginal) effect of the disagreement payoff on profits translates to the upstream seller. Consider, for simplicity, negotiations about a linear input price w in a bilateral monopoly between a single buyer and a single seller who has constant marginal costs and a positive disagreement payoff π_M^o. Key to the identified non-monotonicity in Section 3 is that there are cases when the manufacturer's take-it-or-leave-it offer leaves the retailer's participation constraint slack so that when the manufacturer has all the bargaining power an increase in the retailer's disagreement payoff has no impact on his equilibrium profits. When we consider the buyer's take-it-or-leave-it offer with linear input prices, the seller's participation constraint will be always binding, as the buyer will set the input price equal to marginal costs, so that when the buyer has all the bargaining power the marginal impact of π_M^o on manufacturer's profits is always 1. Thus, this source of non-monotonicity does not translate to a seller with constant marginal costs.

6. Conclusion

In a model of vertically related markets with a single buyer in the downstream market and many sellers in the upstream market I study the impact of upstream mergers when firms negotiate input prices. This note illuminates how the bargaining weight affects the impact of an upstream merger on input prices even when the bargaining power stays constant after the merger. It shows that in general the impact can be non-monotonic and will starkly depend on the relative magnitude of the disagreement payoff of the buyer. I argue that while the focus of this note lies on the impact of mergers, the identified non-monotonicity is more general and also relevant in other contexts where the outside option is affected by changes in market structure, technology, or regulation.

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Appendix: Proofs

Proof of Lemma 1. I show that none of the retailer-manufacturer pairs can increase their respective Nash product by deviating from their equilibrium input price w_i^*. First observe that in the above equilibrium the retailer will purchase the product of manufacturer M_1 as it holds that $w_1^* \leq w_i^*$ and $c_1 < c_i$ for all $i \geq 2$. Consider negotiations between the retailer and a manufacturer M_i with $i \geq 2$. In this bargain it is not possible to increase the value of the respective Nash product since a lower input price $w_i' < w_i^* = c_i$ would result in negative manufacturer profits, which is smaller than the manufacturer's disagreement payoff, whereas a higher $w_i' > w_i^* = c_i$ would have no impact on the Nash product as the retailer continues to sell the product of manufacturer M_1 in the second stage. Next, consider the negotiations between manufacturer M_1 and the retailer. Taking the first-order derivative of the Nash product $NP(w_1) = (\pi_R(w_1) - \pi_R^o)^{1-b}\pi_M(w_1)^b$ between the retailer and M_1 given that the next best offer is given by $w_2^* = c_2$ gives

$$\frac{\partial NP(w)}{\partial w} = (1-b)(\pi_R(w) - \pi_R^o)^{-b}\pi_M^b(w)\frac{\partial \pi_R(w)}{\partial w} + b(\pi_R(w) - \pi_R^o)^{1-b}\pi_M(w)^{b-1}\frac{\partial \pi_M(w)}{\partial w}.$$

Setting $\partial NP(w_1^*)/\partial w = 0$ and rearranging results in the expression in (2). For $0 < b < 1$ the solution w_1^* will be interior, so that this is indeed a maximum, since $\partial NP(w)/\partial w \to \infty$ as $w_1 \to c_1$ and $\partial NP(w)/\partial w \to C$ with $-\infty \leq C < 0$ as $w_1 \to \min\{c_2, \overline{w}\}$ with $\overline{w} = \arg\max_w \pi_M(w)$. **Q.E.D.**

Proof of Lemma 2. Observe that the Nash product $NP = (\pi_R(w) - \pi_R^o)^{1-b}\pi_M(w)^b$ is continuous both in b and in w. Due to the theorem of the maximum it holds that w^* is continuous in b. It remains to show that $d\pi_R^*/d\pi_R^o$ is continuous in w which is sufficient for continuity in b. Recall that $q(p)$ is three times continuously differentiable in p. Thus, p^*, defined by $q(p^*) + (p^* - w)\partial q(p^*)/\partial p = 0$, is differentiable in w with

$$\frac{dp^*}{dw} = \frac{\partial q(p^*)/\partial p}{2\partial q(p^*)/\partial p + (p^* - w)\cdot \partial^2 q(p^*)/\partial p^2}.$$

