

# Revisiting the Effect of Household Size on Consumption Over the Life-Cycle

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## Abstract

In this paper we revisit the role of household size on consumption over the life-cycle. Although the link between these two variables has a strong empirical support, there is no consistent way in which demographics are dealt with in the standard life-cycle incomplete markets model, where the use of *Single Agent* models is commonplace. We study the relationship between the predictions of the *Single Agent* model versus a simple model extension where deterministic changes in household size and composition affect optimal consumption decisions. We provide theoretical results comparing both approaches and study the quantitative effects of household size on life-cycle consumption profiles, consumption smoothing of income shocks and welfare computations. Given that our proposed model can be seen as the simplest stand-in for more explicit models of household interactions (e.g., dynamic collective, multi-agent models) our results show that the use of single agent models is not an innocuous shortcut.

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# 1 Introduction

It has been long established that *humps* in household consumption are closely related to changes in its size and composition over the life-cycle.<sup>1</sup> In the quantitative macroeconomic literature however, a standard approach entails extracting per-adult equivalent consumption facts from household survey data and use them as targets to be replicated by *Single Agent* models, which are also calibrated using per-adult equivalent household or individual worker's income.<sup>2</sup> In this paper we argue that this shortcut is far from innocuous: whether demographics are explicitly considered for agent's decisions or used externally to the model (e.g., as control for estimated moments to be matched) matters for predictions.

Although there are dynamic models which explicitly consider multi-person households, and thus make explicit the non-trivial channels through which consumption decisions might depend on household size and composition,<sup>3</sup> their main drawback is their complexity and lack of a unified theory of the households. This, then translates in difficulties to contrast and compare models and their predictions. In this paper we are not advocating against these models, but providing an intermediate step missing in the literature so far: studying how different the implications of models are, when we consider demographics inside or outside of the model, in a setup that is general enough to accommodate the *Single Agent* model as a special case.

We perform our analysis by extending the standard incomplete markets model by allowing for exogenous, deterministic changes in household size and composition during the life-cycle to affect optimal decisions on consumption and savings in a unitary model approach. We label this as the *Demographics* model<sup>4</sup> (as opposed to the standard *Single Agent* model used in much of the literature). Similar economies have been used to study different questions in the literature: the effects of German reunification on savings behavior in [Fuchs-Schündeln \(2008\)](#), the welfare effects of different bankruptcy laws in [Livshits, MacGee, and Tertilt \(2007\)](#), the optimality of day care

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<sup>1</sup>See for example, [Banks, Blundell, and Preston \(1994\)](#), [Deaton and Paxson \(1994a\)](#), [Attanasio and Weber \(1995\)](#), [Attanasio, Banks, Meghir, and Weber \(1999\)](#), [Gourinchas and Parker \(2002\)](#) [Browning and Ejrnæs \(2009\)](#) among others.

<sup>2</sup>Some recent papers in this vein include [Heathcote, Storesletten, and Violante \(2008\)](#), [Blundell, Low, and Preston \(2008\)](#), [Low and Pistaferri \(2010\)](#), [Fernández-Villaverde and Krueger \(2010\)](#) and [Güvenen and Smith \(2010\)](#).

<sup>3</sup>See for example [Cubeddu and Ríos-Rull \(2003\)](#), [Aiyagari, Greenwood, and Guner \(2000\)](#) and [Mazzocco, Ruiz, and Yamaguchi \(2007\)](#) among others.

<sup>4</sup>This is a setting very similar to the one proposed in [Attanasio, Banks, Meghir, and Weber \(1999\)](#) and [Gourinchas and Parker \(2002\)](#)

subsidies in [Domeij and Klein \(2011\)](#) and to provide a framework to understand partial insurance in [Heathcote, Storesletten, and Violante \(2012\)](#), among others.

Using a simple two period version of our proposed model, we show theoretically that *Single Agent* models produce in general different predictions of per-adult equivalent consumption than the *Demographics* case: agents in *Single Agent* models ignore the fact that the relative price of consumption across periods in which family size is changing might be affected by economies of scale and direct preferences over household size. When the *Demographics* model is specified such that this relative price does not change, the two approaches predict the same per-adult equivalent consumption profiles. However, this result breaks down if income uncertainty is introduced. This is because the presence of economies of scale in consumption alter the resources needed to provide insurance (in the sense of consumption smoothing) if household size changes over time.

In a quantitative exercise, based on a standard model of life-cycle consumption with income uncertainty and incomplete markets, similar to the one in [Storesletten, Telmer, and Yaron \(2004\)](#), we first confirm that this latter mechanism is quantitatively important under the assumption that in the *Demographics* model the relative price of consumption does not change with household size and conclude that in this case it is favorable to work with the *Demographics* instead of the *Single Agent* model. We then investigate the implications for different ad-hoc specifications of the *Demographics* model for (i) per-adult equivalent consumptionlife-cycle profiles, (ii) the degree of consumption smoothing with respect to contemporaneous shocks to income and (iii) welfare implications of a change in a policy that affects borrowing in the economy. Computation of statistics to contrast (i) is straightforward; for (ii) we follow [Blundell, Pistaferri, and Preston \(2008\)](#) and [Kaplan and Violante \(2010\)](#) and calculate model implied *insurance coefficients*, statistics that relate income shocks to changes in consumption; for (iii), we use the model in [Livshits, MacGee, and Tertilt \(2007\)](#), to compare welfare under different bankruptcy laws. These experiments show how the way in which we model the effect of household size on consumption affects statistics usually derived from the standard incomplete markets model. For all the exercises, we find that model predictions depend crucially on the interaction between the degree of economies of scale in the household and how the utility of per-adult equivalent consumption of each household member is valued. For example, the Continental European-style bankruptcy law is associated with higher welfare than the US-style bankruptcy law if households put little weight on each household member's utility and this result

reverses as more weight is placed on each household member’s utility.

Finally, we compare our setup with the preference structure estimated in [Attanasio, Banks, Meghir, and Weber \(1999\)](#) and discuss how to choose preference parameters for quantitative work. For the most commonly used equivalence scales, as e.g. the OECD scale, with low to medium economies of scale, the empirical estimates suggest that *Demographics* models with moderate preferences for household size are closer to the data. Under these parameterizations, household size changes have only a small effect on the relative price of consumption, i.e. in the absence of income uncertainty predictions of the *Demographics* model would be close to those of the *Single Agent* model. However, as we have already argued before, income uncertainty drives a quantitatively substantial wedge between the two models as the presence of economies of scale in consumption alter the amount of resources needed for providing insurance.

The structure of the paper is as follows: in [Section 2](#) we discuss our proposed preferences for the household and present theoretical predictions in a stylized two period framework. In [Section 3](#) we discuss the model we use to quantify these theoretical predictions. In [Section 4](#) we show the quantitative features of the model and the calibration strategy while [Section 5](#) shows our main quantitative results. [Section 6](#) presents an identification exercise and some discussion, while in the last section, we conclude.

## 2 A Two Period Model

### 2.1 Setup

Households live for two periods. Household size is normalized to one in the first period ( $N_1 = 1$ , e.g. a young person living alone) and increases deterministically in the second period ( $N_2 > 1$ , e.g. a child is born). For the theoretical analysis all we need is a change in household size between two periods whereas the quantitative analysis will feature differences in household size and composition, i.e. a distinction between the number of adults and children in the household, and a realistic life-cycle length. Households receive income  $\mathbb{Y}_1$  in the first period and  $\mathbb{Y}_2$  in the second period. We first consider the case when period two income is known and introduce income uncertainty in the next subsection. In any case the household can borrow (up to the natural borrowing constraint) and save at an interest  $r$  which is set to zero. The discount factor is set to one, i.e. from the perspective

of period one the utility in period two is not discounted.

## 2.2 Demographics Model

In our benchmark model household size  $N_t$  affects the marginal utility of consumption, i.e.  $u(\mathbb{C}_t^D, N_t)$ . In particular, we will assume a unitarian framework in which a household decision maker allocates household consumption  $\mathbb{C}_t^D$  optimally to the two periods

$$\max_{\mathbb{C}_1^D, \mathbb{C}_2^D} U = u(\mathbb{C}_1^D, N_1) + u(\mathbb{C}_2^D, N_2) \quad (1)$$

subject to

$$\mathbb{C}_1^D + \mathbb{C}_2^D = \mathbb{Y}_1 + \mathbb{Y}_2. \quad (2)$$

We specify the utility function as

$$u(\mathbb{C}_t^D, N_t) = \delta(N_t) u\left(\frac{\mathbb{C}_t^D}{\phi(N_t)}\right). \quad (3)$$

There is no private consumption. The equivalence scale  $\phi(N_t)$  transforms household consumption into per-adult equivalent consumption, is normalized to one for  $N_t = 1$  and increases in household size by a factor smaller than one.<sup>5</sup>  $\delta(N_t)$  can be best interpreted as aggregating up the individual utilities from per-adult equivalent consumption  $u\left(\frac{\mathbb{C}_t^D}{\phi(N_t)}\right)$  of all household members. E.g. if the household planner assigns each member household member  $i$  a weight  $\bar{\delta}_i \in [0, 1]$  then  $\delta(N_t) = \sum_{i=1}^{N_t} \bar{\delta}_i$ . While there are certainly more elaborate models of the household,<sup>6</sup> we use this formulation as it nests various specifications used in recent contributions in quantitative macroeconomics. [Livshits, MacGee, and Tertilt \(2007\)](#), [Attanasio, Low, and Sanchez-Marcos \(2008\)](#) set

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<sup>5</sup> The three mechanisms through which household size affects the intra-temporal rate of transformation between expenditures and consumption services, and that are captured partially through equivalence scales, are family/public goods, economies of scale, and complementarities, see e.g. [Lazear and Michael \(1980\)](#). As a concrete example, consider the widely used OECD equivalence scale which is given by  $\phi_{OECD} = 1 + 0.7(N_{ad} - 1) + 0.5N_{ch}$  with  $N_{ad}$  being the number of adults and  $N_{ch}$  the number children in the household. This can be interpreted as follows: it takes 1.7\$ of consumption expenditures to generate the same level of welfare out of consumption for a two adult household that 1\$ achieves for a single member household. [Lewbel \(1997\)](#) provides an extensive survey with a particular emphasis on definition, identification, estimation and use of equivalence scales. The scales used in actual policy making are rather simple weighting schemes as e.g. the OECD scale, see [Citro and Michaels \(1995\)](#) for a lengthy discussion. We follow the macroeconomic literature on income and consumption inequality and use these latter equivalence scales.

