Public Policy and Venture Capital Financed Innovation: A Contract Design Approach

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Abstract

The effects of public policy programmes which aim at internalising spill-overs due to successful innovation are analysed in a sequential double-sided moral hazard double-sided adverse selection framework. The central focus lies in analysing their impact on contract design. We show that in our framework only ex post grants are a robust instrument for implementing the first-best situation, whereas the success of guarantee programmes, ex ante grants and some public-private partnerships depends strongly on the characteristics of the project: in certain cases they not only give no further incentives but even destroy contract mechanisms and so worsen the outcome.

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1 Introduction

Policies to foster innovation are widespread as without innovation an economy would stagnate, with obvious negative effects on the welfare of future generations. Financial development is particularly important against the background of financing constraints for young innovative firms with few tangible assets (see e.g. Audretsch and Thurik (2001), Rajan and Zingales (2001)); in particular, the venture capital industry plays a crucial role in the financing of these firms (see Kortum and Lerner (2000)). Thus, it is not surprising that a broad range of public policy programmes aims at fostering innovation by supporting the venture capital industry. The European Investment Fund’s venture capital portfolio, for example, amounts to more than 2.5 billion euros invested in more than 185 venture capital funds. Thus, the EIF has indirectly supported more than 1800 high-tech firms in Europe (see EIB (2004)).

But the crucial question is whether governments should really intervene in the venture capital industry, and if so, how should public policy programmes be designed. This paper aims at answering these questions within a formal framework by focussing on the impact public policy measures have on contract design. Thus, the paper’s purpose is to contribute to filling the gap in research on public policy in the venture capital area - a particularly crucial gap given the importance of the venture capital industry and the high intensity of public support.

We choose this contract design approach because of the prevailing role of explicit contracts in the venture capital industry. Explicit contracts are necessary to solve the severe moral hazard and adverse selection problems which arise from the specific characteristics of this industry, namely the active involvement of venture capitalists in the operations of the start-up firms and the high degree of uncertainty to which venture capitalists are exposed regarding the value of their investments, exacerbated by the small collateral platform from which entrepreneurs operate. Moreover, as the partnership between entrepreneur and venture capitalist is of limited duration (the venture capitalist’s objective is to exit, i.e. to make a return by getting out), it is not possible to solve these conflicts of interest implicitly. In fact, empirical studies confirm the existence of complex and highly
sensitive contracts which do determine not only the cash-flow rights of the contracting parties but also specific control and decision rights of the venture capitalist (e.g. Kaplan and Strömberg (2003)).

We will analyse different public policy programmes which can be observed in the various countries, namely ex post grants, guarantee programmes, ex ante grants, public-private partnerships and public support. Ex post grants can be interpreted as tax breaks and there exists a great variety of ex ante grants. The awards of the “Small Business Innovation Research Program” or those of the “Advanced Technology Program” in the United States may be the most prominent examples. Guarantee programmes can/could be observed in many countries: in Germany, for example, risk transfer was an essential component of many programmes of Kreditanstalt für Wiederaufbau (KfW) until 2004 and there still exists a guarantee programme targeted at investments in later stage firms; in the Netherlands, the “Private Participation Guarantee Order Scheme” was in operation between 1981 and 1995; the Banque du Développement des Petites et Moyennes Entreprises in France and the Austria Wirtschaftsservice Gesellschaft both give specific guarantees. Direct supply of capital by the government is organised in different ways, as can be observed in various current and former programmes of KfW in Germany, of the European Investment Fund, or of the “Small Business Investment Companies Program” in the United States. Finally, public support is not very common but the European Investment Fund, for example, has the “Seed Capital Action Programme” whereby part of the VC’s management costs are covered. We will show that in our framework only ex post grants can implement the first-best situation independently of specification issues. The success of guarantee programmes, ex ante grants and some public-private partnerships, on the contrary, depends strongly on the project characteristics: sometimes they not only give no further incentives to the contracting parties but even destroy contract mechanisms and so worsen the outcome.

To the best of our knowledge, this paper is the first to analyse public policy measures by focussing on their impact on contract design. Indeed, there exist only very few papers dealing with public policy in the venture capital area. In an empirical study, Lerner (1999)
examines the “Small Business Innovation Research Program” in the United States and finds that firms which had received awards were more likely to attract venture capital finance. In a second article, Lerner (2002) reviews the rationales for public policy programmes and comments on their design. Da Rin et al (2004) examine how public policy can contribute to increasing the share of venture capital investments in innovative companies, finding that the availability of exit channels plays an important role and that a reduction of the capital gains tax as well as a reduction of labor regulation yields such an increase. The only theoretical contributions to deal with public policy in a venture capital framework with a moral hazard problem are, to our knowledge, the papers by Keuschnigg and Keuschnigg and Nielsen (see Keuschnigg (2003), Keuschnigg and Nielsen (2003) or Keuschnigg and Nielsen (2004)). They analyse public policy strategies in an equilibrium model but do not explicitly model contract design.

On the contrary, our point of departure is the broad literature on optimal contract design between entrepreneurs and venture capitalists (see Tykvova (2000) for an overview). There are two types of papers: those which determine the optimal contract design in a double-sided moral hazard framework and those which analyse the optimal contract design in an adverse selection framework. It is important to combine these two types to do justice to the complexity of reality, so we construct a basic approach in two steps: first, we determine the optimal contract design in a double-sided moral hazard framework; then, in a second step, we introduce a double-sided adverse selection problem. More importantly, this procedure allows us to represent different development stages of the venture capital industry. Our approach is based on the models of Schmidt (2003) and Houben (2002). These papers offer a good starting point for our analysis as they show that both problems can be solved by adequately designed contracts; moreover, these contracts - convertible preferred stock contracts with specific control and redemption rights - are empirically well confirmed (see Kaplan and Strömberg (2003)). We modify these models so as to address the public policy issues in a tractable manner.

Other theoretical papers on optimal contract design which are close to the framework of the present paper are Casamatta (2003), Repullo and Suarez (2004) and D’Souza
Casamatta (2003) determines the optimal security design in a double-sided moral hazard framework too, but she shows that convertible securities are only optimal with high investments and that the first-best outcome is not reached. Repullo and Suarez (2004), on the other hand, focus on how a double-sided moral hazard problem in the expansion stage influences the security choice at the beginning of the relationship between entrepreneur and venture capitalist. They show that only when no objective performance indicators pertain after the start-up phase does the initial claim of the venture capitalist correspond to a combination of standard non-linear claims which may be interpreted as convertible security. D’Souza (2001), in contrast, shows the optimality of convertible securities in a framework where only the venture capitalist has to expend effort and the entrepreneur gets private information about the state of the project after contracting.

In order to allow for public intervention, we address potential inefficiencies in the innovation process. As mentioned above, we model a double-sided adverse selection problem and a double-sided moral hazard problem and, additionally, introduce a spill-over effect to the rest of the economy which is related to the realisation of innovative projects. For the latter inefficiency, we rely both on empirical insights and on theoretical foundations. We take the OECD’s Science, Technology and Industry Outlook (2001) as starting point: “Although venture capital does not aim at supporting R&D per se, its substantial emphasis on small, high-technology businesses has enabled markets to become considerably more capable of sustaining large, risky investments in R&D in early business stages (...) The influx of venture capital does not necessarily remove the rationale for government support for SMEs because significant discrepancies can still exist between private and social returns to R&D and innovation, even in sectors that receive considerable private capital. Recent research indicates that the most successful government-funded small-business projects have been in industry sectors that boast high levels of private venture capital. This finding suggests that private venture capital signals the presence of significant technological findings in a field, and that government funding can stimulate additional exploitation of those opportunities.” Indeed, the existence of the described spill-over effect is supported by empirical as well as theoretical evidence: Griliches (1992) shows that highly innovative
projects - such as those financed by venture capital - involve a spill-over effect on the rest of the economy. Their social rate of return can be twice as high as their corresponding private rate of return. Now the literature on endogenous growth distinguishes three effects of R&D: the positive effect of innovations on the profitability of other firms as shown by Romer (1986), the so-called consumer-surplus effect, and the business-stealing effect - the extraction of rents of one firm by another - as modelled by Aghion and Howitt (1992). If the first two effects are sufficiently high - as it normally is for highly innovative projects - the spill-over effect on the rest of the economy will be positive.

This paper proceeds as follows: after presenting our framework which is based on the models of Schmidt (2003) in the first part and Houben (2002) in the second part, we analyse the effects of different public policy measures. The fourth section makes some robustness checks. Section five draws together the threads.

2 The Model

We consider a market of many venture capitalists (VCs) looking for profitable investment opportunities and some entrepreneurs (Es) with innovative ideas but without financial resources. Therefore, each E has to convince a VC to invest an amount of $I$ in his project. As the market of VCs is competitive, Es have all the bargaining power ex ante. Moreover, we assume that the VCs and the Es are risk-neutral.

Projects differ only in their inherent innovative value $\alpha$ which is always observable by the contracting parties. Each project can be in three possible states (bad, medium, good) which result from a combination of the market conditions for the innovation and the quality of the E’s idea. The market conditions indicate whether the sales expectancy for the product is good or bad and the quality of the idea refers to the technological quality, i.e. the degree of feasibility. The project is of high quality with probability $p$ or of low quality with probability $(1 - p)$; and the market conditions for the innovation are either good with probability $q$ or bad with probability $(1 - q)$.

If the market conditions are bad and the quality is low, each project will fail and yield
a liquidation value of $l(a)$. The liquidation value depends on the effort exerted by the VC in order to sell off the assets of the company and $l(a)$ is increasing and concave in $a$. Moreover, we assume that the first-best liquidation value $l(a^*_t)$ is smaller than the initial investment $I$.

If the market conditions are bad and the quality is high, or if the market conditions are good but the quality is low - we refer to these cases as the medium state of the project - the project is a so called “living-dead”. This means that while the project may leave a return in the amount of the initial investment $I$ it will never generate a higher cash flow - even if both parties exerted high effort levels.

If the market conditions are good and the quality of the idea is high, the project is in the good state and yields a gross surplus of $\bar{x}$, which depends on three factors: the inherent innovative value of the project $(\alpha)$, the effort spent by the VC $(a)$ and the effort exerted by the E $(e)$. We interpret the effort of the E as effort invested in the technological development of the project and that one of the VC as involving managerial contributions. We assume that efforts are imperfect substitutes in the good state: if at least one contracting party does not exert any effort, the cash flow amounts only to the initial investment $I$ independently of the inherent innovative value:

$$\bar{x}(0, a, \alpha) = \bar{x}(e, 0, \alpha) = \bar{x}(0, 0, \alpha) = I$$

Otherwise, the cash flow function is increasing and concave in both effort levels and inherent innovative value. Effort levels are not complementary at the margin and their marginal impact does not depend on the innovative value $\alpha$ of the project. This is stated formally in the following assumptions.

$$\frac{\partial \bar{x}(e, a, \alpha)}{\partial a} > 0 \quad \text{and} \quad \frac{\partial^2 \bar{x}(e, a, \alpha)}{\partial a^2} < 0$$
$$\frac{\partial \bar{x}(e, a, \alpha)}{\partial e} > 0 \quad \text{and} \quad \frac{\partial^2 \bar{x}(e, a, \alpha)}{\partial e^2} < 0$$
$$\frac{\partial \bar{x}(e, a, \alpha)}{\partial \alpha} > 0 \quad \text{and} \quad \frac{\partial^2 \bar{x}(e, a, \alpha)}{\partial \alpha^2} < 0$$

$$\frac{\partial^2 \bar{x}(e, a, \alpha)}{\partial a \partial e} = 0 \quad \text{and} \quad \frac{\partial^2 \bar{x}(e, a, \alpha)}{\partial e \partial \alpha} = 0$$
The effort of the E and the VC, which is observable but not verifiable, occasions costs in the amount of:

\[ c(e) = \frac{1}{2} \beta e^2 \]
\[ c(a) = \frac{1}{2} \gamma a^2 \] (3)

Furthermore, the project yields non-transferable private benefits \( B \) to the E if and only if he has the control rights over the company in at least one period.

