To deter or to moderate? Alliance formation in contests with incomplete information*

Kai A. Konrad† Florian Morath‡

May 24, 2017

Abstract

We consider two players' choice about the formation of an alliance ahead of conflict in a framework with incomplete information about the strength of the potential ally. When deciding on alliance formation, players anticipate the self-selection of other players and the informational value of own and other players' choices. In the absence of these signaling effects, strong players have an incentive to stand alone, which leads to a separating equilibrium. This separating equilibrium can be destabilized by deception incentives if beliefs are updated on the basis of endogenous alliance formation choices. Weak players may find it attractive to appear strong in order to deter competitors from positive effort choices. Strong players may find it attractive to appear weak in order to give their competitors a false sense of security and then beat them with little effort. Moreover, appearing weak allows players to free-ride when alliances are formed.

Keywords: alliance; incomplete information; endogenous formation; all-pay contest

JEL Classification: D72, D74

*We gratefully acknowledge comments by the participants at the CBESS Conference on 'Contests: Theory and Evidence' May 23-24, 2016, and by two anonymous referees and the editors. The usual caveat applies.

†Corresponding author. Address: Kai A. Konrad, Max Planck Institute for Tax Law and Public Finance, Marstallplatz 1, 80539 Munich, Germany. E-mail: kai.konrad@tax.mpg.de.

‡Goethe University Frankfurt, Theodor-W.-Adorno-Platz 4, 60629 Frankfurt am Main, Germany, and Max Planck Institute for Tax Law and Public Finance, Marstallplatz 1, 80539 Munich, Germany. E-mail: morath@econ.uni-frankfurt.de.

1 Introduction

This paper draws attention to a so-far underresearched aspect of alliance formation: incomplete information about an ally's strength and the strategic implications for self-selection with respect to the decision whether to stand alone or join an alliance, and for the effort choices inside and outside the alliance.

Self-selection based on differences in players' strength was observed in a laboratory experiment by Benenson et al. (2009). They find that stronger players are more likely to abstain from alliance formation and to stand alone. Herbst et al. (2015), in a different experimental setup, also study the role of self-selection in endogenous alliances. Their study is motivated by the example of Wilhelm Tell. He is the protagonist in Friedrich Schiller's theater play and is a strong fighter who refuses to join an alliance. A fighter who is prepared to expend much fighting effort may be better off standing alone than joining an alliance: if he fights for himself, he fully internalizes the return on his effort. If he fights in an alliance, his effort also benefits other members of the alliance, and a strong player is likely to be exploited by weaker alliance members who accommodate their own efforts and free-ride on the efforts of the strong player.[1] Overall, their empirical findings show that Tell's reluctance was appropriate.[2]

It has been widely acknowledged that collective action problems constitute a major strategic problem of alliances. When alliance formation choices are made under incomplete information about others' strengths, why should players self-select into alliances when there is a considerable probability that their ally is not willing or not able to contribute much to the alliance effort? Our analysis shows that weaker players benefit not only when joining an alliance with a strong player. Alliances between weak players can lead to a moderating effect on the opponent's effort, which reduces rent dissipation and increases the players' expected payoffs.[3] As a result,

[1] This argument strongly draws on the theory of private provision of public goods as analyzed by Bergstrom et al. (1986), Nitzan (1991), Davis and Reilly (1999), and Esteban and Ray (2001).

[2] The experiment by Herbst et al. (2015) also offers a theoretical analysis of the Tullock (1980) lottery contest that corresponds to their experimental setup, assuming, however, that all decisions are made under perfect and complete information.

[3] This effect is reminiscent of different types of strategic commitment by which players restrain their future contest effort in order to moderate competitors' effort, as found by Kolmar and Wagener (2013) in the context of the Tullock (1980) lottery contest. They also discuss related contexts in which strategic moderation occurs: Stackelberg leadership (Baik and Shogren 1992) and strategic

weak players prefer alliance formation, regardless of whether their ally is strong or weak. But a moderating effect can also occur for alliances between strong players, given that this reduces the competition among them. We show that the latter effect can also make strong types opt for alliance formation, but only if it is sufficiently likely that their ally will be a strong type. Overall, our analysis suggests an importance of self-selection in the presence of unobservable co-players' characteristics, which results in weak players allying to fight a higher-ranked opponent, coalitions that are sometimes also called "all-up" or "revolutionary" coalitions (Mesterton-Gibbons et al. 2011, Bissonnette et al. 2015).

The formation of alliances has many aspects and has been studied in an extensive literature in economics (see Konrad 2014 for a survey), political science (see, e.g., Morrow 2000 and Fordham and Poast 2016 for surveys and views) and in biology (see Bissonnette et al. 2015 for a survey). The formation of a coalition or an alliance often changes many things, compared to stand-alone play: players may enjoy fighting synergies (Skaperdas 1998, Kovenock and Roberson 2012), may overcome budgetary limitations (Konrad and Kovenock 2009) or create a distributional conflict between them (Katz and Tokadlidu 1996, Konrad 2004, Esteban and Sákovics, 2003). Coalitions or alliances also have many informational aspects. Bearce et al. (2006) highlight that alliance members may learn about each other. The formation of alliances may also take place in a 'larger game' with dynamic and intertemporal aspects. Moreover, the choice as to whether to take part in a coalition may be a costly signal that is informative about other attitudes or personal characteristics (Gintis et al. 2001). Related to this, Aimone et al. (2013) study self-selection choices into groups that non-cooperatively contribute to a group-public good. They find that the choice between groups with high and low individual rates of return on own voluntary contributions may lead to endogenous sorting and cause a positive selection, because free-riders avoid groups with reduced individual rates of return. This also alludes to the issue of who may enter into a coalition with whom.

In many of these problems, clear a priori constraints exist about who may form an alliance with whom: a deer hunter may team up with another deer hunter, but not with the deer. This may make it legitimate to side-step the problem of which coalitions may be formed.[4]

delegation (Konrad et al. 2004).

[4]Coalition formation in a set-up with arbitrary coalitions between a larger number of players generates interesting, but difficult conceptual issues. Hart and Kurz (1983) were among the first

The literature on alliance formation reveals a large variety of purposes or functions, types of internal interaction, types of benefits or costs, and a high degree of complexity. It reveals that there is no simple all-encompassing model of coalitions. Therefore, it is not surprising that real-world examples of alliances include alliances between strong players, between weak players, or asymmetric alliances, depending on the particular environment under study. In line with an approach that isolates and studies single aspects we address here the role of incomplete information about possible alliance members' fighting strength. To isolate this effect, we keep fixed all other aspects that could and often do change when fighting in an alliance, as compared to stand-alone fighting.

To describe possible conflict, we use the theory of all-pay contests as developed by Hillman and Riley (1989), Baye et al. (1996) and Siegel (2009) as the main tool.[5] Our framework derives theory predictions about selection effects that emerge from the heterogeneity of players' strengths and incomplete information about these. The analysis separates pure selection effects and information transmission effects of alliance formation decisions. In a baseline regime in section 2 we isolate the pure selection effects: Players who differ in their motivation to expend effort make their choices about alliance formation, but can safely disregard the informational value of the offers they make in these negotiations. This is because the players' motivation becomes fully revealed to all players once the alliance formation stage is completed. Alliance formation choices may be informative in this regime, but the information has no additional value given that all information asymmetries are resolved at a later stage of the game. We find that strong self-selection incentives are at work even when players do not know the characteristics of their possible alliance partner, and show the existence of a separating equilibrium.

Section 3 considers the robustness of this result along several dimensions. First, we assume that information about strength is not exogenously revealed at the contest stage. Here, we find that a separating equilibrium may cease to exist if players rely

---

to highlight these conceptual issues, particularly the issue of coalition stability. See also Ray and Konishi (2003) and Ray and Vohra (1997, 2015) for more recent contributions. This question is, however, orthogonal to the issue addressed in our paper.

[5] The decisive nature of small differences in contestants' efforts in the all-pay contest has sometimes been criticized on plausibility grounds. However, the robustness of this structure toward contests with some, but small noise has been shown by Alcalde and Dahm (2010) who find that this much larger class of contests has equilibria that are payoff equivalent to those of the all-pay contest.

on the choices of whether to enter into an alliance to make inference about their co-players' strengths. Further departures from the baseline model consider a cooperative choice of effort inside the alliance or allow for public goods aspects of winning.

2 The Benchmark Analysis

2.1 The model framework

Consider a contest with three players A, B, and C. The three players compete for one given prize in an all-pay contest. Player C is a stand-alone player. Players A and B may stand alone but they also have the option to form an alliance at a stage which precedes the contest stage. More precisely, Stage 1 is the coalition formation stage in which players A and B simultaneously choose whether they want to enter into an alliance. The alliance formation choice of player $i \in \{A, B\}$ is denoted by $\lambda_i \in \{1, 0\}$, where $\lambda_i = 1$ means that i opts favorably for the formation of the alliance and $\lambda_i = 0$ means that i opts against alliance formation. Once the choices λ_A and λ_B are made, they become common knowledge and the alliance formation process is completed. The alliance is formed if $\lambda_A = \lambda_B = 1$ and it is not formed otherwise. That is, the alliance is formed if and only if both players A and B want to enter into an alliance. Otherwise, all players stand alone.

Now players enter into Stage 2, the contest stage. Players compete in an all-pay contest without noise. Each player $i \in \{A, B, C\}$ chooses a non-negative effort denoted by $x_i \geq 0$. Choices are made simultaneously and independently. We assume that none of the players are constrained by some finite budget or highest possible effort. The efforts, together with the outcome of the alliance formation stage, determine the players' payoffs.

