Advanced Financial Economics 2
– Financial Contracting –

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– Part 3 –

Liquidity, Free Cash Flow, and Risk Management

Main Questions

- Why do some firms hold liquid assets and lines of credit with financial intermediaries?
- Why do others pay dividends or raise short-term debt?
- What is the optimal maturity structure of corporate liabilities from that perspective?
- How does the need to hold liquidity interact with the size of the firm?
- What is the interrelation between risk management and precautionary liquidity holding?

Intuition

- So far corporate financing was a one shot game

→ Firm investment and financing had the same maturity

- Corporate investments are typically ongoing projects that might generate cash flow also in intermediate term
- Corporate liabilities require some repayment before initial investments mature
- Corporate investments often require additional reinvestments
- Additional investment opportunities might arise before initial investments mature

⇒ A shortfall of the intermediate cash flow from the (re-)investment needs and the contractual intermediate payout generates a liquidity shortage

- Credit rationing limits firms' ability to raise new funds
- Liquidity shortages might endanger the viability of positive NPV firms

Intuition

- Firms have an incentive to insure against liquidity shortages

$\rightarrow$ Corporate liquidity holdings and credit lines serve as insurance

$\rightarrow$ Risk management helps stabilize intermediate cash flow

$\rightarrow$ Longterm debt and equity provides additional insurance

- However, ex-ante funding constraints limit the availability of these instruments

Liquidity risks - Assumptions

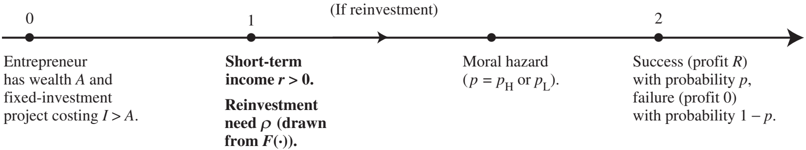

Figure 5.1

In addition to baseline model ...

- a deterministic return $r > 0$ is realized in interim period
- a stochastic reinvestment of size $\rho \in [0, \infty)$ with $F(\rho)$ and $f(\rho)$ being the cumulative distribution and the density function, respectively, is required to continue project
- if project is terminated liquidation return is zero

Liquidity risks - Assumptions

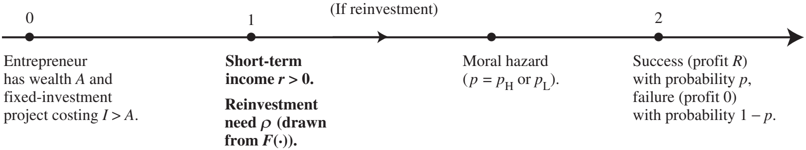

Figure 5.1

Note difference to private verifiable liquidity preferences shock:

1. Reinvestment ensures project viability and is therefore also in investors' interest
2. Effort choice can be made contingent on liquidity injection

Liquidity Risk - Inefficiency of "wait-and-see"

- Assume that entrepreneur does not preserve any funds in $t = 0$ for reinvestments in $t = 1$ and that he does not prearrange any access to liquidity for $t = 1$
- Assume that he invested all his funds A in $t = 0$ in the project to get it started and that he pledged maximum future returns including r to outside investors to attract funding
- Now in the $t = 1$ he faces a reinvestment need of ρ
- By providing the additional liquidity ρ investors can secure their expected contractual repayment
- Consequently, outside financiers are willing to provide additional liquidity up to

$$\rho_0 = p_H \left(R - \frac{B}{\Delta p} \right)$$

- At $t = 1$ the initial outside investment $(I - A)$ is sunk

Liquidity Risk - Inefficiency of "wait-and-see"

- From the perspective of the entrepreneur continuation is beneficial as long as the expected returns exceed the additional costs
- Thus enrepreneurs would reinvest up to

$$p_H R - \rho_1 = 0 \quad \Rightarrow \quad \rho_0 \leq \rho_1 = p_H R$$

$\Rightarrow$ Moral hazard problem reduces pledgability of future returns

- Positive NPV projects are credit (liquidity) rationed and have to be terminated