Finally, we assume that a project in the good state whose cash flow exceeds the initial investment costs - we will refer to these projects as successful good state projects - induces a spill-over effect on the rest of the economy which is given by \( S = \varepsilon \bar{x} \), i.e. the spill-over effect is more pronounced for more innovative projects and the effort choices exert an influence on it.

The time structure of the model is as follows: at \( t_0 \), the government announces its public policy measure. This announcement is costless but binding. The time of execution of the public policy measure depends on the type of the measure. At \( t_1 \), the E makes a take-it-or-leave-it offer to the VC. If the VC accepts, \( I \) is invested. At \( t_2 \), the E chooses his effort level \( e \) and at \( t_3 \), the VC chooses her effort level \( a \).\(^1\) At \( t_4 \), the cash flow of the project is realised and at \( t_5 \), the contract is executed.

As concerns the distribution of information, we analyse two different scenarios in order to be able to focus first on the impact of the double-sided moral hazard problem and the ex ante uncertainty and then on the impact of the double-sided adverse selection problem. But as mentioned before, even more decisive is the fact that these scenarios can be interpreted as different development stages of the venture capital market, because only in an experienced market are the contracting parties able to acquire private information ex ante. Thus, in a first scenario, we assume that no information about the state of the project exists before contracting time (this can be interpreted as the case of an inexperienced market), but that both contracting parties learn the state of the project after the initial investment has sunk (scenario 1, see *). In a second step, we assume that information does

\(^1\)For the reader's convenience in following my argumentation I refer to the E as he/him and to the VC as she/her.
exist about the state of the project - thus we are confronted with an experienced market - but we introduce a double-sided adverse selection problem by assuming that the E and the VC each receive a private signal (scenario 2, see **). As the E developed the innovative idea, we assume that he receives a perfect signal about the technological quality of the project (λ for a high quality and Λ for a low quality project). On the other hand, the VC has gained experience in the venture capital market and thus she has private information about the perspectives of the product market, i.e. she receives a perfect signal about the market conditions for the innovation (η for good and η̄ for bad market conditions).

\begin{align*}
&\begin{array}{c}
t = 0 \\
t = 0.5 \\
t = 1 \\
t = 1.5 \\
t = 2 \\
t = 3 \\
t = 4 \\
t = 5 \\
\end{array} \\
&\begin{array}{c}
\text{Government announces} \\
\text{public policy} \\
\text{programme} \\
\text{(announcement} \\
\text{is binding)} \\
\text{[Signals]} \\
\text{λ and η} \\
\text{**} \\
\text{Take-it-or-leave-it-offer} \\
\text{of contract(s) by E;} \\
\text{Investment} \\
\text{of I by VC} \\
\text{[E and VC} \\
\text{learn the} \\
\text{state of the project]} \\
\text{*} \\
\text{Choice of} \\
\text{effort level e} \\
\text{by E} \\
\text{Choice of} \\
\text{effort level a} \\
\text{by VC} \\
\text{Cash-flow} \\
\text{is realized} \\
\text{Execution} \\
\text{of the} \\
\text{contract} \\
\end{array}
\end{align*}

Figure 1: Model Timeline

* refers to scenario 1, ** to scenario 2

### 2.1 The first-best solution

A first necessary step in the evaluating of public policy programmes is to define the (public) first-best solution as a reference case. Let

\[ W = V(\theta) - \frac{1}{2}\beta e^2 - \frac{1}{2}\gamma a^2 - I + B + S(\theta) \]  

be the net social value of the project according to the state of the project (\(\theta\)) with \(\theta \in \{\text{bad, medium, good}\}\) and

\[ S(\theta) = \begin{cases} 
\varepsilon \bar{x}(e, a, \alpha) & \text{for } \theta = \text{good and } \bar{x}(e, a, \alpha) > I \\
0 & \text{otherwise}
\end{cases} \]  

(5)
The public first-best efficient effort levels for the three different states are then given by:

\[ b^{*}_{SP} = 0 \quad \text{and} \quad a^{*}_{SP} = \frac{v'(a)}{\gamma} \]

\[ m^{*}_{SP} = 0 \quad \text{and} \quad a^{*}_{m,SP} = 0 \quad (6) \]

\[ g^{*}_{SP} = \frac{(1 + \varepsilon)}{\beta} \frac{\partial \bar{x}(e, a, \alpha)}{\partial e} \quad \text{and} \quad a^{*}_{g,SP} = \frac{(1 + \varepsilon)}{\gamma} \frac{\partial \bar{x}(e, a, \alpha)}{\partial a} \]

In addition, it is efficient to give the E control rights in order to conserve the private benefits \( B \). We assume that \( 0 < B < \min[pq \frac{1}{2} \beta e^{*}_{g}; I - L(a^{*}_{b})] \). This implies that bad state projects are not socially profitable\(^2\). Thus, as concerns the investment decision, we know that in scenario 2 an investment is always profitable in the medium and good states and not profitable in the bad state of the project, whereas in scenario 1 an investment is profitable only if it is sufficiently innovative, i.e. \( \alpha \geq \tilde{\alpha}^{SP} \) with:

\[ \bar{x}(e^{*}_{g,SP}, a^{*}_{g,SP}, \tilde{\alpha}^{SP}) = \frac{1}{2} \beta e^{*}_{g,SP} - \frac{1}{2} \gamma a^{*}_{g,SP} \]

\[ + \frac{B}{pq} + S(e^{*}_{g,SP}, a^{*}_{g,SP}, \tilde{\alpha}^{SP}) = I + \frac{(1 - p)(1 - q)}{pq} [I - L(a^{*}_{b,SP})] \quad (7) \]

with \( L(a^{*}_{b,SP}) = I(a^{*}_{b,SP}) - \frac{1}{2} \gamma a^{*}_{b,SP} \) and \( S(e^{*}_{g,SP}, a^{*}_{g,SP}, \tilde{\alpha}^{SP}) \) being the spill-over effect of a project with the inherent innovative value \( \tilde{\alpha}^{SP} \) if public first-best effort levels are chosen.

In a second step, we must determine the private first-best solution. To this end, we understand the first-best solution as having a spill-over effect of zero, as it is obvious that the VC and the E will never consider the spill-over effect, though they may be able to solve the adverse selection and moral hazard problems. The private first-best efficient effort levels for the three different states are then given by:

\[ b^{*} = 0 \quad \text{and} \quad a^{*}_{b} = \frac{v'(a)}{\gamma} \]

\[ m^{*} = 0 \quad \text{and} \quad a^{*}_{m} = 0 \quad (8) \]

\[ g^{*} = \frac{1}{\beta} \frac{\partial \bar{x}(e, a, \alpha)}{\partial e} \quad \text{and} \quad a^{*}_{g} = \frac{1}{\gamma} \frac{\partial \bar{x}(e, a, \alpha)}{\partial a} \]

As \( a^{*}_{b} = a^{*}_{b,SP} \) and \( a^{*}_{m} = a^{*}_{m,SP} \) as well as \( b^{*} = b^{*}_{SP} \) and \( m^{*} = m^{*}_{SP} \), we will use the first representation in order to simplify notation.

\(^2\) Moreover, this assumption guarantees that all privately profitable projects do get to be financed in a private first-best situation despite the limited liability of the E.
With respect to the allocation of control rights, it is efficient to give them to the E. As concerns the investment decision, we know that in scenario 2 all socially profitable projects are individually profitable, too. In scenario 1, however, the economic agents do not take into account the spill-over effect, so the project is only profitable if $\alpha \geq \alpha^{PR}$ with:

$$\bar{x}(e^*_g, a^*_g, \alpha^{PR}) - \frac{1}{2} \beta e^{*2} - \frac{1}{2} \gamma a^{*2} + \frac{B}{pq} = I + \frac{(1 - p)(1 - q)}{pq} [I - L(a^*_b)]$$

(9)

It can be shown that the critical $\alpha^{PR}$ must be higher than $\alpha^{SP}$ in order to fulfill the private profitability condition. Thus, less innovative projects which are still socially profitable are not profitable for the economic agents and will not get financed.

### 2.2 The market solution

It is important to know whether the described private first-best solution can be achieved by the market and, if so, how the concrete contract design will look. Therefore, we determine the behaviour of the economic agents first for scenario 1 (information disclosure after contracting) and then for scenario 2 (ex ante signals).

**No information about the state of the project at contracting time**

First, we consider the case where neither of the two agents has any information about the state of the project at contracting time but where both agents acquire such before choosing their effort levels. The entrepreneur will offer the VC the contract which gives him the highest benefits in $t_5$. As he has all the bargaining power ex ante, the E pays the VC just as much as is necessary to fulfill her participation constraint independently of the financial instrument used: the VC has to be compensated for her initial investment and her effort costs in the good state. Here the E has to take into account the loss of the VC in the bad state which he wants minimised. This implies that the E gets the entire net surplus of the project. We know from the definition of the private first-best that for $a \leq a^*_g$ the necessary effort compensation is always smaller than the rise of the project’s outcome by exerting this effort. Thus, E’s profit is maximised if both agents exert their private first-best effort levels. The question is, however, whether there exists a financial
contract which gives both agents simultaneously an incentive to exert private first-best effort levels.

We know that incentives in the good state can be given only by a compensation scheme which depends on the realised outcome. Thus, debt cannot be efficient. Equity, on the other hand, gives suboptimal incentives because it is impossible to make both parties full residual claimants at the same time. But following the insights of Schmidt (2003), we show that adequately designed preferred stock contracts give efficient incentives to both contracting parties in our sequential double-sided moral hazard framework.

**Proposition 1** The E offers the VC a contract of convertible preferred stock which implements the private first-best situation. This contract guarantees the VC a limited preferred dividend of \( D \) and the right to convert the preferred stock into an equity stake \( s_{1CV}^{CV} \) in \( t_4 \) if the project’s surplus reaches at least the threshold \( z \):

\[
D = C + \frac{(1-p)(1-q)}{pq}[I - L(a^*_p)]
\]

\[
s_{1CV}^{CV} = \frac{C + 0.5\gamma a^*_g^2}{[\bar{x}(e^*_g, a^*_g, \alpha)]}
\]

\[
z = \bar{x}(e^*_g, a^*_g, \alpha)
\]

**Proof 1** See appendix.

Convertible preferred stock contracts implement the private first-best situation because they function as pure debt contracts in the bad and medium states of the project - that which is efficient - yet, at the same time, the conversion option guarantees an efficient outcome in the good state. This is due to the following: the VC is a full residual claimant in the bad state and thus chooses the efficient effort level, while in the good state when debt contracts are no longer efficient the threshold \( z \) guarantees that the VC will not choose a suboptimal effort level and the sequential effort decision structure guarantees that the E has an incentive to choose the threshold \( z \) in such a way that he has also an incentive to choose his private first-best effort level.
Private information at contracting time

Now we consider the case of an experienced market where both agents receive private signals before contracting and are able to reveal their private information by acting in a specific way: in the case of the E, by offering a specific menu of contracts; and in that of the VC, by choosing one specific contract or rejecting the contract. If both parties communicate their private information truthfully, the state of the project can be deduced - if not, the parties must choose their effort levels in a state of uncertainty. But both parties may have incentives to misstate their private information: the E to overstate his private information in order to get bad state projects financed and receive private benefits; the VC to understate her private information in order to get a higher compensation. Therefore, the concrete contract design is crucial; especially, the allocation of control rights seems to play a key role.

Indeed, in order to achieve an efficient outcome, a contingent control allocation is necessary. This is due to the fact that private benefits get lost if the VC holds the control rights and that the E will always have an incentive to overstate if he holds them independently of the state of the project. A contingent control allocation, however, can be achieved neither by debt nor by equity contracts. However, following the insights of Houben (2002), it can be shown that adequately designed preferred stock contracts implement this necessary contingent control allocation by transferring control rights to the VC and by giving the E the possibility to recover by redeeming the preferred stock in the medium but not in the bad state of the project. Thus, truth-revealing by both agents can be achieved without private benefits being lost.