The contest is for a prize that will be attributed to one of the players, but the players can have different valuations of the prize. Player C's prize valuation is $V_C = M$ and is common knowledge. The prize valuation V_i of player $i \in \{A, B\}$ is drawn from the set $\{H, L\}$ with probabilities

$$\Pr(V_i = H) = q \text{ and } \Pr(V_i = L) = 1 - q, \ \ i = A, B,$$

where $q \in (0, 1)$. This probability distribution is the same for V_A and V_B and is common knowledge. At the beginning of Stage 1, V_A and V_B are drawn independently from this distribution; player $i \in \{A, B\}$ privately learns his valuation V_i. At the beginning of Stage 2, and prior to the players' contest effort choices, the true valuations V_A and V_B also become common knowledge. We assume that

$$0 < L < M < H.$$

In Stage 2, if no alliance is formed then A, B, and C compete in a standard three-player all-pay contest, and their efforts decide who wins. The expected payoff of player $i \in \{A, B, C\}$ is given by

$$\pi_i(x_A, x_B, x_C) = \begin{cases} V_i - x_i & \text{if } x_i > x_j \text{ for all } j \neq i, \\ \frac{V_i}{k} - x_i & \text{if } i \text{ ties for the highest effort with } k-1 \text{ others}, \\ -x_i & \text{if } x_i < \max\{x_A, x_B, x_C\}. \end{cases}$$

Hence, the player with the highest effort wins the prize; all others lose. If several players choose the same highest effort, the winner is determined as an outcome of a symmetric lottery between the players expending the same, highest effort. Independent of winning or losing, each player i incurs a cost of his effort, which is normalized to the effort itself.

If an alliance is formed then the efforts of players A and B add to the alliance effort: $x_A + x_B$. This additivity is common in the literature on the voluntary provision of public goods more generally.[6] In the contest the alliance effort $x_A + x_B$ is compared with the effort x_C, which results in expected payoffs that are equal to

$$\begin{aligned} \pi_i(x_A, x_B, x_C) &= \begin{cases} \frac{V_i}{2} - x_i & \text{if } x_A + x_B > x_C, \\ \frac{V_i}{4} - x_i & \text{if } x_A + x_B = x_C, \\ -x_i & \text{if } x_A + x_B < x_C, \end{cases} \quad i = A, B, \\ \pi_C(x_A, x_B, x_C) &= \begin{cases} -x_i & \text{if } x_A + x_B > x_C, \\ \frac{V_C}{2} - x_i & \text{if } x_A + x_B = x_C, \\ V_C - x_i & \text{if } x_A + x_B < x_C. \end{cases} \end{aligned}$$

Player C wins if $x_C > x_A + x_B$. The alliance wins if $x_A + x_B > x_C$, in which case the prize is allocated to either A or B with equal probability. Should $x_A + x_B = x_C$, then victory is attributed to players A and B each with probability $1/4$ and to player

[6] For standard models see Bergstrom et al. (1986), Nitzan (1991), and Davis and Reilly (1999). Additionality of efforts also maps the basic setup of Olson and Zeckhauser (1966) and the more formal work on competing alliances (e.g., Esteban and Ray 2001, Esteban and Sákovics 2003).

C with probability 1/2. The specific tie-breaking probability choices are convenient but not crucial for the results.

We highlight two key assumptions in this setup: the additivity of efforts inside the alliance,[7] and an exogenous and symmetric random division of the alliance's benefits of winning.[8] The additivity of efforts and a constant overall rent of winning are chosen because it makes the alliance play technologically comparable with the stand-alone contest.

We now solve the game recursively. At Stage 2 the true valuations are common knowledge. The players participate in what can be considered a standard all-pay contest with complete information. If A and B have not formed an alliance, Stage 2 is a three-player all-pay contest with linear effort costs and with prize valuations (V_A, V_B, V_C). If A and B entered into an alliance and compete jointly against C, this contest structure is also well-known from the literature. We need to distinguish between these two possible continuation games.

2.2 The contest equilibrium

Before turning to the endogenous formation of an alliance we solve for the contest in the continuation game without an alliance and with an alliance.

Stand-alone play The continuation game with three stand-alone players has an equilibrium that is in mixed strategies of the two players with the highest valuations of the prize. The third player chooses zero effort.[9] The equilibria are known from the literature on the all-pay auction.[10] Using the results in Baye et al. (1996, Theorems

[7]Some of the literature assumes superadditivity (Skaperdas 1998). Another part of the literature assumes partial redundancy or the strong complementarity of intra-alliance efforts as in the best-shot or weakest link models of voluntary contributions to a group public good (Hirshleifer 1983).

[8]Alternatively, the overall benefit for the alliance may be larger than in the stand-alone contest, if winning has aspects of an intra-alliance public good, or the overall benefit may be smaller if the members of the winning alliance have to enter a fight and dissipate resources in an internal conflict.

[9]For $V_A = V_B = L < M = V_C$, there is a multiplicity in the equilibrium in mixed strategies, but all equilibria lead to the same expected payoffs.

[10]For an early treatment see Hillman and Riley (1989). The first full characterization and discussion of existence and uniqueness is in Baye et al. (1996). The concept has been extended along many dimensions, including the heterogeneity of both prize valuation and contribution cost functions (Siegel 2009), incumbency advantages (Konrad 2002, Meirowitz 2008), and various cost externalities (Baye et al. 2012).

		$E(\pi_A)$	$E(\pi_B)$	$E(\pi_C)$
$V_A = L$	$V_B = L$	0	0	$M - L$
$V_A = L$	$V_B = H$	0	$H - M$	0
$V_A = H$	$V_B = L$	$H - M$	0	0
$V_A = H$	$V_B = H$	0	0	0

Table 1: Expected payoffs of players A, B, and C in the three-player stand-alone all-pay auction, as a function of the valuations of A and B, assuming that the valuation of C is $V_C = M$.

1 and 2) we obtain the unique equilibrium expected payoffs as a function of the different prize valuations as summarized in Table 1.

The payoffs in the table have an intuitive interpretation. Only the two players with the highest valuations are active. All but the two strongest players prefer not to bid positive amounts and obtain payoffs of zero. Let, for instance, V_A and V_C with $V_A \geq V_C$ be the two highest valuations for winning. Then, in order to outbid the possibly maximal bid for player C, it is enough for player A to bid V_C, as player C would never bid more than his valuation. This gives a lower bound of $V_A - V_C$ for the expected payoff of player A, and it turns out that this is also this player's expected equilibrium payoff. Should both highest-valuation players have the same valuation, then cut-throat competition between them will drive down their expected payoffs to zero.

Alliance play With an alliance formed, its members individually contribute x_A and x_B to the total alliance effort $x_A + x_B$ and pay for their own efforts. Each alliance player i has a benefit of $V_i/2$ in expectation if the alliance wins.[11]

To calculate the equilibrium for the case $V_A = V_B$ we have to make an additional assumption, because several Nash equilibria exist at the contribution stage. Taking

[11]Depending on the context, alliances may jointly win more than $(V_A + V_B)/2$, particularly if winning has aspects of a public good for them, or they may win less than $(V_A + V_B)/2$, if the former members of the winning alliance have to expend efforts to solve the distributional conflict that emerges for them after winning. We focus on the intermediate case between these and discuss the pure group-public good case separately in section 3.3.

into account that the equilibrium must involve mixed strategies, denote by $F_i(x_i)$ the distribution of player i's effort, $i = A, B, C$. In one Nash equilibrium player B chooses zero effort $x_B \equiv 0$ such that F_B is degenerate and only A and C choose mixed strategies with positive effort choices in expectation, such that alliance effort equals the effort of player A. In another equilibrium the roles of A and B are reversed: $x_A \equiv 0$ and B chooses a non-degenerate bid strategy. The payoffs of A and B in these two equilibria are not the same, and we need to define a rule that determines which equilibrium is played. For this we assume that players A and B inside the alliance costlessly observe the outcome of a random variable θ which can take either value $\theta = a$ or $\theta = b$ with equal probability of $1/2$.[12] The outcome of θ determines which of the two Nash equilibria is chosen. More formally, A chooses a mixed strategy that is described by two cumulative distribution functions $F_{A,\theta=a}(x_A)$ and $F_{A,\theta=b}(x_A)$; one applies if $\theta = a$ and the other applies if $\theta = b$, and analogously for B. For equilibrium it must hold that ($F_{A,\theta}$, $F_{B,\theta}$, and F_C) are mutually optimal replies for each θ. We find the following continuation equilibrium payoffs:

Proposition 1 *Suppose that players A and B form an alliance and have valuations of the prize equal to V_A and V_B, respectively. Then, the symmetric equilibrium payoffs of the continuation game in Stage 2 are as summarized in Table 2.*

A proof of Proposition 1 is in the appendix. The payoffs in the continuation games are used for the main Proposition 2.

The proof characterizes the players' Nash equilibrium mixed strategies in this continuation game. Intuitively, if $V_A > V_B$, then the set of mutually optimal replies is unique and as follows: A and C behave as in an all-pay auction between two players with valuations $V_A/2$ and V_C, and player B's optimal reply to these equilibrium mixed strategies is $x_B \equiv 0$. Note that the strategy of A also determines $F_{AB}(x_A + x_B)$ in this equilibrium. If $V_A < V_B$ the equilibrium mirrors the one described, with A and B switching roles.

In case $V_A = V_B$ there is a multiplicity of equilibria.[13] Two of these are par-

[12] This correlation device is a variant of the equilibrium concept that has been used by Konrad and Leininger (2011) and Konrad and Kovenock (2009).

[13] The efforts x_A and x_B are contributions to a public good for the members of the alliance. With linear payoffs and identical marginal contribution costs and valuations, the logic of Nitzan (1991) applies, making any composition of non-negative contributions to a total equilibrium amount $(x_A + x_B)$ an equilibrium composition. This is the reason for a multiplicity of equilibria.

