Market Power, Price Discrimination, and Allocative Efficiency in Intermediate-Goods Markets*

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Abstract

We consider a monopolistic supplier's optimal choice of two-part tariff contracts when downstream firms are asymmetric. We find that the optimal discriminatory contracts amplify differences in downstream firms' competitiveness. Firms that are larger–either because they are more efficient or because they sell a superior product–obtain a lower wholesale price than their rivals. This increases allocative efficiency by favoring the more productive firms. In contrast, we show that a ban on price discrimination reduces allocative efficiency and, surprisingly, can lead to higher wholesale prices for *all* firms. Consumer surplus and welfare are thus lower.

Keywords: Vertical Control; Input Markets; Robinson-Patman Act

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1 Introduction

A key insight from the literature on price discrimination is that a firm optimally charges a higher price to buyers whose demand is less elastic. This insight, which is useful for firms selling in *final-goods* markets, has also been applied to *intermediate-goods* markets. There, an upstream supplier optimally discriminates between downstream firms on the basis of differences in their *derived demands*. Typically, a downstream firm's derived demand is less elastic if its own final good is more attractive to consumers or if it has a lower marginal cost of operation (i.e., if it is more efficient). By analogy to the case of price discrimination in final-goods markets, therefore, one might think that the supplier should optimally charge the more efficient or otherwise superior firms a higher wholesale price (cf. DeGraba, 1990; Yoshida, 2000). However, by giving higher wholesale prices to the 'more productive' downstream firms, allocative efficiency and thus welfare is reduced.

An assumption in the above-cited literature is that it is optimal for suppliers to offer linear wholesale contracts. While this is without loss of generality if each downstream firm demands at most one unit, unit demands are typically not the case in practice. Even if each *final* consumer purchases at most one unit, the fact that these purchases are aggregated in the downstream firms' derived demands implies that each firm will typically demand multiple units. This allows the supplier to offer more complex contracts to facilitate price discrimination. As is well-known, with nonlinear contracts such as two-part tariffs, the supplier can disentangle the objective of extracting surplus from that of providing downstream firms with the right incentives to choose a given retail price or quantity. We show that once this is taken into account, the supplier's optimal discriminatory wholesale prices no longer *dampen* differences in downstream firms' competitive positions but instead *amplify* these differences. Lower wholesale prices are given to more productive firms. As a result, the more productive firms become even larger and allocative efficiency increases.

A ban on price discrimination in intermediate-goods markets–i.e., a requirement that all downstream firms pay the same marginal prices and fixed fees—would thus lead to a reduction in allocative efficiency when nonlinear contracts are feasible. This is the opposite of what one finds when only linear wholesale contracts are feasible. Another difference is that we find that a ban on price discrimination tends to raise all final-goods prices and thus decrease total output. Unlike in DeGraba (1990) and Yoshida (2000), where the effect of a ban on price discrimination is to raise the wholesale price of the less efficient downstream firm and lower the wholesale price of the more efficient downstream firm, we show, in contrast, that when demand is linear and nonlinear contracts are feasible, the adverse effects of a ban on price discrimination are sufficiently strong that *all* wholesale prices increase, irrespective of the degree of substitution among the downstream firms' final goods. A ban on price discrimination in intermediate-goods markets when nonlinear contracts are feasible thus may reduce welfare on *two* accounts: it may increase the deadweight loss to society due to the higher final-goods prices that are caused by the supplier's higher wholesale prices, and it may shift a larger share of the now smaller total output to downstream firms that are either less efficient or that produce inferior products.

To see why a ban on price discrimination tends to increase wholesale prices and thus also final-goods prices, take the extreme case in which the downstream firms' products are independent in demand. In this case, in order to maximize joint surplus, the supplier will want to set its wholesale price equal to its marginal cost when selling to downstream firms (this avoids the well-known problem of double marginalization). If price discrimination is banned, however, the supplier will want to charge its downstream firms a strictly higher wholesale price. This follows because it then becomes optimal for the supplier to use its now uniform wholesale price as a "metering" device in order to extract more surplus from the firms with the higher derived demands. This insight extends as well to the case where the downstream firms' products are substitutes. There, as noted above, a ban on price discrimination also prevents the supplier from shifting output to the more efficient firms.

The beneficial effects of price discrimination in the intermediate-goods market can be sufficiently strong that welfare may actually be higher when the upstream market is monopolized (and price discrimination is feasible) than if it were perfectly competitive. In the latter case, all downstream firms would be able to purchase their inputs at cost and there would be no favoring of the more efficient firms. In contrast, when the upstream market is monopolized, the monopolistic supplier exercises its market power by raising its wholesale prices above marginal cost whenever downstream firms compete (so as to dampen their competition), thus reducing welfare. At the same time, however, it also chooses its wholesale prices so as to swing more output to the more efficient firms and/or those with superior products, thus increasing welfare. On balance, either effect, the latter effect or the 'dampening-of-competition' effect, can dominate. If price discrimination were not feasible, however, only the dampening-of-competition effect would be operative, and thus welfare would be unambiguously lower when the upstream market was monopolized.

A key assumption in our analysis is that downstream firms can observe their rivals' contracts even when price discrimination is feasible. If contract offers and acceptances were instead private information, the supplier's contract terms to one firm would not affect the rival downstream firms' retail prices or quantities, and thus the supplier would be tempted to choose its terms so as to maximize bilateral joint profits instead of overall joint profits. This gives rise to a potentially severe opportunism problem in which all downstream firms, irrespective of their efficiency and size, are offered wholesale prices equal to the supplier's marginal cost whether or not the supplier is able to price discriminate (cf. O'Brien and Shaffer 1992, 1994; McAfee and Schwartz 1994; and Rey and Verge, 2004).[1]

Since all downstream firms receive the same wholesale prices, the case of unobservable contracts is arguably not ideal as a benchmark for an analysis of the welfare effects of price discrimination in intermediate-goods markets.[2] Whereas this benchmark would predict the absence of discriminatory wholesale prices even when price discrimination is feasible, in contrast, our model predicts that larger downstream firms will obtain lower wholesale prices. In fact, in our model, with two downstream firms, when one firm becomes relatively more competitive (e.g., when its own marginal cost decreases), it becomes optimal for the supplier to increase the other firm's wholesale price. This "waterbed effect" occurs because when the difference in the firms' competitive positions widens, holding wholesale prices constant, the difference in the firms' markups also widens. This increases the attractiveness to the supplier of shifting additional sales to the now more productive downstream firm, and thus efficiency requires that the supplier increase the difference in the two firms' wholesale prices. This ensures that both firms' market shares are again set to match the differences in their marginal costs or in the attractiveness of their products.[3]

[1] An important finding in this literature is that a ban on price discrimination mitigates the supplier's opportunism problem by making contracts observable, which, albeit for entirely different reasons than in our model, leads also to an increase in all (marginal) wholesale prices (cf. O'Brien and Shaffer, 1994).