**Proposition 2** The E offers the VC the following truth-revealing contract(s):

- If he receives the bad signal $\Delta$, he offers a contract which implies a preferred fixed payment $D_b$ and which is redeemable at a price $P_b$ at $t_4$ if control rights are given to the VC:

\[
D_b = I \\
\]

\[
P_b = D_b
\]
- If he receives the good signal $\bar{\lambda}$, he offers the following menu of contracts:

1. A redeemable voting preferred stock contract: the VC gets control and receives a limited preferred dividend of $D_m$, but the E gets the right to redeem the preferred stock at a price $P_m$ at $t_4$, with

   \[
   D_m = I \\
   P_m = D_m
   \]

2. A convertible preferred stock contract: the VC does not get control, but receives a limited preferred dividend of $D_g$ and the right to convert the preferred stock into an equity stake $s_{2CV}$ in $t_4$ if the surplus reaches at least the threshold $z$, with

   \[
   D_g = I \\
   s_{2CV} = \frac{I + 0.5\gamma a_g^{*2}}{\bar{x}(e_g^*, a_g^*, \alpha)} \\
   z = \bar{x}(e_g^*, a_g^*, \alpha). 
   \]

In addition, these contracts always induce private first-best effort levels.

Proof 2  See appendix.

On the one hand, the E reveals his private information by offering different contracts to the VC and truth-revealing is achieved by the control transfer of the first contract for high quality projects. On the other, the VC reveals her private information by accepting the contract for low quality projects only if she has received the good signal, and by choosing the adequate contract according to her received signal within the contract set for high quality projects. Truth-revealing by the VC is guaranteed through the specific contract design, especially the threshold $z$. Moreover, we know from proposition 1 that preferred stock contracts implement private first-best effort levels in all states of the project.
3 Public Policy

In the previous section, we pointed out that the market is able to solve the problems of moral hazard and adverse selection by using adequately designed financial instruments, namely preferred stock contracts with different features like redemption rights, control rights and conversion clauses. The private first-best situation is therefore reached allowing public policy programmes to focus on internalising the spill-over effect due to successful innovation. As discussed in the introduction, a broad range of public policy programmes exists, and it is important to know their effects on contract design and the individual behaviour if suitable public policy measures are to be identified. In order to concentrate on our analysis, we assume lump-sum financing of public policy programmes.

In what follows, we will determine the optimal design of the public policy measure in order to get all (or at least more) socially profitable projects financed in scenario $1$. To simplify, we assume that the spill-over effect exceeds a specific minimum level: $S(e^*_g, a^*_g, \alpha^{SP}) > \frac{(1-p)(1-q)}{pq}[L(a^*_g) - l(0)]$. This assumption guarantees that the range of socially profitable projects with public intervention is never smaller than the range of projects which are individually profitable without intervention.

Moreover, we need to make an assumption about the observability and verifiability of the inherent innovative value $\alpha$ of each project. For the following analysis, we assume that it is not observable by the public institution. In section 3.3, we will expand on two alternative assumptions. It is important to consider that an ex post discrimination is always possible due to the verifiability of the generated surplus.

In section 3.2, we will look at the impact of the defined public policy measures on the contract design and the induced behavior in an experienced market, i.e. in scenario 2.

\[ ^3 \text{We will not consider explicitly renegotiation issues in our analysis, as renegotiation provability is given analogously to proof 1 in the case of ex ante grants and public-private partnerships; there is no scope for renegotiation as concerns guarantee programmes; and with respect to ex post grants and public support, we assume that the bargaining power of the E is sufficiently small to prevent deviation and subsequent renegotiation by the E.} \]
3.1 Public policy in inexperienced venture capital markets

Ex post grants

Ex post grants can be made contingent on the outcome of the project. As the aim of the public intervention is to internalise the spill-over effect, it makes sense to pay the E an ex post grant corresponding to the size of the spill-over effect. Now the question is whether this policy instrument has any influence on the contract mechanisms and if it is possible to internalise the spill-over effect.

Whereas in the bad and medium states of the project, nothing changes as the E does not receive a grant, in the good state of the project, the E has an incentive to increase the project surplus in order to obtain a higher grant from the government. Analogously to proposition 1, we again consider a convertible preferred stock contract with a limited dividend $D_{EPG}$ and a conversion option in an equity stake $s_{1, EPG}$ in $t_4$ if the threshold $z_{EPG}$ is reached. Through the grant, the cash-flow of the project in the good state increases. Thus, it seems reasonable to define the share of the VC as a share of the total cash flow. Solving the modified maximisation problem of the E as in proof 1 yields the following contract, which induces the public first-best situation:

$$D_{EPG} = C = I + \frac{(1 - p)(1 - q)}{pq}[I - L(a^*_b)]$$

$$s_{1, EPG} = C + \frac{1}{2} \gamma a^*_g SP^2$$

$$z_{EPG} = \tilde{x}(e^*_g SP, a^*_g SP, \alpha)$$

Thus, the E adjusts the threshold $z_{EPG}$ in order to guarantee that conversion is possible only if both parties choose their public first-best effort levels. Moreover, as by guaranteeing the VC a share $s_{1, EPG}$ from the total cash flow $(1 + \varepsilon)\tilde{x}(e^*_g SP, a^*_g SP, \alpha)$ he adjusts the conversion stake so that the VC’s participation is binding.

**Proposition 3** Paying the E an ex post grant $EPG = \varepsilon \tilde{x}(e, a, \alpha)$ if the cash-flow exceeds the initial investment costs $I$ gives the E an incentive to offer modified preferred stock contracts which induce public first-best effort levels. All socially profitable projects get financed. Welfare is maximised.
Proof 3 The proof follows directly from the above argumentation.

Guarantee programmes

As pointed out, there exist (or existed) many public policy programmes which aim (aimed) at promoting VC financing by assuming part of the project’s risk. For simplification, we set out that risk is totally assumed by the public institution. This means that the VC gets back her initial investment $I$ even if the project is in the bad state. Now the question is whether this policy instrument has any influence on the contract mechanisms and if it is possible to internalise the spill-over effect.

Guarantee programmes do not increase the outcome of the project in the medium and good states. So they do not give any further incentives to augment effort levels. In the bad state of the project, however, risk is assumed completely by the government. This results in the effort level of the VC decreasing to 0, as she does not have any longer the incentive to minimise the loss. Moreover, it is important to take into account that she is not reimbursed for her forgone effort costs. It is obvious that guarantee programmes worsen the outcome of all projects in scenario 1 that would have been financed without the guarantee programme. Furthermore, guarantee programmes relax the participation constraint of the VC [$PC(VC)_{1}$ of proof 1], and the incentive constraints which ensure that conversion is profitable for both contracting parties [$IC(VC)_{\text{conv}}$ and the $IC(E)_{\text{conv}}$ of proof 1] are slightly modified. Thus, the E will offer the following contract:

$$D_{GP} = I$$

$$s_{1,GP} = \frac{I + \frac{1}{2} \gamma a_{g}^{2}}{\bar{x}(e_{g}^{*}, a_{g}^{*}, \alpha)}$$

(11)

$$z = \bar{x}(e_{g}^{*}, a_{g}^{*}, \alpha)$$

The public institution is interested in getting all projects financed that have a positive net social value. As shown above, in a first-best world this means that all projects whose inherent innovative value amounts at least to $\alpha^{SP}$ should get financed. But if the public policy programme does not give any further incentives, this range shrinks and only projects with an inherent innovative value of $\alpha \geq \alpha^{SP}_{NI}$ continue to be socially profitable, whereby
\( \alpha^{SP}_{NI} \) is given by

\[
\bar{x}(e^*_g, a^*_g, \alpha^{SP}_{NI}) - \frac{1}{2} \beta e^*_g - \frac{1}{2} \gamma a^*_g + \frac{B}{pq} = \\
I + \frac{(1-p)(1-q)}{pq} [I - L(a^*_g)] - S(e^*_g, a^*_g, \alpha^{SP}_{NI}) \tag{12}
\]

However, as guarantee programmes not only give no further incentives in the good state but even result in a decrease of the VC’s effort level in the bad state, the range of projects with a positive net social value decreases further; thus only projects with an inherent innovative value of \( \alpha \geq \alpha^{GP} > \alpha^{SP}_{NI} \) should get financed. The minimum level is given by:

\[
\bar{x}(e^*_g, a^*_g, \alpha^{GP}) - \frac{1}{2} \beta e^*_g - \frac{1}{2} \gamma a^*_g + \frac{B}{pq} = \\
I + \frac{(1-p)(1-q)}{pq} [I - l(0)] - S(e^*_g, a^*_g, \alpha^{GP}) \tag{13}
\]

As to the private investment decision, we know that projects get financed if the following condition holds:

\[
\bar{x}(e^*_g, a^*_g, \alpha) - \frac{1}{2} \beta e^*_g - \frac{1}{2} \gamma a^*_g + \frac{B}{pq} \geq I \tag{14}
\]

Thus, to ensure financing of all these projects: \( S(e^*_g, a^*_g, \alpha^{GP}) = \frac{(1-p)(1-q)}{pq} [I - l(0)] \).

If \( S(e^*_g, a^*_g, \alpha^{GP}) > \frac{(1-p)(1-q)}{pq} [I - l(0)] \), not all socially profitable projects will get financed and if \( S(e^*_g, a^*_g, \alpha^{GP}) < \frac{(1-p)(1-q)}{pq} [I - l(0)] \), too many projects will get financed.

**Proposition 4** Guarantee programmes bring about a decrease in the effort level of the VC to 0 in the bad state of the project, which implies a worsened outcome for all projects which would have been financed without introduction of the guarantee programme. More, though never all, socially profitable projects will get financed if the spill-over effect is relatively large in comparison to the expected loss. In the inverse case, even socially unprofitable projects get financed. The total welfare effect depends on the parameters.

**Proof 4** The proof follows directly from the above argumentation.
Ex ante grants

It is important to distinguish between ex post and ex ante grants - ex post grants can depend on the outcome while ex ante grants are just fixed support payments. Thus, ex ante grants do not give any further incentives in the medium and good states. But they do relax the E’s limited liability constraint: he will be able to pay the VC additionally the amount of the obtained ex ante grant independently of the state of the project. Therefore, as in the case of guarantee programmes, ex ante grants may have negative incentive effects in the bad state: if the fixed dividend \(D\) of the VC is guaranteed independently of the effort she exerts \((EAG + l(0) \geq D)\), she will choose \(a_b = 0\) (case 1); if \(D\) is guaranteed only if at least \(\hat{a}_{EAG} \in (0, a_b^*)\) is chosen \((EAG + l(\hat{a}_{EAG}) = D)\), she will choose \(\hat{a}_{EAG}\) (case 2); finally, if \(D\) is never guaranteed or only if she chooses her private first-best effort level \((EAG + l(a_b^*) \leq D)\), she will choose her private first-best effort level (case 3).

The concrete contract design can be determined analogously to proof 1 and depends on the outcome of the bad state, which depends in turn on the size of the spill-over effect and the optimal ex ante grant. The public institution does not know the inherent innovative value \(\alpha\) of each project and so will make available the same grant to all projects. Consequently, it will choose the ex ante grant in such a way that all projects with a positive net social value get financed and the incentive effect is the same for all projects.

It is important to take into account that the range of socially profitable projects depends on the effort level chosen by the VC in the bad state: in case 1, the least innovative project to be still socially profitable is the one with the inherent innovative value \(\alpha_{EAG1}\); in case 2, it is the one with \(\alpha_{EAG2}\); and in case 3, the one with \(\alpha_{EAG3} = \alpha_{SP}\). The cut-offs are given by:

\[
\bar{x}(e_g, a_g, \alpha_{EAG1}) = \frac{1}{2} \beta e_{g}^{*2} - \frac{1}{2} \gamma a_{g}^{*2} + \frac{B}{pq} = I + \frac{(1-p)(1-q)}{pq} [I - l(0)] - S(e_g^*, a_g^*, \alpha_{EAG1})
\]

\[
\bar{x}(e_g, a_g, \alpha_{EAG2}) = \frac{1}{2} \beta e_{g}^{*2} - \frac{1}{2} \gamma a_{g}^{*2} + \frac{B}{pq} = I + \frac{(1-p)(1-q)}{pq} [I - L(\hat{a}_{EAG})] - S(e_g^*, a_g^*, \alpha_{EAG2}) (15)
\]
We can see that $\alpha_{EAG1} > \alpha_{EAG2} > \alpha_{EAG3} = \alpha_{SP1} > \alpha_{SP}$. The optimal size of the ex ante grants as well as the optimal contract design is shown in section 3 in the appendix.