		$E(\pi_A)$	$E(\pi_B)$	$E(\pi_C)$
$V_A = L$	$V_B = L$	$\frac{1}{16}\frac{L^2}{M}$	$\frac{1}{16}\frac{L^2}{M}$	$M - \frac{L}{2}$
$V_A = L$	$V_B = H \leq 2M$	$\frac{1}{8}\frac{HL}{M}$	0	$M - \frac{H}{2}$
$V_A = L$	$V_B = H > 2M$	$\frac{H-M}{H}\frac{L}{2}$	$\frac{H}{2} - M$	0
$V_A = H \leq 2M$	$V_B = L$	0	$\frac{1}{8}\frac{HL}{M}$	$M - \frac{H}{2}$
$V_A = H > 2M$	$V_B = L$	$\frac{H}{2} - M$	$\frac{H-M}{H}\frac{L}{2}$	0
$V_A = H \leq 2M$	$V_B = H \leq 2M$	$\frac{1}{16}\frac{H^2}{M}$	$\frac{1}{16}\frac{H^2}{M}$	$M - \frac{H}{2}$
$V_A = H > 2M$	$V_B = H > 2M$	$\frac{1}{2}H - \frac{3}{4}M$	$\frac{1}{2}H - \frac{3}{4}M$	0

Table 2: Expected payoffs of players A, B, and C if A and B are in an alliance and C is a stand-alone player, as a function of the valuations of A and B, assuming that the valuation of C is $V_C = M$.

ticularly interesting:[14] Equilibrium strategies in one of these are given by $x_A \equiv 0$ such that $F_{AB}(x_A + x_B) = F_B(x_B)$ and by B and C choosing non-degenerate mixed strategies as in a two-player contest with valuations $V_B/2$ and V_C. As is confirmed in the appendix, these choices of A, B, and C are mutually optimal replies to each other – as is required for a Nash equilibrium. Also there is a mirror equilibrium in which A and B simply switch roles. As each of these two equilibria is equally plausible, we make it a random draw (an outcome of a random variable θ that takes values from the set$\{a, b\}$) which equilibrium is chosen by A and B and give each choice an equal probability of one half. The random variable θ allows A and B to coordinate on which equilibrium is played and causes symmetric expected payoffs for A and B in case they have symmetric valuations.[15] For more details see Appendix A.1.

In each contest equilibrium with alliance formation, the expected payoffs for the

[14] These two equilibria emerge as the limits of the cases $V_B > V_A$ for $V_B \searrow V_A$ and $V_A > V_B$ for $V_A \searrow V_B$. These two equilibria do not exhaust the set of equilibria, but it is important to note that other equilibria for which both players A and B choose non-degenerate strategies typically require more coordination between A and B.

[15] Note that $F_{AB}(x_A + x_B)$ needs to be uniform - except for a possible mass point at zero - to make player C indifferent for effort choices on the equilibrium support. The sum of two symmetric non-degenerate random distributions cannot be a uniform distribution. Hence, an equilibrium in symmetric and independent mixed strategies by A and B does not exist.

active players have the same intuition as for the stand-alone contest, but are based on a lower benefit of winning for the active alliance player due to prize sharing inside the alliance. For instance, if $V_A = H$ and $V_B = L$, player A is the active alliance player in equilibrium and bids at most $H/2$. If his competitor C's valuation $V_C = M$ is smaller than $H/2$, the active alliance player can ensure a payoff $H/2 - M$ by bidding $x_A = M$, which is also his payoff in the mixed strategy equilibrium. If $M > H/2$, the active player earns zero expected payoff. In both cases, the inactive alliance player B wins with some positive probability (due to the effort choices of the other member of the alliance); combined with zero effort this gives the player a positive expected payoff.

Of particular interest are the cases $V_A = V_B = L$ and $V_A = V_B = H$. Here, a random variable θ selects among the two equilibria in which one of the alliance players is active and the other is inactive and bids zero. Within each of these two equilibria, the same logic as before applies to the equilibrium payoffs so that the active player $i \in \{A, B\}$ earns a positive expected payoff if and only if $V_i/2 > M$. The inactive player, however, always benefits from the active player's effort choice and gets a positive expected payoff. Taking into account that each of the two alliance players is in the active role with probability $1/2$ yields the equilibrium payoffs in Table 2.

One of the surprising insights from the payoffs summarized in Table 2 is that alliance players do well compared to stand-alone play (Table 1). Even when $V_A = V_B = L < V_C$, and even though this generates a free-rider problem inside the alliance, the members of the alliance achieve a positive expected payoff. The intuition behind this result is that players benefit from the possibility to free-ride within the alliance. If $V_A = V_B = L$ then one of the players $i \in \{A, B\}$ becomes the active player, competes, and gets an expected payoff of zero in equilibrium. The other, passive player j, however, can free-ride and enjoy a windfall; the passive player j's expected payoff is $V_j/2$, multiplied by the probability that the active alliance player will win against player C. In expectation, each of the players A and B is in the active or inactive role with an equal probability of $1/2$. Note that the formation of an alliance comes with a reduction in the efforts of A and B, which is complemented by a reduction in the expected effort by the stand-alone player C. This strategic effect benefits players A and B, even for $V_A = V_B = L < V_C$.[16]

[16]The result is reminiscent of what has been found in the context of strategic delegation in the all-pay contest by Konrad et al. (2004). They showed that symmetric players may delegate their right to take part in an all-pay contest, such that one player chooses a delegate with a very high

As seen from a comparison of tables 1 and 2, players can also lose when forming an alliance, compared to the three-player stand-alone all-pay contest. Whether they gain or lose depends on their own valuation of winning relative to the valuation of the other alliance member, and on the valuation of the stronger alliance player compared to the valuation of the stand-alone player C. Weak players (with $V_i = L$) always gain in expectation from the formation of the alliance. Strong players (with $V_i = H$) lose when forming an alliance with a weak player, compared to stand-alone play. But strong players tend to gain when forming an alliance with another strong player. Therefore, under incomplete information about the co-player's type, this causes players' self-selection to depend on the anticipated self-selection of other players.

2.3 Alliance formation decisions

When A and B decide whether they are willing to enter into an alliance ($\lambda_i = 1$) or not ($\lambda_i = 0$), they anticipate the contest outcomes that emerge in the continuation games for all possible combinations of prize valuations. They also anticipate that an alliance is formed if and only if both players opt for it, that is, $(\lambda_A, \lambda_B) = (1, 1)$. When deciding on alliance formation, player $i \in \{A, B\}$ knows his own valuation V_i and the probability distribution of the valuation of $j \in \{A, B\}$, $j \neq i$, that is, i knows that $V_j = H$ with probability q and $V_j = L$ with probability $1 - q$. The players also know that the true prize valuations of all players will become common knowledge once players enter into the contest stage.[17] We state the equilibrium choices in the next proposition that is the first main result:

Proposition 2 *For the equilibrium continuation payoffs characterized in Tables 1 and 2, the following choices characterize perfect Bayesian equilibrium behavior in*

valuation, but the other player delegates to a delegate whose valuation is much lower than the player's actual valuation. Hiring a delegate causes credible bid-shading which, in turn, causes a reduction in the competitor's effort choice; the overall effect is also beneficial for the player who hires the delegate with a low valuation.

[17]The complete information assumption about the contest stage is important. It avoids all the complications emerging from how the alliance formation choices are interpreted by the different players, in particular, how the choices affect players' beliefs about other players' true valuations of the prize. The common knowledge assumption in Stage 2 makes the second stage of the game straightforward and separates self-selection effects from signaling aspects. Section 3.1 considers the implications for self-selection when departing from this assumption.

Stage 1:
(i) Equilibrium type 1: Players $i \in \{A, B\}$ choose $\lambda_i = 1$ if $V_i = L$ and $\lambda_i = 0$ if $V_i = H$.
(ii) Equilibrium type 2: Players $i \in \{A, B\}$ choose $\lambda_i = 1$. This equilibrium exists if $q \geq \tilde{q}(H, M)$ where

$$\tilde{q}(H,M) := \begin{cases} \frac{16\left(\frac{H}{M}-1\right)}{\left(\frac{H}{M}\right)^2+16\frac{H}{M}-16} & \text{if } H \leq 2M, \\ \frac{2\frac{H}{M}}{4\frac{H}{M}-3} & \text{if } H > 2M. \end{cases} \tag{1}$$

Proposition 2(i) shows the existence of a separating equilibrium in which the weak players opt for the formation of an alliance and the strong players opt out, even if the players do not know the type of their possible co-players inside the alliance.[18] This result offers a theoretical basis for the experimental findings on self-selection and corroborates the "Wilhelm-Tell effect" of strong types rejecting alliance formation offers in a Bayesian equilibrium with incomplete information about player-types. It identifies a self-selection tendency for alliances that holds more generally, according to which very strong players opt out if they cannot be sure of their potential ally's strength.

Bissonnette et al. (2015, p. 8) distinguish three configurations of within-group coalitions against a single target individual that may be formed by two partners and allude to field evidence. They call alliances between coalition members whose status outranks the status of the target "all-down." Coalitions between more low-ranked players are called "all-up" and coalitions are called "bridging" if one coalition member outranks the target and the other ranks lower than the target. Our analysis can offer a theory explanation for a possible prevalence of "all-up" coalitions when incomplete information about others' types is important.

In Proposition 2(ii), the other two types of coalitions may be observed as well. More precisely, there is a possible multiplicity of equilibrium in a subset of the parameter range.[19] While the equilibrium with self-selection always exists, an equilibrium

[18] Note that there is no equilibrium for which players $i \in \{A, B\}$ choose $\lambda_i = 0$ if $V_i = L$ and $\lambda_i = 1$ if $V_i = H$. These choices contradict the strict dominance of $\lambda_i = 1$ in case of $V_i = L$ if the probability of an H-type choosing $\lambda_i = 1$ is positive.

[19] In addition to the equilibria in Proposition 2, there is a trivial equilibrium in which $i \in \{A, B\}$ chooses $\lambda_i = 0$ independent of V_i, as none of the players can affect the alliance formation decision if the other player chooses $\lambda_j = 0$. This equilibrium is not robust to a number of refinements such as

with universal opting in can also occur, provided that the share of strong players is sufficiently high. Consider, for instance, the extreme case in which all players A and B are strong. Then, a player who chooses to opt in knows that he will be in an alliance with another strong player. This is preferable to being in a stand-alone contest against this strong player. Hence, an equilibrium with universal opt-in exists if there are sufficiently many strong types within the population.

The equilibria in Proposition 2 offer insights into the incentives of players as regards the formation of an alliance. One effect is a direct disincentive effect of alliance formation. Players who enter into an alliance have to "share" the winner prize (where sharing occurs here in a probabilistic sense). This makes winning less attractive and reduces their general motivation to expend effort. For players with a low valuation this disincentive effect is not a major cost since they already expect a zero payoff in the three-player stand-alone contest. In contrast, the disincentive effect is a true disadvantage for a player who is the only player with the highest valuation of the prize. His expected payoff is high in the three-player stand-alone contest (equal to the difference between his and the second-highest prize valuation), but the disincentive from prize-sharing makes this player a much weaker competitor in an alliance and tends to reduce his payoff.