[2] As the legislative history of the Robinson-Patman Act shows, the Congressional intent behind banning price discrimination in intermediate-goods markets was to create a level playing field among downstream competitors. At the time of the Act's passage in 1936, small and independent firms were competing against large chain stores that were receiving lower marginal prices on the same products (cf. ABA 1980).

[3] The "waterbed effect" is also observed in Inderst and Valletti (2009), who study linear wholesale

Our assumption of a monopolistic supplier is standard in the literature on price discrimination in intermediate-goods markets. We follow Katz (1987), DeGraba (1990), Yoshida (2000), and Cowan (2007) in also assuming that the supplier can make take-it-or-leave-it offers, which arguably can be justified on the grounds that for antitrust purposes the consideration of price discrimination in intermediate-goods markets is primarily relevant if the supplier enjoys a dominant position.[4] Two exceptions in this literature are O'Brien and Shaffer (1994) and O'Brien (2008), who look at price discrimination in intermediate-goods markets when downstream firms can bargain. O'Brien and Shaffer (1994) assume that contracts are private information when price discrimination is feasible. O'Brien (2008) looks at the welfare effects of price discrimination when downstream firms have outside options and can bargain, but restricts attention to linear contracts.[5] The case of bargaining when contracts are observable and nonlinear has not been examined.

The rest of the paper is organized as follows. Sections 2 and 3 deal with price discrimination and uniform pricing for the case in which downstream firms differ in their marginal cost of operation. Section 4 extends results to the case in which derived demands differ because firms face asymmetric final demand. Section 5 offers some concluding remarks.

2 Price Discrimination

The Model

We consider a downstream market in which two firms $i = 1, 2$ are active. Downstream firms have constant own marginal cost c_i and set prices p_i.[6] There is a single supplier in the upstream market. The supplier's own constant marginal cost is normalized to zero.

pricing in a setting in which downstream firms have access to an alternative supply option (as in Katz, 1987). In contrast, with linear demand, linear contracts, and without binding outside options, DeGraba (1990) shows that each downstream firm's wholesale price is *independent* of the marginal cost of its rival.

[4] For a contrasting perspective, see Marx and Shaffer (2007), where an upstream firm sells to two competing downstream firms and the downstream firms are the ones making the take-it-or-leave-it offers.

[5] Like us, O'Brien finds that wholesale prices can be higher and thus welfare can be lower when price discrimination is banned. For an application of his model to telecommunications, see O'Brien (1989).

[6] We cast our analysis in a model of price competition where the downstream products are either independent in demand or imperfect substitutes. Our results hold equally well in these cases with quantity competition, and do not depend on whether firms' strategies are strategic complements or substitutes.

We suppose that in order to produce one unit of the final good, each downstream firm requires one unit of the supplier's good as an input, although our results easily extend to any other production technology with fixed proportions. We also suppose that the supplier can make observable take-it-or-leave-it offers. In particular, we assume the supplier can specify for each firm a fixed fee F_i and a constant per-unit wholesale price w_i. As will become clear in what follows, this restriction to two-part tariffs (as a particular form of nonlinear contract) is without further loss of generality as long as the contracts are observable and price discrimination is permitted.[7] Working with a two-part tariff, we thus obtain for each downstream firm the respective overall marginal cost $k_i := w_i + c_i$.

In this and the next section, we assume that differences in the downstream firms' derived demands are obtained solely from differences in their marginal costs c_i. We thus stipulate a symmetric demand function $q_i = q(p_i, p_j)$, which is twice continuously differentiable (where positive) with $\partial q_i/\partial p_i < 0$ and $\partial q_i/\partial p_j \geq 0$, for $j \neq i$. (The case in which $\partial q_i/\partial p_j = 0$ holds everywhere corresponds to a situation in which the downstream firms operate in separate markets.) We further assume for convenience that downstream profits, which are given by $(p_i - k_i)q_i$, are strictly quasi concave in p_i. It then follows that as long as $q_i > 0$ is optimal, the first-order conditions with respect to p_i can be written as

$$p_i - k_i = -\frac{q_i}{\partial q_i/\partial p_i}. \tag{1}$$

Optimal Wholesale Prices

Denote equilibrium profits by π_i. Since the supplier optimally extracts these profits from the downstream firms by setting $F_i = \pi_i$, its choice of wholesale prices w_i ensures that industry profits are maximized. Assuming that the program to maximize industry profits,

$$\Omega := \sum_{i=1,2} (w_i q_i + \pi_i) = \sum_{i=1,2} (p_i - c_i) q_i,$$

is strictly quasi concave in final-goods prices p_i, we have the first-order conditions

$$\sum_{j=1,2} (p_j - c_j) \frac{\partial q_j}{\partial p_i} + q_i = 0. \tag{2}$$

Using conditions (1) and (2) to solve for the optimal w_i then yields the following result.

[7] That is, our results would be unchanged if price discrimination were feasible and one were to allow instead any nonlinear tariff, $T_i(q_i)$, where q_i is the respective quantity chosen by downstream firm i.

Lemma 1 *With price discrimination, the optimal wholesale prices w_i satisfy*

$$w_i \;=\; (p_j - c_j)\left(-\frac{\partial q_j/\partial p_i}{\partial q_i/\partial p_i}\right) \;\geq\; 0. \tag{3}$$

This says that the wholesale price charged to firm i should equal the joint profit margin of firm j and the supplier from sales of firm j's product, $p_j - c_j$, times the diversion ratio between firm i and j's products at the equilibrium final-goods prices, where the diversion ratio is defined as the fraction of sales lost by firm i that are gained by firm j when i's final-goods price increases.[8] Intuitively, the higher is the diversion ratio between products, and the higher is the joint profit margin of the supplier and firm j from sales of j's product, the more attractive it is for the supplier to favor firm j. For a given price w_j, the supplier shifts sales away from firm i and toward firm j by increasing i's wholesale price w_i.

Lemma 1 is useful for determining which firm, if any, will receive a lower wholesale price. Clearly, if the two downstream firms are equally efficient, so that $c_1 = c_2$, then each firm's contribution margin $p_i - c_i$ will be the same under symmetry, as will each firm's diversion ratio. It follows that the supplier will choose $w_1 = w_2$ in this case as there is no reason to favor one firm over the other. The supplier's optimal wholesale prices will also be the same in the case of independent goods or separate markets. In that case, $\partial q_j/\partial p_i = 0$ implies that the diversion ratios are zero for all p_i and p_j, and thus from (3) we have that $w_i = 0$ for both firms. More generally, however, the optimal wholesale prices will not be the same for both firms, as diversion ratios will typically not be zero when firms compete nor will the joint profit margins typically be the same across firms.