**Proposition 5**  
Ex ante grants may bring about a decrease in the effort level of the VC in the bad state, implying a worsened outcome for all projects that would have been financed without introduction of ex ante grants. This effect is the more pronounced, the larger the spill-over effect. All positive net value projects will get financed. The total welfare effect depends on the parameters.

**Proof 5**  
The proof follows directly from the above argumentation and the table given in section 3 of the appendix.

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**Public-private partnerships**

The notion of public-private partnership covers a large variety of organisational forms. Three characteristics may be of special importance for our analysis. First, we assume that the public institution and the VC act always as one institution towards the E. Second, the public institution will provide financial input whereby we will consider two different support schemes, namely re-financing and co-financing. Third, it may be possible that the public institution provides some sort of know-how, access to networks or managerial skills. But in order to focus on the impact of re- and co-financing schemes, we will omit the aspect of public know-how support for the moment and treat it in detail in the next section.

In a first step, we will analyse re-financing schemes by distinguishing two cases: the case where the claims of the VC and the public institution have the same rank and the one where the VC has a liquidation preference. Assume that the public institution accepts an equally ranked credit claim $D^{PI} = I^*$ in return for its contribution to the investment costs in the amount of $I^*$ and that, for simplicity’s sake, the interest rate is $r=0$. The solution is analogous to proof 1. As concerns the effort decision of the VC and the E, the only change is that the VC is no longer a full residual claimant in the bad state of the project meaning that she will choose the suboptimal effort level $\hat{a}^{NLP}_{PPP}(I^*) = \frac{I^* - t^* - v(a)}{\gamma}$. This implies a worsened outcome for all projects that would have been financed without
public intervention.

With respect to the concrete contract design, the E has to take into account the modified $PC(VC)'_1$ which implies that:

$$C = I + \frac{(1-p)(1-q)}{pq}[I - I^* - L(\hat{\alpha}^{NLP}) + \frac{I^*}{I}[I(\hat{\alpha}^{NLP})]]$$  \hspace{1cm} (16)

This means in turn that all projects with

$$\bar{x}(e^*_g, a^*_g, \alpha) - \frac{1}{2} \beta e^*_g - \frac{1}{2} \gamma a^*_g + \frac{B}{pq} \geq C$$  \hspace{1cm} (17)

will get financed. It can be shown that $\frac{\partial C}{\partial T} < 0$: the higher the contribution of the public institution, the more projects get financed. As concerns the range of socially profitable projects, we know that projects must be sufficiently innovative because public-private partnerships not only give no further incentives in the medium and good states but even worsen the incentives in the bad state: $\alpha \geq \alpha^{NLP}_{PPP} > \alpha^{SP}_{NI}$ whereby $\alpha^{NLP}_{PPP}$ is given by

$$\bar{x}(e^*_g, a^*_g, \alpha^{NLP}_{PPP}) - \frac{1}{2} \beta e^*_g - \frac{1}{2} \gamma a^*_g + \frac{B}{pq} =$$

$$I + \frac{(1-p)(1-q)}{pq}[I - L(\hat{\alpha}^{NLP})] - S(e^*_g, a^*_g, \alpha^{NLP}_{PPP})$$  \hspace{1cm} (18)

The optimal $I^*$ is thus given by

$$I^* = \frac{pqI}{(1-p)(1-q)[I - l(\hat{\alpha}^{NLP})]} S(e^*_g, a^*_g, \alpha^{NLP}_{PPP})$$  \hspace{1cm} (19)

Note, however, that as $I^*$ must be smaller than $I$, the spill-over effect must be sufficiently small to ensure that all socially profitable projects get financed:

$$S(e^*_g, a^*_g, \alpha^{NLP}_{PPP}) < \frac{(1-p)(1-q)}{pq} [I - l(0)]$$  \hspace{1cm} (20)

Let us now analyse the second case, namely the re-financing scheme with a liquidation preference by the VC. In this case, the concrete contract design and the effort decision of the VC in the bad state of the project depends on the relation between her contribution to the investment costs ($I - I^*$) and the different possible liquidation values, which in turn depends on the size of the spill-over effect of the least innovative but still socially profitable project. It can be shown that for relatively small spill-over effects the VC will
continue to exert her private first-best effort level in the bad state. Therefore, all projects with an inherent innovative value $\alpha \geq \zeta_{NI}^{SP}$ are socially profitable. In this case, the public institution should choose $I^*$ proportional to the spill-over effect in order to get all socially profitable projects financed.

$$S(e^*_g, a^*_g, \zeta_{NI}^{SP}) \leq \frac{(1-p)(1-q)}{pq}[I - l(a^*_b)]$$

$$I^* = \frac{pq}{(1-p)(1-q)} S(e^*_g, a^*_g, \zeta_{NI}^{SP})$$

If the spill-over effect is larger, it can be internalised if the contribution of the public institution to the investment costs is sufficiently high but not too high as to guarantee that the VC will exert the suboptimal effort level $\hat{a}_{PPP}^{LP} \in [0, a^*_b)$. The minimum inherent innovative value $\zeta_{PPP}^{LP}(\hat{a}_{PPP}^{LP})$ is thus higher than $\zeta_{NI}^{SP}$:

$$\frac{(1-p)(1-q)}{pq}[I - l(a^*_b)] < S(e^*_g, a^*_g, \zeta_{PPP}^{LP}) \leq \frac{(1-p)(1-q)}{pq}[I - l(0)]$$

$$I^* = I - l(\hat{a}_{PPP}^{LP}(S))$$

with $S(e^*_g, a^*_g, \zeta_{PPP}^{LP}) = \frac{(1-p)(1-q)}{pq}[I - l(\hat{a}_{PPP}^{LP})]$.

If the spill-over effect is larger, all socially profitable projects will never get financed.

In a second step, let us now analyse co-financing schemes. With co-financing, the public institution takes a part of the investment costs $(1 - \tau)$ for a share $(1 - \tau)$ of the partnership’s return. Obviously, the effort decision by the VC in the bad state of the project is changed analogously to the case of re-financing schemes and depends on whether the VC has a liquidation preference or not. Moreover, when modifying the good state contract, it is important to take into account that the public institution receives the share $(1 - \tau)$ of the VC’s effort compensation in the good state. In order to avoid this effect, many programmes allow for an additional compensation agreement between the VC and the public institution. In what follows, we assume that the VC does receive a compensation of the forgone effort costs if the claim is converted. This implies that $C$ must be modified to $C_{LP}^{PPP}$ and $C_{NLP}^{PPP}$ depending on whether the VC has a liquidation preference or not:

$$C_{NLP}^{PPP} = I + \frac{(1-p)(1-q)}{pq}[I - L(\hat{a}_{NLP}) + \frac{(1-\tau)}{\tau} \gamma \hat{a}_{NLP}^2] \quad (21)$$
\[ C_{LP}^{PPP} = I + \frac{1}{pq} \left[ \frac{1}{\tau} - \frac{I(a^*_P)}{\tau} \right] \quad \text{for} \quad \tau \geq \frac{I(a^*_S)}{T} \]

\[ I + \frac{1}{pq} \left[ \frac{1}{\tau} \gamma^2 \right] \quad \text{for} \quad \frac{I(0)}{T} < \tau < \frac{I(a^*_S)}{T} \]

otherwise

Obviously, without a liquidation preference for the VC, \( C_{LP}^{PPP} \) is always higher than in the case without intervention. This implies in turn that the range of financed projects is shrinking. Co-financing schemes without a liquidation preference are thus welfare-decreasing\(^4\).

With a liquidation preference, on the contrary, \( C_{LP}^{PPP} \) is always smaller than in the case without intervention. If the spill-over effect is sufficiently small, i.e. \( S(e_g^*, a_g^*, \alpha_{PI}^{SP}) \leq \frac{(1-p)(1-q)}{pq} \left[ \frac{I}{l(a^*_k)} \right] - 1 \), all projects with an inherent innovative value of \( \alpha > \alpha_{PI}^{SP} \) are socially profitable and will get financed. For larger spill-over effects the range of socially profitable projects is smaller (due to the decrease of the VC’s effort level in the bad state) though still larger than in the case without intervention. Whereas here all socially profitable projects get still financed, this is no longer the case if the spill-over effect is too large \((S(e_g^*, a_g^*, \alpha_{PI}^{PPP}) > \frac{(1-p)(1-q)}{pq} \left[ I - l(0) \right])\). Thus, as can be seen, the mechanism of co-financing schemes with a liquidation preference is similar to the mechanism of re-financing schemes. We will therefore restrict ourselves to re-financing schemes in what follows.

**Proposition 6** Re-financing schemes without (with) a liquidation preference bring about always (with a relatively large spill-over effect) a decrease in the effort level of the VC in the bad state of the project which worsens the outcome of all projects with \( \alpha \geq \alpha_{PI}^{PR} \). Such schemes enlarge the range of projects financed. The welfare effect depends on the parameters (is positive for small spill-over effects).

In order to enlarge the range of financed projects with co-financing schemes, the VC must have a liquidation preference. Then with a relatively small (large) spill-over effect, the welfare effect is positive (depends on the parameters because of the decrease in the effort level of the VC in the bad state). Otherwise, co-financing schemes are welfare-decreasing.

**Proof 6** The proof follows directly from the above argumentation.

\(^4\) An alternative specification of co-financing allowing a spread between the actual claim of the public institution and its contribution to the investment costs could make co-financing schemes without a liquidation preference profitable. To the best of our knowledge, this is not common practice though.
Public support

Public support, such as technical assistance or access to networks, may decrease the effort costs of VCs and Es. As to the VC’s costs, we assume that they decrease only for managerial contributions but not for selling off the assets in the bad state. Furthermore, we assume that public support does not produce a lasting effect like e.g. training programmes.

The public institution wants to give the contracting parties the incentive to increase their effort levels to public first-best ones. Therefore, public support programmes should decrease the effort costs of both parties to $\beta^*$ and $\gamma^*$ respectively, with

$$\beta^* = \frac{\beta}{1 + \varepsilon} \quad \text{and} \quad \gamma^* = \frac{\gamma}{1 + \varepsilon}$$

Such a public policy measure has a positive influence on all projects independently of their inherent innovative value.

In order to induce the VC to really bring to bear the public first-best effort level in the good state, the E will offer modified convertible preferred stock contracts, whereby he adjusts the equity stake $s_{CV}^{PS}$ and the threshold $z_{PS}$:

$$s_{CV}^{PS} = \frac{C + \frac{1}{2} \gamma^* a_g^{SP} \frac{\gamma^* a_g^{SP}^2}{\bar{x}(e_g^{SP}, a^g^{SP}, \alpha)}}{\bar{x}(e_g^{SP}, a_g^{SP}, \alpha)}$$

$$z_{PS} = \bar{x}(e_g^{SP}, a_g^{SP}, \alpha)$$

The following projects will get financed:

$$\bar{x}(e_g^{SP}, a_g^{SP}, \alpha) - \frac{1}{2} \beta e_g^{SP^2} - \frac{1}{2} \gamma a_g^{SP^2} + \frac{B}{pq} \geq I + \frac{(1-p)(1-q)}{pq}[I - L(a^*_b)]$$

$$- S(e^*_g, a^*_g, \alpha) + \varepsilon[I + \frac{(1-p)(1-q)}{pq}[I - L(a^*_b)] - \frac{B}{pq}]$$

If the private benefits are sufficiently small ($B < pqI + (1-p)(1-q)[I - L(a^*_b)]$) and the spill-over effect is sufficiently large, then the last term is always positive though smaller than the spill-over effect: public support enlarges the range of financed projects but never ensures financing of all socially profitable projects. We assume that this is the case.

**Proposition 7** Public support gives both contracting parties the incentive to choose public first-best effort levels in all states. More, though never all, socially profitable projects will get financed. Public support is welfare-increasing.
3.2 Public policy in experienced venture capital markets

In this section, we will analyse the impact of the above defined public policy measures for experienced venture capital markets, i.e. for scenario 2 of our framework. We know that in scenario 2 all socially profitable projects are also individually profitable and get financed. Consequently, public policy should aim only at giving further incentives in good states. We know from section 3.1. that only ex post grants and public support are able to provide these incentives. In fact it can be shown that in the case of ex post grants, according to the size of the spill-over effect, the E has an incentive to offer a modified good state contract which induces public first-best effort levels on the part of both contracting parties. The same is true for the case of public support. With respect to the other public policy measures, we know that they will not be able to increase welfare in scenario 2. We have to analyse, however, whether they have only distributional effects or even decrease welfare. The latter will be the case if the truth-revealing mechanism is destroyed.