There is also a less obvious, beneficial effect from alliance formation. The alliance players are substitutes in making effort contributions. As a consequence, they contribute less effort in expectation and win with lower probability. But anticipating the free-riding problem within the alliance, the stand-alone player C also expends less effort. This strategic effect on C's effort mitigates the decrease in the alliance's win probability. For weak players, the positive effect of lower effort cost dominates the lower win probability such that the overall effect of alliance formation is always positive. For strong players, the overall effect of alliance formation is positive if and only if the probability that their alliance partner is also a strong type is sufficiently high. In this case, alliance formation helps avoid the fierce competition with other strong players in stand-alone contests, and alliance members enjoy a windfall from a similar effort moderation effect that makes alliance formation the dominant choice for weak players. However, if they knew their potential ally's type, strong players would dislike the formation of an alliance with a weak player as the weak player

the possibility that players make small mistakes in their choices of λ_i, or that an alliance is formed even for $(\lambda_i, \lambda_j) = (1, 0)$ with a very small probability.

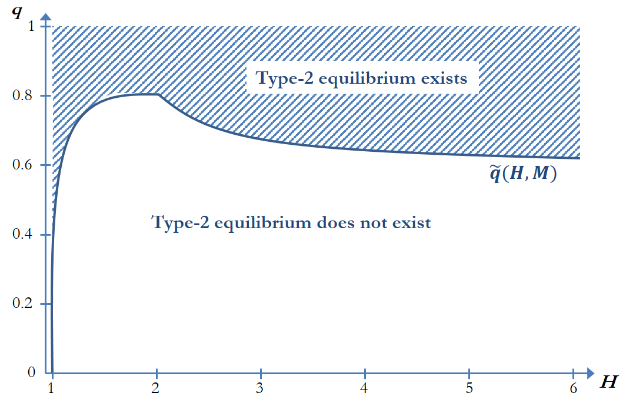

Figure 1: The area above the curve $\tilde{q}\,(H, M)$ represents the range of combinations (H, q) for which an equilibrium type 2 exists where both weak and strong players choose to form an alliance (setting $M = 1$).

free-rides on them.

Figure 1 illustrates the parameter range in which strong players, under incomplete information on the strength of their possible co-player in the alliance, can be willing to enter into an alliance. We consider variations of H and q and set $M = 1$ for this purpose.[20] The range in which the equilibrium with universal opting-in (Proposition 2(ii)) exists is the range above the curve $\tilde{q}\,(H, M)$. Hence, "all-down" or "bridging" coalitions can occur if the share of strong types is high and the strong types are considerably stronger than the other types so that avoiding the competition with another strong type becomes more profitable. In contrast, with complete information on the potential ally's type, "all-down" coalitions between strong types would occur independently of the underlying parameter values, and "bridging" coalitions between asymmetric players would never be an equilibrium outcome.

To gain further intuition for the parameter constellations under which strong types can be part of an alliance, suppose that the share of strong types is fixed and consider the comparative statics properties when changing H. If H is very close to M, the strong type's expected payoff is close to zero when standing alone, as his advantage over the other players is only very small. In this case, the strong type would be better off opting for an alliance formation: he expects a positive payoff

[20] This is a normalization and without loss of generality. The contest problem can, for any given M, be transformed into an equivalent problem with a set of valuations $((L/M), 1, (H/M))$.

if (and only if) his ally turns out to be strong as well. This is why the share of high types has to be above some threshold (defined in (1)) for the equilibrium in Proposition 2(ii) to exist.

When H increases, a strong type also realizes a higher positive payoff when fighting alone, so his decision on alliance formation depends on the trade-off between the loss in payoffs when the ally is weak and the gain when avoiding competition with a strong type; this is why q must again be above some threshold.[21] This threshold becomes lower when the strong type becomes very strong (H becomes large compared to M) so that strong types also realize a high payoff when in an alliance with a weak type and can, in an alliance, avoid the competition with other strong types. In the setup considered, the minimum share $\tilde{q}$ of high types required for the existence of the equilibrium in Proposition 2(ii) converges to $1/2$ when H/M approaches infinity.

3 Departures from the Baseline

This section discusses the robustness of the fully separating equilibrium in Proposition 2. We consider (i) a change in the assumptions about information at Stage 2, (ii) cooperative effort choices among alliance players, and (iii) higher joint payoff from winning in an alliance, due to public-good properties of the prize.

3.1 Moderation or deterrence?

Suppose the strength of players A and B is not always publicly observed at the beginning of the contest stage and consider the robustness of the separating equilibrium in Proposition 2. Equilibrium analysis of the all-pay contest becomes difficult if more than two heterogeneous players are partially informed about each other, and to our knowledge no general solutions for this case exist.[22]

More specifically, we assume that player C never observes the type of players A and B, but players A and B can observe each other's type if and only if $\lambda_A = \lambda_B =$

[21] The non-monotonicity of the threshold illustrated in Figure 1 is analytically caused by the distinction of whether a strong player, weakened by the prize-sharing in an alliance, still remains "stronger" than the stand-alone player, that is, whether $H/2$ is smaller or larger than M).

[22] For results in a framework with two players see Amann and Leininger (1996), Konrad (2004), and Morath and Münster (2013).

1.[23] Suppose all players expect that all other players j will choose $\lambda_j = 1$ if $V_j = L$ and $\lambda_j = 0$ if $V_j = H$. Is it then optimal for them to enter into an alliance if $V_i = L$ and opt out if $V_i = H$? The question is whether the insight (i) in Proposition 2 is robust in a framework in which the choices (λ_A, λ_B) influence players' beliefs.[24]

Proposition 3 *Suppose that players who are jointly in an alliance can observe each other's valuation. Then there is a perfect Bayesian separating equilibrium in pure strategies in Stage 1 with $\lambda_i = 0$ if $V_i = H$ and $\lambda_i = 1$ if $V_i = L$ if and only if*

$$H \leq \frac{16M^2}{16M - L} \tag{2}$$

and

$$(1-q)\left(\frac{H+L}{2} - M\right) \geq q\,(H - M)\,. \tag{3}$$

The proof of this proposition is in the Appendix. We find that the parameter range in which the separating equilibrium exists becomes small. The possible separating outcome suffers from countervailing incentives. Unlike many incentive problems where only one type aims at mimicking the other type, players with valuations L and H both have an incentive to deceive the other players, which makes the problem non-standard. A weak player may want to appear strong for reasons of deterrence (i.e., to make the adversary choose zero effort), and a strong player may want to appear weak for reasons of moderation (i.e., to induce the adversary to make lower bids if the adversary bids positive amounts). Even though the game structure and the information assumptions are quite different, these considerations are reminiscent

[23] Intra-alliance information exchange is a seemingly natural assumption. Players who form an alliance may mutually learn about their types. In political sciences, Bearce et al. (2006) argue that alliances may serve the purpose of internal information exchange. Similarly, it seems natural to assume that the formation of alliances in other areas, such as R&D joint ventures, also increases the information flow between the players who enter into an alliance. Konrad (2012) analyzes the role of information exchange in contests between players with tight budget constraints.

[24] The "trivial" equilibrium with $\lambda_A = \lambda_B = 0$ continues to exist (since players cannot individually induce the formation of an alliance) but remains unstable. The choice of λ_i can, however, influence beliefs about players' types; thus, the existence of the "trivial" equilibrium requires appropriate off-equilibrium beliefs. On the other hand, a perfect Bayesian equilibrium in which $\lambda_i = 0$ if $V_i = L$ and $\lambda_i = 1$ if $V_i = H$ still does not exist in this framework. While the choices of (λ_A, λ_B) would be fully revealing in this candidate equilibrium, weak types would be strictly better off when deviating to alliance formation.

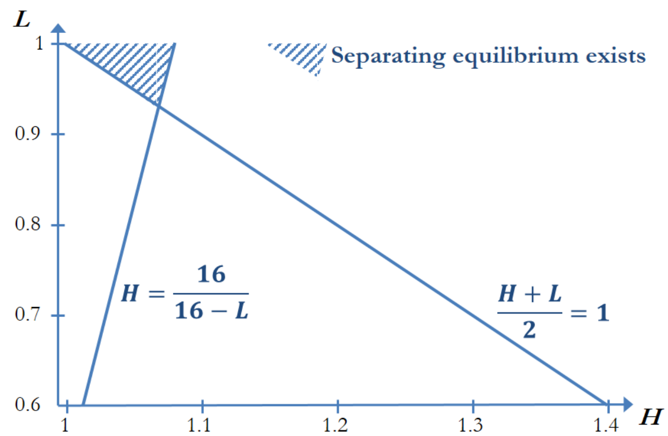

Figure 2: The shaded area represents a range of combinations (H, L) for which the separating equilibrium exists if q is sufficiently small, setting $M = 1$.

of Fudenberg and Tirole (1984) whose seminal paper studies strategic investment in an entry game. There, players prefer to deter others from entry, but would like to soften competition once entry has occurred.

In our framework, a weak player A (with $V_A = L$) gains by being perceived as a strong player: In the subsequent three-player stand-alone contest, this makes C "exit" (that is, abstain from expending positive effort) with a positive probability; moreover, B also abstains from expending positive effort if $V_B = L$. This deception incentive is larger the larger the ratio between H and M, as this ratio determines C's equilibrium exit probability. Formally, condition (2) is the no-deviation condition for weak types and states that valuation H of strong types cannot be too large; otherwise, appearing strong becomes too attractive for weak types.

A strong player A (with $V_A = H$) gains when being perceived as a weak player: He benefits from the bid moderation of C that is induced if an alliance with B is formed, and A deceives a strong player B in the case of stand-alone play (which happens with probability q). In the latter case, B believes that A is weak and bids only a small amount, thinking that this will be enough to win. This, in turn, gives A the opportunity to win against B and C with comparatively little effort and yields a strong advantage from appearing weak. This latter effect must not be too likely for the separating equilibrium to be stable, as is captured by condition (3) as the no-deviation condition for strong types.

The deception incentives both for weak and for strong players are quite strong and the conditions in Proposition 3 narrow down the parameter set that describes the existence of the separating equilibrium. However, the parameter set is non-empty. To see this, let q become very small, in which case condition (3) becomes $(H+L)/2 - M \geq 0$. Figure 2 illustrates the separating hyperplanes that determine the parameter range for which the separating equilibrium exists, normalizing M to $M = 1$. The two relevant non-deviation conditions (2) and (3) restrict existence of the separating equilibrium to the area above the two straight lines.