<sup>6</sup>See for example, [Mazzocco, Ruiz, and Yamaguchi \(2007\)](#), [Hong and Ríos-Rull \(2007\)](#) and [Hong and Ríos-Rull \(2009\)](#)

$\delta(N_t) = 1$  such that the household planner maximizes per-capita utility in each period. [Fuchs-Schündeln \(2008\)](#) and [Laitner and Silverman \(2008\)](#) use  $\delta(N_t) = \phi_t$ , [Heathcote, Storesletten, and Violante \(2012\)](#) use the number of adults in the household and [Domeij and Klein \(2011\)](#) the total number of household members. [Attanasio, Banks, Meghir, and Weber \(1999\)](#), and [Gourinchas and Parker \(2002\)](#) employ CRRA preferences and use a more general taste shifter

$$u(C_t, N_t) = \exp(\xi_1 N_{ad} + \xi_2 N_{ch}) \frac{C_t^{1-\alpha}}{1-\alpha} \quad (4)$$

which for given preference parameters  $\alpha$ ,  $\xi_1$ ,  $\xi_2$  and a specific equivalence scale maps directly into a corresponding  $\delta$ . In fact, in Section 6 we will use the preference parameter estimates from [Attanasio, Banks, Meghir, and Weber \(1999\)](#) to back out  $\delta$  for various equivalence scales.

### 2.3 *Single Agent Model*

A more common approach than the previously described one is to assume that households consist only of a single member and consequently household size has no affect on the marginal utility of consumption. At least since the work by [Attanasio and Weber \(1993\)](#) and [Attanasio and Browning \(1995\)](#) it is well understood that household size changes are important for understanding the patterns of household consumption over the life-cycle. In order to keep on working with the *Single Agent*, a response to these findings is to clean household consumption data for household size and household composition, i.e. demographic, effects. One popular approach is to divide these data by an equivalence scale which transforms total household consumption into a per-adult equivalent consumption, against which the predictions of the *Single Agent* are then compared, see e.g. [Blundell, Low, and Preston \(2008\)](#), [Heathcote, Storesletten, and Violante \(2008\)](#) or [Low and Pistaferri \(2010\)](#).<sup>7</sup> To ensure consistency between the model and the data, income fed into the model is cleaned as well for household size and household composition effects. One popular approach, in particular in the inequality literature, is to divide household income by the same equivalence scale used for consumption as done by [Cutler and Katz \(1992\)](#), [Krueger and Perri \(2006\)](#), [Blundell,](#)

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<sup>7</sup>There are obviously other methods to create per-adult equivalent information from household data. One alternative method is to estimate household size/composition effects directly from micro data using least squares regressions, see e.g. [Aguiar and Hurst \(2009\)](#). Although studying heterogeneity in household size/composition (which is a pre-requisite to understand the regression methodology) is beyond the scope of our paper, this approach generates adjustments that can be trivially converted to an ad-hoc equivalence scales.

Low, and Preston (2008), Meyer and Sullivan (2010), and the 2010 special issue of the Review of Economic Dynamics (Krueger, Perri, Pistaferri, and Violante (2010)). This strategy could be interpreted as dividing the per-period household budget constraint by the equivalence scale. One alternative is to use only the household heads income as done e.g. by Heathcote, Storesletten, and Violante (2008), Low and Pistaferri (2010), and Kaplan (2012). We adjust income with the factor  $\kappa(N_t)$  as a stand-in for these different empirical strategies to obtain a per-adult equivalent income. In any case  $\kappa(N_t)$  is normalized to one for a household of size one and the household chooses per-adult equivalent consumption  $c_t^S$  to solve the following optimization problem

$$\max_{c_1^S, c_2^S} U = u(c_1^S) + u(c_2^S) \quad (5)$$

subject to

$$c_1^S + c_2^S = \frac{Y_1}{\kappa_1(N_t)} + \frac{Y_2}{\kappa_2(N_t)}. \quad (6)$$

Comparing the two setups, it should become evident what we meant in the introduction, with the *Single Agent* model abstracting from household (changes) inside the model, but being 'calibrated' in a fashion that controls for these effects outside the model. Household effects enter only via the budget constraint. In contrast, in the *Demographics* model household size (changes) affect utility directly but not via the budget constraint. It is straightforward to make the optimal consumption allocations comparable. The *Single Agent* model directly predicts per-adult equivalent consumption because the household receives a per-adult equivalent income. In the *Demographics* model, household consumption is predicted which can be easily transformed into per-adult equivalent consumption by deflating with the equivalence scale  $\phi$ , i.e.  $\frac{C_1^D}{\phi_1} = \frac{C_1^S}{1}$  and  $\frac{C_2^D}{\phi_2}$ .

It is important to mention that Attanasio, Banks, Meghir, and Weber (1999) highlight the importance of household size changes or demographics, via affecting the marginal utility, for explaining the life-cycle profile of household consumption. Our key contribution is to analyze in how far the recent practice of using the *Single Agent* model and focusing on per-adult equivalent consumption is a good approximation to the per-adult equivalent consumption predicted by the *Demographics* model. Our theoretical results will focus on per-adult equivalent consumption profiles, i.e.  $\frac{C_2^D/\phi_2}{C_1^D}$  and  $\frac{c_2^S}{c_1^S}$  (or alternatively per-adult equivalent consumption growth), rather than levels.

## 2.4 Consumption Profiles

**Proposition 1.** *The per-adult equivalent consumption profile in the Demographics model and Single Agent model coincide only if  $\delta_2 = \phi_2$ .*

This result can be immediately read of from the two Euler equations for the *Demographics* model (7) and *Single Agent* model (8):

$$u'(\mathbb{C}_1^D) = \frac{\delta_2}{\phi_2} u' \left( \frac{\mathbb{C}_2^D}{\phi_2} \right) \quad (7)$$

$$u'(c_1^S) = u'(c_2^S). \quad (8)$$

In both first-order conditions only per-adult equivalent consumption appears. As a reminder, for the *Single Agent* model the consumption levels  $c_1^S$  and  $c_2^S$  in fact reflect per-adult equivalent consumption because income as an input to the optimization problem has already been adjusted for household size. For the *Demographics* model it is obvious in the second period as the household receives the (marginal) utility from per-adult equivalent consumption  $\frac{\mathbb{C}_2^D}{\phi_2}$  which is however also true in the first period because household size is one in period one ( $\phi_1 = 1$ ).

Equation (8) predicts a flat per-adult equivalent consumption profile for the *Single Agent* model. The per-adult equivalent consumption profile in the *Demographics* model is however only flat if  $\delta_2 = \phi_2$  but upward sloping if  $\delta_2 > \phi_2$ , i.e.  $\mathbb{C}_1^D = \frac{\mathbb{C}_2^D}{\phi_1} < \frac{\mathbb{C}_2^D}{\phi_2}$ , while the opposite is true for  $\delta_2 < \phi_2$ . The intuition behind this result can be best explained when decomposing the benefit of consuming one additional unit of household consumption in the second period in the *Demographics* model which

1. is associated with the marginal utility of per-adult equivalent consumption in period two  $\left[ u' \left( \frac{\mathbb{C}_2^D}{\phi_2} \right) \right]$
2. accrues to all household members reflected through the multiplication by the weighting factor  $[\delta_2]$
3. has to be divided by the equivalence scale  $[\phi_2]$  because each household member does not get the full unit to consume but only the fraction  $\frac{1}{\phi_2}$ .

As an example, consider the case of the weighting factor being equal to household size, i.e.

$\delta_2 = N_2$ . The larger household size in period two provides an incentive to allocate more consumption to period two because the household enjoys a larger utility from consuming as each unit of per-adult equivalent consumption is weighted by  $\delta_2 = N_2$ . However, in period two every unit of consumption has to be shared with more people which is reflected through the division with the equivalence scale  $\phi_2$ . This in turn reduces the incentive to allocate more consumption to period two. Since for all empirically estimated equivalence scales (see [Fernández-Villaverde and Krueger \(2007\)](#)),  $\phi_2 < N_2$ , the case of  $\delta_2 = N_2$  implies that per-adult equivalent consumption in period two exceeds per-adult equivalent consumption in period one. Relative to period one, the absolute loss in consumption in period two (because of the sharing across household members) is outweighed by the fact that each household member enjoys the extra per-adult equivalent consumption. Interestingly, such a configuration may provide an additional explanation for the hump observed in per-adult equivalent consumption documented in [Fernández-Villaverde and Krueger \(2007\)](#). The ratio  $\frac{\delta_2}{\phi_2}$  can be interpreted as changing the effective discount factor in the Euler equation or alternatively as changing the relative price of per-adult equivalent consumption between two periods whenever there is a change in household size. This channel is absent in the *Single Agent* model.

Note that this result is completely independent from  $\kappa_2$  which is used to construct the per-adult equivalent income feed in the *Single Agent* model. This is of course not true for consumption levels for which  $\kappa_2$  crucially matters, see [Appendix A.1](#).

## 2.5 Consumption Profiles and Income Uncertainty

This section introduces income uncertainty. Period two income can take two values:  $\mathbb{Y}_{2,l}$  with probability  $p_l$  and  $\mathbb{Y}_{2,h}$  with probability  $p_h = 1 - p_l$ , where  $\mathbb{Y}_{2,h} > \mathbb{Y}_{2,l}$ . Households are only allowed to borrow as much as they can repay for sure, i.e. at most  $\mathbb{Y}_{2,l}$ . The implied Euler equations for the *Demographics* model [\(9\)](#) and *Single Agent* model [\(10\)](#) are given by:

$$u'(\mathbb{C}_1^D) = \frac{\delta_2}{\phi_2} \sum_{i=l,h} p_i u' \left( \frac{\mathbb{Y}_1 - \mathbb{C}_1^D}{\phi_2} + \frac{\mathbb{Y}_{2,i}}{\phi_2} \right) \quad (9)$$

$$u'(c_1^S) = \sum_{i=l,h} p_i u' \left( \mathbb{Y}_1 - c_1^S + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \right) \quad (10)$$

We will however consider only the case of  $\delta_2 = \phi_2$  because under this configuration and without income uncertainty the per-adult equivalent consumption profiles are the same in both approaches, see Proposition 1.

**Proposition 2.** *In the presence of income uncertainty and for  $\delta_2 = \phi_2$ , the per-adult equivalent consumption profiles in the Demographics and Single Agent model for the low and high income shock coincide only if  $\kappa_2 = 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1^D}$  and  $\mathbb{Y}_1^D > 0$ .*

The proof of Proposition 2 is given in Appendix A.2. The key distinction between the two modelling approaches is the rate of transformation of per-adult equivalent consumption between period one and two: it is one in the *Single Agent* model and  $\frac{1}{\phi_2}$  in the *Demographics* model. In the *Demographics* model giving up one unit of period **one** (per-adult equivalent) consumption yields only  $\frac{1}{\phi_2}$  units of period **two** per-adult equivalent consumption, or conversely giving up one unit of period **two** per-adult equivalent consumption yields  $\phi_2$  units of period **one** (per-adult equivalent) consumption. Hence, in per-adult equivalent terms it is more expensive (cheaper) to save (borrow) in the *Demographics* model compared to the *Single Agent* model. If the household in the *Demographics* model optimally does neither save nor borrow, this difference does not materialize and the per-adult equivalent consumption profiles are the same if  $\kappa_2 = \phi_2$ . This can be seen directly from the Euler equations Equations (9) and (10); with  $\delta_2 = \phi_2$  and  $\kappa_2 = \phi_2$  even the optimal per-adult equivalent consumption levels, and not only the profiles, are the same:<sup>8</sup>

$$c_1^S = \mathbb{C}_1^D \text{ and } c_{2,i}^S = \frac{\mathbb{C}_{2,i}^D}{\phi_2} = \frac{\mathbb{Y}_{2,i}}{\phi_2} \quad \forall i = l, h. \quad (11)$$

The allocation (11) satisfies the budget constraint and, assuming that  $\mathbb{C}_1^D = \mathbb{Y}_1$  is the solution to the maximization problem in the *Demographics* model, the Euler equation for the *Single Agent* model (10). If the household in the *Demographics* model is a saver, Proposition 2 states that the critical value of  $\kappa_2$  for which the per-adult equivalent consumption profiles in the two models are the same has to be lower than  $\phi_2$ . Lowering  $\kappa_2$  increases effective income uncertainty which lowers insurance (in the sense of consumption smoothing) and therefore accommodates for the higher costs of providing insurance (in terms of giving up period one consumption) in the *Demographics* model.<sup>9</sup>

<sup>8</sup>Note that this is also true without income uncertainty.