Linear Demand

To gain more intuition, suppose both firms compete in the same market and that $c_1 < c_2$. Suppose also that $w_1 = w_2$, so that differences in the firms' competitive positions, as expressed by k_i, are entirely due to differences in the firms' own marginal costs. Then, with linear demand, it is well known that the more efficient firm will sell the larger quantity, and thus, from (1) and using symmetry, the more efficient firm will have the strictly higher margin: $p_1 - k_1 > p_2 - k_2$. Since marginal derivatives $\partial q_i/\partial p_i$ and $\partial q_j/\partial p_i$ are constant when demand is linear, it then follows from (3) that $w_1 = w_2$ is in fact *not* optimal. Instead, the supplier should compensate for the more efficient firm's choice of a higher

[8] The diversion ratio is commonly used in antitrust analysis in merger cases (cf. Shapiro, 1996).

margin than its rival by setting $w_1 < w_2$, thereby shifting sales to the more profitable downstream firm. This reasoning extends as well to more general demand functions when changes in the marginal derivatives $\partial q_i/\partial p_i$ and $\partial q_j/\partial p_i$ play only a second-order role.[9]

To make the case with linear demand more explicit, we start from the quadratic utility of a representative consumer,

$$U = a_1 q_1 + a_2 q_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2b q_1 q_2), \tag{4}$$

with $a_i > 0$ and $0 \leq b < 1$. Maximizing $U - \sum_i p_i q_i$ with respect to q_i yields the indirect demand function $p_i = a_i - q_i - b q_j$, from which we obtain

$$q_i = \frac{a_i - b a_j}{1 - b^2} - \frac{1}{1 - b^2} p_i + \frac{b}{1 - b^2} p_j. \tag{5}$$

In order to characterize equilibrium prices, it is convenient (and without loss of generality) to rescale each firm's demand by the factor $1 - b^2$. Together with the definitions $\alpha_i := a_i - b a_j$ and $\gamma := b$, it then follows that direct demands can be written as[10]

$$q_i = \alpha_i - p_i + \gamma p_j. \tag{6}$$

Solving for the Nash equilibrium prices given each firm's profit, $(p_i - k_i)q_i$, yields

$$p_i = \frac{\alpha(2 + \gamma) + 2k_i + \gamma k_j}{4 - \gamma^2}. \tag{7}$$

It follows from (7) that p_i is increasing in k_i and k_j. Since γ is bounded above by one, however, we can see that the increase in p_i is larger for a given marginal increase in k_i. In particular, this implies that the supplier can indirectly control final-goods prices through its choice of wholesale prices, and thereby it can shift relative sales between the two firms.

Making use of Lemma 1 and setting $\alpha_i = \alpha$, we obtain the optimal wholesale prices[11]

$$w_i = \alpha \frac{\gamma}{2(1 - \gamma)} - c_j \frac{\gamma}{2}. \tag{8}$$

[9] What is generally sufficient for the first part of the argument to hold (the more efficient firm has a higher markup) is that a firm's pass-through rate be less than one, while in the case of strategic substitutes the rival firm responds less to a reduction in the other's marginal cost than the other firm does itself.

[10] It should be noted, however, that when changing the degree of substitutability or introducing heterogeneity in demand in what follows, we do so directly via the respective primitives a_i and b.

[11] It can be shown that both firms will be active in equilibrium if and only if $c_2 < \alpha + \gamma c_1$. This condition is obtained by substituting (8) into the equilibrium quantities $q_i = \frac{\alpha(2+\gamma)-(2-\gamma^2)k_i+\gamma k_j}{4-\gamma^2}$.

Since firm i's wholesale price is decreasing in its rival's marginal cost, it follows that i's wholesale price will be lower than j's wholesale price if and only if firm i is more efficient.

Substituting (8) and (7) into (6), and solving for each firm's equilibrium quantity, we see that the more efficient firm will obtain a larger share of downstream sales as a result of its lower wholesale price than it would have obtained had wholesale prices been equal.

Summarizing our findings in this section thus far, we have the following proposition.

Proposition 1 *When demand is linear and price discrimination is feasible, the monopolistic supplier will offer a lower wholesale price to the more efficient downstream firm. In equilibrium, this results in a higher share of sales for the more efficient downstream firm, and thus, allocative efficiency increases when the supplier is able to price discriminate.*

Proposition 1 contrasts with previous findings in the literature which show that with linear demand, simple price discrimination by a monopolistic supplier results in a higher wholesale price to the more efficient downstream firm and thus reduces welfare (cf. DeGraba, 1990; and Yoshida, 2000). The more efficient downstream firm is less price sensitive and thus is given a worse deal than the less efficient, more price sensitive downstream firm. As Proposition 1 implies, however, this conclusion is misleading when more complex means of price discrimination such as two-part tariffs are feasible. With two-part tariffs, the monopolistic supplier's preferences are reversed. The more efficient downstream firm receives a lower wholesale price because price discrimination with respect to the quantities that each downstream firm purchases is now feasible. With a fixed fee to extract surplus, the supplier is free to choose its wholesale price to induce the overall joint-profit maximizing quantities, which results in a lower marginal price for the lower-cost firm. Allocative efficiency is thus higher, not lower, with price discrimination than with uniform pricing.

With linear demand, as we have seen, downstream market adjustments alone do not go far enough in aiding a firm whose costs have decreased. Starting from a situation of equal costs, a decrease in firm 2's cost will cause both firms' final-goods prices to decrease, with firm's 2's price decreasing by more than firm 1's price. Sales will thus shift in favor of firm 2, but not by as much as they should have from the perspective of overall joint profit maximization. This imbalance is corrected when the upstream supplier is allowed to exercise its market power by engaging in price discrimination, because then relative outputs can further be shifted by adjusting wholesale prices (from (8), we see that firm i's wholesale price increases if firm j becomes more efficient and decreases if firm j becomes less efficient). This effect, which is sometimes referred to as the 'waterbed effect', says that as firm j becomes more competitive, wholesale prices will be chosen to further amplify the firms' cost difference, thus causing firm i's position to further deteriorate and allowing firm j to become even more competitive. Moreover, it follows that since

$$w_j - w_i = \frac{\gamma}{2}(c_j - c_i), \tag{9}$$

the difference in wholesale prices and thus the impact of the effect will grow larger as the products become closer substitutes. But despite the negative connotations that are sometimes attributed to it, the waterbed effect in this case enhances allocative efficiency.