The upper area in which the separating equilibrium exists is a range in which both H and L are not too different from M. Intuitively, if the players are rather similar, both deterrence and moderation incentives are less profitable as the competition remains fierce in both cases. But if the players' valuations are very asymmetric, appearing weak and strong, respectively, crucially affects the other players' effort choices so that deception becomes attractive and the fully separating equilibrium ceases to exist.

3.2 Collusive alliances

In the main model of section 2, an alliance changes the rules about how the prize is awarded, but the interaction remains fully non-cooperative and each player remains an independent strategic decision-maker. This assumption about non-cooperative effort contributions is grounded in much of the work on alliances that was inspired by Olson and Zeckhauser (1966). However, alliances may sometimes, and to some extent, succeed in installing a governance structure to overcome free-riding, such that the alliance can act as one strategic player. In this case the alliance chooses aggregate alliance effort so as to maximize the collective benefit of the alliance. In addition, such a governance structure must determine the rules by which the prize is allocated inside the alliance, how the aggregate effort is assigned to the alliance members, whether or not alliance members can make side payments and, in our context, what the information structure applies inside the alliance and between the alliance and the stand-alone player in case an alliance is formed. This shows that departures from the classical free-riding approach may lead to one of a whole set of different structures.

To see that the results need not change dramatically for such a departure, we study collusive alliances for one set of assumptions. More precisely, we assume as in section 2 that signaling aspects are absent: The valuations of players A and B (which are private information in stage 1) become common knowledge at the contest stage. Moreover, just as in the previous sections, we assume that in case the alliance wins, players A and B each receive the prize with probability $1/2$, just as in the previous sections. Finally, we assume that the expected payoffs of both A and B are non-negative and that symmetric players (with $V_A = V_B$) contribute symmetrically to alliance effort.

Remark 1 *Suppose that if an alliance is formed, players A and B choose their efforts so as to maximize the joint expected payoffs. Then, the following choices characterize perfect Bayesian equilibrium behavior in Stage 1:*
(i) Equilibrium type 1: Players $i \in \{A, B\}$ choose $\lambda_i = 1$ if $V_i = L$ and $\lambda_i = 0$ if $V_i = H$;
(ii)Equilibrium type 2: Players $i \in \{A, B\}$ choose $\lambda_i = 1$ if and only if q is sufficiently high.

Details on Remark 1 are in the appendix. Note that the joint payoff maximization of alliance players does not necessarily increase the total expected payoff of players A and B when forming an alliance. On the contrary, if alliance players are rather weak then the disincentive effect and the free-riding possibilities within the alliance make them achieve a strictly higher payoff than if alliance players "cooperated" and maximized joint payoffs. As in a number of other contexts (e.g., Salant et al. 1983), the transformation of two independent strategic players into one strategic player is not necessarily to the advantage of these players.

The qualitative predictions of Proposition 2 are robust with respect to collusion inside the alliance. There is an equilibrium where weak types select into alliances and strong types select into stand-alone fighting. Moreover, if the share of high types is sufficiently large then there is an additional equilibrium in which players A and B opt for alliance formation independently of their respective type.[25] This suggests that the intuition about particularly strong players being better-off standing alone is a more robust result that emerges for several sets of assumptions that can be made in the context of alliance formation.

[25] The exact threshold for q depends on the cost-sharing arrangement within a collusive alliance.

		$E(\pi_A)$	$E(\pi_B)$	$E(\pi_C)$
$V_A = L$	$V_B = L$	$\frac{1}{4}\frac{L^2}{M}$	$\frac{1}{4}\frac{L^2}{M}$	$M - L$
$V_A = L$	$V_B = H$	$\frac{2H-M}{2H}L$	$H - M$	0
$V_A = H$	$V_B = L$	$H - M$	$\frac{2H-M}{2H}L$	0
$V_A = H$	$V_B = H$	$H - \frac{3}{4}M$	$H - \frac{3}{4}M$	0

Table 3: Expected payoffs of players A, B, and C if A and B are in an alliance and C is a stand-alone player, for the case where the prize is a public good for the members of the alliance.

3.3 If the intra-alliance prize is a public good

The baseline model assumed that a winner i in the stand-alone contest wins the full prize and values it by V_i, and that the prize is divided between the alliance members if they win it as an alliance. Many other assumptions could be reasonably made. For instance, the prize allocation in the stand-alone contest could be as in the baseline, but the prize could be a public good for the members of the alliance: both players win the prize if the alliance wins and value it according to their valuations V_A and V_B. In this case, the formation of an alliance generates additional surplus for the alliance players and, everything else given, this makes the alliance more attractive. Compared to the alliance equilibrium in section 2 the expected prize of winning for members of the alliance are V_A and V_B, rather than $V_A/2$ and $V_B/2$. For these prizes, equilibrium payoffs in stage 2 can be calculated straightforwardly along the logic of proposition 1 and are given in Table 3.

Comparing the payoffs in Table 3 with the ones in case of stand-alone play in Table 1 shows that the alliance payoffs weakly dominate the ones without an alliance. So players A and B both find it attractive to choose opting for alliance formation - weak types as well as strong types. This comparison and outcome is, however, less interesting than Proposition 2, because the setup doubles the possible rents from alliance play by assumption.

If there is no rivalry between A and B, both players A and B may also obtain the prizes V_A and V_B if one of them wins, even if they choose efforts independently as stand-alone players. To illustrate, this may be true, for instance, if A and B lobby for the same policy change whereas C fights against this policy change. The situation without an alliance is then very similar to the situation with an alliance, even though their individual efforts x_A and x_B rather than the sum $x_A + x_B$ of these efforts are compared with x_C. Both with alliance formation and with stand-alone play, there is an equilibrium in which only the player among A and B with the higher valuation is active and bids against C. Here, equilibrium payoffs are the same with and without alliance formation so that the question of whether to form an alliance is irrelevant in this case. Formally, the analysis without an alliance is then analogous to a contest with strong identity-dependent externalities.[26]

4 Conclusions

A strong intuition suggests that relative strength matters for players who are considering whether to form an alliance. Weak players may expect to gain from joining an alliance with stronger players. Strong players may find it more attractive to stand alone: Within an alliance, the stronger player may have to contribute the lion's share in effort, and the winner prize may have to be shared with the weaker alliance member. We show that this intuition is correct in the context of all-pay auctions with incomplete information about the players' strengths when forming alliances.

Holding other aspects of alliances such as technological advantages or capacity extensions constant, our analysis suggests that alliances are generally attractive, despite the collective action problem within alliances. For example, when two weak players form an alliance, this does not turn them into a stronger player. Rather, it weakens them further in terms of the total effort they can mobilize.

Where the free-rider problem may seem to be an additional disadvantage, it turns into an advantage in the all-pay contest: it not only reduces the contest effort of the alliance members but also moderates the effort expended by their joint competitor. Overall, the alliance wins less often but has a higher expected payoff than if both weak players stand alone. Similarly, the formation of an alliance weakens two strong players and can make them even weaker than the outside stand-alone player. But alliance formation can still be beneficial, as it prevents the strong players from heavily competing against each other, in which case they would fully dissipate their rents. With respect to the composition of potential alliances, the analysis predicts that due

[26] Such identity-dependent externalities have been discussed for all-pay auctions by Konrad (2006) and by Klose and Kovenock (2015).

to the self-selection of weak players into alliances, "all-up" coalitions of lower-ranked players against a high-value player should be prevalent in environments in which the unobserved heterogeneity of the players is an important feature. Other types of coalitions such as "all-down" coalitions between strong players or asymmetric coalitions between players of different strengths are predicted to be observed only if the population consists of many strong players. Here, a crucial assumption is that strong players can share the resources among themselves. If the resource is indivisible and internal conflict inside the alliance is likely, this would further reduce their incentives to enter into an alliance. For this reason, if information about the allies' types is not revealed prior to the choice of contest efforts, deception incentives both for weak and for strong players make the strategic considerations behind alliance formation considerably more difficult.

A Appendix

A.1 Proof of Proposition 1

Table 2 describes the equilibrium payoffs in the continuation game if a coalition has been formed. To derive these payoffs we have to distinguish two cases.

Case 1: $2V_C > \max\{V_A, V_B\}$. Here, the equilibrium mixed strategies $F_{AB}(x_A + x_B)$ and $F_C(x_C)$ for the alliance AB and for player C, respectively, are given by

$$F_{AB}(x) = \begin{cases} 1 - \frac{\max\{V_A,V_B\}/2}{V_C} + \frac{x}{V_C} & \text{for } x = x_A + x_B \in [0, \max\{V_A, V_B\}/2], \\ 1 & \text{for } x = x_A + x_B > \max\{V_A, V_B\}/2, \end{cases} \tag{4}$$

and

$$F_C(x) = \begin{cases} \frac{x}{\max\{V_A,V_B\}/2} & \text{for } x \in (0, \max\{V_A, V_B\}/2], \\ 1 & \text{for } x > \max\{V_A, V_B\}/2. \end{cases} \tag{5}$$

To confirm that F_{AB} and F_C are mutually optimal replies, first consider player C. With F_{AB} as in (4), for all $x_C \in (0, \max\{V_A, V_B\}/2]$ we get

$$E\left(\pi_C\left(x_C\right)\right) = F_{AB}(x_C)V_C - x_C = V_C - \frac{\max\{V_A, V_B\}}{2} + x_C - x_C,$$

which is independent of x_C. Efforts $x_C > \max\{V_A, V_B\}/2$ lead to a strictly lower payoff; also, $x_C = 0$ yields a lower payoff. Therefore, for player C, any mixed strategy F_C on $(0, \max\{V_A, V_B\}/2]$ is an optimal reply to F_{AB}.