$\Rightarrow$ Entrepreneur has incentive to withhold some liquidity in $t = 0$ to ensure continuation in case of a liquidity shortage

Optimal continuation threshold

- Assume that the entrepreneur can write a contingent contract in $t = 0$ that requires investors to reinvest up to an amount ρ^* in the interim period
- Alternatively, one could also assume that entrepreneur requires additional funds ρ^* from investors in $t = 0$ which he holds in liquidity for reinvestments; if funds are not used they are repaid to investors
- Since funds are abundant entrepreneur retains project's NPV

⇒ His payoff as a function of the maximum liquidity provision:

$$U_b(\rho^*) = r + F(\rho^*)p_H R - I - \int_0^{\rho^*} \rho f(\rho) d\rho$$

Optimal continuation threshold

- From

$$U_b(\rho^*) = r + F(\rho^*)p_H R - I - \int_0^{\rho^*} \rho f(\rho) d\rho$$

follows that

$$\frac{\partial U_b}{\partial \rho^*} = f(\rho^*)p_H R - \rho^* f(\rho^*)$$

$\Rightarrow$ Entrepreneur achieves maximum payoff for $\rho^* = p_H R$

- Entrepreneurs' payoffs increase for $\rho^* \leq p_H R$

Optimal continuation threshold

- An increase in ρ^* also leads to an increase in the expected funding volume
- Thus ρ^* must be feasible given the ex-ante funding constraints
- The pledgable return as a function of the maximum liquidity provision is

$$\mathcal{P}(\rho^*) = r + F(\rho^*)p_H\left(R - \frac{B}{\Delta p}\right) - \int_0^{\rho^*} \rho f(\rho) d\rho$$

- Since

$$\frac{\partial \mathcal{P}}{\partial \rho^*} = f(\rho^*)p_H\left(R - \frac{B}{\Delta p}\right) - \rho^* f(\rho^*)$$

$\Rightarrow$ Pledgable returns are maximized at $\rho^* = p_H\left(R - \frac{B}{\Delta p}\right)$ and decline for $\rho^* \geq p_H\left(R - \frac{B}{\Delta p}\right)$

Optimal continuation threshold - Regime 1

- If the pledgable returns exceeds the funding gap even for a reinvestment policy that ensures continuation of all positive NPV projects ($\rho^* = p_H R$)

$$\mathcal{P}(p_H R) \geq I - A$$

  then the first-best cutoff can be achieved
- Even though investors are sometimes ex-post forced to provide liquidity to continue projects which do not allow them to recoup their reinvestment the overall expected returns from investments are sufficiently high to meet the funding gap

Optimal continuation threshold - Regime 2

- However, if

$$\mathcal{P}(p_H R) < I - A \leq \mathcal{P}\left(p_H\left(R - \frac{B}{\Delta p}\right)\right)$$

  not all positive NPV projects can be continued
- The entrepreneur has to reduce the maximum liquidity injections and terminate those projects with the smallest positive NPV, i.e. with the highest reinvestment needs
- Those are the projects for which the pledgable return falls short of the reinvestment volume
- By terminating those projects the overall pledgable return increases and the entrepreneur can cover the funding gap

Optimal continuation threshold - Regime 2

- In this case the cutoff $\rho^* \in [p_H(R - B/\Delta), p_H R]$ is determined by

$$r + F(\rho^*)p_H\left(R - \frac{B}{\Delta p}\right) - \int_0^{\rho^*} \rho f(\rho) d\rho = I - A$$

Optimal continuation threshold - Regime 3

- However, reducing the liquidity injections and discontinuing projects only increases the pledgable returns for

$$\rho \geq p_H \left(R - \frac{B}{\Delta p} \right)$$

- Reducing the liquidity injection even further and discontinuing projects with $\rho < p_H(R - B/\Delta p)$ would reduce the overall pledgable return since also projects would be terminated whose preserved pledgable return exceeds the reinvestment costs
- Thus for

$$I - A > \mathcal{P}\left(p_H \left(R - \frac{B}{\Delta p} \right) \right)$$

  the project cannot be financed in $t = 0$.

