# Shopping hours and price competition 

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#### Abstract

This paper develops an argument why retail prices may rise in response to the deregulation of opening hours. We make this point in a model of imperfect duopolistic competition. In a deregulated market retailers view the choice of opening hours as a means to increase the degree of perceived product differentiation thus relaxing price competition. If the consumers' preference intensity for time is sufficiently high the equilibrium configuration has asymmetric shopping hours where one retailer stays open for longer than the other. Both retailers charge higher prices than under regulation, and both are strictly better off.


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## 1. Introduction

This paper studies the impact of deregulation of opening hours on price competition at the retail level. Deregulation refers to a change in the legal prescriptions allowing retailers to stay open for longer. ${ }^{3}$ We start from the observation that the retailers'

[^0]choice of opening hours has an important impact on the consumers' decision where to shop. A retailer staying open for longer than its rival attracts additional demand as some consumers find it more convenient to shop at a time when the rival outlet is closed. More importantly, as consumers have a preferred shopping time, outlets selling physically identical products may want to open at different times of the day in order to relax price competition. Thus, the choice of shopping hours may be viewed as a means to increase the degree of perceived product differentiation in the retail market. Could deregulation then lead to higher prices?

The existing theoretical literature on the short-term effects of a deregulation of shopping hours on prices has developed ambiguous predictions on how prices respond. ${ }^{4}$ Clemenz (1990) emphasizes the role of search for equilibrium prices under imperfect information. Deregulation may lead to overall lower prices as longer shopping hours facilitate the comparison of prices. In Morrison and Newman (1983) and Tanguay et al. (1995) prices increase at large stores and fall at small stores. The authors associate small stores with low access time and large stores with high access time. A large store has to charge a lower price in order to attract demand. Deregulation implies a fall in the value consumers attach to access time such that the large stores' locational disadvantage becomes less pronounced. Demand shifts in favor of large stores that increase their price. At small retailers, prices go down.

These two approaches do not treat opening hours as a strategic variable among competitors. In contrast, our paper tackles the question of how prices respond to deregulation in a framework which endogenizes the choice of opening hours. This links the effect of deregulation on price competition to the question of how retailers' choice of opening hours responds to it. Observed price levels after deregulation are then to be interpreted as part of an equilibrium choice of opening hours.

We study this question in a spatial model of duopolistic retail competition with "space" and "time" as dimensions of horizontal product differentiation. We consider two established retailers with given spatial locations. Following deregulation these retailers can choose between three regimes of shopping hours: Daytime, nighttime, or around the clock. For this setting we derive the following important result: If consumers attach great value to time the equilibrium configuration exhibits asymmetric shopping hours. While one retailer uses the additional freedom to open around the clock, the other retailer is active only during daytime. As we abstract from costs incurred when operating longer hours (shift costs), this result is only driven by the retailers' incentive to mitigate price competition. An asymmetric choice ensures that demand is less responsive to price changes. The point of our analysis is that this logic may be stronger than the intuitive argument according to which an outlet must lose when its competitor benefits from a locational advantage. By considering differing (linear) input costs, we are also able to show that the retailer with a cost advantage is more likely to open around the clock.

Recent empirical studies show that, indeed, not all outlets use the additional leeway after deregulation. For Germany, Halk and Träger (1999) compare reported opening

[^1]hours in July and August 1998 to those before deregulation became effective in November 1996. They find that only $39 \%$ of all outlets were open for longer in 1998. This suggests the presence of asymmetric configurations in local markets for which our model provides a rationale. ${ }^{5}$ Moreover, in 1998 larger stores were more likely to stay open for longer than small retailers who rather maintained their old time schedule. Our results are consistent with this regularity, too. Interpreting differing input costs as reflecting shop size the larger outlet is more likely to open longer. However, a surprising implication of our analysis is that the small outlet nevertheless gains from deregulation.

Our model of two-dimensional product differentiation is in the tradition of the pioneering work of Economides (1989), Neven and Thisse (1990), and Tabuchi (1994). From a technical point of view our study extends and complements this literature in at least two respects. As this class of models has become one of the workhorses of the industrial economics of multi-dimensional product differentiation, we feel that our analysis and results are of interest beyond our application to shopping hours.

The first and major novelty is a piecewise uniform distribution of consumer preferences with respect to time. This is meant to capture the empirical fact that most consumers have a preference for daytime shopping. The second novelty concerns the characterization of a product variant. Whereas the literature on multi-dimensional product differentiation represents a product as a point in a given characteristics space we allow retailers to choose their hours of business from a menu of convex time intervals. Hence, a retailer's product variant is characterized by a point in geographical space and an interval in the time space. ${ }^{6}$

The remainder of this paper is organized as follows. The model is presented in Section 2 . Section 3 studies equilibrium shopping hours in the regulated and the deregulated market. Here, we derive our main result concerning the implications of an asymmetric configuration of shopping hours for price competition. Section 4 introduces cost heterogeneity. Section 5 concludes with some tentative remarks on welfare.

## 2. The model

Consider two dimensions of horizontal product differentiation where the first dimension represents space and the second is time. We may think of the spatial dimension as "Main Street" and of the time dimension as representing 24 hours of business. This suggests an intuitive geometry for our characteristic space: A line à la Hotelling (1929) to capture space and a circle in the spirit of Salop (1979) to model time. ${ }^{7}$

[^2]There are two retailers $i=A, B$. Their location determines the product variants in the market. We denote retailer $A$ 's location by a vector $\mathbf{a}:=\left(a_{1}, a_{2}\right)$ and retailer $B$ 's location by $\mathbf{b}:=\left(b_{1}, b_{2}\right)$. The vectors' first component characterizes the respective retailer's position in space and the second component its position in time. The retailers' location in space is exogenously given at $a_{1}=0$ and $b_{1}=1$, respectively; their location in time is endogenous.

A circle of circumference 2 represents the time of the day. We denote noon by 0 (and 2), 6 p.m. by $\frac{1}{2}$, midnight by 1 , and 6 a.m. by $\frac{3}{2}$. Moreover, we call the arc stretching from $\frac{3}{2}$ to $\frac{1}{2}$ the daytime interval, $\bar{I}$, and the arc stretching from $\frac{1}{2}$ to $\frac{3}{2}$ the nighttime interval, I.

The decision of retailers when to open is a discrete choice between four options: not to open at all, to open either during the daytime or the nighttime interval, and to open during both intervals. Denote the options to stay closed by 0 and to open around the clock by $I=\bar{I} \cup I I$. Then, the possible locations of both retailers in the time dimension are $a_{2}, b_{2} \in\{0, \underline{I}, \bar{I}, I\}$.

Extending shopping hours does not lead to higher (operating) costs. This assumption is made to abstract from such cost issues that are commonly made responsible for our main effects: price increase after deregulation and longer shopping hours particularly at larger shops. In Section 3, we consider retailers with identical marginal cost per unit sold. Without loss of generality this cost is normalized to zero. We denote $R_{i}:=p_{i} D_{i}$ the profit of retailer $i$ where $p_{i}$ and $D_{i}$ are the respective retailer's price and demand. Section 4 introduces differing marginal costs.

There is a continuum of consumers. Without loss of generality we normalize total population to 2 . A consumer is characterized by her address $\mathbf{z}:=\left(z_{1}, z_{2}\right)$ indicating her geographical location $z_{1} \in[0,1]$ on main street and her most preferred shopping time $z_{2} \in[0,2]$ on the circle. Addresses are independently distributed over $[0,1] \times[0,2]$. The distribution is uniform with respect to space and piecewise uniform with respect to time. The mass $1 \leqslant 2 K \leqslant 2$ is uniformly distributed over the daytime interval $\bar{I}$ whereas the remaining mass $2(1-K)$ is uniformly distributed over the nighttime interval $\underline{I}$. Thus, a weak majority prefers daytime shopping.