I obtain with the envelope theorem $\partial \pi_R(w)/\partial w = -q(p^*)$ and therefore $\partial^2 \pi_R(w)/\partial w^2 = -\partial q(p^*)/\partial p \cdot dp^*/dw$. Thus $\partial^2 \pi_R(w)/\partial w^2$ is continuous in w. Next I obtain $\partial \pi_M(w)/\partial w =$

$q(p^*(w)) + (w - c_1) \cdot \partial q(p^*)/\partial p \cdot dp^*/dw$ and

$$\frac{\partial^2 \pi_M(w)}{\partial w^2} = 2\frac{\partial q(p^*)}{\partial p}\frac{dp^*}{dw} + (w - c)\left[\frac{\partial^2 q(p^*)}{\partial p^2}\left(\frac{dp^*}{dw}\right)^2 + \frac{\partial q(p^*)}{\partial p}\frac{d^2 p^*}{dw^2}\right]$$

where

$$\frac{d^2p^*}{dw^2} = \left[\left(\frac{\partial^2 q(p^*)}{\partial p^2}\right)^2 \frac{dp^*}{dw}(p - w) - \frac{\partial q(p^*)}{\partial p}\left[\left(\frac{dp^*}{dw} - 1\right)\frac{\partial^2 q(p^*)}{\partial p^2} + (p^* - w)\frac{\partial^3 q(p^*)}{\partial p^3}\frac{dp^*}{dw}\right]\right] \\ \times \left[2\frac{\partial q(p^*)}{\partial p} + (p^* - w)\frac{\partial^2 q(p^*)}{\partial p^2}\right]^{-2}$$

which are all continuous in w. Taken together, the continuity of $d\pi_R^*/d\pi_R^o$ in b follows.

Next I show that NP has strictly decreasing differences in w and π_R^o for $0 < b < 1$ which implies $dw_1^*/d\pi_R^o \leq 0$, cf. Theorem 2.3 in Vives (2000), and thus $d\pi_R^*/d\pi_R^o \geq 0$. Denote with $NP(w, \pi^o)$ the Nash product as a function of the input price w and the disagreement payoff π^o and suppose that $\overline{w} \geq w_h > w_l \geq c_1$ and $\pi_h^o > \pi_l^o$ with $\pi_R(w_h) \geq \pi_h^o$. Then

$$\begin{aligned} &NP(w_h, \pi_h^o) - NP(w_l, \pi_h^o) - [NP(w_h, \pi_l^o) - NP(w_l, \pi_l^o)] \\ =&\pi_M(w_h)^b \left[(\pi_R(w_h) - \pi_h^o)^{1-b} - (\pi_R(w_h) - \pi_l^o)^{1-b}\right] \\ &- \pi_M(w_l)^b \left[(\pi_R(w_l) - \pi_h^o)^{1-b} - (\pi_R(w_l) - \pi_l^o)^{1-b}\right] \end{aligned}$$

which is strictly negative. This follows from $\pi_M(w_h)^b > \pi_M(w_l)^b$ and the fact that

$$\left[(\pi_R(w_h) - \pi_h^o)^{1-b} - (\pi_R(w_h) - \pi_l^o)^{1-b}\right] < \left[(\pi_R(w_l) - \pi_h^o)^{1-b} - (\pi_R(w_l) - \pi_l^o)^{1-b}\right] < 0$$

as $\pi_R(w_l) > \pi_R(w_h)$ and since a power function with an exponent $0 < 1 - b < 1$ is strictly concave.