Optimal continuation threshold - Graphical solution

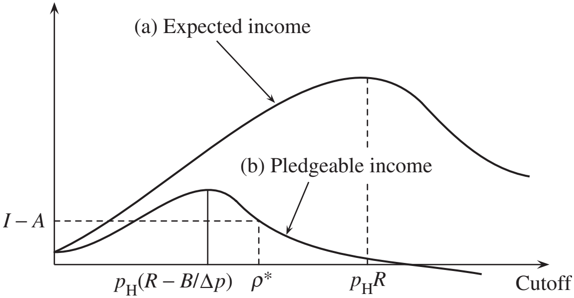

Figure 5.2 Optimal continuation policy:
(a) $U_b + I = r + F(\rho^*)[p_H R] - \int_0^{\rho^*} \rho f(\rho)\,d\rho$;
(b) $\mathcal{P}(\rho^*) = r + F(\rho^*)[p_H(R - B/\Delta p)] - \int_0^{\rho^*} \rho f(\rho)\,d\rho$.

Optimal continuation threshold - Implementation

- It is important to note that for $\rho^* > r$ it is irrelevant
  - whether the short-term cash flow r is pledged to outside financiers while at the same time the investors are obliged to reinvest ρ^* or
  - whether the cash flow r is kept in the firm for reinvestments and topped up by $\rho^* - r$ if needed but distributed to investors in $t = 2$ if not needed
- However, pledging the cash flow r to the entrepreneur is inefficient because entrepreneur would often consume out of cash holdings instead of long-term project returns which would strengthen his incentives and increase pledgable returns
- Problem: Liquidity provisions (credit lines) by outside investors must be credible: Whenever $\rho > \rho_0$ investors have an incentive not to abide commitment to provide additional liquidity

Optimal continuation threshold - Cash-rich firms

- If $r > \rho^*$ the liquidity available from cash flow exceeds the optimal reinvestment volume
- Thus the firm is cash rich; i.e. firm has excess liquidity
- If entrepreneur can use liquidity at his discretion he reinvests up to

$$r = \rho = p_H R$$

- This reduces pledgable return and might make ex-ante funding impossible
- To prevent this, excess liquidity has to be repaid to investors

Optimal continuation threshold - Cash-rich firms

- Short term debt d is an efficient means to drain excess liquidity:
$$d = r - \rho^*$$
- Long-term debt repayments are simply given by the maximum pledgable returns
$$D = R - \frac{B}{\Delta p}$$
- Firms with a high funding gap $(I - A)$ require high pledgable returns and thus can afford only a low ρ^*
- Thus cash rich firms with a high initial funding gap (weak initial balance sheet) must drain more liquidity and need to refinance more with short-term debt, i.e. have a shorter maturity structure

$\Rightarrow$ Highly indebted firms more likely to borrow short term

Optimal continuation threshold - Cash-rich firms

- In principle the liquidity drain could also be achieved by dividend payments
- However, stockholders in that case must be in a position to enforce the optimal dividend repayment d
- At the same time it must be ensured that stockholders cannot extort a higher dividend
- Stockholders would push for a higher dividend because

$$\overline{d} = r - p_H \left(R - \frac{B}{\Delta p} \right) > d$$

Liquidity-scale trade-off

- In the fixed investment case financially constraint firms use additional funds (higher pledgable returns, higher A) available in $t = 0$ always to increase liquidity holdings and continuation
- But this is an artefact resulting from the fact that there is no alternative use for additional funds
- Assuming a variable size investment project liquidity holdings (or prearranged lines of credit) reduce the investment volume
- Thus the benefits of additional liquidity holdings, i.e. the continuation of additional projects, must be traded off against the lower initial investment volume

Liquidity-scale trade-off - Assumptions

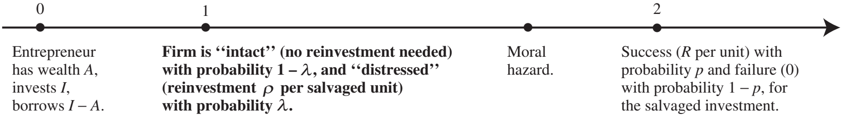

Figure 5.3

In addition to baseline model with variable investment size...