Consumers have a conditional indirect utility function $V_{i}(\mathbf{z}), i=A, B$. A consumer buying at $A$ has utility equal to (a similar expression holds for a consumer purchasing from retailer $B$ )

$$
V_{A}(\mathbf{z}):=S-p_{A}-t_{1} z_{1}-t_{2} \operatorname{dist}\left(z_{2}, a_{2}\right)
$$

where $S$ denotes the gross surplus all consumers enjoy from either variant and $p_{A}$ is the price charged by retailer $A$. The parameter $t_{1}$ has the usual interpretation as the transport cost per unit of distance to make a return trip to a shop. Given the outlets' location on main street $z_{1}$ measures the geographical distance to be covered when purchasing at $A$.

The parameter $t_{2}$ stands for the salience coefficient associated with the time dimension. The term $\operatorname{dist}\left(z_{2}, a_{2}\right)$ measures the closest distance between a consumer located at $z_{2}$ and the time interval during which outlet $A$ stays open. Obviously, this distance is equal to zero when $z_{2}$ is part of this time interval. Otherwise it


Fig. 1. The unfolded characteristic space.
is the absolute value of the difference between $z_{2}$ and the closest boundary of this interval. ${ }^{8}$

Consumers have unit demands. Moreover, $S$ is assumed to be large enough to exclude aggregate demand effects. The demand for variant $A$ is then defined by the mass of consumers for whom variant $A$ is weakly preferred to $B$, i.e., $V_{A}(\mathbf{z}) \geqslant V_{B}(\mathbf{z})$.

The combined characteristic space may be viewed as a right cylinder with the unit segment as its altitude, retailer $A$ located on the lower rim, and retailer $B$ on the upper rim. Suppose, we cut open the cylinder at "midnight" and unfold it so that the morning hours ( $0 \mathrm{a} . \mathrm{m}$. until noon) appear to the left and the afternoon hours (noon until 12 p.m.) appear to the right. Our assumption that consumers are symmetrically distributed around noon implies that the characteristic space can be subdivided into two identical subspaces: $[0,1] \times[1,2]$ ("competition before noon") and $[0,1] \times[0,1]$ ("competition after noon"). This is illustrated in Fig. 1.

Retailers are constrained to charge a single price, regardless of the shopping time. ${ }^{9}$ Given the symmetry before and after noon retailers' profit-maximizing behavior in one subspace will mimic the behavior in the other. Therefore, we solve the model as if there was only "competition after noon". The relevant subspace is the unit square $[0,1] \times[0,1]$ with mass 1 of consumers. To fix ideas, we consider the unit square in a position such that its horizontal dimension represents space and its vertical dimension time. Outlet $A$ is located at the left endpoint of main street and $B$ at the right endpoint. The daytime interval corresponds to $\left[0, \frac{1}{2}\right]$ and contains mass $K$, the nighttime interval is $\left[\frac{1}{2}, 1\right]$ and has mass $1-K$ (see Fig. 2).

Retailers choose their opening hours in view of its impact on price competition. This translates into a two-stage game. At the first stage retailers simultaneously determine their location in time. After having observed these locations they simultaneously compete in prices at stage two.

[^3]

Fig. 2. The right square.

## 3. Equilibrium shopping hours

### 3.1. The regulated market

Regulation restricts the retailers' location choice to $a_{2} \in\{0, \bar{I}\}$ and $b_{2} \in\{0, \bar{I}\}$. The indifferent consumer location for which $V_{A}(\mathbf{z})=V_{B}(\mathbf{z})$ satisfies

$$
\begin{equation*}
p_{A}+t_{1} z_{1}+t_{2} \operatorname{dist}\left(z_{2}, a_{2}\right)=p_{B}+t_{1}\left(1-z_{1}\right)+t_{2} \operatorname{dist}\left(z_{2}, b_{2}\right) \tag{1}
\end{equation*}
$$

Consider the configuration $a_{2}=b_{2}=\bar{I}$. Consumers with a preference for daylight shopping $\left(0 \leqslant z_{2} \leqslant \frac{1}{2}\right)$ buy at their preferred moment in time and do not incur a disutility in the time dimension as $z_{2} \in \bar{I}$. Accordingly, (1) becomes $p_{A}+t_{1} z_{1}=p_{B}+$ $t_{1}\left(1-z_{1}\right)$ and consumers indifferent between shopping at $A$ or $B$ are located at

$$
\begin{equation*}
\hat{z}_{1}\left(p_{A}, p_{B}\right):=\min \left\{1, \max \left\{0, \frac{1}{2}+\frac{p_{B}-p_{A}}{2 t_{1}}\right\}\right\} \tag{2}
\end{equation*}
$$

Next consider consumers with $\frac{1}{2} \leqslant z_{2} \leqslant 1$. In order to minimize the disutility incurred with respect to the time dimension they buy at $6 \mathrm{p} . \mathrm{m}$. and incur a disutility equal to $t_{2}\left(z_{2}-\frac{1}{2}\right)$ independently of whether they shop at $A$ or $B$. The latter term cancels out on both sides of (1) so that we obtain an indifferent consumer location as in (2).

Hence, demand for $A$ is given by

$$
D_{A}\left(p_{A}, p_{B}\right)= \begin{cases}0 & \text { if } p_{A}-p_{B}>t_{1} \\ \hat{z}_{1} & \text { otherwise } \\ 1 & \text { if } p_{A}-p_{B}<\left(-t_{1}\right)\end{cases}
$$

while $D_{B}\left(p_{A}, p_{B}\right)=1-D_{A}\left(p_{A}, p_{B}\right)$. This coincides with the standard one-dimensional Hotelling model. We thus get the following well-known results. ${ }^{10}$

Proposition 1. With regulated shopping hours we obtain equilibrium prices $p_{A}^{\mathrm{s}}=p_{B}^{\mathrm{s}}=t_{1}$ and equilibrium profits $R_{A}^{\mathrm{s}}=R_{B}^{\mathrm{s}}=t_{1} / 2$.

### 3.2. The deregulated market

In what follows, we first characterize the outcome of all symmetric and asymmetric location configurations. As to the latter we confine attention to very large values of $K$. More precisely, we focus on values of $K$ in the neighborhood of $K=1$ so that only very few consumers prefer nighttime shopping. ${ }^{11}$ Subsequently, we turn to the study of equilibrium configurations.

### 3.2.1. Symmetric location configurations

Following deregulation two more symmetric configurations may arise. First, if $a_{2}=$ $b_{2}=I$ both retailers open for 24 hours. As no consumer incurs a disutility in the time dimension the latter drops out on both sides of (1) and demand functions coincide with those under regulation. Second, if $a_{2}=b_{2}=\underline{I}$ both retailers open during the nighttime interval. This configuration is the mirror image of the case under regulation. We thus have the following result.

Proposition 2. The outcome of any symmetric location configuration $a_{2}=b_{2}$ where both shops are open is unique and coincides with the one under regulation.

The intuition is straightforward. Under symmetry outlets open during the same time interval and product variants are perceived as homogeneous with respect to time. As a consequence, retailers' demands coincide for any symmetric configuration. Equilibrium prices only reflect competition along the geographical dimension.

### 3.2.2. Asymmetric location configurations

Our focus on a neighborhood of $K=1$ implies that only those asymmetric configurations constitute a reasonable candidate for an asymmetric location equilibrium that involve one retailer to open around the clock and the other to open during the daytime. Keeping in mind that there are two such configurations let $A$ be the retailer that opens around the clock, i.e. consider the configuration with $a_{2}=I$ and $b_{2}=\bar{I}$. In what follows we derive the corresponding demand functions and prove the existence of a unique price equilibrium. By symmetry these results hold for the case $a_{2}=\bar{I}$ and $b_{2}=I$, too.

[^4]

Fig. 3. Case 1 (left) and Case 2 (right).