<sup>9</sup>We think of effective income uncertainty as income uncertainty relative to expected income as measured by the coefficient of variation:  $CV^S = \frac{S.D.(\mathbb{Y}_1 + \mathbb{Y}_{2,i}/\kappa_2)}{\mathbb{Y}_1 + \sum_{i=l,h} p_i \mathbb{Y}_{2,i}/\kappa_2} = \frac{S.D.(\mathbb{Y}_{2,i})}{\kappa_2 \mathbb{Y}_1 + \sum_{i=l,h} p_i \mathbb{Y}_{2,i}}$ .

The opposite logic applies to the case of borrowing in the *Demographics* model.<sup>10</sup>

We will now illustrate this mechanism with a specific example:  $p_l = p_h = 0.5$ ,  $Y_{2,l} = 1 - \epsilon$  and  $Y_{2,h} = 1 + \epsilon$ ,  $\kappa_2$  is set equal to  $\phi_2$ . For this setup the Euler equations (9) and (10) are given by

$$u'(C_1^D) = \frac{1}{2} \left[ u' \left( \frac{Y_1 - C_1^D}{\phi_2} + \frac{Y_2(1 - \epsilon)}{\phi_2} \right) + u' \left( \frac{Y_1 - C_1^D}{\phi_2} + \frac{Y_2(1 + \epsilon)}{\phi_2} \right) \right] \quad (12)$$

$$u'(c_1^S) = \frac{1}{2} \left[ u' \left( Y_1 - c_1^S + \frac{Y_2(1 - \epsilon)}{\phi_2} \right) + u' \left( Y_1 - c_1^S + \frac{Y_2(1 + \epsilon)}{\phi_2} \right) \right] \quad (13)$$

As a starting point assume that it is optimal for the household in the *Demographics* model to neither borrow nor save in period one, i.e.  $C_1^D = Y_1$ . For this allocation our choice of  $\kappa_2 = \phi_2$  equals the critical value stated in Proposition 2 and (as also outlined above) the two models predict the same per-adult equivalent consumption profiles. Consider now an increase in income uncertainty ( $\epsilon$ ). With concave utility this induces the households in both models to reduce period one consumption and accumulate precautionary savings. The response is stronger in the *Demographics* model, i.e.

$$\frac{\partial C_1^D}{\partial \epsilon} \Big|_{C_1^D=Y_1} < \frac{\partial c_1^S}{\partial \epsilon} \Big|_{c_1^S=Y_1} < 0. \quad (14)$$

Assume that period one consumption in both models would be decreased by exactly the same amount  $x$ , i.e. there is no difference between the marginal utilities of per-adult equivalent consumption in period one. As a consequence, the expected marginal utility of per-adult equivalent consumption in period two in the *Demographics* model will be larger than in the *Single Agent* model: in the *Demographics* model the reduction of period one consumption by  $x$  increases period two per-adult equivalent consumption only by  $\frac{x}{\phi_2}$  but by  $x$  in the *Single Agent* model. In order to satisfy Equation (12), period one consumption in the *Demographics* model has to decrease more than in the *Single Agent* model.<sup>11</sup> This highlights the argument made before: it takes more resources in the *Demographics* model to provide additional consumption insurance which is the key difference between the two models that materializes in the presence of income uncertainty.

Under some additional restrictions, see Appendix A.2.1, we show analytically that the *Demographics* model implies less consumption smoothing or insurance manifesting itself in steeper

<sup>10</sup>Compared to the *Single Agent* model, the household in the *Demographics* model needs to sacrifice less insurance in the second period to achieve a certain level of period one per-adult equivalent consumption. This is accommodated by  $\kappa_2 < \phi_2$  in the *Single Agent* model and implying lower effective income uncertainty.

<sup>11</sup>Condition (14) is derived formally in Appendix A.2.1.

per-adult equivalent consumption profiles (in absolute terms) for both the low and high income shock. Decreasing  $\kappa_2$  to the critical value stated in Proposition 2 increases effective income uncertainty in the *Single Agent* model: this permits less consumption smoothing and hence results in steeper per-adult equivalent consumption profiles (in absolute terms) and thus reestablishes the identity of the per-adult equivalent consumption profiles in the *Demographics* and *Single Agent* model.

### 3 Quantitative Model

Here we present a standard incomplete markets life-cycle model, which follows closely the ones in Storesletten, Telmer, and Yaron (2004) and Kaplan and Violante (2010). For simplicity we abstract from population growth and general equilibrium. In the model, households start their economic life in period  $t_0$  with zero assets. During their working life until period  $t_w$  they receive a stochastic income in every period. There is no labor supply choice. From period  $t_w + 1$  onwards households are retired and have to live from their accumulated savings during working life and social security benefits. Life ends with certainty at age  $T$  but there is an age specific survival probability  $\zeta_t$ . Households do not leave bequests and cannot die with debt. Households have access to a risk-free bond  $a$  which pays the interest rate  $r$  and can borrow at the same interest rate up to the natural borrowing constraint, i.e. an age specific level  $a_{min,t}$  of debt that they can repay for sure. As in Storesletten, Telmer, and Yaron (2004), we introduce fair annuity markets so that savings of the dying population are redistributed equally among the surviving members of their cohort.

In the *Demographics* model, household size changes over the life-cycle deterministically as in Attanasio, Banks, Meghir, and Weber (1999) and Gourinchas and Parker (2002) and is homogenous across all households. The maximization problem is given by

$$\max_{\{a_{t+1}\}_{t=t_0}^{T-1}} E_0 \sum_{t=t_0}^T \left( \prod_{j=t_0}^{j=t} \zeta_j \right) \beta^{t-t_0} \delta_t u \left( \frac{c_t}{\phi_t} \right) \quad (15)$$

subject to (16)

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + (1-\tau)y_t \quad \forall t \leq t_w \quad (17)$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + p(\bar{y}) \quad \forall t_w < t \leq T \quad (18)$$

$$a_{t+1} \geq a_{min,t} \quad (19)$$

where  $\delta$  and  $\phi$  are functions of household size and its composition ( $N_{ad,t}$  and  $N_{ch,t}$ ) over the life-cycle.<sup>12</sup> Pre labor tax income  $y_t$  is stochastic during the working life, i.e. as long as  $t \leq t_w$ , it is given by the following process:

$$\ln y_t = \varrho_t + \epsilon^F + z_t + \epsilon_t^{Tr} \quad (20)$$

where  $\varrho_t$  is an age-dependent, exogenous experience profile (common to all individuals),  $\epsilon^F$  is a fixed effect drawn by households at the beginning of economic life from a normal distribution with mean zero and variance  $\sigma_F^2$ ,  $z_t$  is a permanent shock to labor income, with

$$z_t = \rho z_{t-1} + \epsilon_t^P \text{ with } \epsilon_t^P \sim N(0, \sigma_P^2) \quad (21)$$

and  $\epsilon^{Tr} \sim N(0, \sigma_{Tr}^2)$  is a transitory shock. Finally, we assume that the social security system is financed through linear labor taxes  $\tau$ .

During retirement (for  $t_w < t \leq T$ ) households receive age independent social security contributions  $y_t = p(\bar{y})$  that depend on the realization of income over the working life:  $\bar{y} = \frac{1}{t_w - t_0 + 1} \sum_{j=t_0}^{t_w} y_j$ .

The Euler equation to this problem is given by

$$\frac{\delta_t}{\phi_t} u' \left( \frac{c_t}{\phi_t} \right) = \beta(1+r) \frac{\delta_{t+1}}{\phi_{t+1}} E_t \left[ u' \left( \frac{c_{t+1}}{\phi_{t+1}} \right) \right] \quad (22)$$

The structure of the *Single Agent* problem is very similar. Demographics do not affect the

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<sup>12</sup>Note that in contrast to Section 2, we now denote all variables with lower case letters.

utility function while income  $y_t$  is deflated by household size through equivalence scales  $\kappa_t$ :

$$\max_{\{a_{t+1}\}_{t=t_0}^{T-1}} E_0 \sum_{t=t_0}^T \left( \prod_{j=t_0}^{j=t} \zeta_j \right) \beta^{t-t_0} u(c_t) \quad \text{subject to} \quad (23)$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + (1-\tau) \frac{y_t}{\kappa_t} \quad \forall t \leq t_w \quad (24)$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + \frac{p(\bar{y})}{\kappa_t} \quad \forall t_w < t \leq T \quad (25)$$

$$a_{t+1} \geq a_{\min,t}, \quad (26)$$

with  $y_t$  following the same process as for the *Demographics* model given by Equations (20) and (21). The Euler equation to this problem is given by

$$u'(c_t) = \beta(1+r)E_t[u'(c_{t+1})] \quad (27)$$

In line with studies that investigate jointly income and consumption inequality, see e.g. [Cutler and Katz \(1992\)](#), [Krueger and Perri \(2006\)](#), [Meyer and Sullivan \(2010\)](#), and the 2010 special issue of the Review of Economic Dynamics, we use the same equivalence scale for computing per-adult equivalent income and per-adult equivalent consumption ( $\kappa_t = \phi_t$ ).

## 4 Quantitative Features of the Model

A model period is one year. Agents start life at age 25, retire when 65 and live until age 95 after which they die with certainty. The common profile for survival probabilities comes from the National Center for Health Statistics.<sup>13</sup> To maintain comparability across models, we keep some parameters fixed: we set the interest rate at 2%, the discount factor  $\beta$  at 0.96 and work with CRRA preferences and  $\alpha = 2$ :<sup>14</sup>

$$u = \delta(N_{ad,t}, N_{ch,t}) \frac{\left( \frac{c_t}{\phi(N_{ad,t}, N_{ch,t})} \right)^{1-\alpha}}{1-\alpha}. \quad (28)$$

Of course, for the *Single Agent* model  $\delta$  and  $\phi$  are not part of the utility function. In the next section, we use as a benchmark the square root scale  $\phi_t^{SQR} = \sqrt{N_{ad} + N_{ch}}$ . This scale is almost

<sup>13</sup>[http://www.cdc.gov/nchs/data/lifetables/life90\\_2acc.pdf](http://www.cdc.gov/nchs/data/lifetables/life90_2acc.pdf)

<sup>14</sup>These are common choices in the literature. In Appendix B we consider the case of  $\beta(1+r) = 1$ .

identical to the 'Mean' scale in [Fernández-Villaverde and Krueger \(2007\)](#) which is their preferred choice.<sup>15</sup> Our results are similar when we use other equivalence scales, so we skip them from the presentation.<sup>16</sup>

As for utility weights, we remain agnostic and compare three cases: (i)  $\delta_t = 1$  represents the case when households do not value household size; (ii)  $\delta_t = N_t = N_{ad,t} + N_{ch,t}$  is the opposite, since households always enjoy having more members; and (iii) an intermediate case when  $\delta_t = \phi_t$ , i.e., we let the utility weight take the same value as the equivalence scale.