- with prob. $(1 - \lambda)$ no reinvestment required; per unit expected return is pR
- with prob. λ a reinvestment of fixed size ρ per unit initially invested is required
- with reinvestment the expected return pR of the respective unit is retained
- without reinvestment unit has zero expected return

Liquidity-scale trade-off - Equilibrium

- Assume that no liquidity is withheld
- For the variable investment case manager behaves if

$$\Delta p R_b \geq BI$$

- Since projects are only finished with prob. $(1-\lambda)$ the equilibrium investment volume is determined by

$$(1-\lambda)p_H\left(R-\frac{B}{\Delta p}\right)I = I - A,$$

i.e. pledgable returns are equal to the funding gap;

$$\Rightarrow I = \frac{A}{1-(1-\lambda)p_H\left(R-\frac{B}{\Delta p}\right)}$$

Liquidity-scale trade-off - Equilibrium

- The expected payoffs of the entrepreneur are thus given by

$$
\begin{aligned}
U_b^0 &= [(1-\lambda)p_H R - 1]I \\
&= \frac{(1-\lambda)p_H R - 1}{1-(1-\lambda)p_H\left(R-\frac{B}{\Delta p}\right)}A \\
&= \frac{p_H R - \frac{1}{1-\lambda}}{\frac{1}{1-\lambda} - p_H\left(R-\frac{B}{\Delta p}\right)}A
\end{aligned}
$$

- The expression $1/(1-\lambda)$ are the expected costs of having a completed project
- Since projects are only completed with prob. $(1-\lambda)$, only an expected investment of $1/(1-\lambda)$ leads to 1 completed project.

Liquidity-scale trade-off - Equilibrium

- Assume now instead that manager withholds sufficient liquidity to withstand the liquidity shock
- In that case the funding costs per unit of investment is $1 + \lambda\rho$
- Equilibrium investment is therefore given by

$$(1 + \lambda\rho)I - A = p_H \left(R - \frac{B}{\Delta p} \right) I$$

$$\Rightarrow I = \frac{A}{(1 + \lambda\rho) - p_H \left(R - \frac{B}{\Delta p} \right)}$$

Liquidity-scale trade-off - Equilibrium

- The expected payoffs of the entrepreneur are thus given by

$$
\begin{aligned}
U_b^1 &= [p_H R - (1+\lambda\rho)]I \\
&= \frac{(p_H R - (1+\lambda\rho))}{(1+\lambda\rho) - p_H\left(R - \frac{B}{\Delta p}\right)} A
\end{aligned}
$$

- Again the expression $(1+\lambda\rho)$ gives the expected unit costs of effective investment
- This is the expected effective amount that needs to be invested in order to have one unit of completed investment

Liquidity-scale trade-off - Equilibrium

- Comparing U_b^0 and U_b^1 shows that

$$U_b^1 > U_b^0$$
$$\frac{p_H R - (1+\lambda\rho)}{(1+\lambda\rho) - p_H\left(R - \frac{B}{\Delta p}\right)} A > \frac{p_H R - \frac{1}{1-\lambda}}{\frac{1}{1-\lambda} - p_H\left(R - \frac{B}{\Delta p}\right)} A$$

iff

$$1 + \lambda\rho \leq \frac{1}{1-\lambda}$$
$$(1-\lambda)\rho \leq 1$$

$\Rightarrow$ Entrepreneurs' payoffs are maximized if the expected per unit costs of effective investment are minimized

- It is optimal to insure against liquidity shocks if the liquidity shock is low and likely

Continuous liquidity shocks - Motivation

- The assumption with a binomial liquidity shock does not allow to derive an interior solution for liquidity holdings
- Either firms buffer all liquidity shocks or none
- No marginal trade-off can be studied