Demand: Consumers with a preference for nighttime shopping ( $z_{2} \geqslant \frac{1}{2}$ ) appreciate the additional hours of business offered by $A$. Intuitively, these consumers may want to benefit from the advantage to incur zero disutility in the time dimension when purchasing at $A$ even at the expense of higher transportation costs. At a given pair of prices, $A$ 's demand will therefore rise. This materializes in a shift of the marginal consumer location in this market segment as (2) now becomes $p_{A}+t_{1} z_{1}=p_{B}+t_{1}(1-$ $\left.z_{1}\right)+t_{2}\left(z_{2}-\frac{1}{2}\right)$, yielding the marginal consumer location

$$
\begin{equation*}
\hat{z}_{2}\left(p_{A}, p_{B}, z_{1}\right):=\min \left\{1, \max \left\{\frac{1}{2}, \frac{p_{A}-p_{B}-t_{1}+t_{2} / 2}{t_{2}}+\frac{2 t_{1}}{t_{2}} z_{1}\right\}\right\} . \tag{3}
\end{equation*}
$$

The additional option to shop at $A$ during the nighttime interval does not affect the decision where to shop of those consumers with a preference for daytime shopping. The location of the marginal consumer in this market segment is therefore unaffected. As a consequence, the boundary between the demand for $A$ and $B$ is $\hat{z}_{1}$ of (2) for $z_{2} \leqslant \frac{1}{2}$ and $\hat{z}_{2}$ of (3) for $z_{2} \geqslant \frac{1}{2}$. How does this modification affect the functional form of demand? Our focus on the neighborhood of $K=1$ allows to restrict attention to pairs of prices for which $p_{A}-p_{B} \leqslant t_{1}$. If this condition does not hold then $\hat{z}_{1}=0$ and retailer $A$ serves only consumers located on $z_{2} \geqslant \frac{1}{2}$. Intuitively, such a price setting behavior is not optimal when only very few consumers have a preference for nighttime shopping.

Fig. 3 illustrates the two cases that may arise. They differ as in Case $1\left(p_{A}, p_{B}, t_{1}, t_{2}\right)$ is such that the marginal consumer segment $\hat{z}_{2}($.$) and the z_{2}=1$ locus intersect to the left of or at the upper right corner whereas in Case $2 \hat{z}_{2}($.$) intersects the z_{1}=1$ locus below that corner.

Case 1: Suppose that $p_{A}-p_{B} \geqslant t_{2} / 2-t_{1}$. Then the demands of both retailers are

$$
\begin{equation*}
D_{A}=\hat{z}_{1}\left(p_{A}, p_{B}\right)+(1-K) \frac{t_{2}}{8 t_{1}} \quad \text { and } \quad D_{B}=1-D_{A} . \tag{4}
\end{equation*}
$$

Compared to the symmetric configuration $A$ attracts more consumers at a given pair of prices. Indeed, it is easy to see that $t_{2} / 8 t_{1}$ is the surface of the shaded triangle in

Fig. 3. It contains those consumers for which it is now advantageous to buy at $A$ in spite of higher transport cost. The corresponding mass is $(1-K) t_{2} / 8 t_{1}$.

Case 2: Let prices satisfy $t_{2} / 2-t_{1}>p_{A}-p_{B} \geqslant-t_{1}$. Then, a simple geometrical argument shows that

$$
\begin{equation*}
D_{B}=K\left(1-\hat{z}_{1}\left(p_{A}, p_{B}\right)\right)+(1-K)\left(1-\hat{z}_{1}\left(p_{A}, p_{B}\right)\right)^{2} \frac{2 t_{1}}{t_{2}} \text { and } D_{A}=1-D_{B} \tag{5}
\end{equation*}
$$

Clearly, $D_{B}$ is a weighted average of those consumers located on $z_{2} \leqslant \frac{1}{2}$ and $z_{2} \geqslant \frac{1}{2}$.
Price Equilibrium: The following proposition characterizes the unique price equilibrium.

Proposition 3. Suppose $a_{2}=I$ and $b_{2}=\bar{I}$ and that $K$ is sufficiently close to one. Then we have a unique price equilibrium $\left(p_{A}^{*}, p_{B}^{*}\right)$.

If $t_{2} \leqslant 2 t_{1}$, Case 1 applies and

$$
\begin{equation*}
p_{A}^{*}=t_{1}+\frac{(1-K) t_{2}}{12} \quad \text { and } \quad p_{B}^{*}=t_{1}-\frac{(1-K) t_{2}}{12} \tag{6}
\end{equation*}
$$

If $t_{2}>2 t_{1}$, Case 2 applies and

$$
\begin{align*}
& p_{A}^{*}=2 t_{1} \frac{1-K\left(1-\hat{z}_{1}\right)-2(1-K)\left(1-\hat{z}_{1}\right)^{2} t_{1} / t_{2}}{K+4(1-K)\left(1-\hat{z}_{1}\right) t_{1} / t_{2}}  \tag{7}\\
& p_{B}^{*}=2 t_{1}\left(1-\hat{z}_{1}\right) \frac{K+2(1-K)\left(1-\hat{z}_{1}\right) t_{1} / t_{2}}{K+4(1-K)\left(1-\hat{z}_{1}\right) t_{1} / t_{2}} \tag{8}
\end{align*}
$$

Proof. See Appendix.
To understand Proposition 3 consider $K=1$, i.e. all consumers have a preference for daytime shopping and to open beyond $\bar{I}$ does not attract additional demand. Accordingly, (6)-(8) coincide with the unique equilibrium prices of the symmetric configurations ( $p_{A}^{\mathrm{s}}, p_{B}^{\mathrm{s}}$ ). Proposition 3 claims that a unique price equilibrium also exists in a neighborhood of $\left(p_{A}^{\mathrm{s}}, p_{B}^{\mathrm{s}}\right)$.

The equilibrium relates to either Cases 1 or 2 depending on the parameters $t_{1}$ and $t_{2}$. Case 2 applies if consumers' preference intensity for time is relatively strong, i.e. if $t_{2} / t_{1}$ is sufficiently high. To see this observe that for $K$ close to $1, p_{A}^{*}$ and $p_{B}^{*}$ of Proposition 3 are almost identical. Setting $t_{2}>2 t_{1}$ and $p_{A}=p_{B}$ in (3) we see that $\hat{z}_{2}($.$) intersects the z_{1}=1$ locus below the upper right corner of the unit square. This is precisely the "definition" of Case 2.

Observe also that for $K=1$ product variants appear to be only differentiated with respect to space. With $K<1$ some consumers prefer nighttime shopping. To the latter, product variants appear differentiated with respect to time and space. Accordingly, $t_{1}$ and $t_{2}$ show up in the equilibrium prices of Proposition 3.

Before turning to the derivation of equilibrium locations, it is in order to comment briefly on our focus on high values of $K$. For Case 2, we were not able to determine an explicit solution for equilibrium prices. Such problems are well known to arise in models of multi-dimensional product differentiation (see, e.g. Neven and Thisse, 1990, p. 185). Typically, the literature addresses them using either numerical methods or by
considering only limit cases where, in our setting, either $t_{2} / t_{1} \rightarrow 0$ or $t_{1} / t_{2} \rightarrow 0$ (see, e.g., Gilbert and Matutes, 1993). Our approach is based on the computation of the unique price equilibrium for $K=1$ that is independent of whether (and how many) retailers open at night. Then, we use total derivatives to identify the first order effects on prices and profits when $K$ changes at $K=1$. This allows for clear-cut predictions about when prices and payoffs are higher or lower in an asymmetric configuration compared to the symmetric one, at least for $K$ close to 1 . As our stylized model is not meant to mirror a particular retail market, but mainly serves to isolate and discuss causal effects, we think that this method is well suited to approach a well-known problem.

### 3.2.3. First-stage equilibrium

Following deregulation, shops can open at times that are more convenient for some consumers. If one shop does not use this opportunity, it is easy to see that in the absence of shift costs the other shop strictly prefers to do so and opens around the clock. The question is whether such asymmetric configuration can be beneficial for both retailers relative to a symmetric one.

Suppose $A$ opens around the clock and $B$ sticks to daytime opening hours. This configuration puts $B$ at a locational disadvantage as the resulting equilibrium demands satisfy $D_{B}^{*}<\frac{1}{2}<D_{A}^{*}$ (This is shown in the proof of Proposition 4). Consequently, this strategy is $B$ 's best reply only if the ensuing equilibrium price $p_{B}^{*}$ is sufficiently higher than under symmetric locations thus compensates for the loss in demand. Our main result is that this is indeed the case if consumers' preference intensity for time is sufficiently strong. To develop the intuition it is convenient to state the following structural property of our model.