Unfortunately, allowing the supplier to exercise its market power and engage in price discrimination can also be costly for welfare. As can be seen from the profit-maximizing wholesale prices in (8), for all $\gamma > 0$, the supplier will choose $w_i > 0$ (which exceeds its normalized marginal cost). In doing so, the supplier dampens downstream competition and elevates final-goods prices above what they would have been, thus preventing joint profits from dissipating to final-goods consumers. Although this benefits the supplier, consumers are harmed. This 'dampening-of-competition' effect, like the waterbed effect, becomes larger (both wholesale prices increase) as the goods become closer substitutes.[12]

General Welfare Implications of Supplier Market Power

The two effects that follow from allowing a monopolistic supplier to exercise its market power and engage in price discrimination, the dampening-of-competition effect ($w_i > 0$) and the waterbed effect ($w_i < w_j$ if and only if $c_i < c_j$), have opposing consequences for welfare. The dampening-of-competition effect leads to a decrease in consumer surplus all else being equal, while the waterbed effect leads to an increase in allocative efficiency.

To bring out this tension more formally, we compare the outcome with a monopolistic supplier to that when there is perfect competition upstream. In the latter case, both downstream firms would be able to purchase their inputs at marginal cost, which would benefit consumers by leading to lower final-goods prices, but there would also be no favoring of the more efficient firm. Moreover, one would still be (only) in a second-best situation because the downstream firms would have market power. In contrast,

[12] Using the primitives $\alpha = a(1-b)$ and $\gamma = b$, it can be shown that $dw_n/db = (a - c_j)/2 > 0$.

a monopolistic supplier both reduces welfare by inducing higher final-goods prices and increases welfare by improving productive efficiency for a *given* level of output produced.

Given these conflicting effects, the impact of upstream market power on welfare will generally be ambiguous. As is intuitive, however, a clear-cut case for when welfare is higher with a monopolistic supplier arises when total demand is not sensitive to (marginal) wholesale price changes, at least over the relevant range of prices. This holds, for instance, in the ("workhorse") Hotelling model when the market is fully covered not only under competition but also in the monopoly case, which in turn holds when "transportation" costs are sufficiently low. We prove this in the appendix to the following proposition, where we also show that with linear demand derived from the utility function in (4), the opposite result holds, such that welfare is lower when upstream market power is exercised.

Proposition 2 *The welfare gain that arises from increased allocative efficiency when upstream market power is exercised can more than offset the welfare loss that arises from higher wholesale prices. This holds, in particular, in the case of Hotelling competition when the market is fully covered. On the other hand, when demand is linear and derived from the utility function in (4), welfare is lower when upstream market power is exercised.*

Proof. See Appendix.

It is noteworthy that this identified benefit from market power, which–at least to our knowledge–is new to the literature, only applies if market power is exerted in the *intermediate-goods* market, but does not apply to market power that may be exerted by a multi-product firm in the final-goods market. It is also noteworthy that if the monopolistic supplier were not able or allowed to engage in price discrimination, only the dampening-of-competition effect would be operative and thus welfare would be unambiguously lower if the upstream market were monopolized than if there were perfect competition upstream.

3 Uniform Pricing

Optimal Uniform Wholesale Price

Consider now the effects of a ban on price discrimination, where the expressed purpose is to create a level playing field for the smaller downstream firm. One way to model this is to assume that the supplier must offer the same two-part tariff contract (F, w) to both firms.

Another way to model this is to assume that the supplier can offer a menu of two-part tariff contracts, as long as the same menu is offered to both firms. In what follows, we adopt the former approach and assume that menus of contracts are not allowed. We do so for two reasons. First, allowing the supplier to offer a menu of contracts, even though it must offer the same menu to both downstream firms, would still allow it to engage in price discrimination, albeit only indirectly.[13] Second, menus of two-part tariff contracts may in practice be deemed to be discriminatory by courts and hence violate antitrust laws, especially if the large quantities that must be purchased to obtain a lower wholesale price are not commercially viable for small buyers (i.e., if the lower wholesale price is not deemed to be "functionally available to all").[14] In contrast, when the supplier offers a single two-part tariff contract, as we assume here, both downstream firms, irrespective of the volume they purchase, will be facing the same marginal wholesale price and will thus be competing on the same "level playing field," as required by the spirit of provisions such as in the Robinson-Patman Act (for the US) or in Article 82 (for the European Union).

Assuming it is still optimal to serve both firms when price discrimination is banned (with linear demand, the respective conditions will be made explicit below), the optimal choice of F will, for a given w, not be sufficient to extract all surplus from both firms. Instead, the more efficient firm will be left with positive rent. To see what effect this has on equilibrium wholesale prices, it is instructive to begin by considering first the extreme case where the downstream firms are in *separate* markets (i.e., when their products are independent in demand). Denoting the dependency of each firm's profit on its own marginal cost by the function $\pi(k)$, with $c_1 < c_2$, the supplier's objective function is then

$$\psi := \Omega - [\pi(k_1) - \pi(k_2)] . \tag{10}$$

[13] Any outcome in the final-goods market that can be obtained when the same menu of contracts is offered to both firms can also be obtained with a *single* nonlinear tariff, which maps procured quantity into the total payment that each downstream firm must make. Thus, a ban on menus of contracts, to have any effect, must also effectively rule out the offering of such general nonlinear tariffs by the supplier.

[14] According to *Antitrust Law Developments*, (2007: 518-519), courts have held that if the lower price was in fact made available to the allegedly disfavored buyer, then there is no violation of the law. However, as the courts have made clear, this defense includes two conditions. "First, the competing purchasers must know that the lower price is available. Second, most competing customers must be able to obtain the lower price, such that the lower price is functionally–and not merely theoretically–available to them."

Recall that Ω denotes industry profits, and note that the difference $\pi(k_1)-\pi(k_2)$ captures the more efficient firm's rent. Thus, in the absence of the ability to price discriminate, the supplier will want to distort w away from that which would maximize industry profit. In particular, differentiating (10) with respect to w, the first-order condition requires that

$$\frac{d\Omega}{dw} - \frac{d\left[\pi(k_1) - \pi(k_2)\right]}{dw} = 0. \tag{11}$$

Since Ω is assumed to be strictly quasi concave and as $d\Omega/dw = 0$ holds at $w = 0$, given that firms serve separate markets, the direction of distortion will depend on the sign of the bracketed term in (11), which is negative given that $\pi(k)$ is strictly convex in k. To see this, note that $\pi'(k) = -q$ holds from the envelope theorem and thus $\pi''(k) > 0$ holds from the first-order condition for p_i, which implies that $dq/dk < 0$. It follows that for separate markets the optimal uniform wholesale price satisfies $w > 0$.