Now, consider player i as one of the alliance players $i \in \{A, B\}$ and denote by $j \neq i$ the other alliance player. Let F_C and joint effort $x_A + x_B$ be given. For $x_i \in (0, \max\{V_A, V_B\}/2 - x_j)$, an increase in own effort x_i yields a marginal change in the own expected payoff equal to

$$\begin{aligned}\frac{\partial E(\pi_i)}{\partial x_i} &= F_C'(x_i + x_j)\frac{V_i}{2} - x_i = \frac{1}{\max\{V_A, V_B\}/2}\frac{V_i}{2} - 1 \\ &= \begin{cases} 0 & \text{if } V_i = \max\{V_A, V_B\}, \\ -(1 - \frac{V_i}{\max\{V_A, V_B\}}) < 0 & \text{if } V_i < \max\{V_A, V_B\}. \end{cases}\end{aligned}$$

Moreover, for $x_i > \max\{V_A, V_B\}/2 - x_j$, we get $\partial E(\pi_i)/\partial x_i = -1$.[27] Accordingly, a player i with $V_i < \max\{V_A, V_B\}$ yields his maximum expected payoff for $x_i = 0$, and a player i with $V_i = \max\{V_A, V_B\}$ is indifferent to an increase in $x_A + x_B$ inside the equilibrium support of F_{AB}. Therefore, suppose first that $V_i = \max\{V_A, V_B\} > V_j$. Given $F_C(x_C)$, $x_j = 0$ is optimal for player j; for player i, randomization according to $F_{AB}(x)$ is an optimal reply to $F_C(x)$ and $x_j = 0$.

If instead $V_i = \max\{V_A, V_B\} = V_j$ then the coordination device becomes relevant: if i is in the role of contributor, randomization according to $F_{AB}(x)$ is an optimal reply to the combination ($F_C(x)$ and $x_j = 0$). For the non-contributor j, $x_j = 0$ is a best reply to the combination of ($F_C(x)$ by C and an effort of i chosen according to F_{AB}). The correlation device makes sure that i and j can perfectly negatively correlate their decisions to take the contributor role or the non-contributor role.[28] Note also that not contributing is optimal for A if B contributes according to (4) and vice-versa. (It is true that each player prefers the non-contributor role in this case, but only if the other player takes the contributor role, and the contributor choice is individually optimal if the other alliance member chooses the non-contributor role.) For $V_A = V_B$, the correlation device makes sure that each player A and B has a probability of one half of being in the non-contributor role.

Hence, overall and for the different valuation cases the players' payoffs can be stated as

$$E(\pi_i) = 0 \tag{6}$$

[27]Note, at this point, that x_i and x_j are strategically interdependent with regard to whether a given x_i leads to an $x_i + x_j$ inside the equilibrium support.

[28]Such a correlation device has been discussed and used in the context of all-pay contests by Konrad and Leininger (2011) and by Konrad and Kovenock (2009).

for the contributor $i \in \{A, B\}$,

$$E(\pi_j) = \frac{1}{2} \frac{\max\{V_A, V_B\}/2}{V_C} \frac{V_j}{2} \tag{7}$$

for the non-contributor $j \in \{A, B\}$, and

$$E(\pi_C) = V_C - \frac{\max\{V_A, V_B\}}{2}$$

for the stand-alone player C.

Case 2: $2V_C \leq \max\{V_A, V_B\}$. The mixed strategies $F_{AB}(x_A + x_B)$ and $F_C(x_C)$ that are mutually optimal replies are

$$F_{AB}(x) = \begin{cases} \frac{x}{V_C} & \text{for } x = x_A + x_B \in [0, V_C] \\ 1 & \text{for } x = x_A + x_B > V_C \end{cases} \tag{8}$$

and

$$F_C(x) = \begin{cases} 1 - \frac{V_C}{\max\{V_A, V_B\}/2} + \frac{x}{\max\{V_A, V_B\}/2} & \text{for } x \in (0, V_C] \\ 1 & \text{for } x > V_C \end{cases} . \tag{9}$$

To confirm this mutual optimality, consider first player C. Given F_{AB} as in (8),

$$E(\pi_C(x_C)) = F_{AB}(x_C)V_C - x_C = 0$$

for $x_C \in [0, V_C]$, and $E(\pi_C) < 0$ for $x_C > V_C$. Hence, C is indifferent to any $x_C \in [0, V_C]$ and strictly prefers any of these effort levels to any higher effort.

Consider player $i \in \{A, B\}$ and suppose first that $V_i = \max\{V_A, V_B\}$. Assume that $x_j \equiv 0$ (to be justified later). Given $F_C(x_C)$ as in (9) and $x_j = 0$, i's expected payoff for $x_i \in (0, V_C]$ is

$$E(\pi_i(x_i)) = F_C(x_i)\frac{V_i}{2} - x_i = \frac{V_i}{2} - V_C$$

and is smaller elsewhere. Accordingly, i is indifferent between all x_i in the equilibrium support and may choose x_i to generate $F_{AB}(x)$ given $x_j = 0$. To confirm that $x_j = 0$ is optimal to F_C and the choice of i, note that j's expected payoff for $x_j \in [0, V_C]$ is equal to

$$\begin{aligned} E(\pi_j(x_j)) &= \frac{V_j}{2} \int_{x_i=0}^{x_i=V_C} F_C(x_i + x_j) dF_{AB}(x_i) - x_j \\ &= \frac{V_j}{2} \left[\int_0^{V_C - x_j} (1 - \frac{V_C}{V_i/2} + \frac{x_i + x_j}{V_i/2}) \frac{1}{V_C} dx_i + (1 - \frac{V_C - x_j}{V_C}) \right] - x_j. \end{aligned}$$

Thus,

$$\begin{aligned}\frac{\partial E(\pi_j)}{\partial x_j} &= \frac{V_j}{2}\left[-(1-\frac{V_C}{V_i/2}+\frac{V_C-x_j+x_j}{V_i/2})\frac{1}{V_C}+\int_0^{V_C-x_j}\frac{1}{V_i/2}\frac{1}{V_C}dx_i+\frac{1}{V_C}\right]-1 \\ &= \frac{V_j}{2}\frac{1}{V_i/2}\frac{V_C-x_j}{V_C}-1,\end{aligned}$$

which is strictly negative for $V_j \leq V_i$. (Moreover, $\partial E(\pi_j)/\partial x_j = -1$ for $x_j > V_C$.) Thus, $x_j = 0$ is j's best reply to F_C and to a choice of i according to F_{AB}. As in case 1, if $V_i > V_j$, this yields a unique equilibrium in which only i is active and randomizes according to F_{AB}.

If $V_i = V_j$, the correlation device assigns the roles of contributor and non-contributor with equal probabilities to i and j; the contributor randomizes according to F_{AB} and the non-contributor chooses an effort of zero (which is his optimal reply).

Expected payoffs in the candidate equilibrium are

$$E(\pi_i) = \frac{V_i}{2} - V_C \tag{10}$$

for the contributor $i \in \{A, B\}$,

$$E(\pi_j) = \frac{\max\{V_A, V_B\} - V_C}{\max\{V_A, V_B\}}\frac{V_j}{2} \tag{11}$$

for the non-contributor $j \in \{A, B\}$, and

$$E(\pi_C) = 0$$

for the stand-alone player C.

Using the results in cases 1 and 2, the expected payoffs as a function of the players' valuations follow directly. Recall that $V_C = M$. Thus, if $V_A = V_B < 2M$ then (6) and (7) imply that

$$E(\pi_A) = E(\pi_B) = \frac{1}{2}0 + \frac{1}{2}\frac{1}{2}\frac{V_A/2}{V_C}\frac{V_A}{2},$$

which yields $E(\pi_A) = E(\pi_B) = L^2/(16M)$ if $V_A = V_B = L$ and $E(\pi_A) = E(\pi_B) = H^2/(16M)$ if $V_A = V_B = H$. If instead $V_A = V_B = H > 2M$ then (10) and (11) yield

$$E(\pi_A) = E(\pi_B) = \frac{1}{2}\left(\frac{V_A}{2} - V_C\right) + \frac{1}{2}\frac{V_A - V_C}{V_A}\frac{V_A}{2} = \frac{H}{2} - \frac{3M}{4}.$$

If $V_A \neq V_B$ and, for instance, $V_A > V_B$, (6) and (7) yield

$$E(\pi_A) = 0 \text{ and } E(\pi_B) = \frac{1}{2}\frac{V_A/2}{V_C}\frac{V_B}{2} = \frac{1}{8}\frac{HL}{M}$$

in case of $H < 2M$, and (10) and (11) imply that

$$E(\pi_A) = \frac{H}{2} - M \text{ and } E(\pi_B) = \frac{H-M}{H}\frac{L}{2}$$

in case of $H > 2M$. The expected payoff of the stand-alone player is equal to zero if $\max\{V_A, V_B\} > 2M$ and equal to $V_C - \max\{V_A, V_B\}/2$ otherwise.

A.2 Proof of Proposition 2

Part (i): Consider first player i with $V_i = L$. In the candidate equilibrium, i correctly anticipates that $\lambda_j = 1$ if $V_j = L$ and $\lambda_j = 0$ if $V_j = H$. If $\lambda_i = 1$, an alliance is formed with probability $1 - q$ (that is, if $V_j = L$) and a stand-alone contest emerges with probability q. Since the players' true valuation types are common knowledge at the contest stage, player's deviations in stage 1 only affect the choice of the continuation game but do not influence the contest payoffs in the respective continuation games. Thus, using tables 1 and 2, i's expected payoff in the candidate equilibrium is strictly positive, but i gets zero when deviating to $\lambda_i = 0$ in which case a stand-alone contest emerges with probability one.

Consider next player i with $V_i = H$. Since $\lambda_i = 0$ triggers a stand-alone contest, i's expected payoff in the candidate equilibrium is zero if $V_j = H$ and is equal to $H - M$ if $V_j = L$ (compare Table 1). If i deviates to $\lambda_i = 1$, he correctly anticipates that this deviation will become relevant if and only if $V_j = L$. Thus, he still gets zero if $V_j = H$ (in the subsequent stand-alone contest) and gets $\max\{H/2 - M, 0\}$ if $V_j = L$, in which case an alliance is formed. Comparing the expected payoffs under $\lambda_i = 0$ and $\lambda_i = 1$ shows that i strictly prefers $\lambda_i = 0$ if $V_i = H$. Altogether, this yields the "type-1" equilibrium which is supported by equilibrium beliefs that $\lambda_j = 1$ if $V_j = L$ and $\lambda_j = 0$ if $V_j = H$.