Lemma 1. Let demands be given by (4) or (5). Then, for any price equilibrium and $K<1$ it holds that $p_{A}^{*}>p_{B}^{*}$ if and only if $D_{A}^{*}>D_{B}^{*}$.

Proof. Let $\left(p_{A}^{*}, p_{B}^{*}\right)$ be a price equilibrium. Then both prices satisfy

$$
\begin{equation*}
p_{A}^{*}=\frac{-D_{A}^{*}}{\frac{\partial D_{A}^{A}}{\partial p_{A}}} \quad \text { and } \quad p_{B}^{*}=\frac{-D_{B}^{*}}{\frac{\partial D_{B}^{*}}{\partial p_{B}}} \tag{9}
\end{equation*}
$$

From $\partial \hat{z}_{1}(.) / \partial p_{A}=\left(-\partial \hat{z}_{1}(.) / \partial p_{B}\right)$ and $D_{A}\left(\hat{z}_{1}\left(p_{A}, p_{B}\right),.\right), D_{B}\left(1-\hat{z}_{1}\left(p_{A}, p_{B}\right),.\right)$ we infer $\partial D_{A} / \partial p_{A}=\partial D_{B} / \partial p_{B}=-\partial D_{A} / \partial p_{B}=-\partial D_{B} / \partial p_{A}$. Hence, $p_{A}^{*}>p_{B}^{*}$ holds if and only if $D_{A}^{*}>D_{B}^{*}$.

Reconsider the configuration $a_{2}=I$ and $b_{2}=\bar{I}$ with $D_{B}^{*}<\frac{1}{2}<D_{A}^{*}$ if $K<1$. From (9) we see that a rise in $p_{B}^{*}$ above $p_{B}^{\mathrm{s}}$ requires demand to become less responsive to price changes. In Case 1 this is impossible as the responsiveness of demand to price changes is the same as in the symmetric case (from (4) we have $\partial D_{A} / \partial p_{A}=$ $\left.\partial D_{B} / \partial p_{B}=-1 /\left(2 t_{1}\right)\right)$. Thus, $D_{B}^{*}<\frac{1}{2}$ must imply $p_{B}^{*}<p_{B}^{\mathrm{s}}$. In Case 2, however, the responsiveness of demand depends on the configuration. In fact, as we show next we have $-\partial D_{B} /\left.\partial p_{B}\right|_{p_{B}=p_{B}^{*}}<1 /\left(2 t_{1}\right)$ if $t_{2}$ is sufficiently large.

The essential difference between the repsonsiveness of demand in the two cases is as follows. Consider, for instance, a price decrease by $B$. In Case 1 a lower price attracts a new slice of customers across all time preferences. In contrast, in Case 2 a price decrease does not induce customers with a strong preference for nighttime shopping to patronize $B$. In short, as $t_{2}$ increases, outlets that marginally adjust their prices compete for a smaller segment of the market. ${ }^{12}$ For large values of $K$ Lemma 2 gives a formal statement of this intuition.

Lemma 2. Suppose $a_{2}=I$ and $b_{2}=\bar{I}$. Then $\mathrm{d} p_{A}^{*} /\left.\mathrm{d} K\right|_{K=1}<0$, while $\mathrm{d} p_{B}^{*} /\left.\mathrm{d} K\right|_{K=1}<0$ holds if and only if $t_{2} / t_{1}>\frac{5}{2}$.

## Proof. See Appendix.

Lemma 2 provides a local comparison of equilibrium prices associated with symmetric configurations and the asymmetric configuration $a_{2}=I, b_{2}=\bar{I}$. Here, we use that equilibrium prices for symmetric configurations are independent of $K$ and that equilibrium prices under both configurations coincide at $K=1$. The local comparison between the two configurations can then be made using the derivatives with respect to $K$.

As $D_{B}^{*}<\frac{1}{2}$ we conclude from Lemma 2 that retailer $B$ is strictly worse of if it does not open for 24 hours and $t_{2}$ is relatively small. In fact, $t_{2} / t_{1}>\frac{5}{2}$ is a necessary condition to make the retailer prefer to open only at daytime. As this strategy reduces the retailer's demand compared to the symmetric configuration, this condition is, however, not sufficient.

Relying on the same technical device as Lemma 2, Lemma 3 has a sufficient condition.

Lemma 3. Suppose $a_{2}=I$ and $b_{2}=\bar{I}$. Then $\mathrm{d} R_{A}^{*} /\left.\mathrm{d} K\right|_{K=1}<0$, while $\mathrm{d} R_{B}^{*} /\left.\mathrm{d} K\right|_{K=1}<0$ holds if and only if $t_{2} / t_{1}>4$.

Proof. See Appendix.
As conjectured, the condition $t_{2}>4 t_{1}$ in Lemma 3 is strictly stronger than the condition $t_{2} / t_{1}>\frac{5}{2}$ of Lemma 2. Consumers' preference intensity for convenient shopping hours must become sufficiently strong to ensure that also retailer $B$ is strictly better off after deregulation in spite of its locational disadvantage. To increase profits of retailer $B$, its equilibrium price must rise sufficiently with a decrease in $K$ to compensate for the loss in demand. Formally, this reasoning has an interpretation in terms of "direct demand" and "strategic" effects, too. Applying the envelope theorem to $R_{B}^{*}$, we obtain

$$
\begin{equation*}
\left.\frac{\mathrm{d} R_{B}^{*}}{\mathrm{~d} K}\right|_{K=1}=p_{B}^{*}\left[\left.\frac{\partial D_{B}^{*}}{\partial K}\right|_{K=1}+\left.\left.\frac{\partial D_{B}^{*}}{\partial p_{A}^{*}}\right|_{K=1} \frac{\mathrm{~d} p_{A}^{*}}{\mathrm{~d} K}\right|_{K=1}\right] . \tag{10}
\end{equation*}
$$

[^5]This is negative if the direct demand effect

$$
\left.\frac{\partial D_{B}^{*}}{\partial K}\right|_{K=1}=\frac{1}{2}\left[1-\frac{t_{1}}{t_{2}}\right]>0
$$

is overcompensated by the strategic effect

$$
\left.\left.\frac{\partial D_{B}^{*}}{\partial p_{A}^{*}}\right|_{K=1} \frac{\mathrm{~d} p_{A}^{*}}{\mathrm{~d} K}\right|_{K=1}=\left(\frac{1}{2 t_{1}}\right) \frac{4 t_{1}}{3}\left[\frac{7 t_{1}}{4 t_{2}}-1\right]<0
$$

Hence, the condition $t_{2}>4 t_{1}$ ensures that $A$ 's price rises sufficiently.
We are now in the position to derive the equilibrium opening hours after deregulation.

Proposition 4. If $K$ is sufficiently close to 1, equilibrium opening hours are as follows:
(i) If $t_{2} / t_{1}>4$ there exist exactly two location equilibria where either $a_{2}=I, b_{2}=\bar{I}$ or $a_{2}=\bar{I}, b_{2}=I$; equilibrium prices of both retailers are strictly higher than before deregulation.
(ii) If $t_{2} / t_{1}<4$ the unique location equilibrium is $a_{2}=b_{2}=I$; deregulation does not affect prices.

Proof. See Appendix.
The key to this equilibrium structure lies in Lemma 3. Starting from a location configuration under regulation with $a_{2}=b_{2}=\bar{I}$, it is optimal for one retailer, say $A$, to open all day if the other retailer chooses not to do so. Then $B$ 's best reply depends on whether $t_{2} / t_{1}>4$ is satisfied or not. If consumers' preference intensity for time is relatively low $B$ loses in any asymmetric configuration and will therefore mimic retailer $A$ 's move. The equilibrium is symmetric with $a_{2}=b_{2}=I$ and prices and profits before and after deregulation coincide. If $t_{2} / t_{1}>4$, also $B$ gains from an asymmetric configuration and therefore opens only during the day. ${ }^{13}$

Two remarks concerning case (i) are in order. First, case (i) exhibits multiple (asymmetric) equilibria. This problem could be solved by a sequential game where nature decides who moves first. It is then not difficult to show that the first mover chooses to open up around the clock and has strictly higher profits if $K<1$.