Intuitively, under uniform pricing, the supplier uses its wholesale price to extract more surplus from the more efficient firm. This is done through an increase in w. Importantly, while this also reduces the total surplus that the supplier can extract from the less efficient firm, whose participation constraint just binds, the additional surplus that can be extracted from the more efficient firm is strictly larger due to the convexity in profits.

These insights extend beyond the case of separate markets also to the case in which downstream firms compete. For this we can extend previous notation by writing $\pi(k_i, k_j)$ to denote the downstream firms' profits. The optimal choice of w is then determined by

$$\frac{d\Omega}{dw} - \frac{d\left[\pi(k_1, k_2) - \pi(k_2, k_1)\right]}{dw} = 0. \tag{12}$$

The first-order condition (12) reveals two key effects that a ban on supplier price discrimination has when downstream firms compete. First, to maximize industry profit, which would be the case if $d\Omega/dw = 0$, the supplier is constrained to offer a common wholesale price. As we discuss below, this involves a loss in efficiency and thus welfare. Second, if

$$\frac{d\left[\pi(k_1, k_2) - \pi(k_2, k_1)\right]}{dw} < 0 \tag{13}$$

holds, an increase in the common wholesale price w reduces the more efficient firm's profit by more (in absolute terms) than it reduces the profit of the less efficient firm. This effect makes it optimal for the supplier to increase w in order to extract higher overall profits.

Whether the condition in (13) holds for any given demand system depends, in general, on a comparison of various second-order derivatives. Nevertheless, the underlying intuition is the same as it is for separate markets. That is, given that a more efficient firm sells a larger quantity than a less efficient firm, the more efficient firm will be affected on a larger volume base–and thus more strongly–when there is a common wholesale price increase. It is easy to verify that the condition in (13) is indeed satisfied when demand is linear.

Linear Demand

With the linear demand system in (6), the optimal uniform wholesale price is

$$w = \alpha \frac{\gamma}{2(1-\gamma)} + \frac{c_2(4-3\gamma^2) - c_1(4+4\gamma-\gamma^2)}{4(2+\gamma)}. \tag{14}$$

Comparing this outcome to the optimal discriminatory prices given in (8) yields

$$\begin{aligned} w - w_2 &= \frac{4-3\gamma^2}{4(2+\gamma)}(c_2 - c_1), \\ w - w_1 &= \frac{4+4\gamma-\gamma^2}{4(2+\gamma)}(c_2 - c_1), \end{aligned}$$

which are strictly positive in both cases when $c_1 < c_2$ (i.e., when firm 1 is the more efficient downstream firm), as was assumed in the supplier's objective function in (10) and (12).

Proposition 3 *Suppose firm i is more efficient than firm j. Then both firms' wholesale prices may increase when the supplier is unable to price discriminate. This holds, in particular, when the downstream firms are in separate markets, or when demand is linear.*

These results stand in stark contrast to those in the literature on third-degree price discrimination in final-goods markets, and to the results in DeGraba (1990) and Yoshida (2000), where the supplier's profit-maximizing uniform price lies strictly between the otherwise prevailing (linear) discriminatory prices. If in the absence of price discrimination, nonlinear contracts are infeasible, as we assume, then, at least for the cases of separate markets and linear demand, it follows from Proposition 3 that all wholesale prices increase.

Compared to the case with linear contracts, there is also an interesting difference in the comparative statics of the optimal uniform (marginal) wholesale price. With linear contracts only, the optimal uniform wholesale price is strictly higher if *either* of the two firms becomes more efficient. This follows because the optimal uniform wholesale price in this case is based on a weighted average of the elasticities of the two derived demands. In contrast, it follows from inspection of (14) that when two-part tariffs are feasible, the impact of a change in c_i on w depends markedly on the identity of the respective firm i. While making the currently less efficient firm, $i = 2$, more efficient by reducing c_2 results in a lower uniform wholesale price, the opposite result holds if the already more efficient firm, $i = 1$, becomes yet more efficient. In the latter case, a reduction in c_1 leads to an increase in the uniform wholesale price. Intuitively, the smaller is the difference $c_2 - c_1$, the less important is the second term in the supplier's first-order condition (12), which captures the role of a higher w in extracting additional rent from the more efficient firm.

Consumer Surplus and Welfare

Turning to a comparison of consumer surplus and welfare, it is immediate that a ban on price discrimination leads to a reduction in the quantity supplied by the more efficient firm. It may, however, for γ sufficiently large, lead to an increase in the quantity supplied by the less efficient firm.[15] This follows because the wholesale price increase after a ban on price discrimination is strictly larger for the more efficient firm (i.e., $w - w_1 > w - w_2$). However, since both wholesale prices increase when price discrimination is banned compared to the outcome with discriminatory pricing, it follows that the total quantity supplied, $q_1 + q_2$, must be strictly lower. What is more, under uniform pricing, a larger fraction of the total output will be produced by the firm that is less efficient, which further reduces welfare.

To see this more formally, if total output q is produced, then we can show that total welfare is maximized by choosing final-goods prices and, thereby, quantities such that

$$q_1 - q_2 = \frac{1}{1-\gamma}(c_2 - c_1). \tag{15}$$

That is, the efficient difference in quantities and market shares increases with the difference in the firms' own marginal costs $c_2 - c_1 > 0$ and with the degree of substitution $\gamma = b$.

Note next that given any two wholesale prices w_i, equilibrium quantities are such that

$$q_1 - q_2 = \frac{2+\gamma-\gamma^2}{4-\gamma^2}\left((w_2 + c_2) - (w_1 + c_1)\right). \tag{16}$$

With a uniform wholesale price $w_i = w$, and for a given total quantity q, the *efficient* difference in quantities (15) exceeds the difference in (16). (This also follows immediately from the observation that the more efficient firm charges a higher margin, which shifts sales

[15] Substituting w_i from (8) into the equilibrium quantities (cf. footnote 11) yields $q_2 = \frac{\alpha - c_2 + \gamma c_1}{2}$ for the less efficient firm. After the ban, the less efficient firm's quantity is $q_2 = \frac{\alpha}{2} + \frac{c_1(2+3\gamma-\gamma^2) - c_2(6+\gamma-3\gamma^2)}{4(2+\gamma)}$. The net change in quantity after a ban for the less efficient firm is therefore $\Delta q_2 = \frac{(c_2-c_1)(\gamma+3\gamma^2-2)}{4(2+\gamma)}$.

to the less efficient firm.) If, instead, the supplier could optimally set discriminatory prices, so that $w_2 > w_1$, then sales would shift back to the more efficient firm. In particular, on substituting w_i from (9) into (16), we have that under the optimal discriminatory prices,

$$q_1 - q_2 = \frac{2 + \gamma - \gamma^2}{4 - 2\gamma}(c_2 - c_1), \tag{17}$$

which is larger than the difference in (16) but still less than the efficient difference in (15).