## 4.1 Income

We use data from the Current Population Survey, from 1984 to 2003,<sup>17</sup> in particular the March supplements for years 1985 to 2004, given that questions about income are retrospective. We use total wage income (deflated by CPI-U, leaving amounts in 2000 US dollars).

We construct total household income  $W_{i\tau}$  for household  $i$  observed in year  $\tau$ , as the sum of individual incomes in the household for all households with at least one full time/full year worker. The latter is defined as someone who worked more than 40 hours per week and more than 40 weeks per year and earned more than \$2 per hour. To get life-cycle profiles, we estimate the following regression:

$$\log\left(\frac{W_{i\tau}}{\phi_{i\tau}}\right) = D_{i\tau}^{age} \varrho^{age} + X_{i\tau}\gamma + \epsilon_{i\tau} \quad (29)$$

where  $\phi_{i\tau}$  is an equivalence scale,  $D_{i\tau}^{age}$  represents a set of age dummies of the head of household,  $\varrho^{age}$  and  $\gamma$  are estimated coefficients and  $\epsilon$  are estimation errors. We also control for cohort effects and time effects by introducing birth year and year dummies in  $X_{i\tau}$ .<sup>18</sup> From this estimation, we

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<sup>15</sup>For explicit formulations of the different equivalence scales used in empirical consumption literature, see Table 1 in [Fernández-Villaverde and Krueger \(2007\)](#).

<sup>16</sup>Results using different equivalence scales are available upon request.

<sup>17</sup>The choice of sample period is to maintain comparability with the data used in [Fernández-Villaverde and Krueger \(2007\)](#) to analyze different equivalence scales.

<sup>18</sup>Since year dummies are perfectly collinear with age and birth cohort dummies, we follow [Fernández-Villaverde and Krueger \(2007\)](#) and [Aguar and Hurst \(2009\)](#) and include normalized year dummies instead, such that for each year  $\tau$

$$\sum_{\tau} \gamma_{\tau} = 0 \quad \text{and} \quad \sum_{\tau} \tau \gamma_{\tau} = 0$$

where  $\{\gamma_{\tau}\}$  are the coefficients associated to these normalized year dummies. This procedure was initially proposed by [Deaton and Paxson \(1994b\)](#). To compare life-cycle profiles across different cohorts/time periods, we normalize the estimated coefficients associated to age dummies by adding the effect of a particular cohort/time. More specifically, we picked the cohort corresponding to the median age at the last observed year.

Table 1: Stochastic Income Process:

$\rho$	$\sigma_F^2$	$\sigma_P^2$	$\sigma_{Tr}^2$
0.9989	0.2105	0.0166	0.0630

are interested in the regression coefficients associated with age dummies of the household head (experience profiles in the model). In our exercise below, we smooth such profiles, using a quartic polynomial.

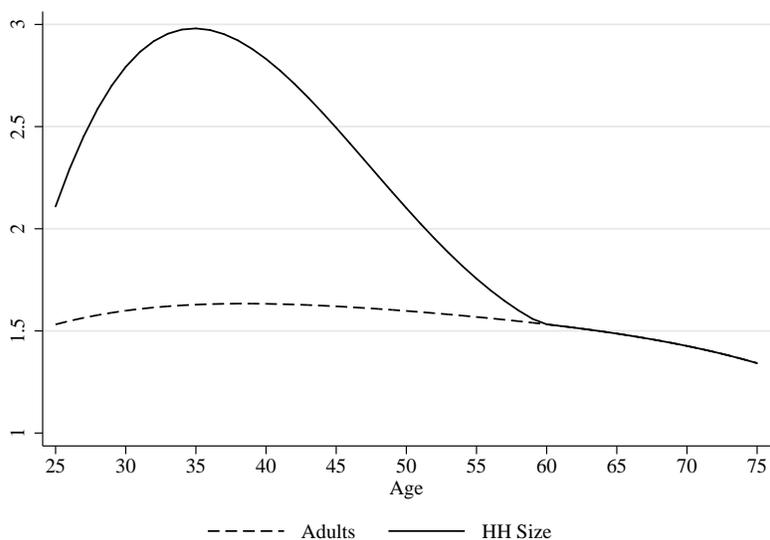
For the *Demographics* model we use household income for the estimation, i.e.  $\phi_{i\tau} = 1 \forall i, \tau$  in Equation (29). To maintain full comparability with our simple theoretical model, we perform an *ex-post equivalization* procedure for the income process in the *Single Agent* model: we use the calibrated income profiles and shocks from the *Demographics* also for the *Single Agent* model (Table 1) and assign the per-adult equivalent income realizations to the *Single Agent* model. Besides making the quantitative model more comparable to the theoretical model, this approach maintains the same shock structure across considered equivalence scales, making the comparison more direct, since no extra 'noise' is being introduced by different volatility parameters. This would be the case for an alternative approach, or an *ex-ante equivalization*: estimating Equation (29) with a particular equivalence scale  $\phi_{i\tau}$ , resulting in different age profiles and calibrated income shocks for the *Single Agent* model. Since then the income would have already been turned into per-adult equivalents *ex-ante*, there would be no need to do so in budget constraint (24).<sup>19</sup>

To parameterize the income process in (21) we pick the parameter estimates from Storesletten, Telmer, and Yaron (2004) (see Table 1). We discretize this calibrated process using the Rouwenhorst method, using 15, 3 and 2 points for the permanent, transitory and fixed effect components respectively. This methodology is specially suited for our case, given the high persistence of the process (see the discussion in Kopecky and Suen (2010)).

After age 65, agents receive social security payments, which we model to mimic the existing payment schedule in the U.S. We use the same function in Kaplan and Violante (2010) and Storesletten, Telmer, and Yaron (2004). For each simulation, we re-scale  $p(\cdot)$  so that the average replacement rate (pension payments over average life-cycle wages) equals 0.45 and compute a value for  $\tau$  so that the social security system is fully funded through a linear labor income tax.

<sup>19</sup>In our computations below, we do not find major differences between the *ex-post* or *ex-ante equivalization* strategies, so we show results only for the former. The results of these exercises are available on request.

Figure 1: Profiles for Household Size and Composition



Note: Both figures are constructed using data from the CPS, 1983-2003.

## 4.2 Family Structure

To compute profiles for family size and composition, we use the March supplements of the CPS for years 1984 to 2003. For each household, we count the number of adults (individuals age 17+) and the number of children: individuals age 16 or less who are identified as being the 'child' of an adult in the household. We compute two separate profiles: one for number of adults and one for number of children. As above, we run dummy regressions to extract life-cycle profiles, where the considered age is that of the head (irrespective of gender) and control for cohort and year effects. After extracting these life-cycle profiles, we smooth them using a cubic polynomial in age, and restrict the number of children to zero after age 60. The results of this procedure are in Figure 1.

## 5 Results

We solve the model by backwards induction using the endogenous grid method as described by Carroll (2006) and Barillas and Fernandez-Villaverde (2007). We then simulate ten thousand

life-cycles and compute aggregate statistics of the variables of interest.<sup>20</sup>

## 5.1 Per-Adult Equivalent Consumption Profiles

In the theoretical section, we showed that life-cycle profiles of per-adult equivalent consumption differ depending on the size of economies of scale and/or utility weights in the household. In this section we show the quantitative differences (we also present the log levels for these figures in Appendix C, for comparison purposes).

### *Demographics Model with $\delta = \phi$ vs. Single Agent Model*

In Figure 2, we show the implied profiles of life-cycle consumption (in logs, normalized by age 25 consumption) for the the *Single Agent* model and the *Demographics* model where  $\delta = \phi$ , i.e., the utility weight is the same as the equivalence scale. Note that both profiles exhibit a hump over the life-cycle, mimicking what is observed in US data and as shown by Deaton and Paxson (1994a), Fernández-Villaverde and Krueger (2007) and Aguiar and Hurst (2009) for example. From Section 2, we know that the latter two profiles are in general different from each other in the presence of income uncertainty and Figure 2 documents that these differences are quantitatively substantial.<sup>21</sup> The prediction for the per-adult equivalent consumption profile can be off by as much as fifty percent: the *Single Agent* model predicts an increase in per-adult equivalent life-cycle consumption of about 10% with respect to age 25, while the *Demographics* model implies an increase of about 15%.

If one believes that changes in household size should not alter the relative price of consumption over time, as done implicitly in the *Single Agent* model and explicitly in the *Demographics* model for the choice of  $\delta = \phi$ , we strongly recommend the use of the *Demographics* model, since in that model economies of scale change the transformation rate of per-adult equivalent consumption between two periods (via saving or borrowing) in the presence of income uncertainty. This is the source of differences between the two models.<sup>22</sup> Further, we could potentially find a  $\kappa$ , different

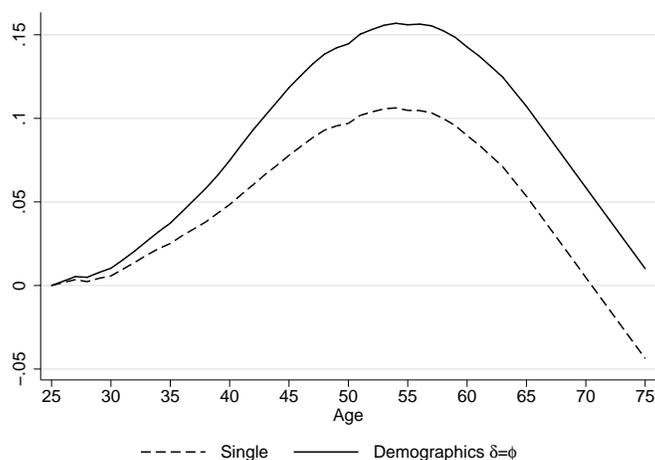
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<sup>20</sup>As discussed in the previous section, we fix the discount factor across models in an ad-hoc fashion ( $\beta(1+r) < 1$ ), in order to maintain comparability across models. In Appendix B, we perform a robustness exercise where ( $\beta(1+r) = 1$ ) and show that our results remain qualitatively unchanged.

<sup>21</sup>Proposition 2 is stricter than our quantitative exercise because it applies to individual consumption profiles as opposed to mean (log) profiles.