Second, case (i) gives rise to equilibria in mixed strategies. ${ }^{14}$ Denote by $\rho$ the probability of retailer $B$ to open around the clock. If both retailers open around the clock or only during daytime, we know that their profits are $R_{i}^{\mathrm{s}}$. For $a_{2}=I, b_{2}=\bar{I}$ denote equilibrium profits of $A$ and $B$ by $R_{A}^{*}$ and $R_{B}^{*}$, respectively. Symmetry implies

[^6]that $A$ earns $R_{B}^{*}$ if $a_{2}=\bar{I}$ and $b_{2}=I$. Then, retailer $A$ is indifferent between $a_{2}=I$ and $a_{2}=\bar{I}$ if $\rho$ satisfies
$$
\rho R_{A}^{\mathrm{s}}+(1-\rho) R_{A}^{*}=\rho R_{B}^{*}+(1-\rho) R_{A}^{\mathrm{s}}
$$
or
\[

$$
\begin{equation*}
\rho=\frac{R_{A}^{*}-R_{A}^{\mathrm{s}}}{R_{A}^{*}+R_{B}^{*}-2 R_{A}^{\mathrm{s}}} . \tag{11}
\end{equation*}
$$

\]

Using symmetry, retailer $A$ must randomize with the same probability $\rho$ to keep $B$ indifferent. Focussing on the neighborhood of $K=1$ both the numerator and the denominator of (11) tend to zero as $K \rightarrow 1$. An application of l'Hôpital's rule gives

$$
\rho=\frac{1}{1+\left.\left[\left(\mathrm{d} R_{B}^{*} / \mathrm{d} K\right) /\left(\mathrm{d} R_{A}^{*} / \mathrm{d} K\right)\right]\right|_{K=1}} .
$$

Proposition 5. If $K$ is sufficiently close to one and $t_{2} / t_{1}>4$ there exists also an equilibrium in mixed strategies, where both retailers randomize between opening around the clock and opening only at daytime. As $K \rightarrow 1$, the symmetric probability for opening around the clock converges to

$$
\begin{equation*}
\rho^{*}=\frac{1}{6} \frac{5\left(t_{2} / t_{1}\right)-8}{\left(t_{2} / t_{1}\right)-2} . \tag{12}
\end{equation*}
$$

Proof. Immediate from the proofs of Lemma 3 and Proposition 4.
Proposition 5 provides additional insights into the role of the ratio $t_{2} / t_{1}$. Clearly, for $t_{2} \rightarrow 4 t_{1}$ it holds that $\rho^{*} \rightarrow 1$. Moreover, differentiating (11) with respect to $\left(t_{2} / t_{1}\right)$ shows that $\rho^{*}$ strictly decreases with this ratio. Hence, the probability for a given retailer to open around the clock and, thus, the probability of both retailers to open around the clock strictly decreases in $t_{2} / t_{1}$. Intuitively, the higher $t_{2} / t_{1}$, the more relaxed is price competition if retailers choose an asymmetric location configuration. In other words, if $t_{2} / t_{1}$ is high, the coordination failure when both shops open for 24 hours becomes increasingly costly.

A limitation of our model is that we allow only for a discrete choice of opening hours. Having a continuous choice of opening hours proved analytically intractable, one reason being that quasi-concavity of profit functions could no longer be guaranteed. We conjecture, however, that our main insights remain valid in such an extended context. The argument is as follows. Suppose that, besides opening during the day, a retailer could continuously choose for how long to open during the night. If one retailer opens around the clock we know for high $K$ and $t_{2} / t_{1}>4$ that the other retailer is strictly better off by opening only during daytime instead of opening for 24 hours, too. This result is independent of whether retailers choose between day- and nighttime shifts or whether their choice is continuous. While it may no longer hold that the other retailer opens only during the day, we know that it does not find it optimal to open for 24 hours.

## 4. The case of differing input costs

Let retailers differ with respect to their constant marginal input costs of serving consumers. To simplify, normalize retailer $A$ 's costs to zero and set $B$ 's equal to $c$. Assume that $A$ 's cost advantage is not too big, i.e. $0 \leqslant c<t_{1} / 2$. This excludes equilibria in which outlet $A$ prices $B$ out of the market. How are our main findings of Proposition 5 affected? ${ }^{15}$

Proposition 6. If $K$ is sufficiently close to 1 equilibrium opening hours are as follows:
(i) If $t_{2}>\left(36 t_{1}^{2}-4 c^{2}\right) /\left(9 t_{1}-9 c\right)$ there exist exactly two location equilibria where either $a_{2}=I, b_{2}=\bar{I}$ or $a_{2}=\bar{I}, b_{2}=I$.
(ii) If $\left(36 t_{1}^{2}-4 c^{2}\right) /\left(9 t_{1}-9 c\right)>t_{2}>\left(36 t_{1}^{2}-4 c^{2}\right) /\left(9 t_{1}+9 c\right)$ the unique location equilibrium is $a_{2}=I, b_{2}=\bar{I}$.
(iii) If $t_{2}<\left(36 t_{1}^{2}-4 c^{2}\right) /\left(9 t_{1}+9 c\right)$ the unique location equilibrium is $a_{2}=b_{2}=I$.

Proof. See Appendix.
To build intuition it is instructive to consider the outcome for $K=1$ with equilibrium prices $p_{A}^{\mathrm{s}}=t_{1}+c / 3$ and $p_{B}^{\mathrm{s}}=t_{1}+2 c / 3$. The horizontal position of the indifferent consumer location is given by $\bar{z}:=\frac{1}{2}+c /\left(6 t_{1}\right)$. Hence, the "address" of indifferent consumers for $K=1$ shifts to the right as $c>0$. If $A$ opens around the clock and $K<1$ this shift makes it easier for $A$ to capture all consumers with sufficiently high preferences for nighttime shopping. From our previous arguments this is essential to ensure that $B$ 's equilibrium price is higher when it opens only during daytime. The further $\bar{z}$ shifts to the right, the "shorter" becomes the marginal consumer segment. Thus, the responsiveness of demand falls and price competition is relaxed. As a consequence, the condition for $a_{2}=I, b_{2}=\bar{I}$ to be an equilibrium under differing input costs is relaxed compared to Proposition 4. More generally, the threshold $t_{2}>\left(36 t_{1}^{2}-4 c^{2}\right) /\left(9 c+9 t_{1}\right)$ strictly decreases in $c$ reaching $t_{2}>4 t_{1}$ for $c=0$.

By the same token, the condition for $a_{2}=\bar{I}, b_{2}=I$ to be an equilibrium is now stricter than the one for the equilibrium with $a_{2}=I, b_{2}=\bar{I}$. Again, this follows as $\bar{z}>\frac{1}{2}$ holds for $c>0$, implying that the marginal consumer segment is "longer" than under symmetric configurations. Thus, if $t_{2} / t_{1}$ takes on intermediate values there is a unique equilibrium where only $A$ opens around the clock.

Despite the multiplicity of equilibria for high values of $t_{2} / t_{1}$, it is fair to say that retailers with a cost advantage are more likely to open around the clock. Associating the cost advantage with shop size this result is consistent with the observation in Halk and Träger (1999) that large stores are more likely to use the leeway in the deregulated market.

[^7]
## 5. Concluding remarks

This paper demonstrates that a short-run implication of a deregulation of shopping hours may be higher prices. This result obtains in an asymmetric equilibrium configuration where one shop opens longer than the other.

The model and the technical analysis were devised to study the strategic aspects of deregulating shopping hours under imperfect competition and to highlight the impact on shops' prices and profits. We have therefore abstained from discussing welfare. Admittedly, an assessment of the short-term welfare effects of deregulation would have to say something about aggregate demand effects and costs components which depend on opening hours. Aggregate demand may well change after deregulation, either in response to price changes or because more flexible shopping hours and/or a reduction of congestion attracts more consumers. Yet, our model of product differentiation does not allow for such effects. As to the second point, it is clear that the desirability of deregulation may be weakened if longer opening hours come at an additional cost.