$\Rightarrow$ Model with variable investment volume and continuum of liquidity shocks

Continuous liquidity shocks - Assumptions

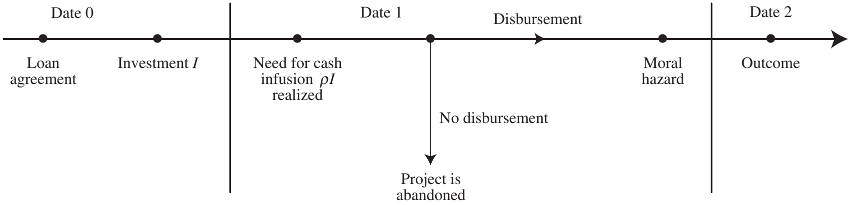

Figure 5.4

In addition to baseline model with variable investment size...

- a cost overrun generates a proportional liquidity need of ρI with ρ distributed continuously according to $F(\rho)$ on $[0, \infty)$ with density $f(\rho)$
- no project return realized if additional costs ρI not invested
- investment size is fixed after $t = 0$

Continuous liquidity shocks - Assumptions

- Investment only positive NPV if effort exerted, i.e. if success probability is p_H
- The project has a positive NPV at least for some continuation policy $\tilde{\rho}$:

$$\max_{\tilde{\rho}} \left\{ F(\tilde{\rho}) p_H R - 1 - \int_0^{\tilde{\rho}} \rho f(\rho) d\rho \right\} > 0$$

Continuous liquidity shocks - Equilibrium

- Optimal cut-off rule ρ^*
- To ensure that effort is exerted:

$$(\Delta p) R_b \geq BI \tag{ICC}$$

- Funding must be feasible:

$$F(\rho^*)[p_H(RI - R_b)] - \int_0^{\rho^*} \rho I f(\rho) d\rho \geq I - A \tag{PC}$$

Continuous liquidity shocks - Equilibrium

- Given competition for projects PC holds with equality
- Inserting ICC in PC and rearranging yields

$$I + I\int_0^{\rho^*} \rho f(\rho)d\rho - F(\rho^*)[p_H(R - B/\Delta p)]I = A$$

- The funding constraint is again given by an equity multiplier:

$$I = \frac{1}{1 + \int_0^{\rho^*} \rho f(\rho)d\rho - F(\rho^*)[p_H(R - B/\Delta p)]}A = k(\rho^*)A$$

- However, here the equity multiplier $k(\rho^*)$ is a function of the optimal continuation policy ρ^*

Continuous liquidity shocks - Equilibrium

- The equity multiplier

$$k(\rho^*) = \frac{1}{1 + \int_0^{\rho^*} \rho f(\rho) d\rho - F(\rho^*)[p_H(R - B/\Delta p)]}$$

is maximized if

$$\int_0^{\rho^*} \rho f(\rho) d\rho - F(\rho^*)[p_H(R - B/\Delta p)]$$

is minimized

Continuous liquidity shocks - Equilibrium

- The first order derivative is

$$\rho^* f(\rho^*) - f(\rho^*)[p_H(R - B/\Delta p)]$$

- Thus the equity multiplier is maximized if projects are continued up to the point where the continuation costs equal the pledgable return

$$\rho^* = p_H \left(R - \frac{B}{\Delta p} \right)$$

- If projects with continuation costs that exceed (fall short of) the pledgable return are continued the first order derivative of the expression is positive (negative) and the equity multiplier declines (increases) if additional project were continued

Continuous liquidity shocks - Equilibrium

- Because of scarcity of projects entrepreneur receives the NPV of firm
- Per unit NPV is given by

$$m(\rho^*) = F(\rho^*)p_H R - 1 - \int_0^{\rho^*} \rho f(\rho) d\rho$$

- It is easy to see that the first order derivative of the per unit NPV is

$$f(\rho^*)p_H R - \rho^* f(\rho^*)$$

- Thus the per unit NPV is maximized for $\rho^* = p_H R$
- The per unit NPV is increasing for $\rho^* < p_H R$ and decreasing for $\rho^* > p_H R$