We know that the E may have an incentive to get bad state projects financed in order to receive private benefits. This is possible only if the VC does not get to know the quality of the project. On the other hand, the E has an incentive to get to know the market conditions as he is worse off without knowing them in the case of high quality projects. Both conditions are fulfilled if the E is able to offer the same set of contracts independently of the signal he receives and without paying a rent to the VC and if these contracts induce truth-revealing by the VC. This implies, in turn, that there must exist a joint contract for bad and medium states which fulfills the VC’s participation constraint. Without public intervention, this will never be the case in our framework due to the limited liability of the E. It can be achieved, however, through public policy measures which relax the E’s limited liability constraint in the bad state of the project.

With guarantee programmes, for example, the VC is protected against losses in bad states, so the E will offer the set of contracts of proposition 2 independently of the signal he receives. Contrary to proposition 2, the first contract will no longer include a control
transfer though. This is due to the fact that the E need not commit to truthfully reveal his private information and that he aims at consuming private benefits in the bad state. It can easily be seen that the VC will continue to truthfully reveal her private information by choosing the first contract if she receives the bad signal and the second contract if she receives the good signal because she may be able to convert. As concerns effort levels, there is no change in the good state of the project: since the E knows the state of the project and chooses his private first-best effort levels and the VC observes the effort level chosen by the E before choosing her effort level, she will know in $t_3$ if the project is in the good state and will therefore choose her private first-best effort level. If the E chooses an effort level of 0, the VC does not know if the project is in the bad or in the medium state. But as mentioned before, the optimal effort level of the VC amounts to 0 in bad state projects too. Thus, the VC will choose her optimal effort level of 0 without knowing the real state of the project.

The same is true in the case of ex ante grants with a relatively large spill-over effect, namely if $EAG + l(0) \geq I$. In this case, the E will be able to compensate the VC in the bad state of the project and so he will offer the same set of contracts as in the case of guarantee programmes independently of the signal he receives. If $EAG + l(0) < I$, the E would have to offer a mark-up on $D_m$ so as to induce the VC to choose the first contract without knowing the state of the project. This may be profitable for the E if he receives the bad signal and his private benefits are relatively high. But if the E receives the good signal, he has an incentive to offer the contracts of proposition 2 in order to save the rent. Thus, he will not offer the same set of contracts independently of the signal he receives and the VC will know the true state of the project and so will not invest in bad state projects. Consequently, with a relatively small spill-over effect, ex ante grants have only distributional effects.

As to public-private partnerships - henceforth, I will always refer to the re-financing alternative - the outcome depends on the distribution of the liquidation value. If the public institution never assumes the total investment costs and has a claim on the liquidation value, the VC will always make losses in the bad state of the project, ensuring
the truth-revealing mechanism. In this case, governmental intervention has only distributional effects. But if the public institution’s claim is subordinated to the claim of the VC, the situation is similar to the situation of ex ante grants: if \( I - I^* \leq l(0) \), the truth-revealing mechanism is destroyed. This is true for a relatively large spill-over effect, i.e. \( S(e_g^*, a_g^*, \Delta_{PPP}^{LP}) \geq \frac{(1-p)(1-q)}{pq}[I - l(0)] \). Otherwise, public-private partnerships with a liquidation preference also have only distributional effects.

### 3.3 Comparison of the different public policy programmes

To sum up, we present an overview of the analysed public policy programmes. We distinguish two cases: the indications without parentheses refer to the case of a large spill-over effect - \( S(e_g^*, a_g^*, \omega) > [1 + \frac{(1-p)(1-q)}{pq}][I - l(0)] \) - while the indications in parentheses refer to the case of a small spill-over effect: \( S(e_g^*, a_g^*, \omega) < \frac{(1-p)(1-q)}{pq}[I - l(0)] \). The project range is indicated with ‘+’ if more, but not all, projects presenting a positive total net value are financed. The indication ‘+++’ refers to the case where all projects with a positive total net value get financed and finally, ‘max’ stands for achievement of the public first-best range: this means that all projects with \( \alpha \geq \alpha^{SP} \) get financed. In the next column, we show the impact of the public policy programme on all projects which would also have been financed without intervention. The last column with respect to scenario 1 points out the total welfare effect of each public policy measure. As mentioned before, we make three different assumptions about the observability of the inherent innovative value by the public institution: non-observability as in the analysis of section 3.1; the observability of two groups of projects with an inherent innovative value larger or smaller than \( \alpha^{PR} \); and the observability of the inherent innovative value \( \alpha \) of each project. Under the second assumption, the public institution would not offer support to all those projects which would have been financed without intervention (i.e. all projects with an inherent innovative value of \( \alpha \geq \alpha^{PR} \)) and whose outcome is not increased by the intervention. Under assumption three, support can decrease with an increase in the innovative value. This

\footnote{For \( \frac{(1-p)(1-q)}{pq}[I - l(0)] \leq S(e_g^*, a_g^*, \omega) \leq [1 + \frac{(1-p)(1-q)}{pq}][I - l(0)] \), the only changes - compared to the case of a large spill-over effect - are the following: as concerns ex-ante grants, the minimum inherent innovative value increases and they are no longer welfare-decreasing in scenario 2.}
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<th>$\alpha^{PR} &gt; \alpha &gt; \alpha_{P_{LP}^{SP}}$</th>
<th>-</th>
<th>?</th>
<th>+</th>
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<td></td>
<td>($\alpha_{P_{LP}^{SP}} \leq \alpha_{SP}^{NI}$)</td>
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<td>($\alpha_{SP}^{NI} \leq \alpha_{SP}^{NI}$)</td>
<td></td>
<td>(?/+)</td>
<td>(+)</td>
<td>(+)</td>
<td>(yes)</td>
<td>(max)</td>
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*: Public-private partnerships refer to the re-financing variant whereby a indicates the one without liquidation preference and b the one with liquidation preference.

**: To reach this outcome: $S(e^*_g, a^*_g, \alpha) \leq [I - l(a^*_g)] + \frac{(1-p)(1-q)}{pq} \cdot [I - L(a^*_g)]$.

***: To reach this outcome, the spill-over effect must be even smaller, i.e. $S(e^*_g, a^*_g, \alpha) \leq \frac{(1-p)(1-q)}{pq} \cdot [I - l(a^*_g)]$. 
increases, in turn, the outcome of these projects because of the limitation of the negative incentive effects\textsuperscript{6}.

As we can see, independently of the size of the spill-over effect and the assumption about the observability of $\alpha$, only ex post grants maximise the welfare in both scenarios and public support in scenario 2 whereas in scenario 1 it continues to be welfare-increasing. Public-private partnerships, ex ante grants and guarantee programmes (with a large spill-over effect) are welfare-increasing if the public institution can classify the projects into at least two groups. If this is not the case, their welfare effect depends on the parameters in scenario 1. It is important to underline that guarantee programmes are always welfare-decreasing in scenario 2, and ex ante grants with a large spill-over effect. Public-private partnerships are only welfare-decreasing in scenario 2 if the spill-over effect is relatively large and the public institution offers the VC a liquidation preference.

### 4 Robustness checks

We will now introduce an alternative specification of the spill-over effect in order to check our results for robustness. It is clear that a spill-over effect dependent on the effort choices can be internalised only by giving additional incentives to the contracting parties. This reduces the set of possible public policy measures. Therefore, we will check our results for a spill-over effect which is independent of the effort levels. Let $S(\alpha)$ be the spill-over effect for a project with the inherent innovative value $\alpha$. Now, private first-best effort levels correspond to public first-best effort levels in all three states of the project. Moreover, it is still efficient to give the $E$ control; in scenario 2, an investment is always profitable in the medium and the good state and not profitable in the bad state of the project; and in scenario 1, an investment is profitable only if it is sufficiently innovative, i.e. if $\alpha \geq \alpha_{\text{rob}}^{SP}$, whereby $\alpha_{\text{rob}}^{SP}$ is given by:

$$\bar{x}(e^*_g, a^*_g, \alpha_{\text{rob}}^{SP}) - \frac{1}{2}\beta e^*_g - \frac{1}{2}\gamma a^*_g + \frac{B}{pq} = I + \frac{(1-p)(1-q)}{pq}[I - L(a^*_h)] - S(\alpha_{\text{rob}}^{SP}) \quad (25)$$

\textsuperscript{6} Note that with this assumption, the above determined optimal support schemes would apply only for the least innovative socially profitable project.
The effects of the different public policy programmes with this alternative specification do not change considerably. Ex post grants continue to be always welfare-maximising in both scenarios. Public-private partnerships with a liquidation preference and ex ante grants are able to maximise the social welfare in scenario 1 with a relatively small spill-over effect, and are otherwise welfare-enhancing if the public institution can identify the mentioned two groups of projects. In this case, public-private partnerships without a liquidation preference and guarantee programmes with a relatively large spill-over effect are also welfare-enhancing. But in scenario 2, guarantee programmes are still always welfare-decreasing, while ex ante grants continue to be welfare-decreasing with a large spill-over effect and public-private partnerships with a large spill-over effect and a liquidation preference by the VC. Moreover, it is evident that public support is no longer an adequate instrument. Thus, introduction of this alternative specification seems to show that our comparative evaluation of the different programmes is quite robust.

A further point we want to comment on is our assumption of risk-neutrality for both contracting parties. We deem the assumption of risk-neutral Es to be unobjectionable as the latter do not invest any private wealth in the project. With respect to the VCs, it could be argued that they can be modelled as being risk-neutral because they hold a diversified portfolio of companies. But the question is whether our results are robust for contracting with risk-averse VCs. The answer is yes. We want to point this out with the example of guarantee programmes in mind, which are often considered as aiming at contracting with risk-averse agents. Without a guarantee programme, we are confronted with the traditional trade-off between giving incentives and offering risk insurance. We will never achieve an optimal risk allocation and projects will get financed only if the project is sufficiently innovative so that the risk premium can be paid by the E in the good state of the project. A guarantee programme ensures an optimal risk allocation, and less innovative but socially profitable projects will get financed now. The mechanisms, however, do not change in comparison to the case of risk-neutral VCs. The same is true for scenario 2: risk aversion does not have any influence if the truth-revealing condition, which eliminates the project’s risk, is achieved.
A last point of the model which could attract criticism is the fact that the double-sided moral hazard problem can be solved privately due to specific assumptions of the model. Moreover, the aim of the paper was to focus on public policy strategies in order to internalise the spill-over effect related to innovation - a stated purpose in many policy agendas. But what would it mean for our results if the double-sided moral hazard problem were not solved by the market? If the private first-best effort levels were not implemented by the market, the government would have to provide further incentives to increase effort levels. As seen above, this can only be reached through ex post grants and public support. The other programmes would have the same effects on the range of financed projects and the contract mechanisms as described, only the effort levels would be more suboptimal than is the case in our framework. The qualitative results about the robustness of ex post grants and the danger of guarantee programmes, ex ante grants and public-private partnerships with a liquidation preference would not change, however: the advantages of ex post grants would even grow as they could also provide these further incentives.

5 Conclusion

We analysed different public policy programmes by attaching special emphasis to their impact on contract design and the resulting effort decisions of both contracting parties. Therefore, we adapted the sequential double-sided moral hazard framework of Schmidt (2003) and the double-sided adverse selection framework of Houben (2002) in order to compare the results of both frameworks and make these models tractable for the purposes of this paper. Furthermore, we allowed for the necessity of public intervention by incorporating a spill-over effect per se which may be proportional to the outcome of the successful project and can then be influenced by both parties through their effort decisions or which may be independent of it. We consider that the analysis of the impact of public policy programmes on contract design is especially important as explicit contracts play a prevalent role in venture capital finance and are quite complex and sensitive.
We found that programme adequacy depends on the size and specification of the spill-over effect and the experience of the VCs and the Es, as only with highly experienced contracting parties is signalling possible. If we are confronted with an inexperienced market and a spill-over effect which is independent of the effort levels, the first-best situation can be reached by ex post grants and, under some conditions, by ex ante grants and public-private partnerships. The effect of guarantee programmes depends on the concrete parameter constellation. If the spill-over effect depends on the effort levels, only ex post grants maximise welfare; but ex ante grants, public-private partnerships and guarantee programmes with a relatively large spill-over effect continue to be welfare-enhancing if the public institution can divide the projects at least into two groups with respect to their inherent innovative value. Otherwise, their welfare effect depends on the concrete parameters. Moreover, public support is always welfare-increasing.