While we leave the incorporation of these phenomena for future research ${ }^{16}$ it nevertheless seems worthwhile to state that our model allows at least for some tentative welfare conclusions. For instance, if both firms incur identical marginal costs one can show that deregulation increases welfare for $K$ sufficiently high. This is immediate when deregulation leads to a symmetric equilibrium where both shops open around the clock. Here prices and the marginal consumer location remain constant while "transportation" costs of the time dimension are reduced for nighttime shoppers. If deregulation leads to an asymmetric equilibrium the marginal consumer location shifts. Even though physical transportation costs increase after regulation, a marginal analysis at $K=1$ shows that this effect is dominated by the reduction in "transportation" costs associated with the time dimension. However, in an asymmetric equilibrium welfare could be further increased if both firms chose to open around the clock.

When retailers' input costs differ an additional effect must be taken into account. In an asymmetric equilibrium the retailer who opens around the clock attracts a larger demand than in the symmetric case (under regulation). If it is the low-cost retailer, who opens around the clock the additional effect increases welfare as aggregate transportation costs decrease. If it is the high-cost retailer, who opens longer (in the case of multiple equilibria), this additional effect reduces welfare and no clear-cut predictions are available.

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[^8]
## Appendix. Proofs

Proof of Proposition 3. The proof proceeds in a series of claims.
Claim 1. Consider $p_{j}$ fixed. Then $R_{i}\left(p_{j}, \cdot\right)$ is strictly quasiconcave and continuously differentiable for all $p_{j}-t_{1} \leqslant p_{i} \leqslant p_{j}+t_{1}$. Moreover, any equilibrium ( $p_{A}^{*}, p_{B}^{*}$ ) where $R_{i}\left(p_{A}^{*}, p_{B}^{*}\right)>0$ for $i=A, B$ and $p_{A}^{*}-p_{B}^{*}<t_{1}$ must satisfy either the first-order conditions of Case 1 or those of Case 2.

Proof. We discuss Cases 1 and 2 in turn. For Case 1 we obtain from (4) the derivatives

$$
\begin{equation*}
\frac{\partial R_{i}}{\partial p_{i}}=\frac{t_{1}+p_{j}-2 p_{i}+(1-K) t_{2} / 4}{2 t_{1}} \tag{13}
\end{equation*}
$$

which yield the first-order conditions in (6) and establish that $R_{i}$ is strictly concave in $p_{i}$. From (5) we obtain for Case 2 the derivatives

$$
\frac{\partial D_{i}}{\partial p_{i}}=-\frac{K+4(1-K) t_{1}\left(1-\hat{z}_{1}\right) / t_{2}}{2 t_{1}}
$$

and

$$
\begin{aligned}
& \frac{\partial R_{A}}{\partial p_{A}}=K \hat{z}_{1}+(1-K)-(1-K)\left(1-\hat{z}_{1}\right)^{2} \frac{2 t_{1}}{t_{2}}-\frac{K+4(1-K) t_{1}\left(1-\hat{z}_{1}\right) / t_{2}}{2 t_{1}} p_{A} \\
& \frac{\partial R_{B}}{\partial p_{B}}=K\left(1-\hat{z}_{1}\right)+(1-K)\left(1-\hat{z}_{1}\right)^{2} \frac{2 t_{1}}{t_{2}}-\frac{K+4(1-K) t_{1}\left(1-\hat{z}_{1}\right) / t_{2}}{2 t_{1}} p_{B}
\end{aligned}
$$

The first-order conditions give rise to the equilibrium prices of (7) and (8). Observe next that $\partial^{2} D_{A} / \partial p_{A}^{2}=-(1-K) /\left(t_{1} t_{2}\right)$ such that

$$
\frac{\partial^{2} R_{A}}{\partial p_{A}^{2}}=2 \frac{\partial D_{A}}{\partial p_{A}}+\frac{\partial^{2} D_{A}}{\partial p_{A}^{2}} p_{A}<0
$$

Regarding $R_{B}$ in Case 2, we obtain that $\partial^{2} R_{B} / \partial p_{B}^{2} \lessgtr 0$ holds iff

$$
\begin{equation*}
-2\left[K t_{2}+2(1-K)\left(t_{1}-p_{B}+p_{A}\right)\right]+(1-K) p_{B} \lessgtr 0 \tag{14}
\end{equation*}
$$

Recall now that we only consider values $p_{B}$ satisfying (i) $p_{B} \geqslant \bar{p}_{B}:=\max \left\{p_{A}-\right.$ $\left.t_{1}, p_{A}+t_{1}-t_{2} / 2\right\}$ and (i) $p_{B} \leqslant \overline{\bar{p}}_{B}:=p_{A}+t_{1}$, where $R_{B}\left(p_{A}, p_{B}\right)>0$ over this interval and $R_{B}\left(p_{A}, \overline{\bar{p}}_{B}\right)=0$. Note next that, as the left-hand side in (14) is strictly increasing in $p_{B}$, there exist at most two values $p_{B}$ at which $\partial R_{B} / \partial p_{B}$ changes sign. If there is no
such value, $R_{B}\left(p_{A}, \cdot\right)$ must be strictly decreasing and therefore strictly quasiconcave. Likewise, if there is only one such value, $R_{B}\left(p_{A}, \cdot\right)$ must first strictly increase and then strictly decrease, making it again strictly quasiconcave. It remains to rule out the case where exactly two values $p_{B}$ exist at which $\partial R_{B} / \partial p_{B}$ changes sign. To argue to a contradiction, denote the respective values by $\bar{p}_{B}<p_{B}^{1}<p_{B}^{2}<\overline{\bar{p}}_{B}$. By the sign of $R_{B}\left(p_{A}, \cdot\right)$ it must hold that $\partial^{2} R_{B} /\left.\partial p_{B}^{2}\right|_{p_{B}=p_{B}^{1}}>0$ and $\partial^{2} R_{B} /\left.\partial p_{B}^{2}\right|_{p_{B}=p_{B}^{2}}<0$, which contradicts (14). Hence, the strict quasiconcavity of $R_{i}, i=A, B$, is established.

Finally, observe that differentiability is only an issue at the boundary between the two cases, i.e. where $p_{A}-p_{B}=t_{2} / 2-t_{1}$. At this point it holds that $\hat{z}_{1}\left(p_{A}, p_{B}\right)=1-t_{2} /\left(4 t_{1}\right)$. (Observe that we can restrict consideration to the case where $0<1-t_{2} /\left(4 t_{1}\right)<1$.) It is then easily checked that $R_{i}$ is in fact smooth at this boundary.

An immediate implication of Claim 1 is that there exists for each retailer a unique best response if prices are restricted accordingly.

Claim 2. There exists at most one equilibrium $\left(p_{A}^{*}, p_{B}^{*}\right)$ where $p_{A}^{*}-p_{B}^{*}<t_{1}$ and $R_{i}\left(p_{A}^{*}, p_{B}^{*}\right)>0$.

Proof. Given the restriction on prices, $\left(p_{A}^{*}, p_{B}^{*}\right)$ must by Claim 1 solve the first-order conditions of either Cases 1 or 2. In Case 1 we obtain by inspection of (13) a single equilibrium candidate. Consider next Case 2 and denote $D^{\prime}=\partial D_{i} / \partial p_{i}$. As $R_{A}$ is strictly concave (over the considered domain), implicit differentiation of the first-order condition yields

$$
\begin{equation*}
\frac{\mathrm{d} p_{B}}{\mathrm{~d} p_{A}}=1-\frac{D^{\prime}}{-D^{\prime}+\left[(1-K) / t_{1} t_{2}\right] p_{A}} \tag{15}
\end{equation*}
$$

where we use that $\partial^{2} D_{A} /\left(\partial p_{B} \partial p_{A}\right)=(1-K) /\left(t_{1} t_{2}\right)$. Similarly, using strict quasiconcavity for $R_{B}$, we obtain from the first-order condition of $B$

$$
\begin{equation*}
\frac{\mathrm{d} p_{B}}{\mathrm{~d} p_{A}}=1-\frac{D^{\prime}}{2 D^{\prime}+\left[(1-K) / t_{1} t_{2}\right]_{B} p_{B}} \tag{16}
\end{equation*}
$$

Consider a possible pair $\left(p_{A}^{*}, p_{B}^{*}\right)$, where the best-response functions for Case 2 intersect. As we are now only considering interior solutions, the second-order condition for $B$ must be satisfied, which yields the requirement $2 D^{\prime}+\left[(1-K) / t_{1} t_{2}\right] p_{B}^{*}<0$. This implies $D^{\prime} /\left(2 D^{\prime}+\left[(1-K) / t_{1} t_{2}\right] p_{B}^{*}\right)>0$, while $D^{\prime} /\left(-D^{\prime}+\left[(1-K) / t_{1} t_{2}\right] p_{A}^{*}\right)<0$, such that $(15)>(16)$. As this must hold at any intersection, the best-response functions for Case 2 may intersect at most once.