Continuous liquidity shocks - Equilibrium

- Entrepreneur's payoff is $U_b = m(\rho^*)k(\rho^*)A$

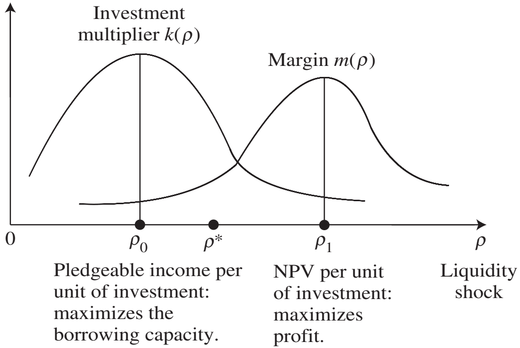

Figure 5.5

- It is maximized for a ρ^* such that: $p_H\left(R - \frac{B}{\Delta p}\right) < \rho^* < p_H R$

Continuous liquidity shocks - Equilibrium

- The overall payoff of entrepreneur is

$$
\begin{aligned}
U_b &= m(\rho^*)k(\rho^*)A \\
&= \frac{p_H R - (1 + \int_0^{\rho^*} \rho f(\rho) d\rho)/F(\rho^*)}{(1 + \int_0^{\rho^*} \rho f(\rho) d\rho)/F(\rho^*) - [p_H(R - B/\Delta p)]} A
\end{aligned}
$$

- So again entrepreneur maximizes his payoffs by minimizing the expected unit costs of effective investment $c(\rho^*)$

$$
U_b = \frac{p_H R - c(\rho^*)}{c(\rho^*) - [p_H(R - B/\Delta p)]} A
$$

with

$$
c(\rho^*) = (1 + \int_0^{\rho^*} \rho f(\rho) d\rho)/F(\rho^*)
$$

Continuous liquidity shocks - Equilibrium

- Before minimizing $c(\rho^*)$ first use partial integration

$$
\begin{aligned}
c(\rho^*) &= \frac{1 + \int_0^{\rho^*} \rho f(\rho) d\rho}{F(\rho^*)} \\
&= \frac{1 + \rho^* F(\rho^*) - \int_0^{\rho^*} F(\rho) d\rho}{F(\rho^*)} \\
&= \rho^* + \frac{1 - \int_0^{\rho^*} F(\rho) d\rho}{F(\rho^*)}
\end{aligned}
$$

Continuous liquidity shocks - Equilibrium

- First order condition:

$$
\begin{aligned}
\frac{\partial c}{\partial \rho^*} &= 1 - \frac{f(\rho^*)\left(1 - \int_0^{\rho^*} F(\rho)d\rho\right) + F(\rho^*)F(\rho^*)}{[F(\rho^*)]^2} = 0 \\
&\Leftrightarrow 1 - \frac{f(\rho^*) - f(\rho^*)\int_0^{\rho^*} F(\rho)d\rho}{[F(\rho^*)]^2} - 1 = 0 \\
&\Leftrightarrow -f(\rho^*) + f(\rho^*)\int_0^{\rho^*} F(\rho)d\rho = 0 \\
&\Leftrightarrow \int_0^{\rho^*} F(\rho)d\rho = 1
\end{aligned}
$$

Continuous liquidity shocks - Equilibrium

- Inserting the first order condition:

$$\int_0^{\rho^*} F(\rho) d\rho = 1$$

in the per unit effective investment costs

$$c(\rho^*) = \rho^* + \frac{1 - \int_0^{\rho^*} F(\rho) d\rho}{F(\rho^*)}$$

shows that in the optimum the expected per unit cost of effective investment is equal to the threshold liquidity shock:

$$c(\rho^*) = \rho^*$$

Continuous liquidity shocks - Equilibrium

- Reinserting in the entrepreneur's payoff function gives:

$$U_b = \frac{p_H R - \rho^*}{\rho^* - [p_H(R - B/\Delta p)]} A$$

- Consequently:

$$p_H R = \rho_1 > \rho^* > \rho_0 = p_H(R - B/\Delta p)$$

Continuous liquidity shocks - In sum

- Since $\rho^* > p_H(R - B/\Delta p)$ it is optimal to continue projects whose pledgable return falls short of the reinvestment need
- Thus for some projects the entrepreneur cannot secure funding of the continuation costs in $t = 1$
- For projects with $\rho^* > p_H(R - B/\Delta p)$ investors cannot recover their reinvestment costs
- They will not provide additional funding in $t = 1$

