

# Contract Theory: Homework I

## Topic: Screening

A monopolistic firm faces a single possible buyer. The buyer can be of two possible types:  $0 < \theta_l < \theta_h$ . The ex-ante likelihood of a high type ( $\theta_h$ ) is denoted by  $0 < \mu < 1$ . When a buyer of type  $\theta$  consumes a product of quality  $y \geq 0$  and pays a transfer  $t$ , then his utility is  $\theta y - t$ . When selling a product of quality  $y$  and receiving a transfer  $t$ , the firm's profit is  $t - y^2/2$ . The firm makes offers that maximize its expected profits.

1. Suppose the buyer's type is commonly known to both the buyer and the firm. Characterize the optimal offer.
2. Suppose now that the buyer's type is privately known to the buyer. Suppose further that the firm wants to ensure that both types accept an offer. Characterize the optimal offers.
3. Suppose still that the buyer's type is privately known to the buyer. When does the firm want to make sure that both types accept an offer and when not?
4. Suppose that the firm can make offers to the buyer before the buyer's type is realized. To be precise, assume the following time sequence: First, the firm can make offers and the buyer must either accept or reject. Then, the buyer learns his type, i.e., only after he has either accepted or rejected the firm's offer. Characterize the optimal offers.
5. Consider now the following game: At a first stage ( $t = 1$ ) the buyer privately learns his type. At a second stage ( $t = 2$ ) the seller can make an offer. If this is accepted by the buyer, the game ends. If the offer is rejected by the buyer, the game proceeds into the final period ( $t = 3$ ). In the final (third) period, the buyer can make an offer to the seller. Suppose that a buyer who does not accept the seller's offer in the second period and proceeds into the third period incurs some small but positive disutility  $d > 0$ . Characterize the (Perfect Bayesian) equilibrium outcome, i.e.: Which contract will either type of buyer obtain?