Having shown that there can be at most one equilibrium where prices solve the first-order conditions of either Cases 1 or 2, it remains to prove that these two equilibrium candidates are mutually exclusive. Recall first that for an equilibrium in Case 2 the slope $\mathrm{d} p_{B} / \mathrm{d} p_{A}$ of the best-response function of $A$ strictly exceeds that of the best-response function of $B$. We can now show that the same holds for Case 1 , which by Claim 1 proves the assertion. To see this, note that implicit differentiation of the best-response function of $A$ in Case 1 yields $\mathrm{d} p_{B} / \mathrm{d} p_{A}=2$, while implicit differentiation of the best-response function of $B$ in Case 1 yields $\mathrm{d} p_{B} / \mathrm{d} p_{A}=\frac{1}{2}$.

Claim 3. If $t_{2} \leqslant 2 t_{1}$, then we can find $\bar{K}$ such that for $\bar{K}<K<1$ there exists a unique price equilibrium with prices given by (6).

Proof. We derive first conditions for existence. By Claim 1, there exists a unique pair of prices satisfying the first-order conditions of Case 1. Calculation reveals that these prices satisfy $p_{A}^{*}-p_{B}^{*}=(1-K) t_{2} / 6$ such that by assumption the conditions of Case 1 are satisfied.

We show next that no firm can profitably deviate. By Claim 1, there is no profitable deviation as long as prices satisfy the conditions of Case 1 or 2 . It thus remains to consider sufficiently high prices $p_{A}$ and sufficiently low prices $p_{B}$ such that $\hat{z}_{1}=0$. Regarding firm $A$, to ensure $D_{A}>0$ the deviating price $p_{A}$ must be bounded from above by $p_{B}+t_{1}+t_{2} / 2$, which implies that the deviation cannot be profitable if $\bar{K}_{1}<K \leqslant 1$ for some $\bar{K}_{1}<1$. Regarding firm $B$, observe first that $R_{B}\left(p_{A}^{*}, p_{B}^{*}\right)$ is continuous in $K$ (for high $K$ ). By offering some $p_{B} \leqslant p_{A}^{*}-t_{1}, B$ 's revenue is bounded from above by $(1-K) t_{2} / 12$, which converges to zero as $K \rightarrow 1$. As $R_{B}\left(p_{A}^{*}, p_{B}^{*}\right)>0$, the deviation is not profitable for $\bar{K}_{2}<K \leqslant 1$ and some $\bar{K}_{2}<1$. Hence, we have proved existence for $K>\max \left\{\bar{K}_{1}, \bar{K}_{2}\right\}$.

We turn next to uniqueness. By Claim 2 there exists no other equilibrium where $p_{A}^{*}-p_{B}^{*}<t_{1}$ and $R_{i}\left(p_{A}^{*}, p_{B}^{*}\right)>0$. It thus remains to cover the case where the price difference is larger than $t_{1}$. As $K \rightarrow 1$ this can be ruled out by standard arguments. Choosing thus $K$ sufficiently large we can ensure that Claim 3 holds.

Claim 4. If $t_{2}>2 t_{1}$, then we can find $\bar{K}$ such that for $\bar{K}<K<1$ there exists a unique price equilibrium with prices given by (7) and (8).

Proof. The argument is omitted as it is analogous to that of Claim 3.
Proof of Lemma 2. By Proposition 2, we can find for a given value of $t_{2} / t_{1}$ a neighborhood of $K=1$ such that there exist a unique price equilibrium which is characterized by the first-order conditions of either Case 1 or 2 . Suppose first that $t_{2} \leqslant 2 t_{1}$, i.e. Case 1 applies. We obtain $\mathrm{d} p_{A}^{*} /\left.\mathrm{d} K\right|_{K=1}=-t_{2} / 12$ and $\mathrm{d} p_{B}^{*} /\left.\mathrm{d} K\right|_{K=1}=t_{2} / 12$. Consider next the case where $t_{2}>2 t_{1}$ such that Case 2 applies. Implicit differentiation of the first-order conditions (7) and (8) yields

$$
\begin{aligned}
& \left.\frac{\mathrm{d} p_{A}^{*}}{\mathrm{~d} K}\right|_{K=1}=2 t_{1}\left[\left.\frac{\mathrm{~d} \hat{z}_{1}}{\mathrm{~d} K}\right|_{K=1}-1+\frac{2 t_{1}}{t_{2}}\left(1-\frac{1}{2}\right)^{2}\right] \\
& \left.\frac{\mathrm{d} p_{B}^{*}}{\mathrm{~d} K}\right|_{K=1}=2 t_{1}\left[-\left.\frac{\mathrm{d} \hat{z}_{1}}{\mathrm{~d} K}\right|_{K=1}+\frac{2 t_{1}}{t_{2}}\left(1-\frac{1}{2}\right)^{2}\right]
\end{aligned}
$$

where we substituted $\left.\hat{z}_{1}\right|_{K=1}=\frac{1}{2}$. By the definition of $\hat{z}_{1}$, we further obtain

$$
\frac{\mathrm{d} \hat{z}_{1}}{\mathrm{~d} K}=\frac{1}{2 t_{1}}\left(-\frac{\mathrm{d} p_{A}^{*}}{\mathrm{~d} K}+\frac{\mathrm{d} p_{B}^{*}}{\mathrm{~d} K}\right)
$$

which yields $\mathrm{d} \hat{z}_{1} /\left.\mathrm{d} K\right|_{K=1}=\left(1-t_{1} / t_{2}\right) / 3$. Hence, we finally obtain

$$
\begin{align*}
& \left.\frac{\mathrm{d} p_{A}^{*}}{\mathrm{~d} K}\right|_{K=1}=\frac{2 t_{1}}{3}\left[-2+\frac{t_{1}}{t_{2}} \frac{7}{2}\right],  \tag{17}\\
& \left.\frac{\mathrm{d} p_{B}^{*}}{\mathrm{~d} K}\right|_{K=1}=\frac{2 t_{1}}{3}\left[-1+\frac{t_{1}}{t_{2}} \frac{5}{2}\right] . \tag{18}
\end{align*}
$$

Using for (17) that $t_{2}>2 t_{1}$ proves $\mathrm{d} p_{A}^{*} /\left.\mathrm{d} K\right|_{K=1}<0$, while the condition for $\mathrm{d} p_{B}^{*} /$ $\left.\mathrm{d} K\right|_{K=1}$ follows from (18).