We next make the comparison of consumer surplus and welfare more formal.

Proposition 4 *When the downstream firms are in separate markets, or when demand is linear, a ban on the supplier's ability to price discriminate weakly reduces both consumer surplus and welfare (strictly reduces them if firm i is more efficient than firm j).*

Proof. Since both wholesale prices increase under the hypothesized circumstances (separate markets or linear demand) when $c_i \neq c_j$ and price discrimination by the supplier is banned, we know that the final-goods prices of the downstream firms will also increase by standard duopoly comparative statics. Thus, both firms' price-cost margins, $p_i - c_i$ will increase. It then follows immediately that consumer surplus must be strictly lower.

Note next that by revealed preference, total industry profits (of the supplier and the two downstream firms) must also be lower because with discrimination the maximum industry profit is obtained whereas without discrimination there is a restriction on the instruments available to capture profits. Taken together, as both consumer surplus and industry profits are strictly lower absent discrimination, welfare strictly decreases. **Q.E.D.**

Once again our results contrast with the results in the literature that are obtained with linear contracts. As shown by DeGraba (1990) and Yoshida (2000), when demand is linear, a ban on price discrimination leads to an increase in consumer surplus and welfare. For welfare, the difference between their results and ours follows immediately from their observation that with linear contracts the discriminatory wholesale price is strictly higher for the more efficient firm, which is the opposite of what we find. For consumer surplus, the reduction in the more efficient firm's wholesale price in their model following a ban on price discrimination more than compensates for the increase in the less efficient firm's wholesale price, and the net effect is an increase in consumer surplus. In contrast, in our model with two-part tariffs, consumer surplus decreases as both wholesale prices increase.

Note finally that we have assumed that the supplier serves both firms whether or not price discrimination is feasible. However, in the absence of price discrimination, the supplier no longer extracts all surplus from both firms, and so this may no longer hold. In the event that the supplier serves only one firm (i.e., the more efficient firm)[16], there would be an additional welfare loss from the imposition of uniform pricing. That the imposition of uniform pricing can reduce consumer surplus and welfare by making it optimal for the supplier to no longer serve all firms (or markets) mirrors analogous results in the extant literature (cf. Katz (1987) for the case of intermediate-goods markets and linear pricing, or the surveys by Armstrong (2005) and Stole (2005) for the case of final-goods markets).

4 Heterogeneous Demand

We now consider the case in which differences in derived demands stem solely from differences in final-goods demands. That is, we now allow consumers to have asymmetric preferences for the two firms' products, but assume the firms' marginal costs are the same ($c_1 = c_2$). We will show that all of our qualitative results are robust to this extension.

Competition in Downstream Markets

Consider again the quadratic utility of a representative consumer (cf. condition (4)). Suppose preferences are such that $a_1 > a_2$. Then, it follows from $\alpha_i = a_i - ba_j$ that $\alpha_1 > \alpha_2$ for the indirect final-goods demand function given in (6). Results with discriminatory pricing mirror those where differences in derived demand are due to differences in firms' own marginal costs. Since the firm with the superior product faces a less elastic (residual) demand, it will, ceteris paribus, want to charge a strictly higher markup $p_i - k_i$. To compensate for this, in order to maximize industry profits, it is therefore optimal for the supplier to offer a strictly lower wholesale price to this firm: $w_1 < w_2$. Formally, we have[17]

$$w_i = \frac{\gamma}{2(1-\gamma^2)}(\alpha_j + \gamma\alpha_i) - c\frac{\gamma}{2}, \tag{18}$$

and thus

$$w_2 - w_1 = \frac{\gamma}{2(1+\gamma)}(\alpha_1 - \alpha_2) > 0,$$

[16] One can show that both firms will be served after the ban if $c_2 < \frac{\alpha(4+2\gamma)+c_1(2+3\gamma-\gamma^2)}{6-3\gamma^2+\gamma}$. Note that this is more restrictive than the respective requirement when price discrimination is feasible, $c_2 < \alpha + \gamma c_1$.

[17] It can be shown that both firms are indeed active under discriminatory pricing if $\alpha_i > c(1-\gamma)$.

where the difference $w_2 - w_1$ is increasing in γ. Interestingly, whereas previously a reduction in c_i led to a higher wholesale price for firm j, we see from (18) that this waterbed effect does not hold if we change α_i. This difference between the two cases is intuitive once it is realized that after an increase in α_i, both prices p_i and p_j must increase in order to maximize industry profits, whereas after a reduction in c_i, industry profits are maximized if firm i's price decreases and firm j's price increases. In all other respects, a ban on price discrimination has the same effect irrespective of whether differences in derived demands arise from differences in the firms' marginal costs or in their intercept terms α_1 and α_2.

Proposition 5 *When demand is linear, price discrimination is feasible, and differences in the firms' derived demands stem solely from differences in their final-goods demands, then $w_1 < w_2$ if and only if firm 1's product has the higher vertical intercept, $\alpha_1 > \alpha_2$. In this case, a ban on the supplier's ability to price discriminate leads to higher wholesale prices for both downstream firms and thus reduces consumer surplus and hence welfare.*

Proof. Equilibrium final-goods prices and quantities are given by

$$\begin{aligned} p_i &= \frac{2\alpha_i + \gamma\alpha_j + 2k_i + \gamma k_j}{4 - \gamma^2}, \\ q_i &= \frac{2\alpha_i + \gamma\alpha_j - (2 - \gamma^2)k_i + \gamma k_j}{4 - \gamma^2}, \end{aligned} \tag{19}$$

whereas for overall joint profit to be maximized, it must hold that

$$p_i = \frac{\alpha_i + \gamma\alpha_j + (1 - \gamma^2)c_i}{2(1 - \gamma^2)}. \tag{20}$$

From the expressions in (19) and (20), it follows that w_i must be given by (18).