Consider the case with $\pi_R^o < \underline{\pi}_R$. When $b = 0$ the Nash product is given by $NP = \pi_R(w) - \pi_R^o$ so that clearly changes in π_R^o will have no impact on π_R^* as the retailer already obtains the highest possible profit. If, on the other hand, $b = 1$, the Nash product reads $NP = \pi_M(w)$. Note that by assumption retailer profits $\underline{\pi}_R = \pi_R(\overline{w})$ are higher than his disagreement payoff such that a marginal increase in π_R^o will have no impact on his payoffs $\pi_R^* = \underline{\pi}_R$. Finally, consider the case with $\pi_R^o \geq \underline{\pi}_R$. When $b = 0$ we can apply the same arguments as above. When $b = 1$ the retailers payoff π_R^* must increase by the same amount as his disagreement payoff π_R^o as to ensure that the negotiations do not break down. Thus, we obtain in this case that $d\pi_R^*/d\pi_R^o = 1$. **Q.E.D.**

Proof of Lemma 3. Consider first the case with linear demand. Then $p^*(w) = (a/d + w)/2$ resulting in $q^*(w) = (a - dw)/2$ and $\overline{w} = c_1/2 + a/(2d)$. Note that by assumption $c_1 < a/d$ we have that $\overline{w} < a/d$ so that $q^*(\overline{w}) > 0$. I further obtain

$$\begin{aligned}
\pi_M(w) &= \frac{1}{2}(w - c_1)(a - dw) \\
\frac{\partial \pi_M(w)}{\partial w} &= \frac{1}{2}\left[(a - wd) - d(w - c_1)\right] \\
\pi_R(w) &= \frac{(a - wd)^2}{4d} \\
\frac{\partial \pi_R(w)}{\partial w} &= -\frac{1}{2}(a - wd).
\end{aligned}$$

Then, the left side of (2) is continuous and increasing in w for $c_1 \leq w \leq \min\{c_2, \overline{w}\}$ as $\pi_M(w)$ is increasing and $\pi_R(w)$ is decreasing in w on this range. Note that at $w = c_1$ the left side is zero. The right side of (2) is given by

$$\frac{b}{1-b}\left[1 - \frac{d(w - c_1)}{a - dw}\right]$$

which is decreasing in $c_1 \leq w \leq \min\{c_2, \overline{w}\}$. When $w = c_1$ it equals $b/(1-b)$ and at $w = \min\{c_2, \overline{w}\}$ it is always strictly smaller than the left side. When $c_2 < \overline{w}$, then the left side is infinite while the right side is still finite. When $\overline{w} < c_2$ then the right side is zero, as $\partial \pi_{M_i}(\overline{w})/\partial w = 0$ whereas the left side is strictly positive. Taken together this proves the claim for linear demand.

Next consider logit demand. The optimal retail price $p^*(w)$ solves

$$1 + \exp(a - dp^*(w)) - d(p^*(w) - w) = 0$$

By applying implicit differentiation I obtain that p^* is increasing in w as

$$\frac{dp^*}{dw} = \frac{1}{1 + \exp(a - dp^*)} = 1 - q(p^*)$$

and thus

$$\frac{dq(p^*(w))}{dw} = \frac{\partial q(p^*)}{\partial p}\frac{dp^*}{dw} = -dq(p^*)(1-q(p^*))^2 < 0.$$

Then I have

$$\frac{\partial \pi_M(w)}{\partial w} = q(p^*)\left[1-(w-c_1)d(1-q(p^*))^2\right].$$

Note that the expression in brackets is strictly decreasing in w, it is strictly positive for $w = c_1$ and strictly negative for sufficiently large w, as $q(p^*)$ approaches zero as w becomes arbitrarily large, so that a unique $\overline{w}$ exists that maximizes the manufacturer's profits.

Next, consider $\pi_R(w)$. I have that $\partial \pi_R/\partial w = -q(p^*) < 0$ and thus the left side of (2) is increasing in $c_1 \leq w \leq \min\{c_2, \overline{w}\}$ and it is zero when $w = c_1$. Now, the right side of (2) is given by

$$\frac{b}{1-b}\left[1-(w-c_1)d(1-q(p^*))^2\right]$$

which is decreasing in w. By the same arguments as above one can show that this expression is always smaller than the left side of (2) when $w = \min\{c_2, \overline{w}\}$. Taken together, these arguments imply that a unique w_1^* exists. **Q.E.D.**