If we have an experienced market and an independent spill-over, we do not need any public intervention in order to reach the first-best situation; but ex post grants and public-private partnerships without a liquidation preference by the VC do not destroy the contract mechanisms and so they have only distributional effects (obviously, this is due to our assumption of lump-sum financing). Guarantee programmes, however, always destroy the truth-revealing mechanism while ex ante grants and public-private partnerships with a liquidation preference by the VC may do it under certain conditions too. If we are confronted with a dependent spill-over effect, only ex post grants and public support are welfare-maximising. Public-private partnerships without a liquidation preference have only distributional effects, but guarantee programmes and sometimes ex ante grants and public-private partnerships with a liquidation preference by the VC continue to destroy the truth-revealing mechanism.

To conclude, we observe that guarantee programmes, ex ante grants and public-private partnerships with a liquidation preference are dangerous as they can destroy the contract mechanisms in the case of an experienced market. Public support is useful in a developed market with dependent spill-overs. But ex post grants are the most suitable instrument, as they guarantee the first-best situation independently of the scenario and the specification.
A Appendix

A.1 Proof of proposition 1:

The E will offer the VC the contract which maximises his profit subject to his limited liability constraint LLC(E), his participation constraint PC(E), the VC’s participation constraint PC(VC) and her incentive constraint IC(VC). The determination of the optimal contracts is by backward-induction. We assume for simplicity’s sake that the initial contract cannot be renegotiated at any point in the relationship. (See V in this proof for a note on the renegotiation issue). Moreover, as a transfer of control rights cannot convey any additional information, we assume that the E will hold these and obtains private benefits independently of the sort of contract chosen.

I. Pure equity

1. The choice of effort levels¹:

\[ IC(VC) : a_{eq}^{g} = \arg \max [s^{eq}\bar{x}(e, a, \alpha) - \frac{1}{2}\gamma a^2] < a_{g}^{*} \]

\[ a_{eq}^{m} = \arg \max [s^{eq}I - \frac{1}{2}\gamma a^2] = a_{m}^{*} \] (1)

\[ a_{eq}^{b} = \arg \max [s^{eq}l(a) - \frac{1}{2}\gamma a^2] < a_{b}^{*} \]

\[ IC(E) : e_{eq}^{g} = \arg \max [(1 - s^{eq})\bar{x}(e, a, \alpha) - \frac{1}{2}\beta e^2 + B] < e_{g}^{*} \]

\[ e_{eq}^{m} = \arg \max [(1 - s^{eq})I - \frac{1}{2}\beta e^2 + B] = e_{m}^{*} \] (2)

\[ e_{eq}^{b} = \arg \max [(1 - s^{eq})l(a) - \frac{1}{2}\beta e^2 + B] = e_{b}^{*} \]

2. The concrete contract design:

The concrete contract design depends on the effort productivity of both contracting parties. The minimum equity stake is determined by the PC(VC):

---

¹ In the bad state, the E’s return does not depend on his effort level so he will not invest. The VC is not a full residual claimant so she will underinvest. In the medium state, the return to both parties is independent of their effort levels. Thus neither of them will invest, which is efficient. As neither the E nor the VC is a full residual claimant in the good state, both parties will underinvest in effort too.
The maximum equity stake is determined by the PC(E) if $B \leq \frac{1}{2}\beta e_{g}^{eq2}$:

$$pq[(1 - s_{max}^{eq})\bar{x}(e_{g}^{eq}, a_{g}^{eq}, \alpha) - \frac{1}{2}\beta e_{g}^{eq2}] + [(1 - p)q + (1 - q)p](1 - s_{max}^{eq})I + [(1 - p)(1 - q)][(1 - s_{max}^{eq})l(a_{b}^{eq})] + B = 0$$ (4)

Otherwise the LLC(E) binds and $s_{max}^{eq} = 1$. Note that the project must be sufficiently innovative in order to fulfill the PC(VC) as well as the PC(E) and the LLC(E) simultaneously.

3. The E’s profit with an equity contract amounts to:

$$\Pi_{eq}^{E} = pq[(1 - s^{eq})\bar{x}(e_{g}^{eq}, a_{g}^{eq}, \alpha) - \frac{1}{2}\beta e_{g}^{eq2}] + [(1 - p)q + (1 - q)p][(1 - s^{eq})I] + [(1 - p)(1 - q)][(1 - s^{eq})l(a_{b}^{eq})] + B$$ (5)

with $s^{eq} \in [s_{min}^{eq}, s_{max}^{eq}]$.

II. Pure debt

1. The choice of effort levels$^2$:

$$IC(VC) : a_{g} = \arg \max [D_{D} - \frac{1}{2}\gamma a^{2}] = \delta < a_{g}^{*}$$

$$a_{m} = \arg \max [I - \frac{1}{2}\gamma a^{2}] = a_{m}^{*}$$ (6)

$$a_{b} = \arg \max [l(a) - \frac{1}{2}\gamma a^{2}] = a_{b}^{*}$$

$$IC(E) : e_{g} = \arg \max [\bar{x}(e, a, \alpha) - \frac{1}{2}\beta e^{2} - D_{D} + B] = e_{g}^{*}$$

$$e_{m} = \arg \max [I - \frac{1}{2}\beta e^{2} - I + B] = e_{m}^{*}$$ (7)

$$e_{b} = \arg \max [l(a) - \frac{1}{2}\beta e^{2} - l(a) + B] = e_{b}^{*}$$

$^2$ We anticipate the LLC(E) and know that the PC(VC) implies that $D_{D} > I$. Thus, in the bad state of the project, the VC is a full residual claimant and invests efficiently, whereas the E will not invest, which is efficient too. In the medium state of the project, the return to both parties is independent of their effort levels and neither of them will invest. In the good state, the E is a full residual claimant and so will choose his private first-best effort level if he assumes the VC to exert at least a marginal effort level. This is, indeed, the case if her fixed payment payment $D_{D}$ is larger than $I$, which must be true because otherwise the PC(VC) would never be fulfilled and there would be no financing.
2. The concrete contract design:

As the E has all the bargaining power ex ante, the PC(VC) is binding. Moreover, the LLC(E) and the PC(E) must be considered. Note that the project must be sufficiently innovative to fulfill all three constraints simultaneously. \( D_D \) is determined by:

\[
pq \cdot D_D + [(1 - p)q + (1 - q)p]I + [(1 - p)(1 - q)]L(a^*_b) = I \tag{8}
\]

3. The E’s profit with a debt contract amounts to:

\[
\Pi^E_D = pq[\bar{x}(e^*_g, \delta, \alpha) - \frac{1}{2} \beta e^*_g^2 - D_D] + B \tag{9}
\]

III. Combinations of debt and equity

Consider any debt-equity contract \((D^{eq,D}, s^{eq,D})\). The E will choose \( D^{eq,D} \) and \( s^{eq,D} \) so as to maximise his profit subject to the above mentioned constraints. Debt-equity contracts, however, never induce private first-best effort levels of both contracting parties in all three states of the project. In the good state, the marginal returns to the investments by the E and the VC are still smaller than one. Therefore, both will keep underinvesting. In the bad state of the project, the VC invests efficiently only if \( D^{eq,D} > l(a^*_b) \).

IV. Convertible preferred stock contracts

Let us consider a convertible preferred stock contract with a limited dividend \( D \) and a conversion option in an equity stake \( s^{CV}_1 \) in \( t_4 \) if the threshold \( z \) is reached.

1. The choice of effort levels:

**Bad state**: As for \( a \leq a^*_b, D \geq I > l(a) \), the VC gets the total surplus from the project. Therefore, she has an incentive to invest efficiently. The E knows that he cannot influence the outcome and so chooses an effort level of 0.

**Medium state**: The VC and the E know that the surplus from the project is independent of the effort levels and so both choose an effort level of 0, which is efficient.

**Good state**: In order to induce the VC to exert effort, the E has an incentive to offer a conversion option in \( t_4 \). The conversion option is linked to the threshold \( z \). This threshold indicates the minimum surplus which must be reached for conversion
to be possible. Suppose that the threshold is set at \( z = \bar{x}(e, a, \alpha) \). We consider the following cases:

a) Assume that the E exerts an effort level of \( e = \underline{e} \) and that the VC can guarantee herself the payoff of \( D \) by exerting only the marginal effort level \( \delta \).\(^3\) Obviously, it cannot be efficient for the VC not to convert and to exert a higher effort level, because she would still get only \( D \) but would have to bear the effort costs. In order to be able to exert her conversion option, she must invest \( a \geq \underline{a} \). After conversion, her payoff in the good state amounts to

\[
\Pi_g^{VC} = s_1^{CV} \bar{x}(\underline{e}, \underline{a}, \alpha) - \frac{1}{2} \gamma a^2
\]  

We know that the VC would maximise her profit by choosing

\[
\hat{a}_g^{CV} = \arg \max \left[ s_1^{CV} \bar{x}(\underline{e}, a, \alpha) - \frac{1}{2} \gamma a^2 \right]
\]  

If \( \hat{a}_g^{CV} \geq \underline{a} \), the VC will choose \( \hat{a}_g^{CV} \). Otherwise for \( \hat{a}_g^{CV} \), the threshold is not reached. As the profit of the VC is a concave function in \( a \), the VC will choose \( a = \underline{a} \).\(^4\) Moreover, it must be guaranteed that the VC does have an incentive to choose the conversion option and to exert \( \underline{a} \). This is the case if the VC’s profit with conversion amounts at least to the profit without conversion:

\[
IC(VC)_{conv.} : \quad s_1^{CV} \bar{x}(\underline{e}, \underline{a}, \alpha) - \frac{1}{2} \gamma \underline{a}^2 \geq D
\]  

b) Assume that the E exerts an effort level of \( e > \underline{e} \). In order to be able to exert her conversion option, the VC must invest only \( a \geq \hat{a} \) whereby \( \hat{a} \) is the critical effort level necessary to reach \( z \). The profit of the E continues to be \( (1 - s_1^{CV})z \) but he has to bear higher effort costs. Thus, it is not profitable for the E to exert an effort level \( e > \underline{e} \).

\(^3\) If \( D \) is not guaranteed, she exerts the effort level which maximises her profit. The return will, however, always be smaller than \( D \). Conversion would become even more profitable.

\(^4\) We now assume that \( \hat{a}_g^{CV} < \underline{a} \). This will turn out to be true in what follows. Indeed, this is intuitively obvious, otherwise convertibles would not improve the outcome in comparison to equity contracts with a liquidation preference.
c) Assume that the E exerts an effort level of $e < e^*$. The VC anticipates that the threshold $z$ will not be reached if she exerts $e^*$.

