Contract Theory: Homework II
Topic: Moral Hazard

A) A risk-neutral principal has a project with the following structure: If the project is a success, the output is $y > 0$, if the project fails, the output is zero. The probability of success depends on the agent's effort, which can be high or low, resulting in a high probability of success or a low probability of success, respectively, with $1 > p_h > p_l > 0$. High effort reduces the agent's utility by c, low effort by zero.
Assume that the principal wants to implement high effort and that the agent is risk neutral, but protected by limited liability, i.e., the wage cannot be negative. Further, the value of the agent's outside option is zero.

1. Write down the principal's optimization problem and determine the optimal wage contract $w(y)$, $w(0)$. Calculate the principal's expected compensation costs.
2. Assume now that the agent has to simultaneously exert effort on two projects of the type described above, with stochastically independent success realizations. Determine now the principal's expected compensation costs to induce high effort (i.e., on both projects and with resulting disutility $2c$).
3. Answer the following question and give a short intuition: Is it cheaper for the principal to employ two agents, each working on one of the two projects or one agent working on both projects?

B) The following is another variation of the problem in part A). Consider now a firm and an employee. Both are risk neutral. The employee can exert privately observable effort e, which is a real-valued, non-negative variable. When he is paid the wage w, his utility is $w - a \cdot e^2/2$. Effort results in a "success" with probability e, which generates the contractible payoff $z > 0$ to the firm, and in a failure with probability $1-e$, which generates a payoff of zero. (When $e \geq 1$ the probability of success remains at 1.) When not contracting with each other, the firm and the employee realize zero payoff.

1. Take the following game: In $t = 1$ the firm makes an offer to the employee. In $t = 2$ the employee can accept or reject. In $t = 3$, when accepting, the employee can exert effort e, and in $t = 4$ payoffs are realized.
   (a) Characterize the first-best effort, i.e. the effort that would be socially efficient.
   (b) Characterize the outcome that results under the optimal offer made by the firm. Why is the effort different to the answer to question a)?
   (c) Now consider the same setting with the addition that the agent has an outside option/ reservation value $u \geq 0$. Characterize all cases regarding u. In the interest of reducing the number of cases, assume $z \leq a$.

2. Suppose now that the employee works on two projects and therefore simultaneously chooses in $t = 3$ two effort levels e_1 and e_2. When he receives the monetary payoff (wage) w, his utility is $w - a \cdot (e_1)^2/2 - a \cdot (e_2)^2/2$. Each project results with probability e_n, where $n = 1, 2$, in a success (and thus in payoff $z > 0$, instead of zero, for the firm). The two realizations are independent. Characterize the optimal offer made by the firm and the resulting effort levels.

C) Consider a firm that sells to a retailer. The firm/manufacturer has zero own costs and also the retailer has zero own costs. The manufacturer sets a wholesale price w for each unit that the retailer buys. Given w, the retailer decides which quantity to buy, which we call q, and then sells on the final market. Demand on the final market is described by the linear function: $q = a - p$, where p is the final/retail price. More precisely, consider thus the following game: At $t = 0$ the manufacturer chooses $w \geq 0$. At $t = 1$ the retailer then chooses which quantity to buy and sells this on the final market. [Note: It does not matter here whether you think of the retailer as choosing a quantity q, so that then the price $p = a - q$ is realized, or as choosing a final price p with corresponding quantity $q = a - p$.]

1. Characterize the optimal offer w of the manufacturer. Characterize also what is here the "agency rent".
2. Suppose that the manufacturer could also charge a fixed fee F that the retailer would have to pay when he wants to buy *any* quantity from the manufacturer. We stipulate now that the retailer has a reservation value of zero, but that $F > 0$ is possible (i.e., there are no other restrictions to contracts, such as those arising from what we called "limited liability"). What is now the optimal offer (F, w) of the manufacturer? Is there still an "agency rent" ?