<sup>22</sup>In fact, if we remove income uncertainty, consumption profiles relative to age 25 of the *Single Agent* and *Demographics* models coincide

Figure 2: Per-Adult Equivalent Consumption (logs) Relative to Age 25



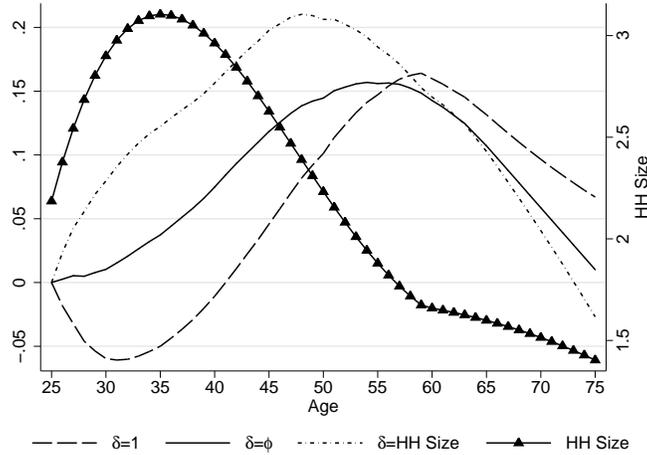
from  $\phi$ , for which the two per-adult equivalent consumption profiles are closer or even identical when there is uncertainty. However, as shown in Proposition 2 this adjustment factor for the *Single Agent* model depends on the consumption predictions by the *Demographics* model and thus would be an arbitrary adjustment and by no means be constant across different calibrations of the *Demographics* model (e.g. if  $\beta$  is varied) and thus not suitable in other applications.

### Different Specifications of the *Demographics* Model

Given the above discussion, in what follows we concentrate in different specifications of the *Demographics* model, their implications and predictions. The different choices of the utility weight imply different relative prices of per-adult equivalent consumption between two periods when household size and composition change. By inspecting Figure 3 (where we also plot household size levels), we see that the value of the utility weight is very important for determining the size and shape of per adult equivalent consumption profiles over the life-cycle. When  $\delta_t = 1$ , per adult equivalent consumption actually decreases in the early part of the life-cycle, tracking the increase in household size in the opposite direction. In this case, consuming when household size is large is only costly because it has to be shared among a lot of members which is however not valued by the household. When  $\delta_t$  equals household size  $N$ , the consumption profile tracks household size more closely, since households want to allocate more consumption to periods when their size is bigger.

Given our assumptions, in all the models depicted in the figure,  $\beta(1+r) < 1$ , which in con-

Figure 3: Per-Adult Equivalent Consumption (logs) Relative to Age 25



junction with the presence of uncertainty, creates a humped shaped consumption profile. As noted above, the steepness of these profiles earlier in life depend on the values of  $\delta$  and these patterns are robust to changes in the values of  $\beta$  and  $r$ , see Appendix B. Related to the steepness of the profiles, the different utility weights imply different incentives to delay consumption, and different ages at which per-adult equivalent consumption peaks. For  $\delta = N$ , consumption peaks around age 46; the opposite scenario is when  $\delta = 1$  and households want to delay consumption as much as possible to allocate it in periods with small households, which produce a peak age of consumption around 60. The intermediate case of  $\delta = \phi = \sqrt{N}$ , has a peak age at 55.

## 5.2 Insurance In Incomplete Markets

Our results show that particular parameterizations of the *Demographics* model have first order effects on the level and timing of per-adult equivalent consumption. In this section we ask a different question: by how much does the degree of consumption smoothing implicit in the calibrated models change across different specifications of household preferences?

Below we compute insurance coefficients for each model, a measure of how much consumption comoves with income shocks. The results in Kaplan and Violante (2010) suggest that the predictions from the standard incomplete markets model under natural borrowing constraints are close to the empirical estimates presented in Blundell, Pistaferri, and Preston (2008). In each model, we

calculate the following statistic

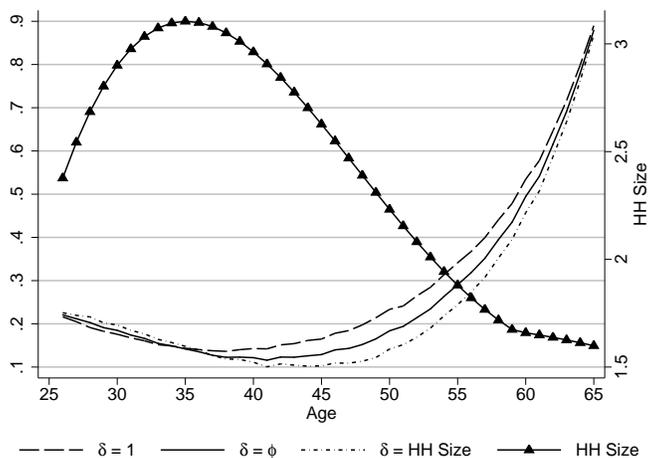
$$\psi_t^x = 1 - \frac{\text{cov}(\Delta \log(c_t/\phi_t), \epsilon^x)}{\text{var}(\epsilon^x)}$$

where  $c_t/\phi_t$  is per-adult equivalent consumption at age  $t$  and  $x = \{P, Tr\}$ , so we calculate the contemporaneous correlation of changes in consumption and permanent ( $P$ ) and transitory ( $Tr$ ) shocks to labor income. The intuition behind this formula is straight forward: the higher the comovement between consumption and unexpected changes in income (represented by the shocks  $\epsilon^x$ ) the lower the willingness to phase out these changes by the household through time and the lower the value of the insurance coefficient.

In Figure 4, we show insurance coefficients by age when we consider the permanent income shock. As a reference, we also plot the value of family size (right axis). The overall shape of these profiles is in line with findings by [Kaplan and Violante \(2010\)](#), where the increasing nature of this coefficient responds mainly to the accumulation of assets by agents over the life-cycle: with more assets, agents can better insulate their consumption when there are unexpected, permanent shocks to their labor income. From the figure we also see that the different *Demographics* models imply similar shapes for the profile of insurance coefficients, but markedly different levels of it. On the one hand, we have the case where  $\delta = 1$ , households do not value consumption much when they are numerous, producing higher asset accumulation early on, which in turn increases the ability to smooth consumption later in the life-cycle. When the utility weight is equal to the household size, we get the opposite effect: consumption is optimally allocated to periods when the size of the household is high, producing less savings and a lesser ability to self insure against permanent income shocks later in life.

Figure 5 shows the same exercise for the case of transitory income shocks. In this case, we find more pronounced differences between the different specifications of the *Demographics* model. In general, the insurance coefficients for transitory shocks are declining in age, because of the horizon effect, as pointed out by [Kaplan and Violante \(2010\)](#): the longer the remaining lifespan, the more time there is to receive offsetting shocks. However, the *Demographics* model introduces changing preferences for consumption across time which implies markedly different profiles of this insurance coefficient around the time household size peaks, resulting in an inverted U shape for this profile,

Figure 4: Insurance coefficients across models (permanent income shocks)

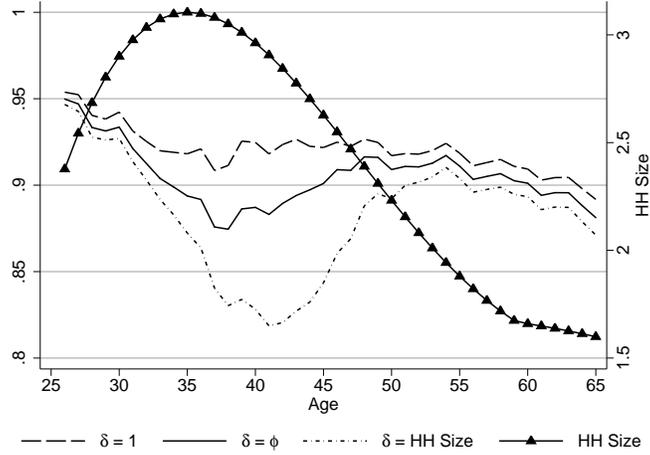


with a bigger 'inverted-hump' the higher the utility weight  $\delta$ . In particular, for  $\delta = N$ , households want to increase consumption early in life so that it syncs with the hump in household size. Given the increasing nature of profiles in earnings, they can only achieve this through borrowing earlier in life. This leads to temporary income shocks being not phased out around the time when household size peaks, since houses may be either hitting their borrowing constraint (in case of a negative shock) or consuming all the extra income (from positive shocks). Once household size starts to decrease, households are less 'desperate' to consume and smooth out temporary income fluctuations more.

### 5.3 Welfare Comparisons: An Example

Livshits, MacGee, and Tertilt (2007) (henceforth LMT) conduct a quantitative analysis of two different consumer bankruptcy arrangements: a US-style system where debtors can fully discharge their debt via bankruptcy without seizure of future earnings (labeled FS, for 'Fresh Start') and a continental European-style where bankruptcy does not discharge debt but only restructures a consumer's debt payments and limits the amount of (future) earnings that can be garnished (NFS, or 'No Fresh Start'). LMT show that the welfare comparison between the two bankruptcy regimes is sensitive both to the nature and magnitude of idiosyncratic shocks and to life-cycle considerations, in particular the life-cycle profile of household size. This latter aspect provides the link to our analysis.

Figure 5: Insurance coefficients across models (transitory income shocks)



The basic setup in LMT is the same as in the previous section: a life-cycle model where households face permanent and transitory income shocks, household size changes deterministically with age and the utility function is specified as in Equation (28) with  $\delta_t = 1$  and  $\phi_t \approx$  Square Root Scale.<sup>23</sup> However, in their setup, households have the option to default on household debt taking as given a bankruptcy rule.<sup>24</sup> The default option introduces a partial contingency of the otherwise noncontingent loans by giving households the possibility to lower the face value of their debt via bankruptcy. This provides a greater insurance against bad income and expenditure realizations and hence increases the household’s ability to smooth consumption across states. On the other hand, the limited ability to commit to future debt repayment limits the ability to smooth consumption over time.

The more generous bankruptcy option under FS (a fully discharge of all debt and no seizure of future earnings) provides more insurance against bad states but also comes along with tighter borrowing constraints which therefore allows less consumption smoothing over time. In the benchmark scenario, welfare (measured as the equivalent consumption variation) in the FS economy exceeds welfare relative to the NFS economy marginally (by 0.06%). As LMT phrase it: “This implies that

<sup>23</sup>LMT do not exactly use the Square Root scale but the mean scale from Fernández-Villaverde and Krueger (2007) which is very close to the Square Root Scale.