Part (ii): Since players i with $V_i = L$ get zero payoff in the stand-alone contest but a strictly positive expected payoff when an alliance is formed, weak players do not want to deviate from the candidate equilibrium. Hence, consider a strong player with $V_i = H$ and consider first the case of $H \leq 2M$. In the candidate equilibrium,

i correctly anticipates that an alliance will be formed with probability one, which yields an expected payoff equal to

$$q\frac{1}{16}\frac{H^2}{M} + (1-q)\,0$$

since $V_j = H$ with probability q and $V_j = L$ with probability $1-q$. If i deviates to $\lambda_i = 0$, a stand-alone contest emerges (independent of λ_j) in which i's expected payoff is equal to

$$q0 + (1-q)\,(H-M)$$

(compare Table 1).[29] Thus, i does not want to deviate from the candidate equilibrium if and only if

$$q\frac{1}{16}\frac{H^2}{M} \geq (1-q)\,(H-M)\,,$$

which is equivalent to $q \geq \tilde{q}$ in (1) for $H \leq 2M$.

Now consider the choice of player i with $V_i = H$ in case of $H > 2M$. In the candidate equilibrium, i's expected payoff is

$$q(\frac{1}{2}H - \frac{3}{4}M) + (1-q)\,(\frac{H}{2} - M).$$

If i deviates to $\lambda_i = 0$, then a stand-alone contest emerges in which i's expected deviation payoff is again

$$q0 + (1-q)\,(H-M)\,.$$

Thus, i does not want to deviate from $\lambda_i = 1$ if and only if

$$q\left(\frac{1}{2}H - \frac{3}{4}M\right) \geq (1-q)\,\frac{H}{2},$$

which is equivalent to $q \geq \tilde{q}$ in (1) for $H > 2M$. The "type-2" equilibrium is supported by beliefs that $\lambda_j = 1$ both for $V_j = L$ and for $V_j = H$.

A.3 Proof of Proposition 3

If all players believe that the other player's choice λ_j is truthful and fully revealing j's valuation type, we need to show that deviations from the candidate equilibrium

[29] Note that j's off-equilibrium beliefs following a deviation of i are irrelevant for whether a deviation is profitable since all players' valuations become common knowledge at the contest stage.

are not profitable for the players given these beliefs. First we state as a benchmark the equilibrium expected payoffs of A and B conditional on their valuation. Then we determine deviation payoffs and compare the benchmark payoffs with the deviation payoffs.

Consider the benchmark payoffs in the candidate equilibrium. If all players' choices are truthful, all players enter Stage 2 as in an all-pay contest with complete information. Suppose first that $V_i = H$, $i \in \{A, B\}$. Then, $\lambda_i = 0$ and Stage 2 is an all-pay contest among three stand-alone players. Here, i's expected payoff is zero if $\lambda_j = 0$ (and $V_j = H$) for $j \in \{A, B\}$, $j \neq i$. Expected payoff is equal to $H - M$ if $\lambda_j = 1$ (and $V_j = L$). Thus, player i's expected payoff at Stage 1 when λ_j has not yet been chosen is

$$E(\pi_i \mid V_i = H) = (1-q)(H-M). \tag{12}$$

Along similar lines, if $V_i = L$ and $\lambda_i = 1$, then i's expected payoff in stage 1 when λ_j has not been chosen is

$$E(\pi_i \mid V_i = L) = q \cdot 0 + (1-q)\frac{1}{16}\frac{L^2}{M}. \tag{13}$$

Suppose that A and B can observe each other's valuation if they are in an alliance. (a) Let $V_i = L$ but $\lambda_i = 0$. Then a three-player stand-alone contest emerges in which the deviating player i is mistakenly taken by j and C as a player with $V_i = H$. The deviating player i's effort choice x_i in this contest depends on the type of player j which is truthfully revealed by j's choice of λ_j.

(aa) If $\lambda_j = 0$ then i expects $F_j(x_j) = x_j/H$ in the equilibrium support $x_j \in [0, H]$, and $x_C \equiv 0$. Player i's optimal reply to these effort choices is $x_i \equiv 0$, yielding a deviation payoff of zero.

(ab) If $\lambda_j = 1$ then i expects $x_j \equiv 0$ and $F_C(x) = (1-M/H+x/H)$ for $x \in [0, M]$. For $x_i = 0$, i's expected payoff depends on the tie-breaking rule since C chooses zero effort with strictly positive probability. For $x_i > 0$, we get

$$\frac{d\pi_i}{dx_i} \leq \frac{1}{H}L - 1 < 0.$$

Thus, i would like to choose a minimal amount of effort to win at least with the probability that $x_C = 0$. To avoid the problem of the non-existence of a best reply function for i in a continuous strategy space, we assume that ties are broken in favor of player i in this case, which yields an optimal deviation effort of i equal to $x_i = 0$ and an expected deviation payoff equal to $(1 - M/H)\,L$.[30]

At the point when λ_j has not been chosen, i's expected deviation payoff is thus

$$q \cdot 0 + (1-q)\left(1 - \frac{M}{H}\right)L. \tag{14}$$

Comparing (13) and (14) shows that a player i with $V_i = L$ does not want to deviate from $\lambda_i = 1$ if and only if

$$\frac{L}{16M} \geq \frac{H-M}{H}, \tag{15}$$

which is equivalent to (2).

(b) Let $V_i = H$ but $\lambda_i = 1$. Then i is mistakenly perceived by C as a player with $V_i = L$. The belief of j about i depends on whether an alliance is formed, which, again, depends on λ_j.

(ba) If $\lambda_j = 0$ (and hence $V_j = H$) then a three-player stand-alone contest emerges. In this contest i correctly expects to compete against one player j with $V_j = H$ and one player C with $V_C = M$. Players j and C mistakenly think that $V_i = L$ and hence expect $x_i \equiv 0$; by Baye et al. (1996), the equilibrium mixed strategies of j and C are $F_j(x_j) = x_j/M$ and $F_C(x_C) = 1 - (M/H) + (x_C/H)$ with equilibrium support $[0, M]$. The best response of the deviating player i to these anticipated choices is $x_i = M$ and leads to a payoff of $(H - M)$.

(bb) If instead $\lambda_j = 1$ (and hence $V_j = L$) then the alliance is formed. Both i and j learn of each other's type ($V_i = H$, $V_j = L$). Player C expects to play against an alliance with $V_A = V_B = L$. The equilibrium that emerges based on these beliefs has $F_C(x_C) = x_C/(L/2)$, $x_j = 0$, and $x_i = L/2$. Since the alliance wins with probability one, i's payoff is $H/2 - L/2$. Thus, i's expected deviation payoff at the point where λ_j has not yet been chosen is

$$q\,(H-M) + (1-q)\left(\frac{H}{2} - \frac{L}{2}\right). \tag{16}$$

Comparing (12) and (16) shows that deviations for players i with $V_i = H$ are not profitable if and only if

$$(1-q)\left(\frac{H+L}{2} - M\right) \geq q(H-M), \tag{17}$$

[30] Note that this deviation payoff is the limit of i's payoff when choosing $x_i = \varepsilon > 0$ and when ε approaches zero.

			$E(\pi_A+\pi_B)$	$E(\pi_C)$
$V_A=L$	$V_B=L$		0	$M-L$
$V_A=H$	$V_B=L$	and $H+L\leq 2M$	0	$M-\frac{H+L}{2}$
$V_A=H$	$V_B=L$	and $H+L>2M$	$\frac{H+L}{2}-M$	0
$V_A=L$	$V_B=H$	and $H+L\leq 2M$	0	$M-\frac{H+L}{2}$
$V_A=L$	$V_B=H$	and $H+L>2M$	$\frac{H+L}{2}-M$	0
$V_A=H$	$V_B=H$		$H-M$	0

Table A.1: Expected payoffs if players A and B are in an alliance and choose efforts so as to maximize their joint expected payoffs, as a function of the valuations of A and B, assuming that the valuation of the stand-alone player C is $V_C=M$.

or, equivalently, (3) holds. Altogether, the separating equilibrium exists for parameters (H,M,L,q) if and only if (2) and (3) hold.

To see that the corresponding parameter set is non-empty suppose that $q\to 0$ and $L\to M$. Then, condition (3) becomes $H\geq M$ which is true by assumption. Condition (2) becomes $H\leq 16M/15$ and is, hence, true if H is sufficiently close to M. Thus, the separating equilibrium exists if H and L are both sufficiently close to M and q is small.

A.4 Proof of Remark 1

Suppose that if an alliance is formed, alliance members will coordinate on a total effort x_A+x_B which maximizes the total expected payoff of the alliance given by

$$\pi_A+\pi_B=\begin{cases}\frac{V_A+V_B}{2}-(x_A+x_B) & \text{if the alliance wins,}\\ -(x_A+x_B) & \text{otherwise.}\end{cases}$$

Note that in equilibrium only the sum x_A+x_B will be determined; the individual payoffs π_A and π_B depend on how the effort is shared. Using the standard logic of the all-pay auction for two strategic players with valuations $V_{AB}=V_A/2+V_B/2$ and V_C, the unique equilibrium follows from Baye et al. (1996), with expected equilibrium payoffs as given in Table A.1.

Before deriving the equilibrium alliance formation choices we compare the total expected alliance payoff as given in Table A.1 to the expected payoffs of an alliance which chooses the efforts non-cooperatively under complete information (Proposition 2; Table 2). Here, we have to distinguish different cases, but it is straightforward to verify that total expected payoff of a collusive alliance (as in Table A.1) is strictly higher than the total expected payoff under non-cooperative effort choices (as in Table 2) if and only if at least one of the alliance players A and B has a high valuation and H is sufficiently large.

Part (i): Suppose first that $V_i = L$. Then i's expected payoff when standing alone is zero and i will get at least zero when forming an alliance. In fact, if $V_j = H > 2M - L$, $j \in \{A, B\}$, $j \neq i$, then the total expected alliance payoff under joint payoff maximization is strictly positive; depending on how i and j share the effort cost (depending on the exact combination (x_A, x_B)), i can achieve a strictly positive payoff. Thus, players i with $V_i = L$ are at least weakly better off for $\lambda_i = 1$ than for $\lambda_i = 0$. Now suppose that $V_i = H$. If $V_j = H$, then $\lambda_j = 0$ in the candidate equilibrium and i's choice is inconsequential since no alliance is formed. If $V_j = L$, however, i gets $H - M$ when standing alone and at most $(H + L)/2 - M < H - M$ when forming an alliance which maximizes joint payoffs. Therefore, for $V_i = H$, i strictly prefers $\lambda_i = 0$ over $\lambda_i = 1$. Altogether, this shows that the type-1 equilibrium always exists.