$\Rightarrow$ Non-binding promises to provide $\rho^* I$ liquidity are not credible

- In order to continue those projects entrepreneur must either
  1) be granted a non-revokable credit line of $\rho^* I$
  2) hoard $\rho^* I$ liquidity exante by raising these additional funds in $t = 0$

Liquidity hoarding - Discussion

- When entrepreneur raises $I(1+\rho^*)$ funds exante investors need to ensure that entrepreneur indeed invests I in the project and exactly holds $\rho^* I$ in cash
- Problem: Soft budget constraint might induce entrepreneur to overinvest in project:

1. Soft budget constraint: Investors will always agree expost to provide $\rho_0 = p_H(R - B/\Delta p)$ additional funds, no matter what the initial investment volume was
2. Expost (in $t = 1$) it is always better for investors to adapt entrepreneur's share to the actual level of investment

$\rightarrow$ Investors need to ensure also expost that entrepreneur exert effort

$\rightarrow$ Entrepreneur always gets a share of $BI/\Delta p$

Liquidity hoarding - Discussion

- Given that the entrepreneur invests the entire funds $I^* = I(1+\rho^*)$ in $t = 0$ in the project he receives a higher payoff iff

$$\begin{aligned} F(\rho^*) p_H \left(\frac{B}{\Delta p} I\right) &< F(\rho_0) p_H \left(\frac{B}{\Delta p} I^*\right) \\ \Leftrightarrow F(\rho^*) &< F(\rho_0)(1+\rho^*) \end{aligned}$$

  with $\rho_0 = p_H(R - B/\Delta p)$

- If B is sufficiently small, ρ_0 is high and close to ρ^* and $F(\rho_0)$ is close to $F(\rho^*)$

$\Rightarrow$ If the agency costs are not too large, projects are likely to be continued anyway; thus entrepreneur has an incentive to overinvest

$\Rightarrow$ Investors must monitor the liquidity holdings

Liquidity hoarding - Discussion

- But entrepreneur can also have an incentive to underinvest in assets and overhoard liquidity
- This is true if an additional unit of cash holding allows the entrepreneur to continue additional projects with a high probability
- I.e., if the density $f(\rho^*)$ is high around ρ^*

$\Rightarrow$ If entrepreneur receives a fixed claim $B\bar{I}/\Delta p$ in the case of success entrepreneur always has an incentive to underinvest and hold too much liquidity

Liquidity shocks - In sum

- The optimal continuation policy suffers from a time inconsistency problem
- Contingent financing contracts required to overcome this
- But contingent financing contracts (liquidity hoarding with contingent pay-out policy, credit lines etc.) require too much information

Benefits from hedging - Intuition

- Model with continuous liquidity shocks and variable investment provides a rational for corporate risk management
- Firms use cash flow to implement optimal reinvestment policy
- Hedging instruments allow firms to limit cash flow risks and implement optimal continuation policy with certainty
- Hedging instruments:
  1) Future contracts and swaps as insurance against price fluctuations (due to e.g. commodity price, interest rate or exchange rate risks)
  2) Factoring and securitization hedges e.g. default risks
  3) Insurances against fire, death or theft
- Here benefits of hedging are not a result of claim holders' demand for insurance

Benefits from hedging - Intuition

- Here hedging is beneficial because it reduces the uncertainty about the liquidity shortage
- It can reduce uncertainty regarding r and ρ
- Optimal reinvestment policy can be implemented ex-ante ($t = 0$)
- Need for contingent financing contracts to implement optimal reinvestment policy reduced