Under uniform pricing, the supplier's profits are again given by $\Omega - (\pi_2 - \pi_1)$, where it can be shown that $\pi_i = q_i^2$. This program is concave in w and is maximized at

$$w = \frac{\alpha_1(4 - 4\gamma + 3\gamma^2) - \alpha_2(4 - 8\gamma + \gamma^2)}{4(2 - \gamma - \gamma^2)} - c\frac{\gamma}{2}.$$

From this expression for w, together with (18), it follows that

$$\begin{aligned} w - w_2 &= (\alpha_1 - \alpha_2)\frac{4 + 4\gamma - \gamma^2}{4(1+\gamma)(2+\gamma)}, \\ w - w_1 &= (\alpha_1 - \alpha_2)\frac{4 - 3\gamma^2}{4(1+\gamma)(2+\gamma)}, \end{aligned}$$

which are both strictly positive as long as $\alpha_1 > \alpha_2$. That consumer surplus is strictly lower under uniform pricing follows as $w > w_i$ implies that both prices p_i are higher. Finally, the argument for why welfare is lower is analogous to that in the proof of Proposition 4. **Q.E.D.**

Separate Markets

By specializing to the case of separate markets, we can extend our results beyond linear demand. In this case, some structure is needed to specify how markets differ in size. Suppose therefore that demand in each of the two separate markets is derived from a representative consumer who has utility $U(q_i - \lambda_i)$, where $q_i \geq 0$ is the quantity consumed of firm i's final good, $\lambda_i \geq 0$ is a shift parameter in firm i's market, and $q_i - \lambda_i \geq 0$. Cowan (2007) used this utility function to analyze the welfare effects of third-degree price discrimination in final-goods markets. We will compare our results to his results below.

Utility maximization implies that the indirect final-goods demand can be written as $p_i = U'(q_i - \lambda_i)$, and thus that firm i's direct demand can be written as $q_i = \lambda_i + g(p_i)$.[18] As in Cowan, we assume there exists a choke price $\bar{p}$ such that for all $p_i > \bar{p}$, $q_i = 0$ and $g(p_i) = 0$, for all $p_i = \bar{p}$, $q_i = \lambda_i + g(\bar{p}) \geq 0$, and for all $p_i < \bar{p}$, $g(p) > 0$. For prices above the choke price, demand is zero. For prices below the choke price, demand is positive.

Note that with this demand and at equal prices, firm 1 serves a larger market than firm 2 if and only if $\lambda_1 > \lambda_2$. Without loss of generality, we assume that $\lambda_1 > \lambda_2$ holds. It is also convenient to suppose that the monopoly pricing problem is strictly quasiconcave.

When price discrimination is feasible, the supplier will charge $w_i = 0$ and use its fixed fee to extract the downstream firms' surplus. To see that the uniform wholesale price in the absence of price discrimination is, instead, strictly positive, which mirrors our results in the previous sections, note that from (11) we only need to show that $d\pi_1/dw < d\pi_2/dw$. That is, we need to show that an increase in w reduces the profits of the firm with the larger final-goods (and thus also derived) demand by more. As we show in the proof of the following proposition, this is the case whenever, holding w constant, a firm facing a larger market (higher λ) ends up selling a higher quantity at the respective optimal price.

[18] One can think of demand as consisting of λ_i committed purchasers who each buy at most one unit, and price-sensitive consumers who each buy more than one unit with aggregate demand given by $g(p_i)$.

A sufficient condition for this to be the case is that, for all p where $q > 0$, it holds that

$$g' + pg'' < 0. \tag{21}$$

Proposition 6 *Suppose firms serve separate markets, and that final-goods demands, as obtained from maximization of a representative consumer's utility, are given by $q_i = \lambda_i + g(p_i)$, where $\lambda_1 > \lambda_2 \geq 0$ represents a shift factor. Then if (21) holds, a ban on price discrimination raises both wholesale prices and reduces consumer surplus and welfare.*

Proof. To show that both wholesale prices will increase if (21) holds and price discrimination is banned, it suffices to show that $d\pi_1/dw < d\pi_2/dw$, or in other words, that

$$\frac{d^2\pi}{dkd\lambda} < 0, \tag{22}$$

where we have omitted the subscripts from $k = c+w$ as well as from λ. Since $d\pi/dk = -q$ by the envelope theorem, (22) follows if and only if $dq/d\lambda > 0$, where we have used the fact that strict quasiconcavity implies that each downstream firm's optimal price and thus also quantity are unique. We next implicitly differentiate the first-order condition $q'(p-k) + q = 0$, which becomes $g'(p)(p-k) + (\lambda + g(p)) = 0$, to obtain $dp/d\lambda = -\frac{1}{2g'+(p-k)g''}$. This can then be substituted to ultimately obtain $\frac{dq}{d\lambda} = 1 + g'(p)\frac{dp}{d\lambda}$, which is

$$\frac{dq}{d\lambda} = \frac{g' + (p-k)g''}{2g' + (p-k)g''}. \tag{23}$$

As the denominator of (23) represents the second-order condition, $\frac{dq}{d\lambda} > 0$ follows from (21). Precisely, it holds surely whenever $g'' \leq 0$, while for $g'' > 0$ we have to invoke (21).

8Finally, the claim that a ban on price discrimination reduces consumer surplus and welfare if (21) holds follows from the same argument as in the proof of Proposition 4. **Q.E.D.**

Once again, our results stand in contrast to those in the existing literature. In this case, the relevant comparison is with Cowan (2007), who concludes that price discrimination lowers welfare for all commonly used demand functions as long as all markets are served. Proposition 6 shows that this finding does not extend to the case of price discrimination in intermediate-goods markets when the upstream firm can offer two-part tariff contracts.

5 Concluding Remarks

From a positive perspective, our paper takes a novel look at price discrimination in intermediate-goods markets. We find that when the supplier's goal is to maximize industry profits, the supplier will adjust wholesale prices not only to dampen downstream (intrabrand) competition, but also to shift final sales among firms. We showed how these adjustments can lead to lower wholesale prices for those downstream firms that are larger in size—either because they are more efficient or because their products are more desirable to consumers. The supplier's optimal choice of discriminatory wholesale prices thus amplifies the competitive advantage of those firms relative to that of their smaller rivals.

This implication distinguishes our model and results empirically from those in the extant literature on price discrimination by a monopolistic supplier. When contracts are linear, it has been shown that the more efficient or otherwise more competitive firm will receive the higher wholesale price, contrary to our model where this firm receives the lower wholesale price. And, in the case in which two-part tariffs are feasible but unobservable and the supplier maximizes bilateral profits, it has been shown that all downstream firms will be offered wholesale prices that equal the supplier's marginal cost and there is no price discrimination even when such discrimination is feasible. In empirical work, it may be possible to distinguish broadly among these three regimes: that of linear contracts, and that of observable or unobservable nonlinear contracts (e.g., two-part tariffs). This distinction among regimes may also be valuable for antitrust purposes in gauging the effects of imposing restrictions on the ability of suppliers to engage in price discrimination.