Proof of Lemma 3. Suppose first that Case 1 applies (in a neighborhood of $K=1$ ), which by Proposition 3 holds for $t_{2} \leqslant 2 t_{1}$. Given the explicit characterization of equilibrium prices in (6), it is easily established that $\mathrm{d} R_{A}^{*} /\left.\mathrm{d} K\right|_{K=1}<0$ and $\mathrm{d} R_{B}^{*} /\left.\mathrm{d} K\right|_{K=1}>0$. Consider next Case 2. By the envelope theorem we obtain

$$
\begin{equation*}
\left.\frac{\mathrm{d} R_{B}^{*}}{\mathrm{~d} K}\right|_{K=1}=p_{B}^{*}\left[\left.\frac{\partial D_{B}^{*}}{\partial K}\right|_{K=1}+\left.\left.\frac{\partial D_{B}^{*}}{\partial p_{A}^{*}}\right|_{K=1} \frac{\mathrm{~d} p_{A}^{*}}{\mathrm{~d} K}\right|_{K=1}\right] . \tag{19}
\end{equation*}
$$

Using (5) and (17), we obtain

$$
\begin{aligned}
& \left.\frac{\partial D_{B}^{*}}{\partial K}\right|_{K=1}=\frac{1}{2}-\frac{t_{1}}{2 t_{2}} \\
& \left.\left.\frac{\partial D_{B}^{*}}{\partial p_{A}^{*}}\right|_{K=1} \frac{\mathrm{~d} p_{A}^{*}}{\mathrm{~d} K}\right|_{K=1}=-\left(-\frac{1}{2 t_{1}}\right) \frac{2 t_{1}}{3}\left[\frac{7 t_{1}}{2 t_{2}}-2\right]
\end{aligned}
$$

Substitution yields $\mathrm{d} R_{B}^{*} /\left.\mathrm{d} K\right|_{K=1}=\frac{2}{3} t_{1} / t_{2}-\frac{1}{6}$, which yields the condition in Lemma 3. Likewise, we obtain $\mathrm{d} R_{A}^{*} /\left.\mathrm{d} K\right|_{K=1}=\frac{4}{3} t_{1} / t_{2}-\frac{5}{6}$, which is strictly negative as $t_{2}>2 t_{1}$.

Proof of Proposition 4. We already considered all location configurations where both firms open during the daytime. It thus remains to show that (i) retailers do not want to deviate from the characterized strategies by closing during daytime and (ii) that retailer would wish to deviate from a configuration where they are supposed to close during daytime. At $K=1$ it is intuitive that a retailer opening only at nighttime would be strictly better off by deviating to opening also at daytime, regardless of the opening hours of the competitor. This extends to all sufficiently high values of $K$. We omit a proof of this claims, which is contained in our discussion paper Inderst and Irmen (2001).

## References

Ben-Akiva, M., de Palma, A., Thisse, J.-F., 1989. Spatial competition with differentiated products. Regional Science and Urban Economics 19, 5-19.
Böckem, S., 1994. A generalized model of horizontal product differentiation. Journal of Industrial Economics 42, 287-298.
Clemenz, G., 1990. Non-sequential consumer search and the consequences of a deregulation of trading hours. European Economic Review 34, 1137-1323.
Clemenz, G., 1994. Competition via shopping hours: A case for regulation? Journal of Institutional and Theoretical Economics 150, 625-641.

Degryse, H., 1996. On the interaction between vertical and horizontal product differentiation: An application to banking. The Journal of Industrial Economics 44, 169-186.
de Meza, D., 1984. The Fourth Commandment: Is it Pareto efficient? The Economic Journal 94, 379-383.
Economides, N., 1989. Quality variation and maximal variety differentiation. Regional Science and Urban Economics 19, 21-29.
Ferris, J., 1990. Time, space, and shopping: The regulation of shopping hours. Journal of Law, Economics, and Organization 6, 171-187.
Gilbert, R., Matutes, C., 1993. Product line rivalry with brand differentiation. Journal of Industrial Economics 41, 223-240.
Gradus, R., 1996. The economic effects of extending shop opening hours. Journal of Economics 64, 247-263.
Halk, K., Träger, U., 1999. Wie wirkt das neue Ladenschlußgesetz auf den Einzelhandel? IFO Schnelldienst 1-2/99, 7-13.
Hotelling, H., 1929. Stability in competition. Economic Journal 39, 41-45.
Inderst, R., Irmen, A., 2001. Shopping hours and price competition. CEPR Discussion Paper No. 3001.
Kay, J., Morris, N., 1987. The economic efficiency of sunday trading restrictions. The Journal of Industrial Economics 36, 113-129.
Klemperer, P., Padilla, A., 1997. Do firms' product lines include too many varieties? RAND Journal of Economics 28, 472-488.
Kosfeld, M., 2002. Why shops close again: An evolutionary perspective on the deregulation of shopping hours. European Economic Review 46, 51-72.
Morrison, S., Newman, R., 1983. Hours of operation restrictions and competition among retail firms. Economic Inquiry 21, 107-114.
Neven, D., Thisse, J.-F., 1990. Quality and variety competition. In: Gabszewicz, J., Richard, F., Wolsey, L. (Eds.), Economic Decision Making: Games, Econometrics and Optimization, Contributions in Honour of J. Dreze. North-Holland, Amsterdam.

Salop, S., 1979. Monopolistic competition with outside goods. Bell Journal of Economics 10, 141-156.
Tabuchi, T., 1994. Two-stage two-dimensional spatial competition between two firms. Regional Science and Urban Economics 24, 207-227.
Tanguay, G., Vallée, L., Lanoie, P., 1995. Shopping hours and price levels in the retailing industry: A theoretical and empirical analysis. Economic Inquiry 33, 516-524.


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    ${ }^{2}$ Financial assistance from the Deutsche Forschungsgemeinschaft under the program "Industrieökonomik und Inputmärkte" and research Grant IR 44/1-1 is gratefully acknowledged.
    ${ }^{3}$ In Germany for a long time shops were obliged to close at 6.30 p.m. on most weekdays and at 2 p.m. on most Saturdays. Since November 1996 they are allowed to stay open on weekdays until 8 p.m. and on Saturdays until 4 p.m. Since June 2003 opening hours on Saturdays until 8 p.m. are permitted. Deregulation of opening hours in The Netherlands became effective in June 1996 and allows shops to open from 6 a.m. to 10 p.m. from Monday to Saturday (Gradus, 1996, p. 248).

[^1]:    ${ }^{4}$ We abstract from long-term effects of deregulation that are due to exit or entry. On these effects, see e.g., de Meza (1984), Ferris (1990), and Kay and Morris (1987).

[^2]:    ${ }^{5}$ Kosfeld (2002) presents an evolutionary perspective on why only some shops make use of deregulated opening hours.
    ${ }^{6}$ The choice of time intervals is also at the heart of a study on the usefulness of regulation of shopping hours by Clemenz (1994). The author deals with the monopoly and competitive cases leaving aside modes of competition between those benchmarks. Such a framework is not suited to study the link between imperfect competition in shopping hours and prices which is the focus of the present paper.
    ${ }^{7}$ A similar geometry has previously been used by, e.g., Ben-Akiva et al. (1989) and Degryse (1996).

[^3]:    ${ }^{8}$ In other words, all consumers who decide to buy at $A$ and whose preferred shopping time lies outside of $A$ 's hours of business buy at the first or the last "minute". This pattern of behavior is optimal from the consumers' point of view given that our setup abstracts from congestion effects which could naturally arise if too many consumers visited an outlet at the same time.
    ${ }^{9}$ High menu costs may justify this assumption.

[^4]:    ${ }^{10}$ The equilibrium outcome of our original setup is obtained by simple doubling of $D_{i}^{\mathrm{s}}$ and $R_{i}^{\mathrm{s}}$. Equilibrium prices are unaffected since the marginal conditions remain the same. This reasoning applies to all equilibrium outcomes derived below.
    ${ }^{11} \mathrm{~A}$ motivation for this analytical strategy is given below.

[^5]:    ${ }^{12}$ Though the formal derivation of this result depends on the considered space, we feel that the underlying argument is quite robust. Just imagine that the two firms are located somewhere on an island. As the time dimension becomes sufficiently important, all customers with a high preference for nighttime shopping will shop at $A$ if $p_{A}-p_{B}$ is not too large.

[^6]:    ${ }^{13}$ So far we considered only configurations where both shops open at daytime. It might be argued that retailers should choose non-overlapping opening hours to achieve maximal differentiation, in particular for high values of $t_{2} / t_{1}$. We can prove that this is not optimal given the concentration of preferences on the daytime segment.
    ${ }^{14}$ We like to thank a referee for suggesting this.

[^7]:    ${ }^{15}$ A more detailed discussion and all proofs for the case of differing input costs can be found in Inderst and Irmen (2001).

[^8]:    ${ }^{16}$ On aggregate demand issues one may want to consider a generalized framework of horizontal product differentiation in the spirit of Böckem (1994). An empirically plausible cost structure is used in Clemenz (1994). Another promising route would be to consider behavioral patterns of retail demand such as consumer loyalty (see, e.g., Klemperer and Padilla, 1997).