Consequently, she will exert only a marginal effort level $\delta^6$. The E foresees the behaviour of the VC and the fact that he will be a full residual claimant. Thus, he chooses $e - \delta$ to maximise his profit. But the E is better off without any deviation, i.e. with choosing $e$ if

$$ IC(E)_{conv.} : \quad (1 - s_1^{CV})\bar{x}(e, a, \alpha) - \frac{1}{2}\beta e^2 \geq \bar{x}(e - \delta, \delta, \alpha) - \frac{1}{2}\beta(e - \delta)^2 - D \quad (13) $$

2. The concrete contract design

The maximisation problem facing the E is the following:

$$ \max_{e, a, D, s^{CV}} \Pi^E_{CV} = \frac{pq[(1 - s_1^{CV})\bar{x}(e, a, \alpha) - \frac{1}{2}\beta e^2]}{s.t.} + B $$

$$ LLC(E) : \quad \Pi^E_{CV} \geq 0 $$

$$ PC(VC)_1 : \quad s_1^{CV}\bar{x}(e, a, \alpha) - \frac{1}{2}\gamma a^2 \geq I + \frac{(1 - p)(1 - q)}{pq}[I - L(a_s)] $$

$$ PC(VC)_2 : \quad D \geq I $$

$$ IC(VC)_{conv.} : \quad s_1^{CV}\bar{x}(e, a, \alpha) - \frac{1}{2}\beta e^2 \geq D $$

$$ IC(E)_{conv.} : \quad (1 - s_1^{CV})\bar{x}(e, a, \alpha) - \frac{1}{2}\beta e^2 \geq \bar{x}(e - \delta, \delta, \alpha) - \frac{1}{2}\beta(e - \delta)^2 - D $$

As the E has all the bargaining power ex ante and no other constraint demands a higher $s_1^{CV}$, the $PC(VC)_1$ is binding:

$$ s_1^{CV} = \frac{C + \frac{1}{2}\gamma a^2}{\bar{x}(e, a, \alpha)} \quad \text{with} \quad C = I + \frac{(1 - p)(1 - q)}{pq}[I - L(a_s)] \quad (14) $$

Note that the project must be sufficiently innovative so that the $PC(VC)$, the $LLC(E)$ and the $PC(E)$ are fulfilled. Because of our assumption that $B \leq pq\frac{1}{2}\beta e^2$, the $LLC(E)$ is never binding in equilibrium. The $PC(E)$ and the $PC(VC)$ are fulfilled.

---

5 She will not exert a higher effort level as she is not compensated for the effort costs and therefore will be worse off with conversion.

6 We assume that her claim is guaranteed. If this is not the case, the incentive of the E to deviate would be even less. The same is true for $D=I$. In this case, the claim is guaranteed even by choosing an effort level of 0, which implies that the total surplus will go to the VC.

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for all individually profitable projects and $s_i^{CV} < 1$.

The conditions $PC(VC)_2$, $IC(VC)_{conv}$ and $IC(E)_{conv}$ put only restrictions on $D$. They stipulate:

$$I \leq D \leq C = I + \frac{(1-p)(1-q)}{pq}[I - L]$$

$$D \geq C - [\bar{x}(e, a, \alpha) - \frac{1}{2}\gamma a^2 - \bar{x}(e - \delta, \delta, \alpha)]$$

And therefore

$$\max[C - [\bar{x}(e, a, \alpha) - \frac{1}{2}\gamma a^2 - \bar{x}(e - \delta, \delta, \alpha)], I] \leq D \leq C$$

The concrete $D$ can be chosen freely by the E (see V. for further comments). As concerns the optimal effort levels, no further restrictions must be considered and so we get the following FOCs from the unrestricted problem:

$$\frac{\partial \Pi^E_{CV}}{\partial e} = pq[\frac{\partial \bar{x}(e, a, \alpha)}{\partial e}] - \beta e = 0$$

$$\frac{\partial \Pi^E_{CV}}{\partial a} = pq[\frac{\partial \bar{x}(e, a, \alpha)}{\partial a}] - \gamma a = 0$$

$$\frac{\partial \Pi^E_{CV}}{\partial \alpha} = \frac{1}{\gamma} \frac{\partial \bar{x}(e, a, \alpha)}{\partial \alpha} = a^*_g$$

The E introduces the threshold $z = \bar{x}(e_g^*, a_g^*, \alpha)$. As $\hat{e}_g^{CV} < e_g^*$ and $\hat{a}_g^{CV} < a_g^*$, both parties will choose private first-best effort levels.

3. The E’s profit with a convertible preferred stock contract amounts to:

$$\Pi^E_{CV} = pq[\bar{x}(e_g^*, a_g^*, \alpha) - \frac{1}{2}\beta e_g^* - \frac{1}{2}\gamma a_g^* - C] + B$$

Comparing the E’s profits with the different financial contracts shows that his profit is maximised with a convertible security. Thus, the E will offer the VC the deduced convertible preferred stock contract which implements the private first-best situation.
V. A note on the renegotiation issue:

Schmidt (2003) shows that if renegotiation is possible, neither the bargaining power of the E nor the benefit from renegotiation must be too high to ensure that the convertible security does implement the first-best situation. In what follows, we will look at the impact of possible renegotiation within our framework. Obviously, renegotiation is only an issue in the good state of the project. Let us assume that renegotiation takes place whenever there is scope for an efficiency improvement. Let us further assume that the E and the VC split the surplus from renegotiation proportionately ($\lambda, 1 - \lambda$,) where $\lambda \in [0, 1]$ is the fraction that goes to the E. We know that the E has an incentive to choose his private first-best effort level in the good state only because of the sequential decision structure, which implies a threat by the VC not to convert and not to exert effort if $e < e^*_g$. In this case, however, there exists scope for renegotiation in $t = 2, 5$: the E could adapt the threshold $z$ and the equity stake $s^{CV}_1$ in order to make it profitable for the VC to convert and to exert her private first-best effort level even though the E has chosen $e < e^*_g$.

First, we will determine how a deviation of the E impacts the outcome without renegotiation. We need to distinguish two kinds of projects. The solution is by backward induction. In a first step, we will analyse the effort choice of the VC.

Case 1:  
a) Assume that the VC has a claim $D > I$ and that $\bar{x}(e, \delta, \alpha) > D$. Then the VC will choose a marginal effort level of $\delta$.

b) Assume that the VC has a claim of $D = I$. Then her claim is always guaranteed and she will choose an effort level of 0.

Case 2:  
a) Assume that the VC has a claim of $D > I$ and that $\bar{x}(e, a^*_g, \alpha) \leq D$. Then the VC is a full residual claimant and will choose $a^*_g$.

b) Assume that the VC has a claim of $D > I$ and that $\bar{x}(e, \delta, \alpha) < D < \bar{x}(e, a^*_g, \alpha)$. Then the VC will choose an effort level of $\hat{a}^R$ such that $\bar{x}(e, \hat{a}^R, \alpha) = D$.

In a second step, we will now determine the effort choice made by the E.

Case 1:  
a) The E anticipates that he will be a full residual claimant and choose an effort level of $e^*_g - \delta$. 

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b) The E anticipates the VC’s effort choice and because of the complementarity of efforts, he will choose an effort level of 0 too.

Case 2: The E anticipates that he will never make positive profits and will choose an effort level of \( \delta \) and not of 0 in order to guarantee profitable renegotiation.

In a last step, we need to determine the profits of the E after renegotiation:

Case 1: a) \( \Pi_{E1a} = \bar{x}(e_g^* - \delta, \delta, \alpha) - \frac{1}{2} \beta(e_g^* - \delta)^2 + \lambda[\bar{x}(e_g^* - \delta, a_g^*, \alpha) - \bar{x}(e_g^* - \delta, \delta, \alpha)] - D - \frac{1}{2} \gamma a_g^2 \)

b) There is no scope for renegotiation.

Case 2: a) There is no scope for renegotiation.

b) \( \Pi_{E2b} = \bar{x}(\delta, \hat{a}^R, \alpha) + \lambda[\bar{x}(\delta, a_g^*, \alpha) - \bar{x}(\delta, \hat{a}^R, \alpha)] - D - [\frac{1}{2} \gamma a_g^2 - \frac{1}{2} \gamma \hat{a}^{R2}] \)

Now, we are able to deduce conditions which guarantee that deviation and subsequent renegotiation do not pay for the E. We know that the E’s payoff in the good state without deviation amounts to

\[
\Pi^E_g = \bar{x}(e_g^*, a_g^*, \alpha) - \frac{1}{2} \beta e_g^* - \frac{1}{2} \gamma a_g^2 - C
\]

As concerns case 1a, it is obvious that renegotiation never pays if \( D = C \). This does not change our deduced contract mechanism. Case 2b is slightly more complicated. If \( D = C \), the following condition must hold. Note that \( \hat{a}^R \) is given by \( \bar{x}(\delta, \hat{a}^R, \alpha) = D \).

\[
\lambda[\bar{x}(\delta, a_g^*, \alpha) - D] \leq [\bar{x}(e_g^*, a_g^*, \alpha) - D] - \frac{1}{2} \beta e_g^* - \frac{1}{2} \gamma \hat{a}^{R2}
\]

We assume in what follows that the bargaining power of the E is sufficiently small for him to have never an incentive to deviate to induce renegotiation and that \( D = C \).

**A.2 Proof of proposition 2:**

We will now determine the optimal contracts that maximise the E’s payoff. The solution is by backward induction. As concerns the effort decision, we can anticipate the results from proposition 1. To simplify, we assume again that the initial contract cannot be renegotiated at any point of the relationship. (See IV for renegotiation issues.)
I. Pure equity

We know that in the good state the effort levels amount to $e^g_{eq}(s^g_{eq})$ and $a^g_{eq}(s^g_{eq})$; that in the medium state neither of the contracting parties will exert effort; that in the bad state the E will not exert any effort while the VC will exert an effort level of $a^b_{eq}(s^b_{eq})$.

$\lambda$: The E offers the VC two contracts, $s^g_{eq}$ and $s^m_{eq}$, which should give the VC the incentive to choose $s^g_{eq}$ if she receives $\bar{\eta}$, and $s^m_{eq}$ if she receives $\underline{\eta}$. Thus, the E maximises his profit subject to the participation constraints and the self selection constraints (SSC) of the VC:

$$\max E_{eq} = q[(1-s^g_{eq})\bar{x}(e, a, \alpha) - \frac{1}{2}\beta e^2] + (1-q)[(1-s^m_{eq})I] + (B)$$

$$s.t.$$

$LLC(E)$

$PC(E) : \Pi^E_{eq} \geq 0$

$PC(VC)_g : s^g_{eq} \bar{x}(e^g_{eq}, a^g_{eq}, \alpha) - \frac{1}{2}\gamma a^g_{eq} \geq I$

$PC(VC)_m : s^m_{eq} I \geq I$

$SSC(VC)_g : s^g_{eq} \bar{x}(e^g_{eq}, a^g_{eq}, \alpha) - \frac{1}{2}\gamma a^g_{eq} \geq s^m_{eq} \bar{x}(0, \hat{a}^g_{eq}, \alpha) - \frac{1}{2}\gamma \hat{a}^g_{eq}^2$

$SSC(VC)_m : s^m_{eq} I \geq s^g_{eq} I$

We know that in order to fulfill the $PC(VC)_m$, the equity stake $s^m_{eq} = 1$. This implies that $SSC(VC)_m$ is always fulfilled. We know that if the VC chooses the medium state contract, the E will belief him and will choose an effort level of 0. Because of the substitution relation, the VC will do the same. Using these results, we get that $SSC(VC)_g$ corresponds to $PC(VC)_g$. Thus, we have to consider only $PC(VC)_g$.

The concrete contract $s^g_{eq}$ depends on the effort productivity of both contracting parties. We know that $s^g_{eq} \in [s^g_{eq}(min), s^g_{eq}(max)]$ whereby the corners are given through the PC(VC) and the PC(E)/LLC(E) respectively by:

$$s^g_{eq}(min) = \frac{I + \frac{1}{2}\gamma a^g_{eq}^2}{\bar{x}(e^g_{eq}, a^g_{eq}, \alpha)}$$

$$s^g_{eq}(max) = \min \left[ \frac{\bar{x}(e^g_{eq}, a^g_{eq}, \alpha) - \frac{1}{2}\beta e^g_{eq}^2 + B}{\bar{x}(e^g_{eq}, a^g_{eq}, \alpha)} ; 1 \right]$$
The E offers the VC the contract \( s_{eq}^m \) which gives her the incentive to choose \( s_{eq}^m \) if she receives \( \bar{q} \), and reject the offer if she receives \( q \). As in the bad state the VC can only lose money, truth-revealing is achieved if \( s_{eq}^g \) fulfills the VC’s participation constraint, i.e. for \( s_{eq}^g = 1 \).