Part (ii): From the previous paragraph it already follows that player i with $V_i = L$ is (weakly) better off when forming an alliance than when standing alone. Therefore, we only need to consider the case of $V_i = H$. If $V_j = H$, then in the candidate equilibrium where $\lambda_j = 1$, i can get a strictly positive payoff when forming an alliance but gets zero when deviating to $\lambda_i = 0$. Assuming that A and B share the cost of the alliance effort equally in case of $V_A = V_B$, i's expected payoff in the alliance is $(H - M)/2$ (compare Table A.1).[31] If $V_j = L$, however, i gets at most $\max\left\{\frac{H+L}{2} - M, 0\right\}$ in the candidate equilibrium with $\lambda_i = 1$, but gets $H - M$ if he deviates to $\lambda_i = 0$. Thus, the type-2 equilibrium exists if and only if the probability that j is a low type is sufficiently low (that is, q is sufficiently high).

To determine the exact threshold above which the type-2 equilibrium exists, we have to make an assumption as to which equilibrium is selected in case an alliance is formed and $V_A \neq V_B$. As an example, suppose that if $V_i = H > V_j = L$ then i and j share the effort $x_A + x_B$ according to their relative valuations, that is, $x_i =$

[31] In fact, except for the limit case in which j contributes nothing to total alliance effort, i's expected equilibrium payoff is strictly above zero.

$(x_A + x_B) V_i / (V_A + V_B)$. Then, if $H + L \leq 2M$, we get $E(\pi_A) = E(\pi_B) = 0$. If, however, $H + L > 2M$, total expected alliance payoff is strictly positive; the sharing rule on the effort cost implies that $E(\pi_i) = V_i/2 - MV_i/(V_A + V_B)$. Therefore, if $H + L \leq 2M$ then player i with $V_i = H$ does not want to deviate to $\lambda_i = 0$ if and only if

$$q\left(\frac{H}{2} - \frac{M}{2}\right) + (1-q)\,0 \geq q \cdot 0 + (1-q)(H - M),$$

which is equivalent to $q \geq 2/3$. If instead $H + L > 2M$, i with $V_i = H$ does not want to deviate to $\lambda_i = 0$ if and only if

$$\left(\frac{H}{2} - \frac{M}{2}\right) + (1-q)\,0 \geq q\left(\frac{H}{2} - \frac{H}{H+L}M\right) + (1-q)(H - M),$$

that is, if and only if $q \geq \tilde{q}$ where

$$\tilde{q} = 1 - \frac{(H+L)(H-M)}{H^2 + HL - 2ML + (H+L)(H-M)}.$$

References

[1] Aimone, Jason, A., Laurence R. Iannaccone, Michael D. Makowsky and Jared Rubin, 2013, Endogenous group formation via unproductive costs, Review of Economic Studies, 80, 1215-1236.

[2] Alcalde, José, and Matthias Dahm, 2010, Rent seeking and rent dissipation: A neutrality result, Journal of Public Economics, 94(1-2), 1-7.

[3] Amann, Erwin, and Wolfgang Leininger, 1996, Asymmetric all-pay auctions with incomplete information: The two-player case, Games and Economic Behavior, 14(1), 1-18.

[4] Baik, Kyung Hwan, and Jason F. Shogren, 1992, Strategic behavior in contests - comment, American Economic Review, 82(1), 359-362.

[5] Baye, Michael R., Dan Kovenock, and Casper G. de Vries, 1996, The all-pay auction with complete information, Economic Theory, 8(2), 291-305.

[6] Baye, Michael R., Dan Kovenock, and Casper G. de Vries, 2012, Contests with rank-order spillovers, Economic Theory, 51(2), 315-350.

[7] Bearce, David H., Kristen M. Flanagan, and Katharine M. Floros, 2006, Alliances, internal information, and military conflict among member-states, International Organization, 60(3), 595-625.

[8] Bergstrom, Theodore C., Lawrence Blume and Hal Varian, 1986, On the private provision of public goods, Journal of Public Economics, 29(1), 25-49.

[9] Bissonnette, Annie, Susan Perry, Louise Barrett, John C. Mitani, Mark Flinn, Sergey Gavrilets and Frans B.M. de Waal, 2015, Coalitions in theory and reality: a review of pertinent variables and processes, Behaviour, 152, 1-56.

[10] Benenson, Joyce F., Markovits, Henry, Thompson, Melissa E., Wrangham, Richard W., 2009. Strength determines coalitional strategies in humans. Proc. R. Soc. B: Biol. Sci. 276, 2589–2595.

[11] Davis, Douglas D., and Robert J. Reilly, 1999, Rent-seeking with non-identical sharing rules: An equilibrium rescued, Public Choice, 100(1-2), 31-38.

[12] Esteban, Joan M., and Debraj Ray, 2001, Collective action and the group size paradox, American Political Science Review, 95(3), 663-672.

[13] Esteban, Joan M., and József Sákovics, 2003, Olson vs. Coase: coalitional worth in conflict, Theory and Decision, 55(4), 339-357.

[14] Fordham, Benjamin, and Paul Poast, 2016, All alliances are multilateral: Rethinking alliance formation, Journal of Conflict Resolution, 60(5), 840-865.

[15] Fudenberg, Drew, and Jean Tirole, 1984, The fat-cat effect, the puppy-dog ploy, and the lean and hungry look, American Economic Review, Papers and Proceedings, 74(2), 361-366.

[16] Gintis, Herbert, Eric Alden Smith, and Samuel Bowles, 2001, Costly signaling and cooperation, Journal of Theoretical Biology, 213, 103-119.

[17] Hart, Sergiu, and Mordecai Kurz, 1983, Endogenous formation of coalitions, Econometrica, 51(4), 1047-1064.

[18] Herbst, Luisa, Kai A. Konrad, and Florian Morath, 2015, Endogenous group formation in experimental contests, European Economic Review, 74, 163-189.

[19] Hillman, Arye, and John G. Riley, 1989, Politically contestable rents and transfers, Economics & Politics, 1(1), 17-39.

[20] Hirshleifer, Jack, 1983, From weakest-link to best-shot: the voluntary provision of public goods, Public Choice, 41(3), 371–86.

[21] Katz, Eliakim, and Julia Tokadlidu, 1996, Group competition for rents, European Journal of Political Economy, 12(4), 599-607.

[22] Klose, Bettina, and Dan Kovenock, 2015, The all-pay auction with complete information and identity-dependent externalities, Economic Theory, 59(1), 1-19.

[23] Kolmar, Martin, and Andreas Wagener, 2013, Inefficiency as a strategic device in group contests against dominant opponents, Economic Inquiry, 51(4), 2083-2095.

[24] Konishi, Hideo, and Debraj Ray, 2003, Coalition formation as a dynamic process, Journal of Economic Theory, 110(1), 1-41.

[25] Konrad, Kai A., 2002, Investment in the absence of property rights; the role of incumbency advantages, European Economic Review, 46(8), 1521-1537.

[26] Konrad, Kai A., 2004, Altruism and envy in contests: an evolutionary stable symbiosis, Social Choice and Welfare, 22(3), 479-490.

[27] Konrad, Kai A., 2004. Bidding in hierarchies. European Economic Review, 48(6), 1301-1308.

[28] Konrad, Kai A., 2006, Silent interests and all-pay auctions, International Journal of Industrial Organization, 24(4), 701-713.

[29] Konrad, Kai A., 2012, Information alliances in contests with budget limits, Public Choice, 151(3-4), 679-693.

[30] Konrad, Kai A., 2014, Strategic aspects of fighting in alliances, in: Wärneryd, K. (ed.), The Economics of Conflict, CESifo Seminar Series. MIT Press, Cambridge & London, 1-22.

[31] Konrad, Kai A., and Dan Kovenock, 2009, The alliance formation puzzle and capacity constraints, Economics Letters, 103(2), 84-86.

[32] Konrad, Kai A., and Wolfgang Leininger, 2011, Self-enforcing norms and efficient non-cooperative collective action in the provision of public goods, Public Choice, 146(3-4), 501-520.

[33] Konrad, Kai A., Wolfgang Peters, and Karl Wärneryd, 2004, Delegation in first-price all-pay auctions, Managerial and Decision economics, 25(5), 283-290.

[34] Kovenock, Dan, and Brian Roberson, 2012, Coalitional Colonel Blotto games with application to the economics of alliances, Journal o Public Economic Theory, 14(4), 653-676.

[35] Meirowitz, Adam, 2008, Electoral contests, incumbency advantages, and campaign finance, Journal of Politics, 70(3), 681-699.

[36] Mesterton-Gibbons, Mike, Gavrilets, Sergey, Gravner, Janko, and Erol Akçay, 2011, Models of coalition or alliance formation, Journal of Theoretical Biology, 274(1), 187-204.

[37] Morath, Florian, and Johannes Münster, 2013, Information acquisition in conflicts, Economic Theory, 54, 99-129.

[38] Morrow, James D., 2000, Alliances: Why write them down?, Annual Review of Political Science, 3, 63-83.

[39] Nitzan, Shmuel, 1991, Collective rent dissipation, Economic Journal, 101(409), 1522-1534.

[40] Olson, Mancur, and Richard Zeckhauser, 1966, Economic theory of alliances, Review of Economics and Statistics, 48(3), 266-279.

[41] Ray, Debraj, and Rajiv Vohra, 1997, Equilibrium binding agreements, Journal of Economic Theory, 73(1), 30-78.

[42] Ray, Debraj, and Rajiv Vohra, 2015, The farsighted stable set, Econometrica, 83(3), 977-1011.

[43] Salant, Stephen W., Sheldon Switzer, and Robert J. Reynolds, 1983, Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium, Quarterly Journal of Economics, 98(2), 185-199.

[44] Siegel, Ron, 2009, All-pay contests, Econometrica, 77(1), 71-92.

[45] Skaperdas, Stergios, 1998, On the formation of alliances in conflict and contests, Public Choice, 96(1-2), 25-42.

[46] Tullock, Gordon, 1980, Efficient rent seeking, in: James M. Buchanan, Robert D. Tollison, and Gordon Tullock (eds.), Toward a Theory of the Rent-seeking Society, Texas A&M University Press, College Station, 97-112.