We also compared the effects of price discrimination on consumer surplus and welfare relative to two benchmarks. In one benchmark, we compared the case where price discrimination is feasible to the case where it is not feasible, assuming the upstream market is monopolized, and found that when demand is linear, a ban on the supplier's ability to engage in price discrimination can lead to higher wholesale prices for all downstream firms, irrespective of whether the supplier's contracts are observable (as in our model) or not. This leads to an unambiguous reduction in both consumer surplus and welfare.[19] In contrast, when contracts are linear, a monopolistic supplier will charge a uniform price

[19] With observable contracts the loss in welfare is greater vis a vis the case of unobservable contracts, as a ban on price discrimination also interferes with the supplier's ability to enhance allocative efficiency.

that lies between the higher discriminatory price of a more efficient (larger) downstream firm and the lower discriminatory price of a less efficient (smaller) downstream firm. In this case, it has been shown by others that consumer surplus and hence welfare are higher under a ban on price discrimination as long as the supplier continues to serve both firms.

In the other benchmark, we compared the case of an upstream monopolist to the case of upstream perfect competition. We found that although wholesale prices are above marginal cost in the former case, resulting in a welfare loss, there is a countervailing effect that results in a welfare gain (the upstream monopolist reallocates output downstream to the more efficient downstream firms). Moreover, we found that this effect can more than offset, in some cases, the welfare loss that arises from the exercise of market power. Hence, in these cases, consumer surplus and welfare is actually higher with an upstream monopoly (provided price discrimination is feasible) than it would be under perfect competition.

6 Appendix: Omitted Proof of Proposition 2

Take first the Hotelling case. In the (standard) Hotelling model, costumers are uniformly distributed over "locations" $x \in [0,1]$, while firms $i=1$ and $i=2$ are positioned at the two extremes and sell, apart from this spatial differentiation, homogeneous products. Each customer demands at most one unit, for which he has valuation $v>0$, and has constant, linear "transportation" costs $\tau>0$. When $c_2-c_1<3\tau$ and $v \geq \frac{3}{2}\tau+(c_1+c_2)/2$, there is a unique equilibrium in which the market is fully covered and both firms have positive demand. In this unique equilibrium, one can show that $p_2-p_1=(c_2-c_1)/3$. In contrast, the price difference that would maximize welfare through reducing aggregate "shoe leather costs," given that the market is fully covered, would clearly be $p_2-p_1=c_2-c_1$.[20]

For the case where the upstream market is monopolized, we follow the main text and solve first for the optimal retail prices of a multi-product firm. If the firm wants to cover the whole market, it maximizes $q_1(p_1-c_1)+q_2(p_2-c_2)$, where $q_i=\frac{1}{2}+(p_j-p_i)/(2\tau)$, subject to the constraint that all customers buy, which becomes $p_2+p_1\leq 2v-\tau$. This yields $p_i=v-\frac{1}{2}\tau+(c_i-c_j)/4$ and thus $p_2-p_1=(c_2-c_1)/2$, such that aggregate "shoe leather costs" are strictly lower than in the case with upstream competition. The

[20] Given a price difference of p_2-p_1 and thus a "critical consumer" located at $\widehat{x}=1/2+(p_2-p_1)/(2\tau)$, aggregate "shoe leather costs" are given by $\tau\left[\int_0^{\widehat{x}} x dx+\int_{\widehat{x}}^1 (1-x)dx\right]$.

respective wholesale prices that generate this outcome are obtained from requiring that $p_i = v - \frac{1}{2}\tau + (c_i - c_j)/4$ is equal to $p_i = \tau + (2k_i + k_j)/3$, which yields $w_i = v - \frac{3}{2}\tau - (c_i + 3c_j)/4$ and thus $w_2 - w_1 = (c_2 - c_1)/2 > 0$. Finally, in solving the monopoly problem under the alternative assumption that the market is not fully covered, we obtain that, as stipulated in the Proposition, the market is also covered under monopoly if $v \geq \tau + (c_1 + c_2)/2$.

We turn next to the case with linear demand derived from (4). Denote industry profits by $\Omega = \sum_{i=1,2} q_i(p_i - c_i)$, such that in this case welfare is given by $\omega := \Omega + U$. More particularly, we denote welfare with perfect upstream competition by ω^C and welfare with upstream monopoly by ω^M. Using similar notation for the respective equilibrium wholesale and retail prices, we obtain $w_i^C = 0$ and w_i^M given by (8), and thus from (7), equilibrium prices p_i^M and p_i^C satisfy

$$p_i^M = p_i^C + \frac{2b(a - c_j) + b^2(a - c_1)}{2(4 - b^2)}.$$

Likewise, we obtain for equilibrium retail quantities

$$q_i^M = q_i^C + \frac{b^2\,(a - c_i) + b(b^2 - 2)\,(a - c_j)}{2(1 - b^2)(4 - b^2)}.$$

These values can now be substituted into ω^C and ω^M. After tedious transformations we find that

$$\begin{aligned} & \omega^C - \omega^M \\ = \;& -A^2\left(1 + \frac{b}{4}\right) + B\left[p_1^C - c_1 + \frac{b(b+2)(a - c_1)}{2(4 - b^2)}\right] + B\left[p_2^C - c_2 + \frac{b(b+2)(a - c_1)}{2(4 - b^2)}\right] \\ & + \frac{b\,(2a - c_1 - c_2)\,((bp_2^C - p_1^C)(2 + b))}{(1 - b^2)(4 - b^2)}, \end{aligned}$$

where

$$\begin{aligned} A &:= \frac{b(b + b^2 - 2)\,(2a - c_1 - c_2)}{(1 - b^2)(4 - b^2)}) < 0, \\ B &:= \frac{(b^2 + b - 2)\,(a - c_1)}{2(1 - b^2)(4 - b^2)} < 0. \end{aligned}$$

Hence, to show that $\omega^C - \omega^M < 0$ holds for all $b > 0$ (while it is clearly equal to zero when products are no longer substitutes, given $b = 0$), it is thus sufficient to show that

$$bp_2^C - p_1^M < 0.$$

As is immediate from inspection of the demand function (5), this condition holds if and only if own-price effects are stronger than cross-price effects, which holds with linear demand.[21] **Q.E.D.**

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[21] Formally, this condition is equivalent to the requirement that the derivative of the representative consumer's utility with respect to p_i is strictly negative, given the optimally purchased quantities q_i and q_j.

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