<sup>24</sup>Their model also introduces an additional iid expenditure shock (e.g., uninsured medical bills, divorce costs, or unplanned children) and financial intermediaries charge individual specific interest rates for debtors which depend on the household’s age, borrowing level and current (but not future) income. These interest rates are determined in equilibrium given an exogenous risk-free savings (set to 4%) interest rate and zero expected profits for financial intermediaries.

the benefits from increased smoothing across states outweigh the distortion of intertemporal credit markets” (p412). They show however, that the welfare comparison between the two bankruptcy regimes is sensitive both to the nature and magnitude of idiosyncratic shocks and to life-cycle considerations. In particular, in one experiment they keep household size constant over the life cycle. Implicitly, this corresponds to setting  $\delta_t = \phi_t^{1-\alpha} < 1$  in (28) as in both cases the (marginal) utility of consumption does not any longer depend on household size. As a consequence, the consumption allocated to periods when household size is large (the middle ages) is decreased and the household consumption profile becomes flatter, or put differently, consumption is smoothed more over time. This reverses the welfare comparison from the benchmark setup with  $\delta_t = 1$  and welfare is actually lower under FS (-0.24%) as the tighter borrowing constraints implied by the more generous default option become more important.<sup>25</sup>

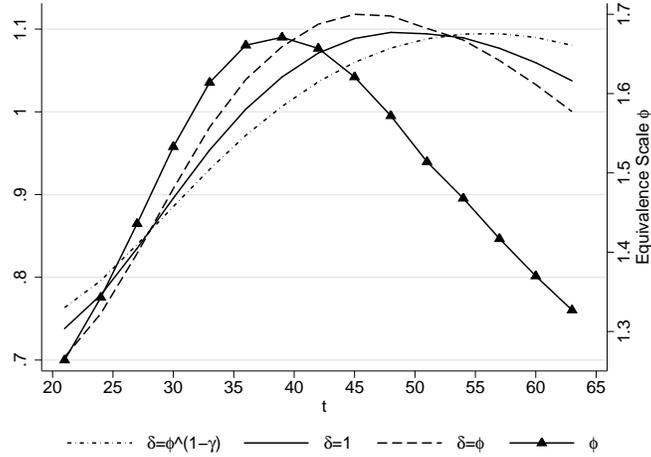
The following exercise underpins further that the choice of  $\delta_t$  is far from being innocuous. We simulate the model in LMT setting  $\delta_t$  equal to  $\phi_t$ . This increases the consumption allocated to periods when household size is large and reinforces the hump in life-cycle consumption, or put differently, consumption is smoothed less over time. Not surprisingly, this reinforces the benefits from FS: *welfare under this regime is now 0.21% larger than under NFS*. For completeness, Figure 6 shows the household consumption profiles for the three different choices of  $\delta_t$  along with equivalence scale used in LMT as a proxy for the evolution of household size.

At first glance, this exercise does not provide any new insights other than an additional quantification of the welfare comparison. The interesting aspect concerns two key model statistics discussed by LMT for the FS economy: these two are essentially the same independently of whether  $\delta_t$  is set to one or to  $\phi_t$ . In particular, the only moment targeted by LMT is a debt-earnings ratio in the US economy of 8.4%. Their benchmark model ( $\delta_t = 1$ ) yields a value of 8.42% and our extension ( $\delta_t = \phi_t$ ) of 8.37%.<sup>26</sup> Note that we used exactly the same technology and preference parameters as LMT for the model simulation with  $\delta_t = \phi_t$ . If we had recalibrated the model, the only non-externally calibrated parameter (the marginal rate of garnishment  $\gamma$ ) would have been essentially the same as in LMT and the welfare comparisons for a given  $\delta_t$  would be left unchanged. In some sense, the model is not really identified, which highlights the importance of the choice of  $\delta$ . We will

<sup>25</sup>It is important to note that LMT do not compare welfare between the two choices of  $\delta_t$  for a given bankruptcy regime, but compare welfare between the two bankruptcy regimes for a given  $\delta_t$ .

<sup>26</sup>Similarly, the default rate which LMT discuss in the context of the model’s performance is 0.709% and 0.704%, respectively. The corresponding values for  $\delta_t = \phi_t^{1-\alpha}$  are 8.8% and 0.711%.

Figure 6: Household Consumption Profiles from LMT for various  $\delta_t$



address this issue the next section.

## 6 Discussion

We have extensively documented that the choice of  $\delta_t$  may have stark quantitative implications for the degree of self-insurance and welfare in life-cycle models. This raises the question of which  $\delta_t$  to choose. Although evidence is scarce, [Attanasio, Banks, Meghir, and Weber \(1999\)](#) (henceforth ABMW) gives us some guidance. They introduce a general taste shifter to capture the impact of household size on the marginal utility of per-adult equivalent consumption (see Equation (4)). For a given coefficient of relative risk aversion, this specification of the utility function coincides with the one used in our exercise (Equation 28), if the following relationship holds:<sup>27</sup>

$$\exp(\zeta_1[N_{ad} - 1] + \zeta_2 N_{ch}) = \frac{\delta(N_{ad}, N_{ch})}{\phi(N_{ad}, N_{ch})^{1-\alpha}}. \quad (30)$$

ABMW log-linearize the Euler equation implied by Equation (4) to estimate the parameters  $\zeta_1$ ,  $\zeta_2$ , and  $\alpha$  from CEX data.<sup>28</sup> With those estimates at hand, one can back out the  $\delta_t$  that solves Equation (30) for a given household size and composition, and a given equivalence scale  $\phi_t$  (henceforth

<sup>27</sup>We normalize the taste shifter from ABMW to one for households of size one.

<sup>28</sup>The regression assumes that households are not borrowing constrained and the coefficient estimates are:  $\zeta_1 = 0.71$ ,  $\zeta_2 = 0.34$  and  $\alpha = 1.57$ .

labeled as  $\delta_{\text{ABMW}}$ ). We consider the seven equivalence scales discussed in [Fernández-Villaverde and Krueger \(2007\)](#).<sup>29</sup> Figure 7 plots the ratio of the two (with  $\delta_{\text{ABMW}}$  in the denominator) against the equivalence scales ordered from the lowest economies of scale (OECD) to the highest economies of scale (Nelson). Figure 7a considers the case of a two adults, two children household. To give a concrete example: for the OECD (NAS) scale  $\delta = \phi$  is 1.2 as large as (nearly identical to) the corresponding  $\delta$  implied by the ABMW estimates. Overall,  $\delta = \phi$  and  $\delta = N_{ad}$  (number of adults in the household) most closely resemble the  $\delta$  implied by the estimates in ABMW for the equivalence scales featuring low to medium economies of scale (OECD to DOC), whereas for the equivalence scales featuring high economies of scale (LM and Nelson)  $\delta = N_{ad} + N_{ch}$  (household size) stands out. Figure 7b shows the life-cycle averages over each ratio using the household size and composition profiles from our quantitative analysis in Section 5, i.e.  $\frac{1}{71} \sum_{t=25}^{95} \frac{\delta_t(N_{ad,t}, N_{ch,t})}{\delta_{\text{ABMW},t}(N_{ad,t}, N_{ch,t})}$ . Deviations from the ABMW estimates of  $\delta$  are smaller for all cases;  $\delta = N_{ad}$  delivers a close fit across all equivalence scales, whereas  $\delta = \phi$  produce better fits only for those scales featuring low to medium economies of scale.

There are two caveats with this comparison: first, it is based on the estimates for both the taste shifter parameters  $\zeta_1, \zeta_2$  and the coefficient of relative risk aversion  $\alpha$  (1.57). Second, if one believes in those estimates, it might be preferable to directly use the functional form assumption made in Equation (4). Apparently, this latter route has not been taken by the literature which preferred to use one of the choices for  $\delta$  discussed before; the coefficient of relative risk aversion is as well often set to another value, most prominently to 2. Nevertheless, this comparison may provide a guideline for the choice of  $\delta$  for a given equivalence scale. In particular, it points to using  $\delta = \phi$  (e.g. [Fuchs-Schündeln \(2008\)](#) or [Laitner and Silverman \(2008\)](#)) or  $\delta = N_{ad}$  (e.g. [Heathcote, Storesletten, and Violante \(2012\)](#)), as opposed to  $\delta_t = 1$  ([Livshits, MacGee, and Tertilt \(2007\)](#)) or  $\delta = N_{ad} + N_{ch}$  (as in [Domeij and Klein \(2011\)](#)).

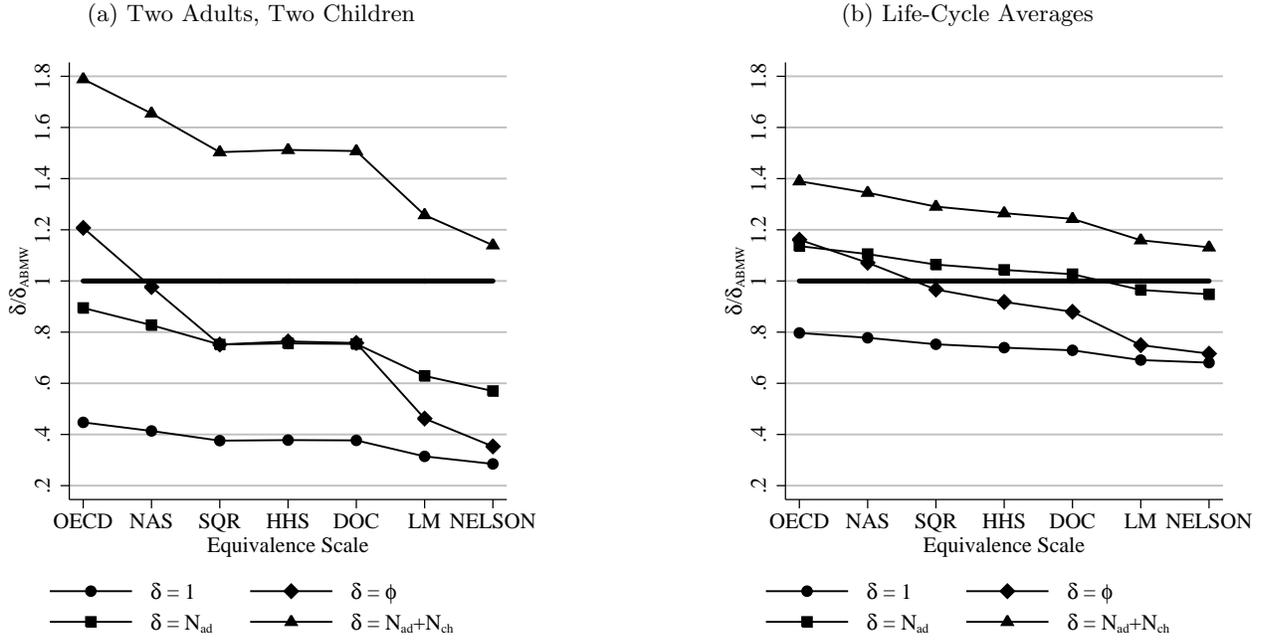
## 7 Conclusions

In this paper we study two alternative ways in which demographics are accounted for in life-cycle models of consumption and savings with incomplete markets. We critically compare the *Single*

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<sup>29</sup>We use the Square Root Scale instead of the 'Mean' scale in [Fernández-Villaverde and Krueger \(2007\)](#) as the two are almost identical.

Figure 7: Ratio  $\frac{\delta}{\delta_{ABMW}}$  for Various Utility Weights and Equivalence scales



*Agent* approach, which uses demographics 'exogenously' (as controls for estimated moments to match by the model) and our proposed *Demographics* model, where household size and composition affects explicitly optimal consumption choices.