But can the VC be sure also that the E reveals his information truthfully too? It can be clearly seen that the E does not have an incentive to understate because to do so would cause the VC to reject the contract in the medium state. But it can be shown that the E does have an incentive to overstate his private information. The following table compares the E’s payoff with truthful revelation and overstatement:

<table>
<thead>
<tr>
<th>state of the project</th>
<th>with truthful revelation</th>
<th>with overstatement</th>
</tr>
</thead>
<tbody>
<tr>
<td>medium</td>
<td>(B)</td>
<td>((1 - s_{eq}^g)I(+B))</td>
</tr>
<tr>
<td>bad</td>
<td>0</td>
<td>(B)</td>
</tr>
</tbody>
</table>

We can see that even giving the control rights to the VC does not prevent the E from overstating, because he will make additional profits in the medium state of the project as \( s_{eq}^g < s_{eq}^m = 1 \). Thus, truth-revealing cannot be achieved with equity contracts.

**II. Pure debt**

We know that the VC will exert her private first-best effort level in the bad state whereas the E will not exert any effort; that in the medium state of the project neither party will exert effort; and that in the good state of the project the VC will exert either no effort or a marginal effort level (depending on the payment of her fixed claim) and the E will exert either no effort or his private first-best effort level respectively.

\( \bar{x} \): The E faces the following maximisation problem:

\[
\max \Pi_d^E = q[\bar{x}(e, a, \alpha) - \frac{1}{2}\beta e^2 - D_g] + (1 - q)[I - D_m] + B
\]

\[
s.t.
\]

\[
LLC(E)
\]

\[
PC(E) : \quad \Pi_d^E \geq 0
\]
\[ PC(VC)_g : \quad D_g - \frac{1}{2} \gamma a^2 \geq I \]
\[ PC(VC)_m : \quad D_m \geq I \]
\[ SSC(VC)_g : \quad D_g - \frac{1}{2} \gamma a^2 \geq D_m \]
\[ SSC(VC)_m : \quad D_m \geq D_g \]

As the E has all the bargaining power ex ante, the PC(VC)s are binding. If \( D_g > I \),
then the payment is only guaranteed in the good state if the VC exerts a marginal
effort level. As the E is interested in a marginal effort level of the VC, \( D_g = I + \delta \)
and \( D_m = I \). Thus, the LLC(E), PC(E) and the SSCs are also fulfilled. The VC
reveals her information truthfully.

\[ \lambda : \text{The E offers one contract } D_m = I. \text{ The VC will accept this offer if she receives } \tilde{\eta}, \]
and will reject the offer if she receives \( \eta \). Thus, the E can induce truthful revelation
of the VC’s private information.

But can the VC be sure that the E also reveals his information truthfully? It can clearly
be seen that while the E does not have an incentive to understate because to do so would
cause the VC to reject the contract in the medium state, he does have an incentive to
overstate in order to get private benefits in the bad state\(^7\). Thus, pure debt contracts
cannot induce truth-revealing either.

**III. Preferred stock**

\( \tilde{\lambda} : \) As concerns the choice of effort levels, the argumentation is the same as in proposi-
tion 1. The E offers a conversion option with a threshold \( z = \tilde{x}(e, a, \alpha) \). Analogously
to proposition 1, it can be shown that the E has an incentive to choose \( e \) and that
the VC has an incentive to choose \( a \) if the E has chosen \( e \) if the following ICs hold:

\[ IC(VC)_{conv} : \quad s^{CV}_2 \tilde{x}(e, a, \alpha) - \frac{1}{2} \gamma a^2 \geq D_g \]
\[ IC(E)_{conv} : \quad (1 - s^{CV}_2) \tilde{x}(e, a, \alpha) - \frac{1}{2} \beta e^2 \geq I - D_g \]

\(^7\) With debt financing, the control rights are with the E. There exists no empirical evidence of an ex
ante control transfer to the VC. But even if truth-revealing could be achieved, the preferred stock
contract would do better as it gives efficient incentives to both contracting parties and as private
benefits do not get lost.
The maximisation problem facing the E is the following:

$$\max \Pi_{\text{E}}^{\text{CV}} = q[(1 - s_2^{\text{CV}})\bar{x}(e, a, \alpha) - \frac{1}{2}\beta \xi^2] + (1 - q)[I - D_m]$$

\[s.t.\]

- $PC(\text{VC})_g: s_2^{\text{CV}} \bar{x}(e, a, \alpha) - \frac{1}{2}\gamma a^2 \geq I$
- $PC(\text{VC})_m: D_m \geq I$
- $SSC(\text{VC})_g: s_2^{\text{CV}} \bar{x}(e, a, \alpha) - \frac{1}{2}\gamma a^2 \geq D_m$
- $SSC(\text{VC})_m: D_m \geq D_g$
- $IC(\text{VC})_{\text{conv}}: s_2^{\text{CV}} \bar{x}(e, a, \alpha) - \frac{1}{2}\gamma a^2 \geq D_g$
- $IC(E)_{\text{conv}}: (1 - s_2^{\text{CV}})\bar{x}(e, a, \alpha) - \frac{1}{2}\beta \xi^2 \geq I - D_g$

It can be shown that if the PC(VC)s are binding ($D_m = I$ and $s_2^{\text{CV}} = \frac{I + \frac{1}{2}\gamma a^2}{\bar{x}(e, a, \alpha)}$), the SSC(VC)$_g$ is also fulfilled. These results show that the SSC(VC)$_m$ and the IC(VC)$_{\text{conv}}$ will demand that payoff without conversion in the good state ($D_g$) must not be bigger than $I$ and, at the same time, the IC(E)$_{\text{conv}}$ will demand that $D_g > D_g(\text{min}) = 2I - [\bar{x}(e, a, \alpha) - \frac{1}{2}\beta \xi^2 - \frac{1}{2}\gamma a^2] \leq I$. The contracts induce truth-revealing by the VC and, as in proposition 1, maximisation yields $e = e_g^*$ and $a = a_g^*$.

\[\Delta\]: The E offers one fixed payment contract which fulfills the $PC(\text{VC})_m$: $D_m = I$. If the VC receives $\bar{q}$, she will not accept the offer as her PC is not fulfilled, while if she receives $\bar{\eta}$, her participation constraint is fulfilled and she will accept the offer.

Her private information is revealed truthfully.

But can the VC be sure that the E also reveals his information truthfully? It is easy to see that he does not have an incentive to understate, because he would lose his private benefits in the medium state of the project\(^8\). The overstatement case is represented in the following table:

<table>
<thead>
<tr>
<th>state</th>
<th>with truthful revelation</th>
<th>with overstatement</th>
</tr>
</thead>
<tbody>
<tr>
<td>medium</td>
<td>($B$)</td>
<td>$I - D_g(+$B$)</td>
</tr>
<tr>
<td>bad</td>
<td>0</td>
<td>($B$)</td>
</tr>
</tbody>
</table>

\[^8\] Even without control rights, he would be indifferent and we assume he would not understate.
Preferred stock contracts afford us the possibility of a contingent control allocation. Control rights can be given to the VC with the preferred stock contract, but the E can get them back either by redeeming the preferred stock contract or by conversion. The redemption price must compensate the VC for his forgone limited dividend $D_m$, thus $P_m = I$.

As the E is wealth-constrained, he will be able to redeem the preferred stock at a price $P_m = I$ only if the project is at least in the medium state. Moreover, conversion is only possible if the threshold $z$ is reached, which can be achieved only for good state projects. This implies that we can construct contracts where the E never holds control rights with overstatement or only does so in the medium or the bad state. At the same time, all these possible contracts guarantee private benefits with truthful revelation. We see from the table that the E must never hold control rights in the bad state with overstatement.

In the medium state with overstatement, there are two possibilities: either the E does not hold control rights with overstatement and $D_g > I - B$, or he holds control rights and $D_g = I$. In both cases, the contracts induce an efficient control allocation and give both agents an incentive to reveal their private information truthfully. Moreover, private first-best effort levels are implemented.

IV. A note on the renegotiation issue:

Houben (2002) shows that the contracts must be slightly modified in the good and medium states in order to make them renegotiation-proof. First, renegotiation is profitable if the VC prevents control transfer back to the E by adopting a certain behavior. The VC may be interested in doing so because he expects to get a share $(1 - \lambda)$ of the renegotiation surplus. One important condition, however, is that the E holds sufficient cash flow to compensate the VC. Thus, in our framework, we do not need any further condition in order to avoid misbehaviour by the VC in the medium state, as the E will not be able to regain control rights even with renegotiation because of his wealth constraint. However, renegotiation is an issue in good state projects with ex ante control transfer. The modified $SSC$s and $IC$s with overstatement are the following:
These conditions demand that the good state contract must be adapted so that the conversion stake is: \( s_{2}^{CV} = \frac{l + (1-\lambda)B + \frac{1}{2}\gamma\alpha^2}{\bar{x}(e^*_g, a^*_g, \alpha)} \). This means that the E has to pay the VC an information rent in order to avoid hold-up.

This problem can be circumvented, however, by forgoing the ex ante control transfer in good state contracts. As seen above, overstatement by the E can be avoided by ex ante control transfer in medium state contracts only and by \( D_g = I \) in good state contracts. Then the VC does not have any hold-up potential. The contracts are renegotiation-proof.

Second, renegotiation may be profitable for the E analogously to scenario 1. In this case, however, the VC’s claim is always guaranteed (as in case 1b of proof 1). Thus, there is no scope for renegotiation in this way.
<table>
<thead>
<tr>
<th>Condition</th>
<th>Expression</th>
<th>$EAG$</th>
<th>$C$</th>
<th>$a_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(e^<em>_g, a^</em>_g, \underline{\alpha}) &gt; \frac{(1-p)(1-a)}{pq}[I - l(0)]$</td>
<td>$S(e^<em>_g, a^</em>_g, \underline{\alpha}) - \frac{(1-p)(1-a)}{pq}[I - l(0)]$</td>
<td>$S(e^<em>_g, a^</em>_g, \underline{\alpha})$</td>
<td>$I$</td>
<td>$0$</td>
</tr>
<tr>
<td>$[I - l(a^<em>_b)] + \frac{(1-p)(1-a)}{pq}[I - L(a^</em>_b)] \leq S(e^<em>_g, a^</em>_g, \underline{\alpha}) &lt; [I - l(a^<em>_b)] + \frac{(1-p)(1-a)}{pq}[I - L(a^</em>_b)]$</td>
<td>$S(e^<em>_g, a^</em>_g, \underline{\alpha}) - \frac{(1-p)(1-a)}{pq}[I - l(\hat{a})]$</td>
<td>$S(e^<em>_g, a^</em>_g, \underline{\alpha}) - \frac{(1-p)(1-a)}{pq}[I - l(\hat{a})] - \frac{(1-p)(1-a)}{pq}[I - l(0)]$</td>
<td>$I + (1-p)(1-q)\gamma_E^{2A}$</td>
<td>$\hat{a}_{EAG}$</td>
</tr>
<tr>
<td>$S(e^<em>_g, a^</em>_g, \underline{\alpha}) &lt; \frac{(1-p)(1-a)}{pq}[I - L(a^*_b)]$</td>
<td>$[1 - (1 - p)(1 - q)]S(e^<em>_g, a^</em>_g, \underline{\alpha}) - \frac{(1-p)(1-a)(1-q)}{pq}[I - L(a^*_b)]$</td>
<td>$1 - (1 - p)(1 - q)S(e^<em>_g, a^</em>_g, \underline{\alpha}) - \frac{(1-p)(1-a)(1-q)}{pq}[I - L(a^<em>_b)] + \frac{(1-p)(1-q)}{pq + (1-p)(1-q)(1-p)(1-a)}[I - L(a^</em>_b)]$</td>
<td>$I_{1} - (1 - p)(1 - q)S(e^<em>_g, a^</em>_g, \underline{\alpha}) + \frac{(1-p)(1-q)}{pq + (1-p)(1-q)(1-p)(1-a)}[I - L(a^*_b)]$</td>
<td>$a^*_b$</td>
</tr>
<tr>
<td>$S(e^<em>_g, a^</em>_g, \underline{\alpha}) &lt; \frac{(1-p)(1-a)}{pq}[I - L(a^*_b)]$</td>
<td>$pqS(e^<em>_g, a^</em>_g, \underline{\alpha})$</td>
<td>$pqS(e^<em>_g, a^</em>_g, \underline{\alpha})$</td>
<td>$I + \frac{(1-p)(1-a)}{pq}[I - L(a^<em>_b)] - (1 - pq)S(e^</em>_g, a^*_g, \underline{\alpha})$</td>
<td>$a^*_b$</td>
</tr>
</tbody>
</table>
References