Using a simple two period version of our proposed model, we show theoretically that *Single Agent* models produce in general different predictions of per-adult equivalent consumption than the *Demographics* case: agents in *Single Agent* models ignore the fact that the relative price of consumption across periods in which family size is changing might be affected by economies of scale and direct preferences over household size. When the *Demographics* model is specified such that this relative price does not change, the two approaches predict the same per-adult equivalent consumption profiles. However, this result breaks down if income uncertainty is introduced. This is because the presence of economies of scale in consumption alter the resources needed to provide insurance (in the sense of consumption smoothing) if household size changes over time.

In a quantitative exercise, based on a standard model of life-cycle consumption with income uncertainty and incomplete markets similar to the one in [Storesletten, Telmer, and Yaron \(2004\)](#), we first confirm that this latter mechanism is quantitatively important under the assumption that

in the *Demographics* model the relative price of consumption does not change with household size and conclude that in this case it is favorable to work with the *Demographics* instead of the *Single Agent* model. We then investigate the implications for different ad-hoc specifications of the *Demographics* model. For all the exercises, we find that model predictions depend crucially on the interaction between the degree of economies of scale in the household and how the utility of per-adult equivalent consumption of each household member is valued. For example, following the work by [Livshits, MacGee, and Tertilt \(2007\)](#) we show that Continental European-style bankruptcy laws are associated with higher welfare than the US-style bankruptcy law if households put little weight on each household member's utility and this result reverses as more weight is placed on each household member's utility.

Finally, we compare our setup with the preference structure estimated in [Attanasio, Banks, Meghir, and Weber \(1999\)](#) and discuss how to choose preference parameters for quantitative work. For the most commonly used equivalence scales, as e.g. the OECD scale, with low to medium economies of scale, the empirical estimates suggest that *Demographics* models with a utility weight equal to the equivalence scale or the number of adults in the household are closer to the data. Under these parameterizations, household size changes have - if at all - only a small effect on the relative price of consumption, i.e. in the absence of income uncertainty predictions of the *Demographics* model would be close to those of the *Single Agent* model. However, as we have already argued before, income uncertainty drives a quantitatively substantial wedge between the two models as the presence of economies of scale in consumption alter the amount of resources needed for providing insurance.

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# Appendix

## A Two Period Model

### A.1 Consumption Levels

**Proposition 3.** *Life-time per-adult equivalent consumption in the Demographics model and Single Agent model coincide only if  $\kappa_2 = 1 + (\phi_2 - 1) \frac{\mathbb{C}_2^D}{\mathbb{Y}_2}$ .*

This result is straightforward to show and based on pure accounting. Life-time per-adult equivalent consumption from the *Demographics* model is given by

$$\mathfrak{c}_D = \mathbb{C}_1^D + \frac{\mathbb{C}_2^D}{\phi_2} = \mathbb{C}_1^D + \overbrace{\frac{\mathbb{Y}_1 + \mathbb{Y}_2 - \mathbb{C}_1^D}{\phi_2}}{=\mathbb{C}_2^D} < \mathbb{C}_1^D + \mathbb{C}_2^D = \mathbb{Y}_1^D + \mathbb{Y}_2^D \quad (31)$$

while in the *Single Agent* model life-time per-adult equivalent consumption equals life-time per-adult equivalent income (see also Equation (6)):

$$\mathfrak{c}_S = c_1^S + c_2^S = y_1 + \frac{\mathbb{Y}_2}{\kappa_2}. \quad (32)$$

Equating Equations (31) and (32) yields the critical value of  $\kappa_2$  stated in Proposition 3. Note that for the case of  $\delta_2 = \phi_2$ , differences in life-time per-adult equivalent income map one to one into differences in per-adult equivalent consumption in each period since the per-adult equivalent consumption profiles are flat.

The intuition behind Proposition 3 can be illustrated best with a concrete example. Assume, household income is zero in the second period ( $\mathbb{Y}_2 = 0$ ), i.e. there is no choice to be made for  $\kappa_2$ , and positive in the first period ( $\mathbb{Y}_1 > 0$ ). In this case life-time per-adult equivalent income in the *Single Agent* model is  $\mathbb{Y}_1$  which by the budget constraint equals life-time per-adult equivalent consumption. For any utility function satisfying the Inada condition period two consumption will be positive such that in turn  $\mathbb{C}_1^D < \mathbb{Y}_1$  and  $\mathbb{C}_1^D + \frac{\mathbb{C}_2^D}{\phi_2} < \mathbb{C}_1^D + \mathbb{C}_2^D = \mathbb{Y}_1$ .

Alternatively, assume that household income is zero in the first period ( $\mathbb{Y}_1 = 0$ ), positive in the second period ( $\mathbb{Y}_2 > 0$ ) and  $\kappa_2 = \phi_2$ . In this case life-time per-adult equivalent income in the *Single Agent* model is  $\frac{\mathbb{Y}_2}{\phi_2}$  which by the budget constraint equals life-time per-adult equivalent consumption. In the *Demographics* model in turn, the household has the income  $\mathbb{Y}_2$  available for consumption. For any utility function satisfying the Inada condition period one consumption will be positive such that  $\mathbb{C}_2^D < \mathbb{Y}_2$ . Given that household size is one in period one, in the calculation of life-time per-adult equivalent consumption in the *Demographics* model only period two consumption is deflated by the equivalence scale. Since  $\mathbb{C}_2^D < \mathbb{Y}_2$ , 'less' in absolute terms is lost through the deflation by the equivalence scale in the calculation of life-time per-adult equivalent consumption in the *Demographics* model compared to the *Single Agent* model.<sup>30</sup>

Essentially, Proposition 3 is the implication of a pure accounting exercise. While looking at life-time per-adult equivalent consumption per se is potentially of less interest, this example highlights that even with a deterministic income the presence of economies of scale in consumption in the

<sup>30</sup>More formally, for  $\mathbb{Y}_1 = 0$  and  $\mathbb{Y}_2 > 0$ ,  $\mathbb{C}_2^D < \mathbb{Y}_2$  implies that  $\mathfrak{c}_D = \mathbb{Y}_2 - \frac{\phi_2 - 1}{\phi_2} \mathbb{C}_2^D > \mathbb{Y}_2 - \frac{\phi_2 - 1}{\phi_2} \mathbb{Y}_2 = \frac{\mathbb{Y}_2}{\phi_2} = \mathfrak{c}_S$ .

*Demographics* model or the adjustment of the budget constraint in the *Single Agent* model may drive a wedge between the two models. These differences in life-time per-adult equivalent consumption are also important in the presence of income heterogeneity. In the *Single Agent* model the timing of income matters as it determines life-time per-adult equivalent income. Even for the same life-time household income  $y_1^A + y_2^A = y_1^B + y_2^B$  but a different timing  $\frac{y_1^A}{y_2^A} \neq \frac{y_1^B}{y_2^B}$  life-time per-adult equivalent incomes differ in the *Single Agent* but not in the *Demographics* model. This implies an artificial inequality in life-time per-adult equivalent consumption in the *Single Agent* model that is not present in the *Demographics* model.

Note that the derivation and implications of Proposition 3 only depend on  $\delta_2$ , or  $\frac{\delta_2}{\phi_2}$ , in so far that it determines  $\mathbb{C}_1^D$  and thus, for a given  $\mathbb{Y}_1$  and  $\mathbb{Y}_2$ , the relationship between the two per-adult equivalent consumption levels.

## A.2 Proof of Proposition 2

$\mathbb{C}_1^{D*} = f(\mathbb{Y}_1, p_l, \mathbb{Y}_{2,l}, p_h, \mathbb{Y}_{2,h}, \phi_2)$  and  $c_1^{S*} = f(\mathbb{Y}_1, p_l, \mathbb{Y}_{2,l}, p_h, \mathbb{Y}_{2,h}, \kappa_2)$  denote the optimal period one consumption in the *Demographics* and *Single Agent* model and the optimal period two consumption allocations implied by the respective budget constraints as well with \*. The per-adult equivalent consumption profiles in the two approaches can only be the same if the optimal allocations satisfy the following condition

$$c_1^{S*} = \eta \mathbb{C}_1^{D*} \text{ and } c_{2,i}^{S*} = \eta \frac{\mathbb{C}_{2,i}^{D*}}{\phi_2} \forall i = l, h \text{ with } \eta > 0. \quad (33)$$

For any strictly concave utility function, condition (33) implies that the Euler equation in the *Single Agent* model holds for  $c_1^{S*}$  if the Euler equation in the *Demographics* model holds for  $\mathbb{C}_1^{D*}$ . This allocation in the *Single Agent* model can however only constitute an optimum if for the low and high income shock the respective budget constraint

$$c_1^{S*} + c_{2,i}^{S*} = \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \forall i = l, h \quad (34)$$

holds with equality, i.e. the allocation is feasible and no resources are wasted. Replacing condition (33) into the budget constraints of the *Single Agent* model (34) yields after some reformulations

$$\mathbb{C}_1^{D*} = \frac{1}{\phi_2 - 1} \frac{1}{\eta} [(\phi_2 - \eta) \mathbb{Y}_1 + (\phi_2 - \eta \kappa_2) \mathbb{Y}_{2,i}] \forall i = l, h. \quad (35)$$

It is important to stress that  $\mathbb{C}_1^{D*}$  has to be taken as an exogenous constant in Equation (35). Equation (35) has to hold for the low and high period two income realization which can only be the case if

$$\eta = \frac{\phi_2}{\kappa_2}. \quad (36)$$

Combining Equations (33) and (36) gives the relationship between the optimal consumption allocations in the two models for which the per-adult equivalent consumption profiles are the same:

$$\kappa_2 c_1^{S*} = \phi_2 \mathbb{C}_1^{D*} \text{ and } \kappa_2 c_{2,i}^{S*} = \mathbb{C}_{2,i}^{D*} \forall i = l, h. \quad (37)$$

Specifically for the second period, Equation (37) is directly interpretable: household consumption in the *Demographics* model ( $C_{2,i}^{D*}$ ) needs to equal *household equivalent* consumption in the *Single Agent* model, i.e. per-adult equivalent consumption multiplied by the factor used to obtain per-adult equivalent income ( $\kappa_2 c_{2,i}^{S*}$ ). Even more importantly, under condition (36) we can solve Equation (35) for the equivalence scale for income  $\kappa_2$  for which the per-adult equivalent consumption in the two approaches are the same:

$$\kappa_2 = 1 + (\phi_2 - 1) \frac{C_1^{D*}}{Y_1}. \quad (38)$$

### A.2.1 Example

We have considered the following specific example:  $p_l = p_h = 0.5$ ,  $Y_{2,l} = 1 - \epsilon$  and  $Y_{2,h} = 1 + \epsilon$ ,  $\kappa_2 = \phi_2$  and  $C_1^{D*} = Y_1$ . This implies that the per-adult equivalent consumption allocations are identical in both models, see the argument in the main text. We first derive the response of period one consumption in both models with respect to an increase in income uncertainty ( $\epsilon$ ):

$$\frac{\partial C_1^D}{\partial \epsilon} = Y_2 \frac{u''(C_{2,h}^D/\phi_2) - u''(C_{2,l}^D/\phi_2)}{2\phi_2 u''(C_1^D) + \sum_{i=l,h} u''(C_{2,i}^D/\phi_2)} \quad (39)$$

$$\frac{\partial c_1^S}{\partial \epsilon} = Y_2 \frac{u''(c_{2,h}^S) - u''(c_{2,l}^S)}{2\phi_2 u''(c_1^S) + \phi_2 \sum_{i=l,h} u''(c_{2,i}^S)} \quad (40)$$

Since period two consumption for the high income shock realization is larger than for the low income realization and  $u'' < 0$ , both derivatives are negative. This in turn implies the claim made in the main text (compare condition 14) that for

$$C_1^{D*} = c_1^{S*} \rightarrow \frac{\partial C_1^D}{\partial \epsilon} < \frac{\partial c_1^S}{\partial \epsilon} < 0 \text{ or } \left| \frac{\partial C_1^D}{\partial \epsilon} \right| > \left| \frac{\partial c_1^S}{\partial \epsilon} \right|.$$

For the consumption choices used in this example, i.e.  $C_1^{D*} = c_1^{S*}$ , the derivatives of the per-adult equivalent consumption profiles w.r.t. to income uncertainty have the following properties. The per-adult equivalent consumption profile for the high income shock will be steeper for the *Demographics* model if the decrease in period one consumption is sufficiently large:

$$\frac{\partial \frac{C_{2,h}^D/\phi_2}{C_1^D}}{\partial \epsilon} > \frac{\partial \frac{c_{2,h}^S}{c_1^S}}{\partial \epsilon} \text{ if } \left| \frac{\partial C_1^D}{\partial \epsilon} \right| > \left| \frac{\partial c_1^S}{\partial \epsilon} \right| \cdot \left( 1 + (\phi_2 - 1) \frac{Y_1}{Y_1 + Y_2(1 + \epsilon)} \right). \quad (41)$$

Similarly, the per-adult equivalent consumption profile for the low income shock will be steeper (in absolute terms) for the *Demographics* model if the decrease in period one consumption is not too large:

$$\frac{\partial \frac{C_{2,l}^D/\phi_2}{C_1^D}}{\partial \epsilon} < \frac{\partial \frac{c_{2,l}^S}{c_1^S}}{\partial \epsilon} \text{ if } \left| \frac{\partial C_1^D}{\partial \epsilon} \right| < \left| \frac{\partial c_1^S}{\partial \epsilon} \right| \cdot \left( 1 + (\phi_2 - 1) \frac{Y_1}{Y_1 + Y_2(1 - \epsilon)} \right). \quad (42)$$

To summarize, if it is initially optimal to consume income in each period, an increase in income uncertainty makes the households to reduce period one consumption and to become savers. The household in the *Demographics* model decreases consumption more than the household in the *Single Agent*

*gle Agent* model. If further conditions (41) and (42) are met, the *Demographics* model displays steeper per-adult equivalent consumption profiles than the *Single Agent* model. Decreasing  $\kappa_2$  to the critical value stated in Proposition 2 increases effective income uncertainty in the *Single Agent* model: this permits less consumption smoothing and hence results in steeper per-adult equivalent consumption profiles (in absolute terms) and thus reestablishes the identity of the per-adult equivalent consumption profiles in the *Demographics* and *Single Agent* model. Clearly, without further specifying the environment in more detail, i.e. a concrete choice of a utility function and income, it is not possible to state whether conditions (41) and (42) are met, individually or jointly.

## B A Different Calibration

In this section we present results for the case when we the discount factor is exactly  $1/(1+r)$  where we keep  $r = 2\%$ . As seen from Figure 8, this calibration of the model leads to the disappearance of the hump in life-cycle consumption, steeper profiles and a smaller gap between the *Single Agent* and *Demographics* model with  $\delta = \phi$ . The same pattern can be found for the different specifications of the *Demographics* model, see Figure 9. As for the implied insurance coefficients, Figures 10 and 11 show that the qualitative findings in the main text remain, but the differences across *Demographics* models shrinks. This comes naturally from the fact that agents accumulate more savings in these economies, given the higher discount factors.

Figure 8: Per-Adult Equivalent Consumption (logs) Relative to Age 25,  $\beta(1+r) = 1$

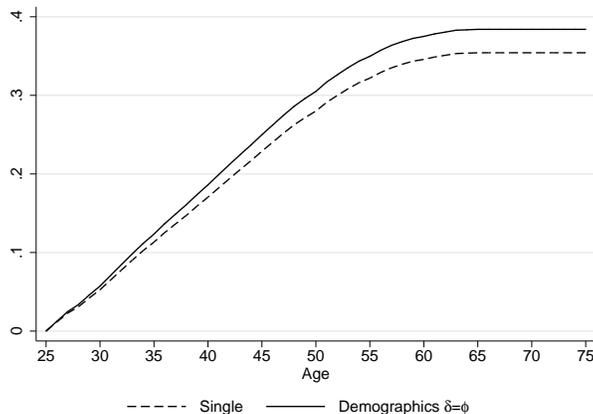


Figure 9: Per-Adult Equivalent Consumption (logs) Relative to Age 25,  $\beta(1+r) = 1$

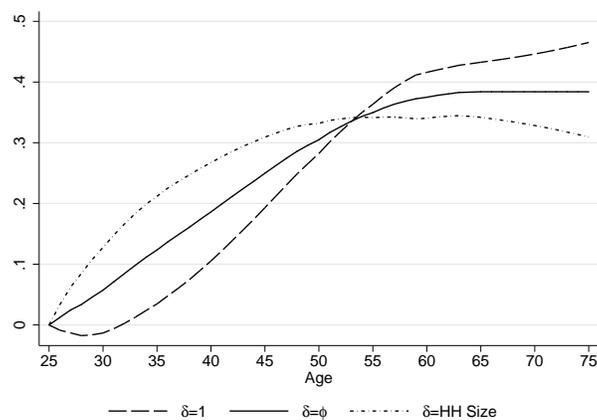


Figure 10: Insurance coefficients across models (permanent income shocks),  $\beta(1+r) = 1$

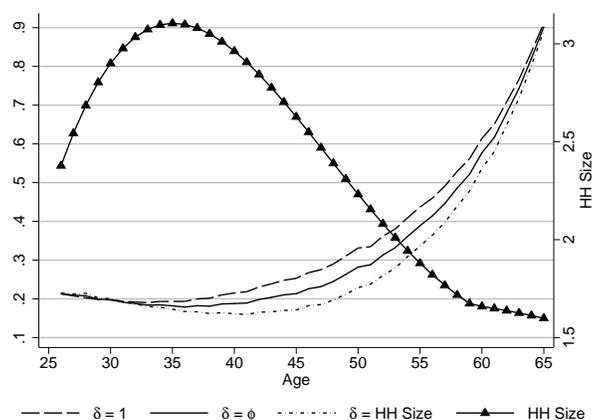
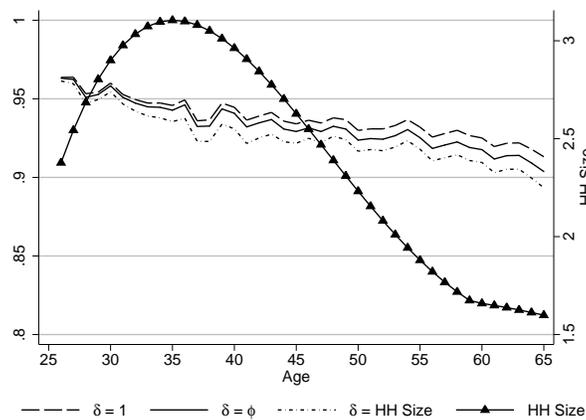


Figure 11: Insurance coefficients across models (transitory income shocks),  $\beta(1+r) = 1$



## C Per Adult Equivalent versus Household Consumption

Figure 12: Per-Adult Equivalent Consumption (logs) Relative to Age 25 (logs)

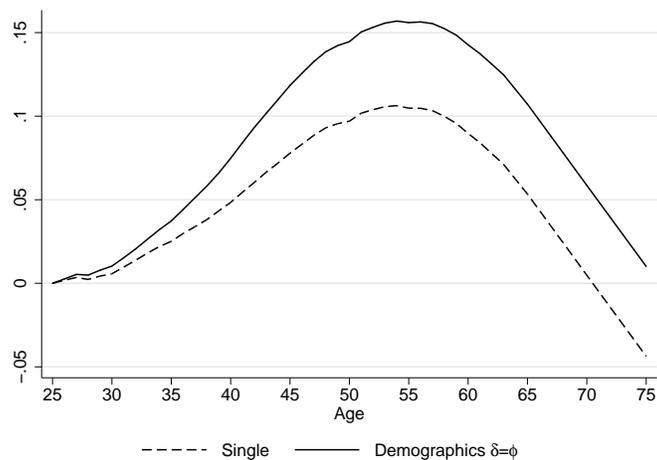


Figure 13: Per-Adult Equivalent Consumption (logs)

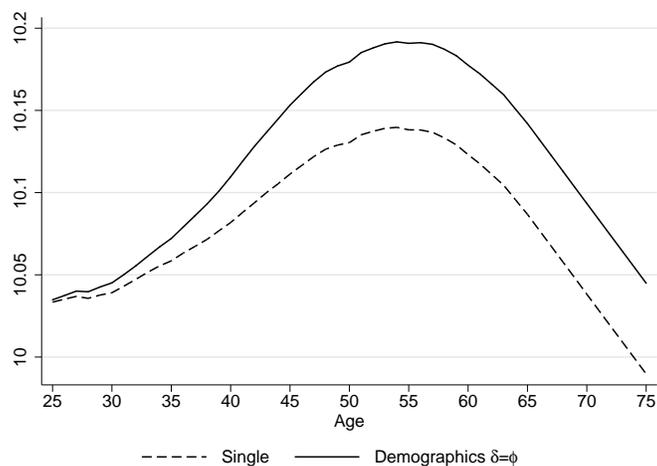


Figure 14: Per-Adult Equivalent Consumption (logs) Relative to Age 25 (logs)

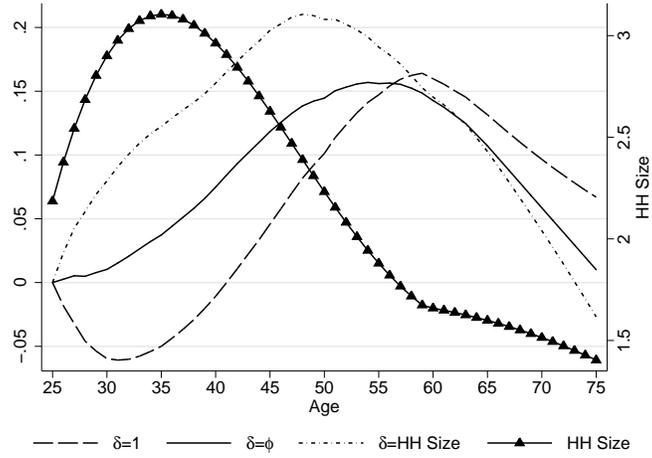


Figure 15: Per-Adult Equivalent Consumption (logs)

